OCR A

A-Level Physics

Exam Board: OCR A

Student Book

The Complete A-Level Course for OCR A

Includes **Free** Online Edition

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How to use this book

Learning Objectives

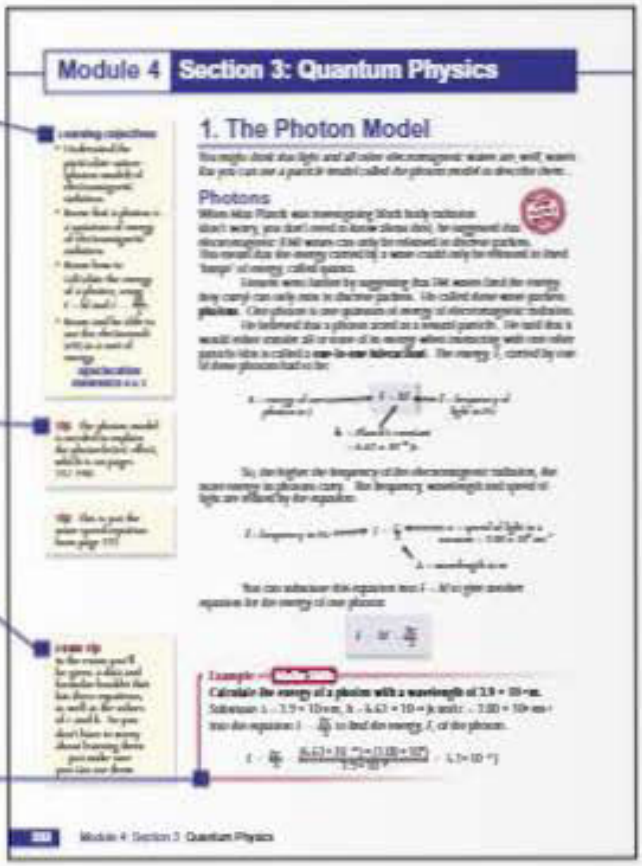
- These tell you exactly what you need to learn, or be able to do, for the exam.
- There's a specification reference at the bottom that links to the OCR A specification.

Tips and Exam Tips

There are tips throughout the book to help with all sorts of things, including exam tips to do with answering exam questions.

Maths Skills

There's a range of maths skills you could be expected to apply in your exams. Examples that show these maths skills in action are marked up like this. There's also a maths skills section at the back of the book.

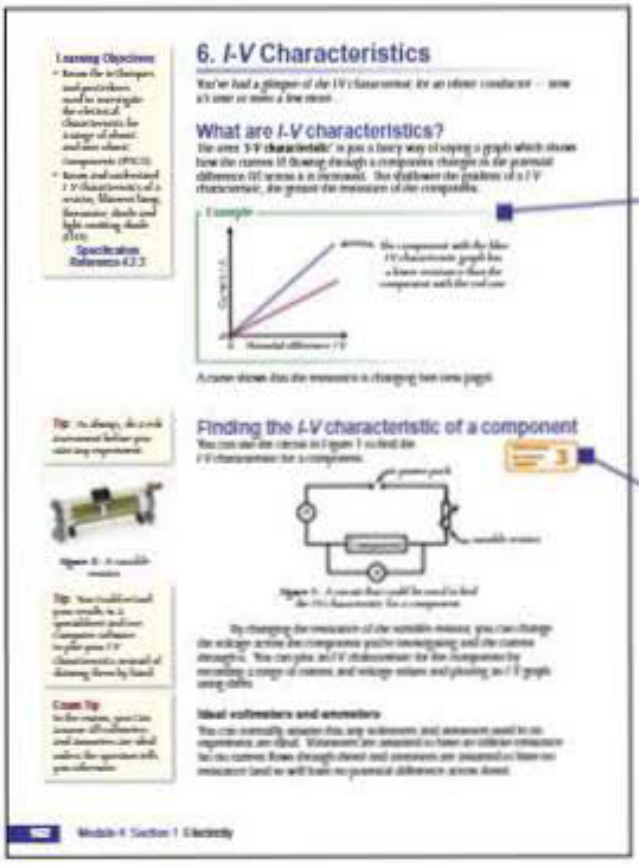


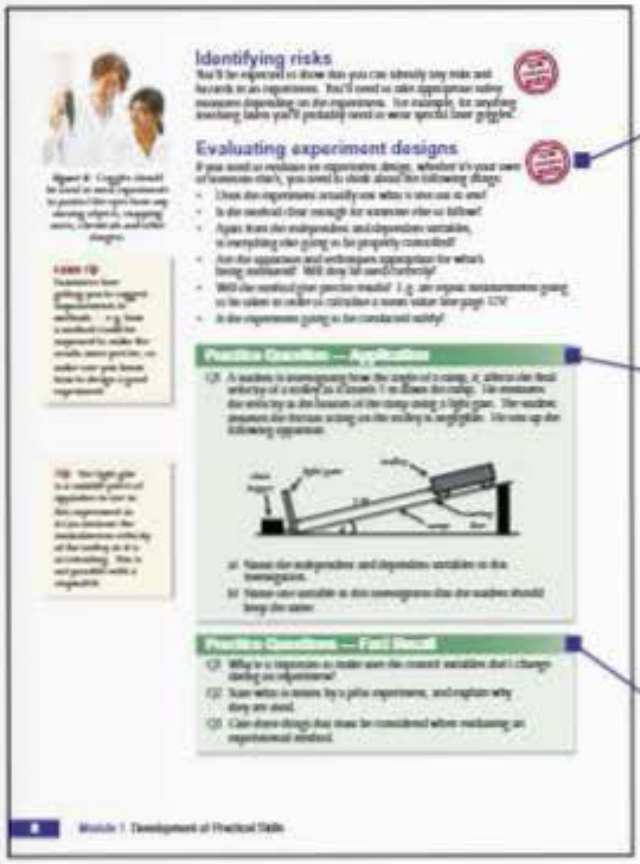
Examples

These are here to help you understand the theory.

Practical Activity Groups

You'll need to show you've mastered some key practical skills in your Practical Endorsement. Information on the skills you need and opportunities to apply them are marked up throughout the book.





How Science Works

- You need to know about How Science Works. There's a section on it at the front of the book.
- How Science Works is also covered throughout the book wherever you see this symbol.

HOW SCIENCE WORKS

Practice Questions — Application

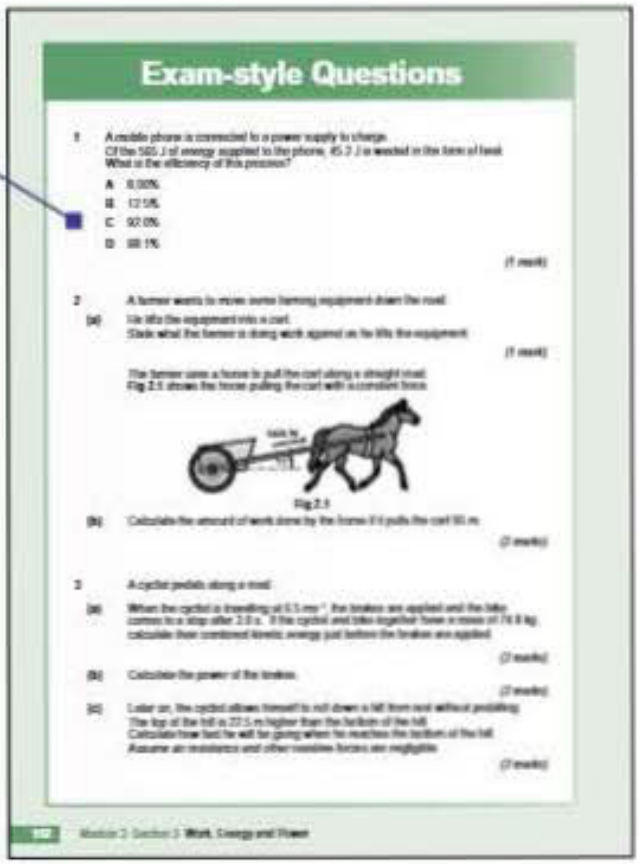
- Annoyingly, the examiners expect you to be able to apply your knowledge to new situations — these questions are here to give you plenty of practice at doing this.
- All the answers are in the back of the book (including any calculation workings).

Practice Questions — Fact Recall

- There are a lot of facts you need to learn — these questions are here to test that you know them.
- All the answers are in the back of the book.

Exam-style Questions

- Practising exam-style questions is really important — you'll find some at the end of each section.
- They're the same style as the ones you'll get in the real exams — some will test your knowledge and understanding and some will test that you can apply your knowledge.
- All the answers are in the back of the book, along with a mark scheme to show you how you get the marks.



Exam Help

There's a section at the back of the book stuffed full of things to help with your exams.

Glossary

There's a glossary at the back of the book full of useful words — perfect for looking up key words and their meanings.

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The Scientific Process

Science tries to explain how and why things happen. It's all about gaining knowledge about the world. Scientists do this by asking questions, suggesting answers and then doing tests to see if they're correct — the scientific process.

Developing and testing theories

A **theory** is a possible explanation for something. Theories usually come about when scientists observe something and wonder why or how it happens. Scientists also sometimes form a **model** too — a simplified picture or representation of a real physical situation. Scientific theories and models are developed and tested in the following way:

- Ask a question — make an observation and ask why or how whatever you've observed happens.
- Suggest an answer, or part of an answer, by forming a theory or a model.
- Make a **prediction** or **hypothesis** — a specific testable statement, based on the theory, about what will happen in a test situation.
- Carry out tests — to provide evidence that will support the hypothesis or refute it (help to disprove it).

Tip: A theory is only scientific if it can be tested.

Tip: The results of one test can't prove that a theory is true — they can only suggest that it's true. They can however disprove a theory — show that it's wrong.

Example

Question: What stops beta radiation?

Theory: Beta radiation will be stopped by a 5 mm thick sheet of aluminium.

Hypothesis: If beta radiation is stopped by a 5 mm thick sheet of aluminium a Geiger-Müller tube and counter will not detect radiation from a beta source if the aluminium sheet is placed between the source and the tube.

Test: Measure the background radiation count rate using the Geiger-Müller tube and counter without a source present. Then, place a beta-emitting source in front of the tube. Record the new count rate. Then, place a 5 mm thick sheet of aluminium between the source and the tube and once more record the count rate. If the count rate has dropped to a level similar to the background count rate, then the beta radiation emitted by the source has been absorbed by the aluminium sheet, and this evidence supports the hypothesis.

Communicating results

The results of testing a scientific theory are published — scientists need to let others know about their work. Scientists publish their results in scientific journals. These are just like normal magazines, only they contain scientific reports (called papers) that use scientific terminology, instead of the latest celebrity gossip.

Scientific reports are similar to the lab write-ups you do in school. And just as a lab write-up is reviewed (marked) by your teacher, reports in scientific journals undergo **peer review** before they're published. The report is sent out to peers — other scientists who are experts in the same area.

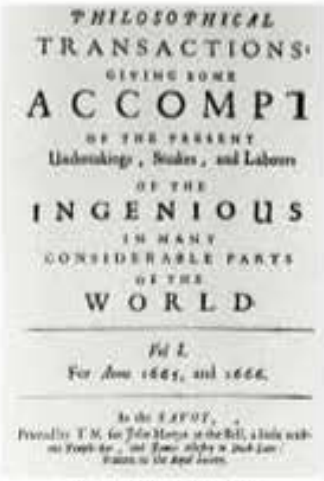

PHILOSOPHICAL
TRANSACTIONS
GIVING SOME
ACCOMPT
OF THE PRESENT
Undertakings, Studies, and Labours
OF THE
INGENIOUS
IN MANY
CONSIDERABLE PARTS
OF THE
WORLD
Vol. I.
For Anno 1665, and 1666.

Figure 1: *The first British scientific journal, 'Philosophical Transactions of the Royal Society', published in 1665.*

Tip: Scientific research is often funded by companies who have a vested interest in its outcomes. Scientists are ethically obliged to make sure that this does not bias their results.

The other scientists go through it bit by bit, examining the methods and data, and checking it's all clear and logical. Thorough evaluation allows decisions to be made about what makes a good methodology or experimental technique. Individual scientists may have their own ethical codes (based on their humanistic, moral and religious beliefs), but having their work scrutinised by other scientists helps to reduce the effect of personal bias (either deliberate or accidental) on the conclusions drawn from the results.

When the report is approved, it's published. This makes sure that work published in scientific journals is of a good standard. But peer review can't guarantee the science is correct — other scientists still need to reproduce it. Sometimes mistakes are made and bad work is published. Peer review isn't perfect but it's probably the best way for scientists to self-regulate their work and to publish quality reports.

Validating theories

Other scientists read the published theories and results, and try to test the theory themselves in order to validate it (back it up). This involves:

- Repeating the exact same experiments.
- Using the theory to make new predictions and then testing them with new experiments.

Tip: Once an experimental method is found to give good evidence it becomes a protocol — an accepted method to test that particular thing that all scientists can use.

Examples

- In 1989, two scientists claimed that they'd produced 'cold fusion' (the energy source of the Sun but without the high temperatures). If it was true, it would have meant cheap energy for the world forever. However, other scientists just couldn't reproduce the results, so the theory of 'cold fusion' couldn't be validated.
- In the 1960s it was proposed that particles like protons were made up of smaller particles called quarks — the quark model (p.424). After this, many experiments were conducted that supported this idea, and so validated the quark model.

How do theories evolve?

If multiple experiments show a theory to be incorrect then scientists either have to modify the theory or develop a new one, and start the testing again. If all the experiments in all the world provide good evidence to back a theory up, the theory is thought of as scientific 'fact' (for now) — see Figure 2. But it will never become totally indisputable fact. Scientific breakthroughs or advances could provide new ways to question and test the theory, which could lead to new evidence that conflicts with the current evidence. Then the testing starts all over again... And this, my friend, is the tentative nature of scientific knowledge — it's always changing and evolving.

Tip: Sometimes data from one experiment can be the starting point for developing a new theory.

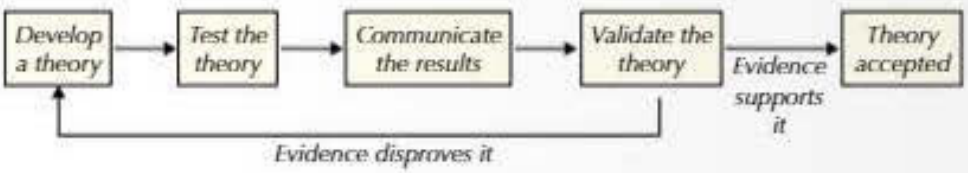


Figure 2: *Flow diagram summarising the scientific process.*

Example

The structure of the atom

It took years and years for the current model of the atom to be developed and accepted.

Dalton's theory in the early 1800s, that atoms were solid spheres, was disputed by the results of Thomson's experiments at the end of that century. As a result, Thomson developed the 'plum pudding' model of the atom, which was later proven wrong by Rutherford's alpha-scattering experiments in the early 1900s. Rutherford's 'nuclear model' has since been developed and modified further to create the currently accepted model of the atom we use today — but scientists are still searching for more accurate models.

Tip: See p.411 for more on the development of the model of the atom.

Collecting evidence

1. Evidence from lab experiments

Results from controlled experiments in laboratories are great. A lab is the easiest place to control **variables** so that they're all kept constant (except for the one you're investigating). This means you can draw meaningful conclusions.

Tip: Pages 23-29 are all about carrying out practicals.

Example

The pressure of a gas

If you're investigating how the volume of a gas affects its pressure, you need to keep all other factors constant. This means controlling things like the temperature of the gas. Otherwise there's no way of knowing that the volume change is what is causing the pressure to change.

2. Investigations outside the lab

There are things you can't study in a lab. And outside the lab, controlling the variables is tricky, if not impossible.

Example

Does living near power lines increase the risk of developing certain cancers?

You could compare the number of cancer cases in a group of people who live near power lines to a group of people who don't. But there are always differences between groups of people. The best you can do is to have a well-designed study using matched groups — choose two groups of people (those who live near power lines and those who don't) that are as similar as possible (same mix of ages, same mix of diets etc.). But you still can't rule out every possibility.

Figure 3: *Studies are ongoing to determine if there is a link between proximity to power lines and the risk of cancer.*

Science and decision making

Scientific knowledge is used by society (that's you, me and everyone else) to make decisions — about the way we live, what we eat, what we drive, etc. All sections of society use scientific evidence to make decisions, e.g. politicians use it to devise policies and individuals use science to make decisions about their own lives.

Tip: Don't get mixed up — it's not the scientists who make the decisions, it's society. Scientists just produce evidence to help society make the decisions.

Example

X-rays are used in dentistry to see images of the teeth inside the human body. They're used to monitor dental health, including seeing when adult teeth or wisdom teeth are coming through.

However, science has found a link between exposure to X-rays and increased risk of cancer. To minimise this risk, there are restrictions on when X-ray images of a patient's mouth can be taken (e.g. not during pregnancy), and how often. As a result, patients don't tend to have X-ray images taken as part of their standard check-ups, unless there is a strong medical reason to do so.

Factors affecting decision making

The scientific evidence we have can be overshadowed by other influences such as personal bias and beliefs, public opinion, and the media.

Tip: People's perception of risk also affects how they make decisions. For example, people may underestimate the risk of things with long-term harmful effects that aren't immediately seen.

Economic factors

Society has to consider the cost of implementing changes based on scientific conclusions. Sometimes it decides the cost outweighs the benefits.

Tip: Economic factors just mean anything to do with money, and social factors just mean anything to do with people.

Example

Building new power plants that use renewable energy resources helps to reduce our contribution to global warming but it costs money. Sometimes the cost of building a new plant is just too much for the government to justify it, especially when government money could be put to more immediate use in, for example, the NHS or schools.

Social factors

Decisions affect people's lives — sometimes people don't want to follow advice, or are strongly against some recommendations.

Examples

- Exposure to UV radiation from tanning beds can lead to cancer. Scientists recommend that people don't use tanning beds, but shouldn't we be able to choose whether we want to use them or not?
- People may not want new wind farms to be built in certain locations, as they believe that the turbines spoil the view.

Figure 4: *Wind farms are often built at sea to reduce the impact on people.*

Environmental factors

Some scientific research and breakthroughs might affect the environment. Not everyone thinks the benefits are worth the possible environmental damage.

Example

Hydroelectricity requires the building of a dam. This often destroys the habitats of plants and animals, so many think that other renewable energy resources should be used instead.

1. Planning an Experiment

You have to do practical work in class as part of your course. You'll be asked about it in exams too, so you need to know how to plan the perfect experiment.

Learning Objectives:

- Know how to design experiments, including how to solve problems set in a practical context.
- Be able to identify variables that must be controlled, where appropriate.
- Be able to select appropriate units for measurements.
- Know how to use a wide range of practical apparatus and techniques correctly.
- Be able to evaluate whether an experimental method is appropriate to meet the expected outcomes.

Specification References 1.1.1 and 1.1.2

Designing an experiment

PRACTICAL ACTIVITY GROUP 11

Before you start planning an experiment, you need to be clear about what you're trying to find out. You might be asked to design a physics experiment to investigate something or answer a question. It could be a lab experiment that you've seen before, or something applied, like deciding which building material is best for a particular job. Either way, you'll be able to use the physics you know from your course and the skills in this topic to figure out the best way to investigate the problem.

Planning experiments

Planning an experiment properly helps to make your results valid, accurate and precise (see p.18). There's plenty of info on the next few pages, but here's a summary of how you go about it...

- Make a prediction or hypothesis — you need to come up with an idea (or question) based on a theory. This is what you'll be trying to prove (or answer) with an experiment.
- Think about the aims of the experiment and identify the independent, dependent and control variables (see below).
- Select appropriate equipment (see page 7) that will give you accurate and precise results.
- Do a risk assessment and plan any safety precautions (see pages 8, 24 and 25).
- Decide what data to collect and how you'll do it (p.9).
- Write out a clear and detailed method — it should be clear enough that anyone could follow it and repeat your experiment exactly.
- Carry out tests — to provide evidence that will support the prediction or refute it.

Tip: There's lots more about how hypotheses are created and the scientific process in How Science Works on pages 1-4.

Variables

You probably know this all off by heart but it's easy to get mixed up sometimes. So here's a quick recap.

A **variable** is a quantity that has the potential to change, e.g. mass. There are two types of variable commonly referred to in experiments:

Independent variable — the thing that you change in an experiment.

Dependent variable — the thing that you measure in an experiment.

Tip: Time is always an independent variable.

Tip: For more on potentiometers see page 180.

Example

You could investigate the effect of varying the potential difference across a filament lamp on the current flowing through it using the circuit shown in Figure 1 below:

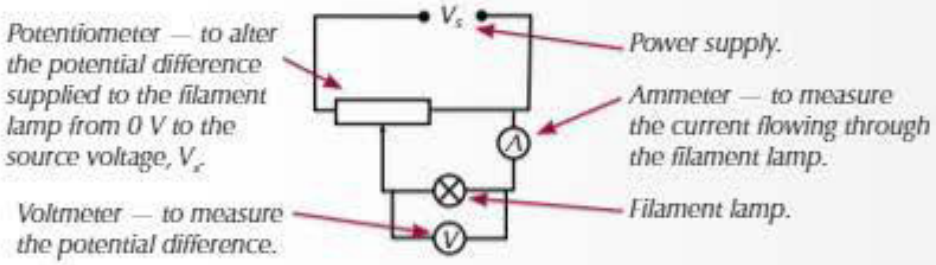

Potentiometer — to alter the potential difference supplied to the filament lamp from 0 V to the source voltage, V_s.
V_s
Power supply.
Ammeter — to measure the current flowing through the filament lamp.
A
Filament lamp.
Voltmeter — to measure the potential difference.
V

Figure 1: *Circuit diagram for measuring the current flowing through a filament lamp.*

- The independent variable will be the potential difference supplied to the lamp.
- The dependent variable will be the current flowing through the lamp.

Apart from the independent and dependent variables, all other variables should stay the same during your experiment. These are known as **control variables**. If control variables are not kept the same, you can't tell whether or not the independent variable is responsible for any changes in your dependent variable, so your results won't be valid (page 18). Control variables that are likely to change during your experiment should be measured to check that they are constant throughout the experiment.

Figure 2: *The filament in a bulb is designed to heat up, so it should be allowed to cool in between repeats to make the experiment a fair test.*

Example cont.

So for the circuit experiment above, the control variables include the length of the connecting leads, the filament bulb used and the temperature of the circuit wires. To keep the temperature constant, you might use small potential differences (and currents) to stop the circuit wires heating up during the experiment, and allow the filament to cool between repeats.

Apparatus and techniques

Before you start, make sure you're measuring things using appropriate units. To choose the correct units, you need to think about what scale your measurements of the independent and dependent variables are likely to be on (e.g. millimetres or metres, milliseconds or hours) and how you're going to analyse the results later.

Example

If you're measuring time, it might be better to use seconds rather than minutes — when you come to processing your results, it'll be easier to work with a result of 73 seconds than a result of 1.217 minutes.

Think about the range you plan on taking measurements over too — e.g. if you're measuring the effect of increasing the force on a spring, you need to know whether you should increase the force in steps of 1 newton, 10 newtons or 100 newtons. Sometimes, you'll be able to estimate what effect changing your independent variable will have, or sometimes a **pilot experiment** might help. In a pilot experiment, the apparatus is set up as it will be used in the actual experiment, and the independent variable is varied to see (roughly) how much the dependent variable changes.

Example

A student is investigating the extension of a test wire using the apparatus in Figure 3. She will add weights to the wire and measure its extension using the marker and the ruler.

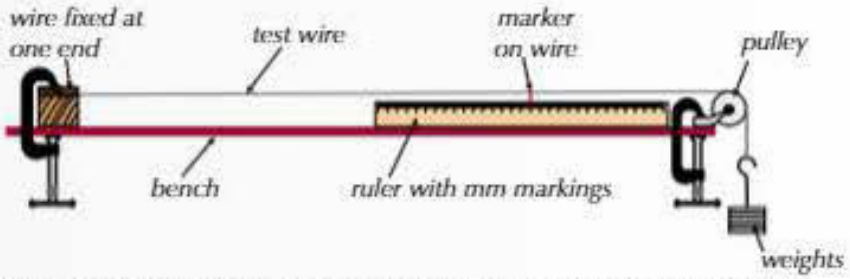


Figure 3: *Experimental set-up for investigating the extension of a test wire.*

She decides to do a pilot experiment to decide on the increments in which she should add weights to the test wire. She is aiming to find out how much weight needs to be added to cause the marker on the wire to move a measurable distance along the ruler that can be clearly recorded, but won't bring the wire close to the point at which it will snap.

Tip: There's more on using this apparatus on page 124.

Tip: The extension being measured here is very small, so you need a long piece of wire for the change to be noticeable. The wire is too long to suspend vertically from a clamp, which is why the pulley is more suitable here.

Tip: Measuring a distance that is small compared to the resolution of your equipment will lead to a high percentage error in your measurements — see page 16.

Considering your measurements before you start will also help you choose the most appropriate apparatus and techniques for the experiment. You want to pick the apparatus that will give you the best results. Make sure you know how to use all the instruments and equipment you've come across in class and can carry out all the techniques too.

Examples

- If you're measuring the length of a spring that you're applying a force to, you might need a ruler. If you're measuring the diameter of a wire, you'd be better off with some callipers.
- If you're measuring a time interval, you could use a stopwatch. If the time is really short (for example if you're investigating acceleration due to gravity of an object as it falls to the floor), you might need something more sensitive, like light gates (see page 66).

Tip: Remember — part of choosing your apparatus and planning your technique will involve thinking about safety precautions. This should all be covered in your risk assessment (see pages 8, 24 and 25).

Make sure you perform all techniques carefully and that any apparatus is set up correctly — this will help to minimise errors which would affect your results. E.g. if you're measuring a length, make sure your eye is level with the ruler when you take the measurement.

Figure 4: *Goggles should be used in most experiments to protect the eyes from any moving objects, snapping wires, chemicals and other dangers.*

Identifying risks

You'll be expected to show that you can identify any risks and hazards in an experiment. You'll need to take appropriate safety measures depending on the experiment. For example, for anything involving lasers you'll probably need to wear special laser goggles.

Evaluating experiment designs

If you need to evaluate an experiment design, whether it's your own or someone else's, you need to think about the following things:

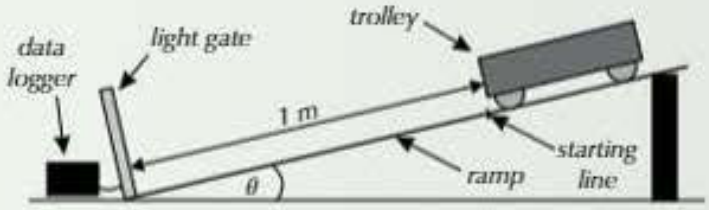
- Does the experiment actually test what it sets out to test?
- Is the method clear enough for someone else to follow?
- Apart from the independent and dependent variables, is everything else going to be properly controlled?
- Are the apparatus and techniques appropriate for what's being measured? Will they be used correctly?
- Will the method give precise results? E.g. are repeat measurements going to be taken in order to calculate a mean value (see page 17)?
- Is the experiment going to be conducted safely?

Exam Tip
Examiners love getting you to suggest improvements to methods — e.g. how a method could be improved to make the results more precise, so make sure you know how to design a good experiment.

Tip: The light gate is a suitable piece of apparatus to use in this experiment as it can measure the instantaneous velocity of the trolley as it is accelerating. This is not possible with a stopwatch.

Practice Question — Application

Q1 A student is investigating how the angle of a ramp, θ, affects the final velocity of a trolley as it travels 1 m down the ramp. He measures the velocity at the bottom of the ramp using a light gate. The student assumes the friction acting on the trolley is negligible. He sets up the following apparatus:



a) Name the independent and dependent variables in this investigation.

b) Name one variable in this investigation that the student should keep the same.

Practice Questions — Fact Recall

Q1 Why is it important to make sure the control variables don't change during an experiment?

Q2 State what is meant by a pilot experiment, and explain why they are used.

Q3 Give three things that must be considered when evaluating an experimental method.

2. Recording and Processing Data

Time to find out how to record your data correctly, and how to do maths-y things with your data like calculating means. The fun doesn't end...

Learning Objectives:

- Be able to record and present observations and data in an appropriate format.
- Understand the terms 'qualitative results' and 'quantitative results'.
- Be able to use significant figures appropriately.
- Be able to process experimental results.
- Be able to use appropriate mathematical skills to process quantitative data.
- Be able to identify anomalies in experimental measurements.

Specification References 1.1.2 1.1.3 and 1.1.4

Tables of data

You need to make a table to write your results in. You should include:

- Space for your independent variable and your dependent variable. You should specify the units in the headers, not within the table itself.
- Space to repeat each measurement at least three times to reduce random error and make sure your final results are more precise (see p.17), and to spot anomalous results (p.10).
- Space for any results from data processing you need to do, e.g. calculating the mean from repeats (see the next page), or calculating speed from measurements of distance and time. (Your table shouldn't include your workings, just the results from your calculations.)

Your table should have the independent variable in the first column, and the dependent variable in the columns next to that. Any results from data processing should be in the furthest right-hand column(s). Figure 1 is the sort of table you might end up with when you investigate the effect of potential difference (p.d.) on current.

P.d. / V	Current / A Run 1	Current / A Run 2	Current / A Run 3	Mean current / A (to 3 s.f.)
0.0	0.000	0.000	0.000	**0.000**
1.0	0.104	0.105	0.102	**0.104**
2.0	0.150	0.151	0.149	**0.150**
3.0	0.188	0.187	0.187	**0.187**
4.0	0.219	0.220	0.193	**0.220**

Figure 1: *Table of results showing the effect of p.d. on current through a resistor.*

Most of the time, you'll be recording numerical values, known as **quantitative data**. Occasionally, you may have to deal with data that can be observed but not measured with a numerical value. This is known as **qualitative data** (see Figure 2). It's still best to record this kind of data in a table to keep your results organised, but the layout may be a little different.

Tip: You should give all of your data to the number of decimal places that you measured to — e.g. 0.000 A, not just 0 A.

Figure 2: *Measuring how the diffraction of water waves varies with gap width will involve recording qualitative data.*

Rounding to significant figures

When using your data in further calculations you need to consider significant figures. The first significant figure of a number is the first digit that isn't a zero. The second, third and fourth significant figures follow on immediately after the first (even if they're zeros).

Example **Maths Skills**

0.6074976 rounds to **0.61** to **2 s.f.** and to **0.607** to **3 s.f.**

When you're doing calculations using measurements given to a certain number of significant figures, you should give your answer to the lowest number of significant figures that was used in the calculation.

The lowest number of significant figures in the calculation is used because the fewer digits a measurement has, the less accurate it is. Your answer can only be as accurate as the least accurate measurement in the calculation.

Exam Tip
You shouldn't give your answer to too few significant figures. So if the lowest number of significant figures in a calculation is 3, then you shouldn't give your answer to 1 or 2 significant figures. You could lose a mark in the exam if you do this.

Example — **Maths Skills**

For the calculation: $1.2 \div 1.85 = 0.648648648...$

1.2 is given to 2 s.f. and 1.85 is given to 3 s.f., so the answer should be given to 2 s.f.: $1.2 \div 1.85 =$ **0.65 (to 2 s.f.)**

Calculating a mean

For many experiments, you'll need to calculate the arithmetic **mean** (average) of some repeated measurements:

$$\text{arithmetic mean (average) of a measurement} = \frac{\text{sum of your repeated measurements}}{\text{number of measurements taken}}$$

Watch out for **anomalous results**. These are ones that don't fit in with the other values and are likely to be wrong. They're usually due to experimental errors, such as making a mistake when measuring (see page 36). You should ignore anomalous results when you calculate averages.

Tip: Just because you ignore anomalous results in your calculations, you shouldn't ignore them in your write-up. Try to find an explanation for what went wrong so that it can be avoided in future experiments.

Example — **Maths Skills**

Look at the table in Figure 1 again — the current at 4.0 V in Run 3 looks like it might be an anomalous result. It's much lower than the values in the other two runs. It could have been caused by the filament lamp being hotter at the end of Run 3 than it was at the end of the first two runs.

The anomalous result should be ignored to calculate the average:

With anomalous result: $(0.219 + 0.220 + 0.193) \div 3 = 0.211$ (to 3 s.f.)

Without anomalous result: $(0.219 + 0.220) \div 2 = 0.220$ (to 3 s.f.)

So the average current at 4.0 V should be 0.220 A (rather than 0.211 A).

Practice Questions — Application

Q1 Calculate the following, giving your answer to the correct number of significant figures:

a) 4.53×3.142 b) $0.315 \div 0.025$

Q2 A scientist carries out an investigation to find the force constant of a spring. She adds different masses to the spring, then records the weight of the masses and the spring's extension. Her results are shown in the table below. Complete the table by calculating the mean extension of the spring for each weight.

Tip: There's more on this spring experiment on page 116.

Weight / N	*Extension / cm*			
	Trial 1	*Trial 2*	*Trial 3*	*Mean*
2.0	*2.9*	*3.2*	*3.1*	
4.0	*5.8*	*8.6*	*5.9*	
6.0	*8.4*	*8.4*	*8.5*	

Practice Questions — Fact Recall

Q1 What is meant by quantitative data?

Q2 What is an anomalous result?

3. Presenting Results

Learning Objective:

- Be able to plot suitable graphs from experimental results, including selecting and labelling axes with appropriate scales, quantities and units.

Specification Reference 1.1.3

When you're planning an experiment, you need to think about the best way to present your results. It all depends on what sort of data you've got...

Types of data

1. Discrete data

You get discrete data by counting. E.g. the number of weights added to the end of a spring would be discrete (see Figure 1). You can't have 1.25 weights. That'd be daft. Shoe size is another good example of a discrete variable — only certain values are allowed.

There are lots of ways to present discrete data, depending on what other data sets you've recorded. Scatter graphs (p.12) and bar charts are often used.

Figure 1: *The number of weights added to a spring is discrete data. The spring's extension is continuous data.*

2. Continuous data

A continuous variable can have any value on a scale. For example, the extension of a spring or the current through a circuit. You can never measure the exact value of a continuous variable.

The best way to display two sets of continuous data is a line graph or a scatter graph (see page 12).

3. Categoric data

A categoric variable has values that can be sorted into categories. For example, types of material might be brass, wood, glass or steel.

If one of your data sets is categoric, a pie chart or a bar chart is often used to present the data — see Figure 2.

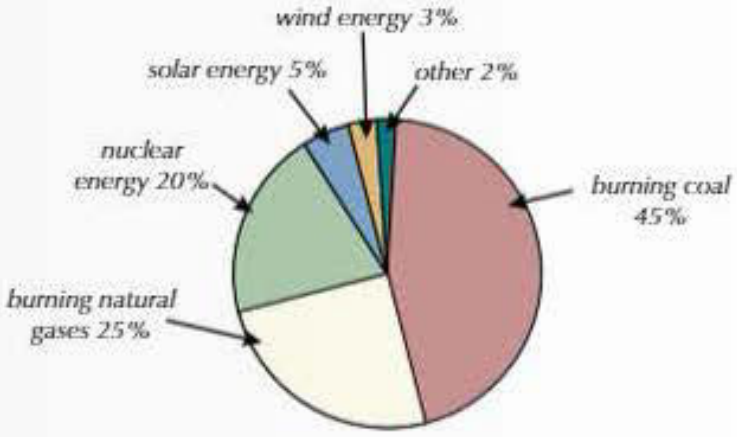


Figure 2: *Categoric data of different types of energy production in a particular country presented on a pie chart.*

4. Ordered (ordinal) data

Ordered data is similar to categoric, but the categories can be put in order. For example, if you classify frequencies of light as 'low', 'fairly high' and 'very high' you'd have ordered data.

A bar chart is often used if one of your data sets is ordered — see Figure 3.

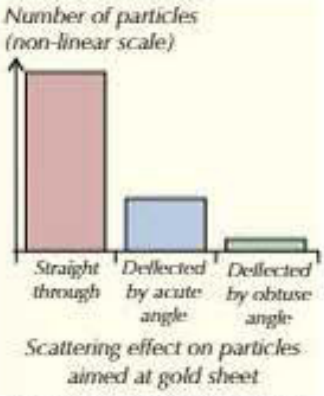

Number of particles (non-linear scale)
Straight through
Deflected by acute angle
Deflected by obtuse angle
Scattering effect on particles aimed at gold sheet

Figure 3: *Ordered data of particle scattering effects presented on a bar chart.*

Tip: Axes increments should go up in sensible amounts that are easy plot, like 1s, 2s, 0.1s, etc. You shouldn't ever use awkward increments like 3s or 7s as any intermediate points between those labels would be very tricky to plot or read.

Tip: You should make sure the scale is labelled with numbers regularly, e.g. at least every three large squares.

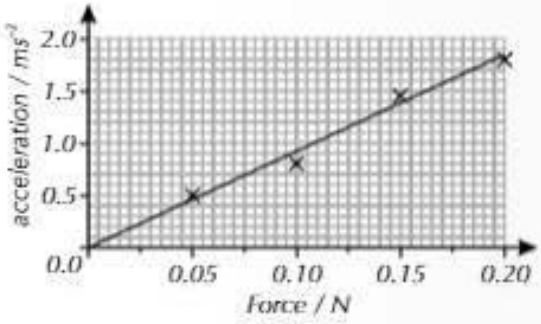
Scatter graphs

You can plot discrete or continuous data on a scatter graph. These are useful for spotting any correlation between two variables (see the next page). Usually, the independent variable goes on the *x*-axis and the dependent variable goes on the *y*-axis. Both axes should be labelled clearly, with the quantity and units. The scales used should be sensible (i.e. they should go up in sensible steps, and should spread the data out over the full graph rather than bunching it up in a corner). If you need to use your graph to measure something, select units for your axes that will let you do this easily (e.g. by measuring the gradient or the intercept, see pages 14 and 15).

Plot your points using a sharp pencil, to make sure they're as accurate as possible. You need to draw a line of best fit for your results. Around half the data points should be above the line, and half should be below it (you should ignore anomalous results). Depending on the data, the line of best fit might be straight or curved.

Example — **Maths Skills**

An experiment is carried out to see how the acceleration of a trolley is dependent on the force acting on it. The results are shown in the graph.



- The force is the independent variable and so is on the *x*-axis.
- The acceleration is the dependent variable and so is on the *y*-axis.
- The line of best fit is a straight line, with 2 points above the line and 2 points below it.

Example — **Maths Skills**

- In the graph on the right, the points show that the velocity is increasing at an increasing rate over time.
- So a curved line of best fit is drawn.

Practice Question — Application

Q1 A car is accelerating along a road. A computer connected to the car measures the velocity of the car every 2.5 s. The results are shown in the table below. Plot a suitable graph for the data.

Time / s	*0.0*	*2.5*	*5.0*	*7.5*	*10.0*	*12.5*	*15.0*
Velocity / mph	*0.0*	*12.8*	*22.4*	*29.0*	*33.3*	*35.8*	*37.0*

Practice Questions — Fact Recall

Q1 Give one way that categoric data can be presented in results.

Q2 Which variable usually goes on the *x*-axis of a scatter graph?

4. Analysing Results

Now you've got your data, you can use your results to describe what happened in the experiment. You can also use your results to calculate more quantities.

Learning Objectives:

- Be able to analyse and interpret qualitative and quantitative experimental results.
- Be able to use appropriate mathematical skills for analysis of quantitative data.
- Be able to interpret suitable graphs from experimental results, including by taking measurements of gradients and intercepts.

Specification Reference 1.1.3

Qualitative data

Analysing qualitative data is a bit more wordy than analysing quantitative data. You need to be able to talk about what your results are showing.

Example

An experiment is carried out to see the diffraction patterns produced by shining a laser through a slit in a piece of card. The size of the slit is varied to see the effect on the diffraction pattern.
A sketch of the results are shown below.

From these results you can see that as the slit gets wider, the pattern gets narrower — there are more fringes in a given area.

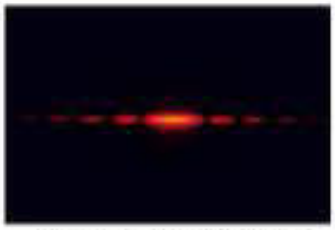


Figure 1: *The diffraction pattern produced when a laser is shone through a slit. There's more on diffraction patterns on p.212.*

Scatter graphs and correlation

Correlation describes the relationship between two variables — usually the independent one and the dependent one. Data can show positive correlation, negative correlation or no correlation (see Figure 2).

Positive correlation
As one variable increases, the other also increases.

Negative correlation
As one variable increases, the other decreases.

No correlation
There is no relationship between the variables.

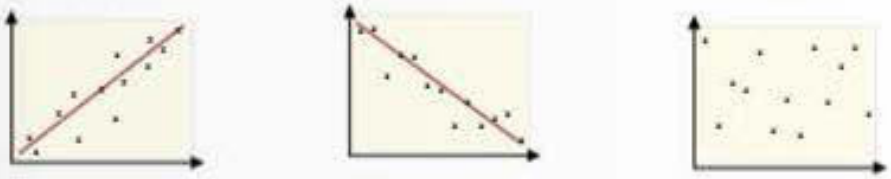


Figure 2: *Scatter graphs showing positive, negative and no correlation.*

Tip: You have to be very careful when drawing conclusions from data like this because a correlation between two variables doesn't always mean that a change in one variable causes a change in the other. There's more on this on page 20.

Linear graphs

If you plot two variables that have a linear relationship, you'll get a straight line of best fit. The equation of a straight line is $y = mx + c$, where m = gradient (slope of the line) and c = y-intercept. This means you can use your graph to work out certain values, and the relationship between your variables.

Proportionality

If you plot two variables against each other and get a straight line that goes through the origin, the two variables are **directly proportional**. The y-intercept, c, is 0, so the equation of the straight line is $y = mx$ where m is a constant. The constant of proportionality, m, is the gradient of the graph.

You can show two variables as being directly proportional using the symbol $\propto$. For example $x \propto y$ means that x is directly proportional to y.

Example — **Maths Skills**

Current and potential difference are directly proportional for ohmic conductors. You can see this by using the circuit in Figure 3. Use the variable resistor to decrease (or increase) the resistance in small equal steps. This changes the amount of current flowing through the ohmic conductor. Take readings of current through it and potential difference across it at each step. Once you have all the data, plot a graph of I against V (Figure 4). The graph you'll get is a straight line going through the origin — so current and potential difference are directly proportional.

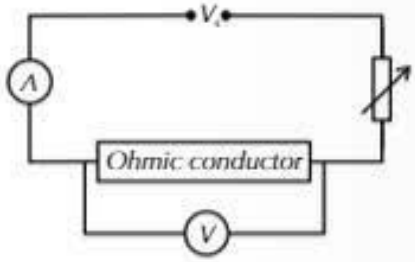


Figure 3: *Circuit for showing the proportional relationship between I and V for an ohmic conductor.*

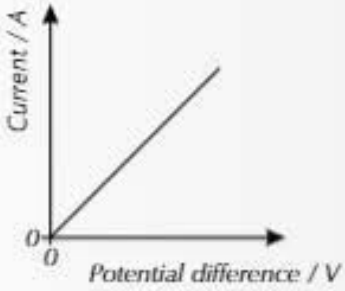


Figure 4: *An I-V graph for an ohmic conductor.*

Tip: For more on this experiment see page 162.

Finding the gradient and *y*-intercept

If you've plotted a linear graph, you can read the gradient and y-intercept straight off it. This means you can work out certain quantities from your graph. You can find the gradient of the straight-line graph using:

$$\text{Gradient} = \frac{\text{Change in } y}{\text{Change in } x}$$

You should draw a triangle to help you find the gradient of a line. Make it as big as possible — it should take up more than half of the line, and use values you can easily read from the graph.

Example — **Maths Skills**

Returning to the example of the ohmic conductor above, you can use the I-V graph to calculate the resistance of the component:

- I and V are directly proportional, so $I = kV$, where k is a constant.
- You know from $V = IR$ that $I = \frac{1}{R}V$.
- $I = \frac{1}{R}V$ is the equation of a straight line where $y = I$, $x = V$, $m = \frac{1}{R}$ and $c = 0$.
- You can work out $\frac{1}{R}$ from the graph — it's the gradient.

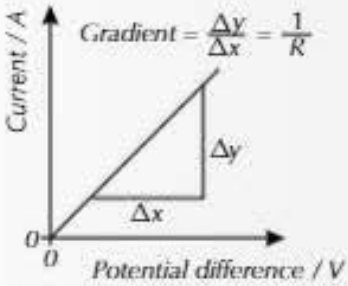


Figure 5: *Using an I-V graph to work out the resistance of an ohmic conductor.*

Tip: To find R, just calculate the inverse of $\frac{1}{R}$ found from the graph.

Tip: The gradient of an I-V graph like this is only equal to $\frac{1}{R}$ if the graph is a straight line through the origin (which is only true for ohmic conductors — see page 159).

Example — **Maths Skills**

The solid line in the graph in Figure 6 shows how the maximum kinetic energy of the photoelectrons from a metal surface varies with the frequency of the light shining on it. You can use a graph like this to find the value of the work function of the metal (ϕ) by extending the graph back to the y-axis.

Rearranging the equation $hf = \phi + KE_{max}$ gives $KE_{max} = hf - \phi$. Since h and ϕ are constants, $KE_{max} = hf - \phi$ is just the equation of a straight line (in the form: $y = mx + c$). You can just read ϕ from the graph — it's the intercept on the vertical axis. You'll just need to continue the line back to the y-axis to find the intercept, then the value of the y-intercept will be $-\phi$.

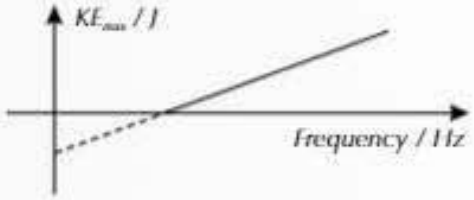


Figure 6: *You can extend the line on a graph to find the y-intercept.*

Tip: For more on work functions, see page 237.

Tip: Here, $y = KE_{max}$, $m = h$, x = frequency and $c = -\phi$.

Tip: If you were to calculate the gradient, you should get h, Planck's constant.

Non-linear relationships

Linear graphs are really easy to work with, but some variables won't produce a straight line if you plot them against each other. You can sometimes change what you plot on the axes so that you get one though.

For example, say two variables are inversely proportional, then $y = \frac{k}{x}$, where k is a constant. If you plot y against x, you'll get a curved graph which shoots off to infinity (Figure 7). It's not very easy to work out the value of k from this graph, but if you plot y against $\frac{1}{x}$ you'll get a lovely straight line (Figure 8) with a constant gradient of k that goes through the origin. This is because the graph plotted is $y = k\left(\frac{1}{x}\right)$, which is just the equation of a straight line in form $y = mx + c$, where $m = k$ and $x = \frac{1}{x}$ (and $c = 0$).

If you want to find the gradient of a point on a curved line of best fit, you can draw a tangent to the curve at that point and then find the gradient of the tangent (see p.480).

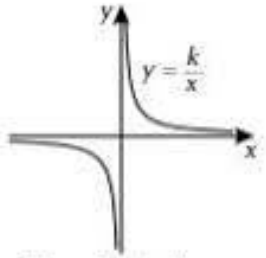


Figure 7: *An inverse proportionality relationship between x and y.*

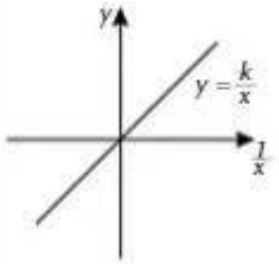
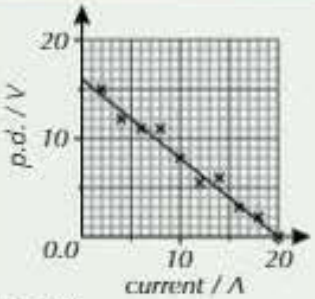


Figure 8: *If $y = \frac{k}{x}$, plotting y against $\frac{1}{x}$ gives a straight line through the origin.*

Practice Question — Application

Q1 A student wants to find the internal resistance of a cell. He sets up a series circuit containing the cell and a variable resistor. He varies the resistance of the variable resistor and records the current through and the potential difference (p.d.) across the cell. A graph of his results is shown on the right.



a) State what type of correlation the graph shows.

The graph illustrates the relation $V = \varepsilon - Ir$.

b) Determine ε, the e.m.f. of the cell.

c) Calculate r, the internal resistance of the cell.

Tip: This experiment is shown on page 173.

Learning Objectives:

- Understand precision and accuracy of measurements and data, including margins of error, percentage errors and uncertainties in apparatus.
- Understand how to evaluate results.
- Be able to refine experimental designs by suggesting improvements to the procedures and apparatus.
- Be able to identify the limitations in experimental procedures.
- Be able to draw conclusions.

Specification Reference 1.1.4

5. Evaluating and Drawing Conclusions

You need to be able to evaluate your results and draw conclusions from them. You also need to be able to evaluate and draw conclusions from other people's data — which is what you're likely to be asked to do in the exams.

Uncertainty

When you're evaluating your results, it's often a good idea to talk about the uncertainty in your data — in other words, the amount of error there might be. The results you get from an experiment won't be completely perfect — there'll always be a degree of uncertainty in your readings or measurements due to limits in the sensitivity of the apparatus you're using. Say you measure the temperature of a beaker of water with an analogue thermometer. You might think you've measured its temperature as 21 °C, but at best you've probably measured it to be 21 ± 0.5 °C. And that's without taking into account any other errors that might be in your measurement.

If you measure a length of something with a ruler, you actually take two measurements, one at each end of the object you're measuring. There is an uncertainty in each of these measurements. E.g. a length of 17.0 cm measured using a mm ruler will have an uncertainty of 0.05 + 0.05 = ± 0.1 cm (page 37).

The ± bit gives you the range in which the true value (the one you'd really like to know) probably lies. For the length above, 17 ± 0.1 cm tells you the true length is very likely to lie in the range of 16.9 to 17.1 cm. The maximum difference between your value and the true value (here 0.1 cm) is sometimes called the **margin of error**.

The smaller the uncertainty in a result or measurement, the smaller the range of possible values the result could have and the more precise your data can be. There are two measures of uncertainty you need to know about:

Figure 1: *This thermometer measures to the nearest °C. Any measurement you take using it will have an uncertainty of ± 0.5 °C.*

Absolute uncertainty — the total uncertainty for a measurement.

Percentage error — the uncertainty given as a percentage of the measurement.

Tip: There's more about different types of errors and how to do calculations with uncertainty on pages 36 and 37.

Example — **Maths Skills**

The resistance of a filament lamp is given as 5.0 ± 0.4 Ω. Give the absolute uncertainty and the percentage error for this measurement.

The absolute uncertainty is 0.4 Ω.

To get the percentage error, just convert this to a percentage of the lamp's resistance: (0.4 ÷ 5.0) × 100 = 8%

Reducing uncertainty

There are a few different ways you can reduce the uncertainty in your results:

Repeating and averaging

One of the easiest things you can do is repeat each measurement several times. The more repeats you do, and the more similar the results of each repeat are, the more precise the data.

By taking an average (mean) of your repeated measurements (see p.10), you will reduce the random error in your result. The more measurements you average over, the less random error you're likely to have. Repeating also allows you to check your data for any anomalous results (see p.10).

Tip: Precision and errors are covered on pages 18 and 36.

Using appropriate equipment

You can also cut down the uncertainty in your measurements by using the most appropriate equipment. The smallest possible uncertainty in a measurement is usually taken to be ± half the smallest interval that the measuring instrument can measure. A micrometer scale has smaller intervals than a millimetre ruler, so by measuring a wire's diameter with a micrometer instead of a ruler, you instantly cut down the random error in your experiment.

Figure 2: *A micrometer is very precise, it gives readings to within 0.01 mm.*

Computers and data loggers can often be used to measure smaller intervals than you can measure by hand and reduce random errors, e.g. timing an object's fall using a light gate rather than a stopwatch. You also get rid of any human error that might creep in while taking the measurements.

There's a limit to how much you can reduce the random uncertainties in your measurements, as all measuring equipment has a **resolution** — the smallest change in what's being measured that can be detected by the equipment. The resolution of the equipment is an example of a limitation of an experiment (see the next page).

Tip: A data logger (or data recorder) is an electronic device that can record data over time using a sensor. They can sometimes be connected to a computer.

Significant figures and uncertainty

You always have to assume the largest amount of uncertainty in data.

Whether you're looking at experimental results or just doing a calculation question in an exam, you must round your results to the same number of significant figures as the given data value with the fewest significant figures. Otherwise you'd be saying there is less uncertainty in your result than in the data used to calculate it.

If no uncertainty is given for a value, the number of significant figures a value has gives you an estimate of the uncertainty. The assumed uncertainty is half the increment of the last significant figure that the value is given to.

Example — **Maths Skills**

2 N only has 1 significant figure, so without any other information you know this value must be 2 ± 0.5 N — if the value was less than 1.5 N it would have been rounded to 1 N (to 1 s.f.), if it was 2.5 N or greater it would have been rounded to 3 N (to 1 s.f.).

Evaluations

Figure 3: *Newton's famous experiment to show that white light is made up of a spectra of colours has been reproduced by scientists and students all over the world. The results are now accepted.*

Tip: It's possible for results to be precise but not accurate, e.g. a balance that weighs to 1/1000th of a gram will give precise results, but if it's not calibrated properly the results won't be accurate.

Before you can make any conclusions, you'll need to evaluate your results to see how convincing they are. You need to be careful about what words you use — precise, valid and accurate may all sound similar, but they all say different things about your results.

1. Precise results

The smaller the amount of spread of your data from the mean, the more precise it is. Precision depends on the amount of random error (p.36) in your readings. A precise result is one that is repeatable and reproducible.

- **Repeatable results** — Results are repeatable if you can repeat an experiment multiple times and get the same results.
- **Reproducible results** — Results are reproducible if someone else can recreate your experiment using different equipment or methods, and get the same results you do.

2. Valid results

A valid result answers the original question using precise data. If you don't keep all variables apart from the ones you're testing constant, you haven't only tested the variable you're investigating — the results aren't valid.

3. Accurate results

An accurate result is really close to the true answer.
You can only comment on how accurate a result is if you know the true value of the result. So you can't assess the accuracy of a result if your measuring something that's unknown or has never been measured before.

Evaluating methods

When you evaluate your method, you need to think about how you could improve your experiment if you did it again. Here are some things to consider:

- Is there anything you could have done to make your results more precise or accurate? Could you do anything to prevent random or systematic errors (p.36)?
- Were the results valid? If not, could you fix this, e.g. by changing the data you're collecting?
- Were there any limitations in your method (see below)?
- Did you take enough measurements to be confident in spotting any anomalies? Should you have taken measurements more frequently?
- Could you have reduced the uncertainty in your results, e.g. by using more sensitive apparatus or equipment?

Experimental limitations

An experiment will always have limitations — this is anything that will affect the overall results. You need to be able to identify the limitations for any experiment (e.g. the resolution of the equipment used, see p.17).

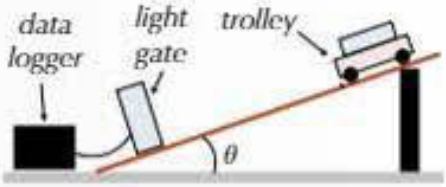
Example

A student needs to measure the length of a piece of wire to use in an experiment. She does this using a ruler that can measure to the nearest millimetre. She measures it to be 9.8 cm. The length of the wire could actually be 9.8127563 cm. But the measurement is limited by the resolution of the ruler.

Other limitations include uncontrollable external variables that contribute to the data.

Example

A student sets up the apparatus as shown to see how varying the angle of the ramp (θ) affects the acceleration of the trolley on the ramp. The acceleration of the trolley is due to gravity, and the student therefore assumes it is constant.



The main limitation in this experiment is the friction acting on the trolley due to the ramp and the air. The friction will increase as the speed of the trolley increases, which means the acceleration of the trolley on the ramp will not be constant. You can reduce the amount of friction acting on the trolley by using a smooth ramp and making the trolley more streamlined, but it is not possible to remove the effects of friction completely.

Tip: For more on the dependence of friction on speed see page 90.

Drawing conclusions

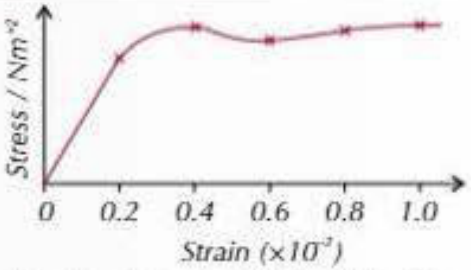
A conclusion explains what the data shows. You can only draw a conclusion if your data supports it. Your conclusion should be limited to the circumstances you've tested it under — if you've been investigating how the current flowing through a resistor changes with the potential difference across it, and have only used potential differences between 0 and 6 V, you can't claim to know what would happen if you used a potential difference of 100 V, or if you used a different resistor.

You also need to think about how much you can believe your conclusion, by evaluating the quality of your results (see previous page). If you can't believe your results, you can't form a strong conclusion.

Tip: Whoever funds the research (e.g. an engineering company) for an investigation may have some influence on what conclusions are drawn from the results, but scientists have a responsibility to make sure that the conclusions they draw are supported by the data.

Example

The stress of a material *X* was measured at strains of 0.002, 0.004, 0.006, 0.008 and 0.010. Each strain reading had an error of 0.001. All other variables were kept constant, and the results are shown on the right.



Tip: For more on interpreting stress-strain graphs see pages 127-130.

A science magazine concluded from this data that material *X*'s yield point is at a strain of 0.005. This could be true — but the data doesn't support this. Because strain increases of 0.002 at a time were used and the stress at in-between strains wasn't measured, you can't tell where the yield point is from the data. All you know is that the yield point is somewhere between 0.004 and 0.006, as the stress drops between these values.

Also, the graph only gives information about this particular experiment. You can't conclude that the yield point would be in this range for all experiments — only this one. And you can't say for sure that doing the experiment at, say, a different constant temperature, wouldn't give a different yield point.

You must also consider the error in the strain readings. The error in each reading is 0.001, which gives a percentage uncertainty of 50% for the lowest strain reading. This means the results might not be accurate.

Tip: Taking more readings in an experiment can allow you to make stronger conclusions. If a measurement had been taken at a strain of 0.005 here, you'd have been able to say more about where the yield point was. It's also good to do a pilot experiment (p.7) to get an idea of roughly where the yield point is before you start.

Correlation and cause

Your data may show a correlation between two variables, but the change in one may not be causing the change in the other. Both changes might be caused by a third variable.

Example

Some studies have found a correlation between exposure to the electromagnetic fields created by power lines and certain ill health effects. So some people argue that this means we shouldn't live close to power lines, or build power lines close to homes. But it's hard to control all the variables between people who live near power lines and people who don't. Ill health in people living near power lines could be affected by many lifestyle factors or even genetics. Also, people living close to power lines may be more likely to believe that any ill health they suffer is due to the EM fields from the power lines if they are aware of the studies.

Practice Questions — Application

Q1 a) What is the uncertainty of a mass balance that measures masses to the nearest 0.2 g?

b) What is the percentage error of a measurement of 50 g made with the mass balance in part a)?

Tip: Remember — if you don't know the uncertainty of the apparatus used to measure a value, you can use the number of significant figures of the value to work out its uncertainty.

Q2 A student is investigating the potential difference across a lamp. He measures the potential difference across the lamp 5 times whilst the current is 5.2 A. His results are shown in the table.

Repeat 1	*Repeat 2*	*Repeat 3*	*Repeat 4*	*Repeat 5*
3.4 V	*3.6 V*	*4.0 V*	*3.9 V*	*3.6 V*

a) What is the uncertainty of the value for current?

b) Calculate the mean potential difference.

Q3 A student is carrying out an experiment to find the resistivity of a wire by measuring its resistance at different lengths. She measures the resistance of the wire every 10 cm, from 10 cm to 60 cm. She uses a millimetre ruler to measure the length and diameter of the wire. She stops her experiment halfway through, and continues with it later the same day when the temperature of the room has dropped. She then plots a graph of wire length against resistance and uses the gradient to find the wire's resistivity.

a) Comment on the validity of her experiment.

b) Give two ways she could improve her experiment.

Practice Questions — Fact Recall

Q1 What is the percentage error of a measurement?

Q2 Give two ways of reducing uncertainty in an experiment.

Q3 What is the meaning of the terms precision and accuracy?

Section Summary

Make sure you know...

- How to design experiments in order to investigate something or answer a question.
- How to solve problems in a practical context by starting with a prediction or hypothesis.
- How to identify the different types of variables in an experiment, including independent, dependent and control variables.
- That control variables must stay the same throughout an experiment.
- How to select the units that data should be recorded in.
- How and when to use different practical apparatus and techniques.
- How to evaluate an experimental method, including whether the experiment will answer the original question and whether the experiment will be valid.
- How to correctly record data in tables.
- That all the measurements of one variable should be recorded to the same number of decimal places.
- What is meant by quantitative data and what is meant by qualitative data.
- What significant figures are and how many significant figures a value should be given to.
- How to calculate the arithmetic mean of the results of repeated measurements.
- What an anomaly is and how to identify one in a set of results.
- How to choose the correct way to present data depending on what the data is.
- How to plot scatter graphs, including choosing the right scales and units for the axes.
- How to draw a line of best fit on a scatter graph.
- How to analyse the results of data that is either qualitative or quantitative.
- That the equation of a linear graph is $y = mx + c$, where m is the gradient and c is the y-intercept.
- How to calculate the gradient and y-intercept of a scatter graph of results, and how to work out what these values represent in an experiment.
- How to find the uncertainty in a measurement from the resolution of the apparatus used.
- What the terms margin of error, absolute uncertainty and percentage error mean.
- That taking more repeats in an experiment or using more appropriate apparatus can reduce the uncertainty of the results.
- How to find the uncertainty of a value based on the number of significant figures it has been given to.
- The terms precision, repeatable, reproducible, valid and accurate and what they mean.
- How to evaluate an experiment and be able to comment on how an experiment could be improved if it was repeated.
- How to identify limitations in an experimental method.
- How to draw conclusions from an experiment based on the evidence available.
- That a correlation between two variables doesn't necessarily mean there is a causal relation between them.

Exam-style Questions

1 The equation that relates final velocity (***v***), initial velocity (***u***), acceleration (***a***) and displacement (***s***) is $v^2 = u^2 + 2as$.

A graph is plotted for an object moving with a constant acceleration, with v^2 on the *y*-axis and ***s*** on the *x*-axis. Which of the following is equal to the acceleration of the object?

A The square root of the *y*-intercept of the graph.

B The *y*-intercept of the graph.

C The gradient of the graph multiplied by 2.

D The gradient of the graph divided by 2.

(1 mark)

2 A student wants to find how the resistance of a thermistor varies with temperature. They set up the circuit shown in **Fig. 2.1**.

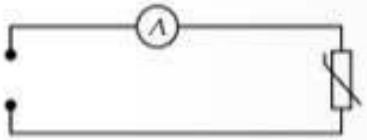


Fig. 2.1

The thermistor is then placed in a beaker of hot water that contains a thermometer. The thermometer has 1 °C graduations. The student then records the current through the thermistor for each 20 °C temperature drop of the water. She will use her results and the potential difference of the power supply to calculate the resistance of the thermistor. She repeats the experiment 6 times. Her results are shown in **Fig. 2.2**.

Temperature / °C	*Current / A*						
	Run 1	*Run 2*	*Run 3*	*Run 4*	*Run 5*	*Run 6*	*Mean*
90	*5.3*	*5.5*	*5.2*	*5.2*	*5.3*	*5.3*	*5.3*
70	*3.1*	*3.3*	*3.3*	*3.0*	*3.1*	*2.9*	*3.1*
50	*1.9*	*1.8*	*2.8*	*2.0*	*2.0*	*2.1*	

Fig. 2.2

a) Calculate the percentage uncertainty for the temperature reading of 90 °C.

(2 marks)

b) Calculate the mean value for the current when the water's temperature was 50 °C. Include the uncertainty of this value in your answer.

(3 marks)

c)* Evaluate the student's experiment. In your answer, you should include suggestions for how the student could improve her experiment.

(6 marks)

* The quality of your response will be assessed in this question.

Practical Skills for the Practical Endorsement

What is the Practical Endorsement?

The Practical Endorsement is assessed slightly differently to the rest of your course. Unlike the exams, you don't get a mark for the Practical Endorsement — you just have to get a pass grade. The Practical Endorsement is split into twelve categories, called Practical Activity Groups (PAGs). Each PAG covers a variety of practical techniques, for example, using an oscilloscope or constructing circuits. All the PAGs are listed on page 28 and the practical techniques are listed on page 29. In order to pass the Practical Endorsement, you'll have to carry out at least twelve experiments, and demonstrate that you can carry out each of the required techniques. You'll do the experiments in class, and your teacher will assess you as you're doing them.

You'll need to keep a record of all the assessed practical activities that you carry out for your Practical Endorsement.

Tip: Throughout this book, experiments and skills that you could use for your Practical Endorsement are marked with a big PAG stamp, like this one:

PRACTICAL ACTIVITY GROUP 7

1. General Practical Skills

PRACTICAL ACTIVITY GROUP 11

The way you do an experiment is important. You may be given a method, or you may have to plan it yourself. It's important that you follow all the steps in a method — this ensures that you work safely, and also makes your results more likely to be precise.

Solving problems in a practical context

Practical experiments are used to solve problems or test whether a theoretical model works in a practical setting. If you're given a method to follow for an experiment, you should carry out each step, as described, in the correct order.

Tip: Experiments are often used to test scientific theories to see if they are true in a practical context.

It's possible you'll be given a problem and asked to solve it using your own knowledge. There's loads of information about how to plan and carry out experiments correctly on pages 5-8. Here's a quick round-up of some of the things you'll need to think about when you plan an experiment:

- First, identify the aim of your experiment.
- Work out how to achieve the aim — you'll often need to identify the **independent** and **dependent variables** for the investigation.
- Identify all the control variables (and how to control them).
- Think about how to make your data as **precise** and **accurate** as possible.

Tip: Variables include things like temperature, time, mass and volume.

Tip: Precise results are results that don't vary much from the mean. Accurate results are close to the true value.

Implementing experiments

Once your experiment is planned, you can carry it out, working carefully to make sure your results are precise. Relevant practical techniques will be covered in this book as they come up during the course. All of these techniques should be carried out safely (see pages 24-25) and correctly.

Recording and analysing data

As you carry out your experiment, you should record your results in a well laid-out table, leaving space for any data analysis you might want to do later. Your table should have a heading for each column, where any units should be included. See pages 9-10 for more on recording and analysing data.

Tip: Any recorded results that don't fit in with the rest of the data are called anomalous results. If you can work out that they were due to a mistake when carrying out your experiment, you can leave them out of your data analysis.

Tip: A correlation between two variables can be positive (as one variable increases, the other increases), negative (as one variable increases, the other decreases), or there may be no correlation present.

Tip: Take a look back at pages 13-15 for more information about how to draw and interpret graphs.

Tip: When evaluating your experiment, you could comment on the uncertainty of your measurements and work out the absolute uncertainty or percentage error. The smaller these values are, the more precise your results will be. Look back at pages 16-17 for how to work them out.

Tip: A hazard is anything that has the potential to cause harm or damage. The risk associated with that hazard is the probability of someone (or something) being harmed if they are exposed to the hazard.

Figure 1: *Safety warning on a piece of electrical equipment.*

Presenting data

Presenting your data can make it easier for you to understand your results and spot any trends. How you present your data will depend on the type of data and what you want to find out. You'll have covered all this on pages 11-12, but you'll mostly want to present your results on a scatter graph.

Drawing a scatter graph lets you show how two variables are related (or correlated). You can draw a line (or curve) of best fit on the scatter graph to help show the trend in your results. The trend is called the **correlation**.

You may even be able to use software to process your data and generate graphs for you, especially if you've used a data logger in your experiment.

Conclusions and evaluations

When you've collected and analysed your data, it's time to wrap up your experiment with some conclusions and an evaluation. Conclusions explain what your results showed. They need to be supported by the data you collected, and shouldn't make sweeping generalisations. If you've found a correlation between two variables, you should be cautious about claiming that the change in one has caused the change in the other — there may be another factor that is causing both variables to change.

An evaluation is a chance for you to look at what you did and think about how you could have improved your method to improve the **validity**, accuracy and precision of your results. You should consider how well your results address the original aim of your experiment, how you could reduce errors in your results, and whether you need to repeat your experiment further to show your results can be reproduced.

Minimising risks

Many physics experiments have risks associated with them. These can include risks associated with the equipment you're using, as well as risks associated with, say, using radioactive materials.

When you're planning an experiment, you need to identify all the hazards and what the risk is from each hazard — this is a risk assessment. A risk assessment includes working out how likely it is that something could go wrong, and how serious it would be if it did go wrong. You then need to think of ways to reduce these risks.

One precaution that's worth taking in most experiments is to wear goggles. These will protect the eyes from moving objects, snapping wires, chemicals and other dangers.

Other precautions you need to take might depend on specific risk factors associated with particular experiments or materials.

Electricity

Some of the experiments that you are expected to do during your course involve electrical circuits. When working with electricity, always take the following steps to avoid shocks, overheating or damage to equipment:

- Make sure you turn the power supply off while you're adjusting your equipment, and then only turn the power on when you're ready to actually carry out the experiment.
- Never get electrical equipment wet. Wet electrical equipment can lead to short circuits, creating a risk of fire or electric shock.

- Always set up your equipment carefully, following all the instructions you've been given. And pay attention to any safety warnings on the equipment itself — see Figure 1 on the previous page for an example.
- Be very careful when working with capacitors. Even when not connected to a circuit, they can store enough charge to give a shock. Make sure you connect them the right way round in the circuit too, or you could damage the capacitor.

Tip: Some capacitors can catch fire or explode if connected the wrong way round.

Lasers

Lasers can be very dangerous because laser light is focused into a very direct, powerful beam of monochromatic light which can easily damage someone's eyesight.

When carrying out any experiments involving lasers, it's especially important to wear special laser goggles. To make sure you don't cause damage while using lasers, make sure you:

- Never shine the laser towards a person.
- Remember to turn it off when it's not in use.
- Wear laser safety goggles.
- Avoid shining the laser beam at a reflective surface.
- Have a warning sign on display (see Figure 2).

Figure 2: *Laser warning signs should be used to alert people of the danger of laser beams.*

Radioactive materials

Ionising radiation (e.g. alpha, beta or gamma radiation) can damage living cells. If a cell's DNA is damaged, there's a risk it may become cancerous. So when you're carrying out experiments involving radiative materials, remember how dangerous they can be and follow these precautions:

- Radioactive sources should be kept in a lead-lined box when they're not being used.
- Always wear gloves when working with radioactive materials.
- Radioactive materials should only be picked up using long-handled tongs or forceps.
- You must take care not to point them at anyone, and always keep a safe distance from them.

Figure 3: *Hazard symbol to indicate the presence of radioactive substances.*

Tip: Your teacher should closely supervise any use of radioactive sources.

Glassware

Many pieces of equipment that you may use during your course are made of glass. Broken glass can cause serious injury, so you should take care to transport glass items safely and check them for cracks and flaws before you use them. Any broken pieces of glass should be moved away from the work area immediately and disposed of in an appropriate container (not the normal waste bin).

Tip: The CLEAPSS® website has student safety sheets, and your school or college may have CLEAPSS® Hazcards® you can use. These are all good sources of information if you're writing a risk assessment.

Appropriate clothing

When working in the lab, you should make sure that you are wearing sensible clothing to reduce the risk of injury, e.g. open shoes or sandals won't protect your feet against things that have been accidentally dropped. You should also wear a lab coat to protect your skin and clothing. It's all about using your common sense really.

2. Keeping Scientific Records

When you carry out experiments, it's important to keep records of everything you do. The records should be detailed and clear enough that a complete stranger would be able to read them and understand what you did.

Records of scientific experiments

Throughout your A-Level Physics course, you should keep a record of all the experiments you carry out, the results you obtain and the solutions to any data analysis you do. This could be done in a physical lab book, or kept in folders on a computer. However you choose to keep your records, the information for each experiment should include:

- The aim of the experiment.
- A detailed method for how you carried out the experiment, including any safety precautions you had to take.
- The results of your experiment, clearly set out in a table. The results may be hand-written, or a print-out of data collected by a data logger.
- Any other important observations you made whilst carrying out your experiment, for example, anything that went wrong or anything you did slightly differently from how it was described in the method.
- The solutions to any analysis you did on your results, or any graphs drawn using your results. These should be clearly labelled to show what analysis has been done or what graph has been drawn.
- Citations of any references you used.

Tip: Try to keep all your lab reports in the same place — write them in the same book, or keep them in the same folder on your computer. That way you'll know where everything is.

Tip: A clear and detailed method is important, as it could be used by another scientist who is trying to reproduce your results.

Tip: There's loads more detail about making observations, recording data and analysing your results in Module 1.

Sources of information

PRACTICAL ACTIVITY GROUP 12

It's possible you'll have to do some research to find out information before you get started with an experiment. Useful sources of information include:

Websites

Using the Internet for research is really convenient, but you have to be slightly wary as not all the information you find will be true. It's hard to know where information comes from on forums, blogs and websites that can be edited by the general public, so you should avoid using these. Websites of universities or other respected institutions (e.g. the Institute of Physics), provide lots of information based on reliable scientific sources. To decide whether a website gives reliable information, think about the following things:

- Who has written the information — was it a scientist, a teacher, or just a member of the public?
- Whether or not anyone will have checked the source — articles on websites for scientific organisations will have people reading through the information and checking all the facts. Information on forums or blogs is likely to have been written by an individual, and won't necessarily have been thoroughly checked.
- What the purpose of the website is — if it's a website all about physics, then it's likely whoever has written it will know quite a lot. If it's a website where you can also find out how to make a laser gun from objects you'd find in a typical garden shed, then the depth and quality of the information may not be enough.

Tip: If you're unsure whether the information on a website is true or not, try and find the same piece of information in a different place. The more sources you can find for the information, the more likely it is to be correct.

Textbooks

Your school or public library is likely to have textbooks covering specific areas of physics in a lot of detail.

Scientific papers

You can find papers in online catalogues (e.g. arxiv.org), as well as in journals that are often available in public libraries.

The source you do your research from needs to give the right level of information. It's no good trawling through a scientific paper if you're just looking for the speed of light — you'll probably end up wading through lots of complicated information that you don't need to understand. Equally, if you're researching the theory behind an experiment, you want a source that gives enough detail. A GCSE textbook will probably be too simplistic — you're better off finding a book that deals specifically with the subject in a library instead.

Tip: Scientific papers are checked by other scientists who are experts in the subject of the paper. This is called peer review (see page 1).

Using references and making citations

PRACTICAL ACTIVITY GROUP 12

It sounds obvious, but when you're using the information that you've found during your research, you can't just copy it down word for word. Any data you're looking up should be copied accurately, but you should rewrite everything else in your own words.

When you've used information from a source, you need to cite the reference properly. **Citations** allow someone else to go back and find the source of your information. This means they can check your information and see you're not making things up out of thin air. Citations also mean you've properly credited other people's data that you've used in your work.

Citations are included in the main text of a report and are usually written in brackets after the relevant piece of information. They can either include the entire reference or link the information to a list of references at the end of the report (e.g. using a number — see Figure 1). **References** for each piece of information may include the title of the book, paper or website where you found the information, the author and/or the publisher of the document and the date the document was published.

Referencing a website: include the author(s), year, title [online], date accessed, URL.

Referencing a book: include the author(s), publication year, book title, edition, publisher's location and publisher.

Referencing a paper: include the author(s), publication year, title of the paper, the journal it was published in, the volume number and page numbers.

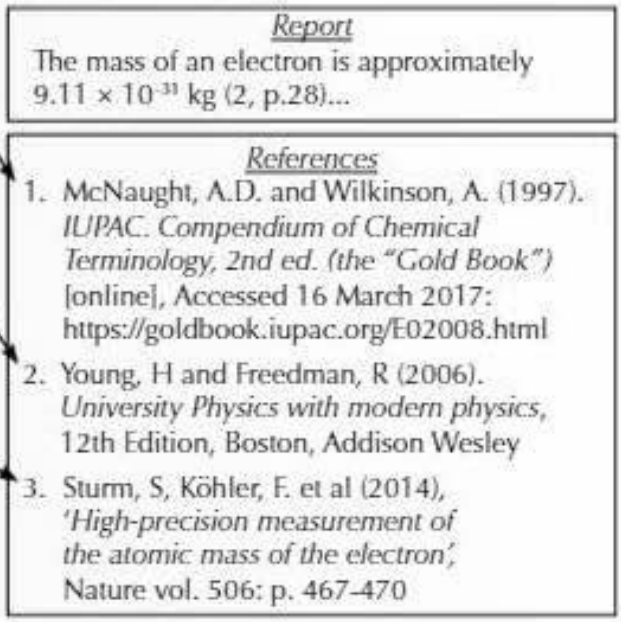

Report
The mass of an electron is approximately 9.11 × 10⁻³¹ kg (2, p.28)...
References
1. McNaught, A.D. and Wilkinson, A. (1997). IUPAC. Compendium of Chemical Terminology, 2nd ed. (the "Gold Book") [online], Accessed 16 March 2017: https://goldbook.iupac.org/E02008.html
2. Young, H and Freedman, R (2006). University Physics with modern physics, 12th Edition, Boston, Addison Wesley
3. Sturm, S, Köhler, F. et al (2014), 'High-precision measurement of the atomic mass of the electron', Nature vol. 506: p. 467-470

Figure 1: *Example of a citation in the main text of a report and the corresponding references document.*

Tip: There are lots of slightly different ways of referencing sources, but the important thing is that it's clear where you found the information.

Tip: You should include page numbers with your citation if you quote directly from the text or copy a diagram.

Tip: Scientific papers are often written by large groups of people. References often include the name of just a small number of these authors, followed by 'et al', which means 'and others'.

3. Practical Activity Groups

This section tells you all the Practical Activity Groups you'll be expected to have carried out for A-Level Physics, as well as the techniques included in them. They'll be covered in more detail as they crop up throughout the book.

PAGs

There are 12 Practical Activity Groups (PAGs) that you should have covered by the end of your A-Level course. These are shown in the table below, along with an example of the type of activity you could carry out for each one.

PAG		Example activity
1	Investigating motion	Determining the acceleration of free fall (see pages 51-52).
2	Investigating properties of materials	Determining Young's Modulus for a metal (see pages 124-125).
3	Investigating electrical properties	Determining the resistivity/conductivity of a metal (see pages 156-157).
4	Investigating electrical circuits	Investigation of potential divider circuits (see page 180).
5	Investigating waves	Determination of the wavelength of light with a diffraction grating (see page 328).
6	Investigating quantum effects	Determination of Planck's constant using LEDs (see pages 234-235).
7	Investigating ionising radiation	Absorption of α or β or γ radiation (see pages 429-430).
8	Investigating gases	Determining an estimate of absolute zero using variation of gas temperature with pressure (see pages 259-260).
9	Investigating capacitors	Determining how the current through a circuit containing a capacitor varies as the capacitor charges (see page 356).
10	Investigating simple harmonic motion	Investigating the factors affecting the period of a simple harmonic oscillator (see pages 287-289).
11	Investigation	Apply investigative approaches and methods to your practical work (see page 23).
12	Research skills	Researching online for further information on a topic (e.g. see page 26).

Tip: PAG11 and PAG12 are more general than the others — there are lots of areas in the course where these might be covered.

Tip: Research skills include being able to cite sources of information. There's more about citations on p.27.

Practical techniques

As part of each PAG you'll be expected to show that you can carry out certain techniques, such as:

- use of appropriate analogue apparatus to record a range of measurements (to include length/distance, temperature, pressure, force, angles and volume) and to interpolate between scale markings
- use of appropriate digital instruments, including electrical multimeters, to obtain a range of measurements (to include time, current, voltage, resistance and mass)
- use of methods to increase accuracy of measurements, such as timing over multiple oscillations, or use of fiducial markers, set square or plumb line
- use of a stopwatch or light gates for timing
- use of calipers and micrometers for small distances, using digital or vernier scales
- correctly constructing circuits from circuit diagrams using d.c. power supplies, cells, and a range of circuit components, including those where polarity is important
- designing, constructing and checking circuits using d.c. power supplies, cells, and a range of circuit components
- use of a signal generator and oscilloscope, including volts/division and time-base
- generating and measuring waves, using microphone and loudspeaker, or ripple tank, or vibration transducer, or microwave/radio wave source
- use of a laser or light source to investigate characteristics of light, including interference and diffraction
- use of ICT such as computer modelling, or data logger with a variety of sensors to collect data, or use of software to process data
- use of ionising radiation, including detectors
- applying investigative approaches and methods to practical work
- using online and offline research skills
- correctly citing sources of information

All of these techniques should be covered across all of the PAGs.

Tip: Each individual PAG won't cover every single one of these techniques, but you should have covered all the techniques you need to know by the end of your course.

Learning Objectives:

- Know that physical quantities have a numerical value and a unit.
- Know the Système Internationale (S.I.) base quantities and their units – mass (kg), length (m), time (s), current (A), temperature (K), amount of substance (mol).
- Know derived units of S.I. base units.
- Be able to check the homogeneity of physical equations using S.I. base units.
- Understand units listed in this book.
- Know prefixes and their symbols and be able to use them to indicate decimal submultiples or multiples of units – pico (p), nano (n), micro (μ), milli (m), centi (c), deci (d), kilo (k), mega (M), giga (G), tera (T).

Specification Reference 2.1.2

1. Quantities and Units

For absolutely everything in Physics, you need to make sure your values are in the correct units. The next few pages show you the correct units for the most common quantities, as well as how to use prefixes with them.

S.I. units

A **physical quantity** is something that you can measure. Every physical quantity has a numerical value and a unit. Each time you measure something, or do a calculation, you need to give the correct units of the quantity.

You might need to convert quantities into the right units before using a formula. The Système International (S.I.) includes a set of **base units** from which lots of others are derived. There are seven base units in this system, but you only need to know these six:

Quantity	*S.I. base unit*	*Symbol*
Length	*metre*	*m*
Mass	*kilogram*	*kg*
Time	*second*	*s*
Current	*ampere*	*A*
Temperature	*kelvin*	*K*
Amount of a substance	*mole*	*mol*

Figure 1: *Quantities and their S.I. base units.*

Homogeneity of units

The units in any equation must always be equivalent (in terms of S.I. base units) on both sides. This is called **homogeneity of units**. You can use this rule to work out **S.I. derived units** — other units derived from the S.I. base units.

For example, density is found using $\rho = \frac{m}{V}$. The unit of mass, m, is kilograms, kg. The unit of volume, V, is the unit of length (m) cubed, m^3. So density, ρ, is in 'kilograms per metre-cubed', $kg\,m^{-3}$.

Example — **Maths Skills**

Show that the S.I. derived unit for speed is ms^{-1}.

You know that speed = distance ÷ time

Distance is a length, so its S.I. base unit is the metre, m.
The base unit of time is the second, s.

To find the unit for speed, just put the units for distance and time into the equation for speed: $m \div s = ms^{-1}$.

The S.I. derived units you need to know will be covered throughout the book.

All S.I. derived units can be expressed purely in terms of S.I. base units. But some S.I derived units have their own names and symbols. Here are some of examples of these S.I. derived units that you'll come across quite often:

Quantity	S.I. derived unit	Symbol	Written in S.I. base units
Energy, work, heat	joule	J	kgm^2s^{-2}
Resistance	ohm	Ω	$kgm^2s^{-3}A^{-2}$
Potential difference, e.m.f.	volt	V	$kgm^2s^{-3}A^{-1}$
Charge	coulomb	C	As
Force, weight	newton	N	$kgms^{-2}$
Power	watt	W	kgm^2s^{-3}
Pressure, stress	pascal	Pa	$kgm^{-1}s^{-2}$
Frequency	hertz	Hz	s^{-1}

Figure 3: *Quantities and their S.I. derived units.*

You can also use the homogeneity of units to help you to check your working if you have to combine or rearrange equations.

Figure 2: *Alessandro Volta, the man after whom the 'volt' was named. All of the derived S.I. units in Figure 3, and some of the base units in Figure 1, are named after famous physicists.*

Tip: You don't need to remember all of the combinations of units in the last column of Figure 3. If you need them, you're better working them out using the formulas you know.

Examples — Maths Skills

Rearrange $P = I^2R$ for I. Check the homogeneity of your answer.

Rearrange $P = I^2R$ to get: $I = \sqrt{\frac{P}{R}}$

To check the units are the same on both sides, substitute the units for each quantity into the equation, then cancel down:

I is current in amperes, P is power in watts, or kgm^2s^{-3}, and R is resistance in ohms, or $kgm^2s^{-3}A^{-2}$.

$$A = \sqrt{\frac{kgm^2s^{-3}}{kgm^2s^{-3}A^{-2}}} = \sqrt{\frac{1}{A^{-2}}} = \sqrt{A^2} = A$$

There are only amperes left on both sides of the equation, so the equation is homogeneous.

For an object moving with uniform acceleration, $v = u + at$ and $s = \frac{1}{2}(u + v)t$.

Combine these equations to get an equation for s in terms of u, v and a. Check the homogeneity of your answer.

Rearrange $v = u + at$ to get: $t = \frac{v - u}{a}$

Then substitute this into $s = \frac{1}{2}(u + v)t \Rightarrow s = \frac{1}{2}(u + v)\left(\frac{v - u}{a}\right)$

Then simplify the equation: $s = \frac{1}{2a}(v^2 - u^2)$

Again, substitute the units for each quantity into the equation and cancel down (you can ignore any numbers, e.g. the 2):

s is a length in metres, v and u are velocities in ms^{-1} and a is acceleration in ms^{-2}.

$$m = \frac{1}{ms^{-2}}((ms^{-1})^2 - (ms^{-1})^2) = \frac{1}{ms^{-2}}(m^2s^{-2} - m^2s^{-2}) = m$$

There are only metres left on both sides of the equation, so the equation is homogeneous.

Tip: The S.I. base unit kilogram, kg, is the only one that already has a prefix. Most of the time you'll need to convert into kg to do a calculation, not g.

Figure 5: *A coulombmeter measuring in nC. Often, measured values will already have a prefix, defined by the apparatus, so make sure you check before recording anything.*

Prefixes

Physical quantities come in a huge range of sizes. **Prefixes** are scaling factors that let you write very big or small numbers without having to put everything in standard form (p.471). Again, the Système International (S.I.) defines some standard prefixes. The S.I. prefixes that you need to know are:

Prefix	Multiple of unit
pico (p)	1×10^{-12}
nano (n)	1×10^{-9}
micro (μ)	1×10^{-6}
milli (m)	$0.001\ (1 \times 10^{-3})$
centi (c)	$0.01\ (1 \times 10^{-2})$
deci (d)	$0.1\ (1 \times 10^{-1})$
kilo (k)	$1000\ (1 \times 10^{3})$
mega (M)	1×10^{6}
giga (G)	1×10^{9}
tera (T)	1×10^{12}

Figure 4: *Common S.I. prefixes.*

Prefixes can tell you the order of magnitude of a quantity. This gives you a rough idea of the quantity's size, which is useful if you're using it to estimate another value. For example, a length of 1 m is 3 orders of magnitude greater than a length of 1 mm.

Tip: If you get confused with converting between prefixes, it might be easier to first convert to the unit without a prefix and then convert to the new prefix. So for this example, you'd multiply 0.247 by 1×10^{6} first to change the quantity into W. Then you'd divide by 1×10^{3} to convert the quantity into kW.

Example — **Maths Skills**

Convert 0.247 megawatts into kilowatts.

1 MW = 1×10^{6} W and 1 kW = 1×10^{3} W

So the scaling factor to move between MW and kW is:

$(1 \times 10^{6}) \div (1 \times 10^{3}) = 1 \times 10^{3}$

So 0.247 MW = $0.247 \times 1 \times 10^{3}$ kW = 247 kW.

If you need to convert between prefixes for area or volume, be very careful. For example, you could be asked to convert from m^2 to cm^2. In this case it's not enough to multiply the quantity by 1×10^{-2}, you would need to multiply the quantity by $(1 \times 10^{-2})^2$.

It's the same for converting between volumes, for example from nm^3 to mm^3, you would multiply by $(1 \times 10^{-6})^3$ and not just by 1×10^{-6}.

Tip: When converting, you should check your answer is sensible. A centimetre is smaller than a metre, so there are more of them in a given value. So if you're converting from m to cm (or m^3 to cm^3) the number should get bigger.

Example — **Maths Skills**

Convert 0.083 m^3 into cm^3.

1 cm = 1×10^{-2} m, so the scaling factor between m and cm is $1 \div (1 \times 10^{-2}) = 1 \times 10^{2}$.

So to convert from m^3 to cm^3, multiply by $(1 \times 10^{2})^3$.

$0.083 \times (1 \times 10^{2})^3 = 83\ 000\ cm^3$

You need to have a rough idea of the size of each S.I. base unit and S.I. derived unit in this book, as well as prefixes, so that you can estimate quantities using them. There's more about estimating quantities on the next page.

Practice Questions — Application

Q1 **A wave has a frequency of 0.3 THz.**
State the frequency of the wave in Hz.

Q2 **The momentum of an object is found using $p = mv$, where p is the momentum, m is the object's mass and v is the velocity of the object.**
Find the S.I. derived unit of momentum.

Practice Questions — Fact Recall

Q1 What is the S.I. unit of temperature?

Q2 Give two S.I. derived units.

Q3 Give the multiple of the unit for the following prefixes:

a) pico

b) centi

c) mega

Learning Objective:

- Be able to make estimates of physical quantities listed in this book.

Specification Reference 2.1.1

2. Making Estimates

Estimates are a just informed guesses. They're a good way to check your calculations and to make sure your experiments are correct too.

Estimating values

Estimating values can be tricky, especially for random situations you're not likely to come across in real life. The easiest thing to do is to compare the situation with one you are more familiar with. Figure 1 shows some typical values for things you come across in everyday life.

	Typical value
Mass of a person	*70 kg*
Mass of a car	*1500 kg*
Height of a man	*1.8 m*
Walking speed	*1.5 ms^{-1}*

Figure 1: *Common values for everyday situations.*

Tip: Lots of everyday speeds are given in miles per hour (mph) instead of metres per second, which can make estimates difficult. To (roughly) change from mph to ms^{-1}, divide the value by two. E.g. a car travelling at 60 mph travels at roughly 30 ms^{-1}.

Figure 2: *Common objects that have a mass of 1 kg are a large bag of sugar and a litre bottle of water.*

Example — **Maths Skills**

A cyclist goes for a ride. Estimate the kinetic energy of a cyclist as he travels at a constant, typical speed.

The equation for kinetic energy is $E_k = \frac{1}{2}mv^2$.

E_k is kinetic energy, m is mass and v is velocity.

So you need to make an estimate of the cyclist's total mass and velocity.

Using information from Figure 1, the mass of a person is 70 kg and walking speed is about 1.5 ms^{-1}.

The mass of a road bike is roughly 10 kg.

A cyclist probably travels at around 10 mph, or 5 ms^{-1}.

Substitute these estimates into the equation:

$E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times (70 + 10) \times 5^2 = \frac{1}{2} \times 80 \times 25 = 1000$ J

So the kinetic energy of the cyclist is about 1000 J.

Tip: There are a few ways you could estimate the cyclist's speed. If you think you have a good idea, you can just use that as an estimate, but if you're struggling, you could try comparing cycling to walking, or driving at 30 mph.

You can also use estimates to quickly check that you've not made mistakes during a calculation.

Example — **Maths Skills**

A handbag costs £5.22 to make. It is sold for £29.99. Calculate the profit made if 8 handbags are sold.

Profit for one = £29.99 – £5.22 = £24.77

Total profit = £24.77 × 8 = £198.16

Using estimates, you can quickly check if this number is roughly correct:

Profit for one is roughly £30 – £5 = £25,

Total profit is roughly £25 × 8 = £200

The two answers are very close, so £198.16 is probably correct.

Tip: It will depend on the question, but most estimates should be rounded to 1 significant figure.

Making predictions

You may also be asked to use estimates to make predictions about how the outcome of an experiment would change, were one of the variables to change.

Examples

The equipment used in Figure 3 is used to measure the speed of sound, *v*.

The tuning fork creates a sound wave at a set frequency, *f*.

The distance between the fork and the water is increased until the wave reaches its first point of resonance. At this point the distance is equal to one quarter of the wavelength.

An error is made whilst measuring this distance. The actual distance is smaller than the value that has been recorded.

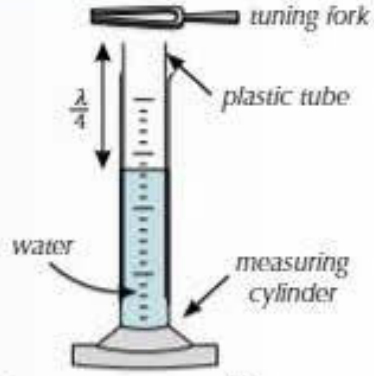


Figure 3: *Set-up used for finding the speed of sound.*

Discuss how this error will affect the result of the experiment.

This error will mean that the actual value of the wavelength of the sound wave will be smaller than the result of the experiment. As $v = f\lambda$, this will also mean that the actual value of the speed of sound will be smaller than that calculated.

The resistivity of a wire is investigated. The diameter of the wire is measured and its cross-sectional area calculated. Its resistance for given lengths is recorded, and a graph of resistance against length is produced. An error is made whilst recording the diameter of the wire.

The diameter recorded is smaller than the actual value.

Discuss how this will affect the results of the experiment.

Cross-sectional area, $A = \pi r^2 = \pi\left(\frac{\text{diameter}}{2}\right)^2$

So if the actual value of the diameter is larger than the recorded value, the actual cross-sectional area is larger too.

Resistivity, ρ, is equal to $\frac{RA}{l}$. So if the actual value of A is larger than that calculated from the results, the actual resistivity of the wire is higher than the value that will be calculated.

Tip: There's more about this experiment on page 226.

Tip: Resistivity is covered in more detail on page 155.

Practice Questions — Application

Q1 Estimate:

a) The height of a two-storey house.

b) The gravitational potential energy of a child 3 m above the ground in a tree. (Gravitational potential energy = mass of object × g × height of object above ground, where $g = 9.81\ \text{ms}^{-2}$.)

Q2 The force acting on a ball as it is kicked is investigated. The ball's mass, its change in velocity and the time taken to change velocity are measured. This is used to calculate the force acting on the ball. The change in the ball's velocity is incorrectly measured. The actual value is larger than the one recorded. Describe how this would affect the calculated value of the force acting on the ball.
(Acceleration = change in velocity ÷ time; Force = mass × acceleration)

Tip: If you're struggling with Q1a) think about how high a room is, compared to the height of a person.

3. Errors and Uncertainties

Learning Objectives:

- Understand systematic errors (including zero errors) and random errors in measurements.
- Know how errors affect precision and accuracy.
- Be able to calculate absolute and percentage uncertainties when data are combined by addition, subtraction, multiplication, division and raising to powers.
- Be able to calculate percentage difference.

Specification Reference 2.2.1

Scientists always have to include the uncertainty of a result, so you can see the range the actual value probably lies within. Dealing with error and uncertainty is an important skill — you need to make sure that you know and try to minimise the uncertainty of your experimental results.

Types of error

Every measurement you take has an experimental uncertainty (see page 16), caused by two types of error:

Random errors

Random errors cause readings to be spread about the true value due to the results varying in an unpredictable way. They affect precision (see page 18).

They can just be down to noise, or because you're measuring a random process such as nuclear radiation emission. You can get random errors in any measurement and you can't completely prevent them, no matter how hard you try.

If you measured the length of a wire 20 times, the chances are you'd get a slightly different value each time, e.g. due to your head being in a slightly different position when reading the scale. It could be that you just can't keep controlled variables exactly the same throughout the experiment. Or it could just be the wind was blowing in the wrong direction at the time.

Example

You could investigate the force constant (see page 113) of a particular rubber band using the apparatus in Figure 1.

The force constant of the rubber band increases with temperature. If the surrounding temperature changes, it could introduce a random error.

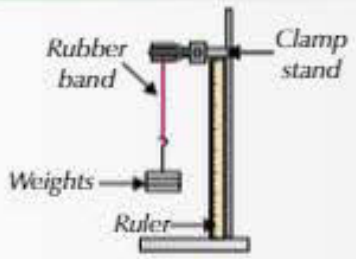


Figure 1: *An experiment to find the force constant of a rubber band.*

Tip: All sorts of things are affected by temperature, from the properties of a material to the current flowing in a circuit.

Using more sensitive apparatus can reduce the size of random errors, which makes your results more precise. Repeating your experiment and taking an average (mean) of your repeated measurements (see page 10) will also reduce the effect of random error. The more measurements you average over, the less effect random error is likely to have in your final result.

Systematic errors

Systematic errors usually cause each reading to be different to the true value by the same amount i.e. they shift all of your measurements. They affect the accuracy of your results (see page 18).

You usually get systematic errors because of the environment, the apparatus you're using or your experimental method, e.g. using an inaccurate clock, repeated identical mistakes in measurements, etc.

The problem is often that you don't know systematic errors are there. You've got to spot them first to have any chance of correcting for them. They're annoying, but there are things you can do to reduce them if you manage to spot them (see next page).

If you suspect a systematic error, you should repeat the experiment with a different technique or apparatus and compare the results.

Tip: A newton meter that always measures values 1 N greater than they should be will shift all your results up by 1 N — this would introduce a systematic error due to the apparatus used.

Example — continued

Look back at the investigation of the force constant on the previous page (see Figure 1). If the ruler is not correctly lined up to the top of the piece of rubber, all the extension measurements would be shifted by the same amount. This would introduce a systematic error due to your experimental method.

You can **calibrate** your apparatus by measuring a known value. If there's a difference between the measured and known value, you can use this to correct the inaccuracy of the apparatus, and so reduce your systematic error.

Calibration can also reduce **zero errors** (caused by the apparatus failing to read zero when it should do, e.g. when no current is flowing through an ammeter) which is a type of systematic error.

Tip: To calibrate a set of scales you could weigh a 10.0 g mass and check that it reads 10.0 g. If these scales are precise to the nearest 0.1 g, then you can only calibrate to within 0.05 g. Any measurements taken will have an uncertainty of ± 0.05 g.

Combining uncertainties

When you do calculations involving values that have an uncertainty, you have to combine the uncertainties to get the overall uncertainty for your result.

Adding or subtracting

When you're adding or subtracting data you add the absolute uncertainties (p.16).

Example — **Maths Skills**

A wire is stretched from 0.3 ± 0.1 cm to 0.5 ± 0.1 cm. Calculate the extension of the wire.

First subtract the lengths without the uncertainty values:

$0.5 - 0.3 = 0.2$ cm

Then find the total uncertainty by adding the individual absolute uncertainties:

$0.1 + 0.1 = 0.2$ cm

So, the wire has been stretched 0.2 ± 0.2 cm.

Tip: Be very careful. Even if you subtract the data, you add the uncertainties (combining uncertainties should always make the uncertainty increase).

Multiplying or dividing

When you're multiplying or dividing data, you add the percentage uncertainties, p.16.

Example — **Maths Skills**

A force of 15 ± 3% N is applied to a stationary object which has a mass of 6.0 ± 0.3 kg. Calculate the acceleration of the object and state the percentage uncertainty in this value.

First calculate the acceleration without uncertainty:

$\boldsymbol{a} = \boldsymbol{F} \div m = 15 \div 6.0 = 2.5$ ms^{-2}

Next, calculate the percentage uncertainty in the mass:

$$\%\text{ uncertainty in } m = \frac{0.3}{6.0} \times 100 = 5\%$$

Add the percentage uncertainties in the force and mass values to find the total uncertainty in the acceleration:

Total uncertainty = 3% + 5% = 8%

So, the acceleration = 2.5 ± 8% ms^{-2}

Tip: Don't forget to convert all the uncertainties to percentages before you combine by multiplying or dividing — see page 16 for how.

Tip: You're given the area of a circle as πr^2 in the data and formulae booklet.

Raising to a power

When you're raising data to a power, you multiply the percentage uncertainty by the power.

Example — Maths Skills

The radius of a circle is $r = 40 \pm 2.5\%$ cm. What will the percentage uncertainty be in the area of this circle, i.e. πr^2?

The radius will be raised to the power of 2 to calculate the area.

So, the percentage uncertainty will be $2.5\% \times 2 = 5\%$

Percentage difference

Tip: Be careful to not confuse this with percentage uncertainty.

If you know the true value of what you're investigating you can measure the accuracy of your result using **percentage difference**. This is the difference between your experimental value and the accepted value, expressed as a percentage of the accepted value.

$$\text{percentage difference} = \frac{\text{experimental value} - \text{accepted value}}{\text{accepted value}} \times 100$$

Tip: 'Accepted' means that the value is currently believed to be true by the scientific community as it has lots of evidence supporting it. It is sometimes called the true value.

Example — Maths Skills

The speed of light in a vacuum is 299 792 458 ms^{-1}.
In an experiment, it is measured to be 299 790 000 ms^{-1}.
What is the percentage difference of these results?
Give your answer to 2 significant figures.

$$\text{percentage difference} = \frac{\text{experimental value} - \text{accepted value}}{\text{accepted value}} \times 100$$

$$= \frac{299\,790\,000 - 299\,792\,458}{299\,792\,458} \times 100$$

$$= -0.000008199... \times 100 = -0.00082\% \text{ (to 2 s.f.)}$$

Tip: The sign of the percentage difference shows you if the experimental value was too big or too small.

Practice Questions — Application

Q1 The speed, v, of a sound wave is found using $v = f\lambda$. In an experiment, the frequency, f, is measured to be $125 \pm 1\%$ Hz. The wavelength, λ, is measured to be $2.72 \pm 1.5\%$ m. Calculate the speed of the sound wave and the percentage uncertainty in this value.

Q2 The length, L, of each side of a cube is 4.0 ± 0.05 cm. Calculate the volume of the cube and the percentage uncertainty in this value.

Practice Questions — Fact Recall

Q1 Give one way to reduce:
a) random errors
b) systematic errors

Q2 Describe how to find the uncertainty in the result of adding two pieces of data that both have an uncertainty.

Q3 State the equation you would use to calculate the percentage difference. Explain what percentage difference shows.

4. Graphical Representations of Uncertainties

Learning Objectives:

- Understand and use graphical treatment of errors and uncertainties, including absolute and percentage uncertainties.
- Recall the conventions used for labelling graph axes and table columns.
- Be able to draw lines of best fit and worst lines.

Specification References 2.1.2 and 2.2.1

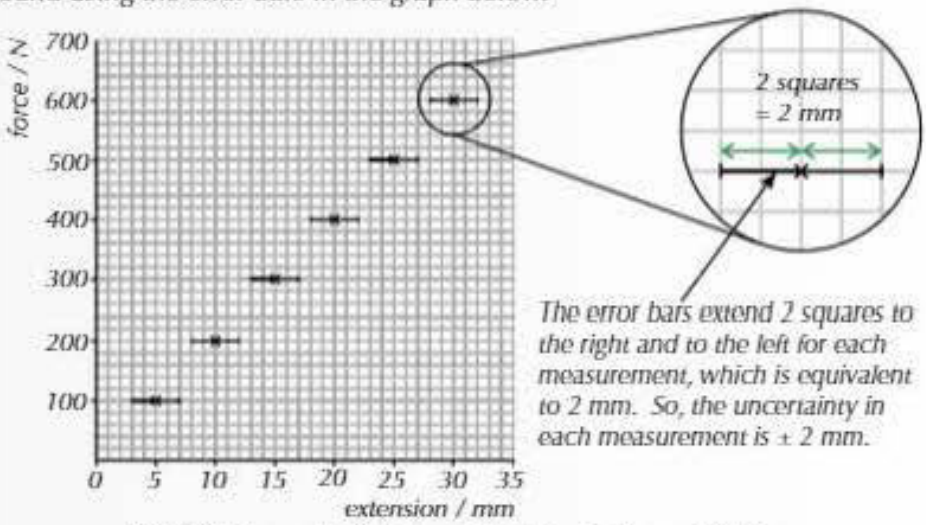
You can show the uncertainties in your data on any graphs that you plot. These graphs can be used to see both the uncertainty in each measurement, as well as to calculate the uncertainty in your final result.

Error bars

Most of the time, you work out the uncertainty in your final result using the uncertainty in each measurement you make. When you're plotting a graph, you show the uncertainty in each measurement by using error bars to show the range the point is likely to lie in. You can have error bars for both the dependent and the independent variable.

Example **Maths Skills**

The error in measuring the extension of material *X* can be found using the error bars in the graph below.



The error bars extend 2 squares to the right and to the left for each measurement, which is equivalent to 2 mm. So, the uncertainty in each measurement is ± 2 mm.

Figure 1: *A graph of force against extension for material X.*

Whenever you're drawing a graph, make sure you follow the conventions covered on page 12. Make sure that the axes of graphs are written in the form "quantity / unit". The same is also true for table headings — see Figure 2.

Trial	Length ± 5% / m
1	0.80
2	0.81
3	0.80

Figure 2: *If the uncertainty is the same for all values in a table, it's sometimes written in the table heading instead of by each value. Look out for this when drawing graphs or using data from a table.*

Measuring uncertainty of final results

Normally when you draw a graph you'll want to find the gradient or intercept. You can find the uncertainty in that value by using **worst lines**.

Draw a line of best fit for your data and use this to calculate your result. Then, draw lines of best fit which have the maximum and minimum possible slopes for the data and still go through all of the error bars. These are the worst lines for your data.

Tip: Your line of best fit (page 12) should always go through all of the error bars.

For example, you can calculate k, the force constant of the object being stretched, from the gradient of the graph in Figure 3 — here it's about 20 000 Nm^{-1}. The the pink and blue lines in Figure 3 are the worst lines for your data.

Tip: Be careful — sometimes error bars are calculated using a set percentage of uncertainty for each measurement, and so will change depending on the measurement.

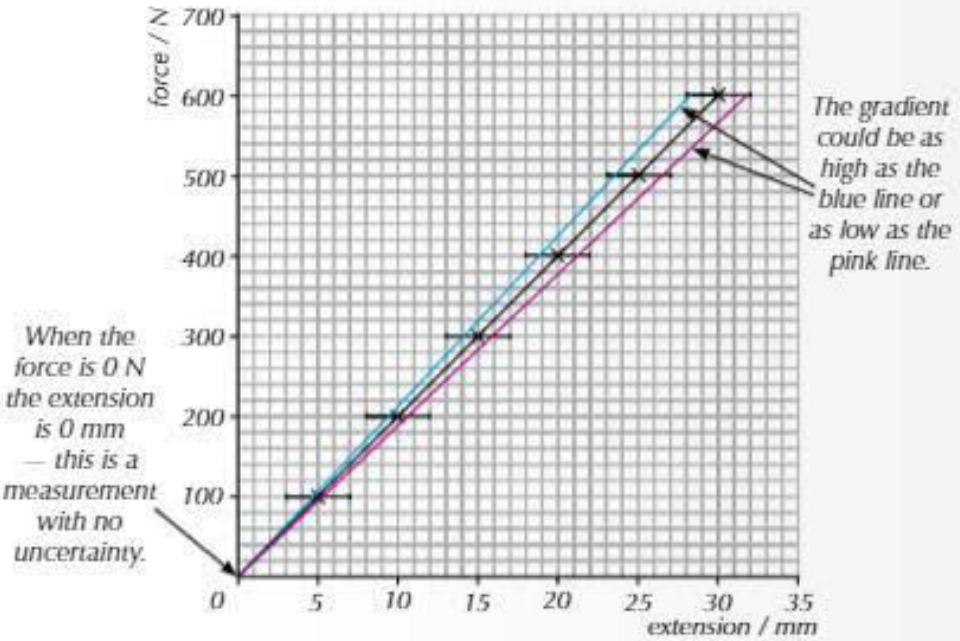


Figure 3: *The maximum and minimum slopes possible through the error bars.*

Tip: If a point doesn't have an error bar (like the origin in Figure 3) you should treat it as you would a normal point whilst drawing a line of best fit. The worst lines you draw should go through or near to it.

Calculate the worst gradient — the gradient of the slope that is furthest from the gradient of the line of best fit. In Figure 3, the gradient of the blue line is about 21 000 Nm^{-1} and the gradient of the pink line is about 19 000 Nm^{-1}, so you can use either.

The uncertainty in the gradient is given by the difference between the best gradient (of the line of best fit) and the worst gradient — here it's 1000 Nm^{-1}. So this is the uncertainty in the value of the force constant. For this object, the force constant is 20 000 ± 1000 Nm^{-1} (or 20 000 Nm^{-1} ± 5%).

Similarly, the uncertainty in the y-intercept is just the difference between the best and worst intercepts (although there's no uncertainty in Figure 3 since the best and worst lines both go through the origin).

Practice Question — Application

Q1 The table below shows the temperature of a beaker of water as it is heated over time.

Time / s	Temperature / °C
0	21 ± 1
30	28 ± 1
60	35 ± 1
90	43 ± 1
120	50 ± 1

a) Draw a temperature-time graph for the water.

b) Calculate the gradient of the graph. State the uncertainty in your answer.

5. Scalars and Vectors

Vectors are quantities with both a size and a direction. You need to be able to add them to find a resultant vector or split them into components.

Learning Objectives:

- Understand the difference between scalar and vector quantities.
- Be able to add and subtract vectors.
- Be able to use a vector triangle to determine the resultant of any two coplanar vectors.
- Be able to resolve a vector into two perpendicular components; $F_x = F\cos\theta$; $F_y = F\sin\theta$.

Specification Reference 2.3.1

The difference between scalars and vectors

- A **scalar** quantity has no direction — it's just an amount of something, like the mass of a sack of potatoes.
- A **vector** quantity has magnitude (size) and direction — e.g. the velocity of a car is its speed and direction.

You need to know some examples of scalar and vector quantities — like the ones given in the table below.

Scalars	Vectors
length/distance, speed, mass, temperature, time, energy	displacement, velocity, force (including weight), acceleration, momentum

Finding the resultant vector

Adding or subtracting two or more vectors is called finding the **resultant vector**. There are two ways of doing this you need to know about.

For both methods, you should start by drawing a diagram. Draw the vectors 'tip to tail'. If you're doing a vector subtraction, draw the vector you're subtracting with the same magnitude but pointing in the opposite direction.

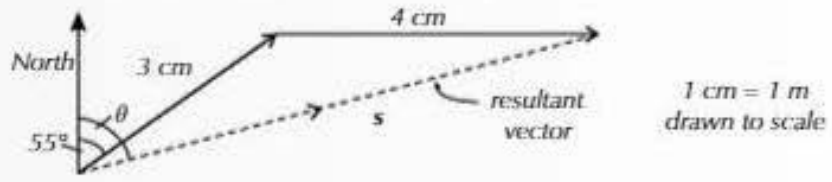
Finding resultant vectors using scale diagrams

You can find the resultant vector of any two vectors by drawing a **scale drawing** of them 'tip-to-tail' and then measuring the length and angle of the resultant vector on the diagram.

Example — **Maths Skills**

A man walks 3.0 m on a bearing of 055° then 4.0 m east. Find the magnitude and direction (to the nearest degree) of his displacement, *s*.

Start by drawing a scale diagram for how far the man walked using a ruler and a protractor:



Then just measure the missing side with a ruler and the missing angle with a protractor: $s = 6.7$ cm and $\theta = 75°$ (to the nearest degree).

So, using the scale from the drawing, the man's displacement is 6.7 m, on a bearing of 075°.

Tip: A bearing is a three digit angle measured clockwise from north in degrees.

Tip: You could also calculate this using the sine and cosine rules — see page 488.

Tip: The man has walked 7 m in total, but his displacement is less than 7 m. Displacement gives the position relative to the start point (see page 48).

Finding resultant vectors using trigonometry

If two vectors are perpendicular to each other, like in Figure 1, you can calculate the size and angle of the resultant vector using trigonometry, by drawing a vector triangle.

Tip: Printed vectors are normally bold (**a**). When they're handwritten, you normally write them underlined ($\underline{a}$) or with an arrow above them ($\vec{a}$).

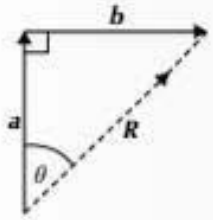


Figure 1: *A right-angled triangle representing two vectors '**a**' and '**b**', and the resultant vector '**R**'.*

You can calculate the size of the resultant vector ***R*** using the formula:

$$R = \sqrt{a^2 + b^2}$$

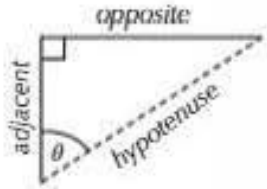
Tip: This is from Pythagoras' theorem. You might remember this from GCSE maths.

You can calculate the size of the angle θ in degrees using the formula:

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

Tip: a and b are the sizes of vectors ***a*** and ***b*** respectively.

For any right-angled triangle where you know two sides, you can work out the size of an angle with one of the formulas below. A handy way to remember them is SOH CAH TOA (see page 487).



$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp} \qquad \tan\theta = \frac{opp}{adj}$$

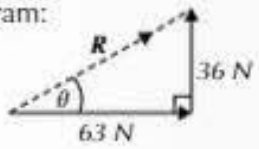
Figure 2: *SOH CAH TOA for a right-angled triangle.*

Figure 3: *Boats won't just travel in a straight line if they try to sail straight across a river — there is a force on the boat from the river. So it will experience a resultant force that makes it travel on a longer path.*

Example — Maths Skills

A remote-controlled aeroplane flies with a driving force of 63 N east while being pushed north by the wind by a 36 N force.
What is the resultant force on the aeroplane?

Start by sketching a diagram:



Then find R:

$R = \sqrt{63^2 + 36^2} = 72.5603... = 73$ N (to 2 s.f.)

Then find θ:

$\theta = \tan^{-1}\frac{36}{63} = 29.744... = 30°$ (to 2 s.f.)

So the resultant force ***R*** is a 73 N force, 30° anticlockwise from east.

Tip: You may be given angles in degrees or radians. Make sure you know how to change your calculator between the two.

Resolving vectors into components

Resolving vectors is the opposite of finding the resultant — you start from the resultant vector and split it into two components at right angles to each other. You're basically working backwards from the examples on pages 41 and 42.

Resolving is dead useful because two perpendicular components of a vector don't affect each other. This means you can deal with the two directions completely separately. When you have to add another vector that affects only one of the components, you can just ignore the other.

Tip: The components are normally horizontal and vertical. If you're working with an object on an inclined plane, it might be easier to use components that are parallel and at right angles to the plane (see next page).

Resolving a vector into horizontal and vertical components

The components of a vector are perpendicular to each other, so they form a right-angled triangle with the vector.

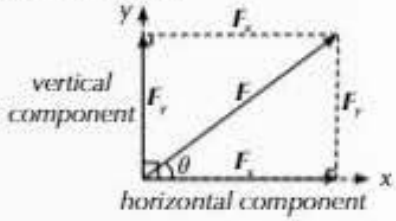


Figure 4: *The vector* F *and its horizontal component* F_x *and vertical component* F_y.

You just need to use a bit of trigonometry to find the components of the vector in each direction:

You get the horizontal component F_x like this:

$$\cos\theta = \frac{F_x}{F}$$

$$F_x = F\cos\theta$$

...and the vertical component F_y like this:

$$\sin\theta = \frac{F_y}{F}$$

$$F_y = F\sin\theta$$

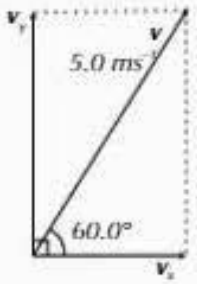
Tip: This uses SOH CAH TOA as well (p.42).

Tip: In these formulae, θ is measured from the horizontal.

Example — **Maths Skills**

A hot air balloon is travelling at a velocity of 5.0 ms^{-1} at an angle of 60.0° up from the horizontal. Find the vertical and horizontal components of its velocity.

First, sketch a diagram:



The horizontal component v_x is:

$$v_x = v\cos\theta = 5.0\cos 60.0° = 2.5 \text{ ms}^{-1}$$

The vertical component v_y is:

$$v_y = v\sin\theta = 5.0\sin 60.0° = 4.3301... = 4.3 \text{ ms}^{-1} \text{ (to 2 s.f.)}$$

Exam Tip
$\cos 60° = \sin 30° = 0.5$. Remembering this may save time in the exam.

Tip: Turning the paper to an angle can help you see what's going on better in vector problems on a slope.

Exam Tip
Examiners like to call a slope an 'inclined plane'.

Tip: Weight always acts vertically downwards (see page 76).

Tip: Angle rules were used to work out that the angle between W and W_2 was 30° too.

Tip: You'll need to resolve forces when it comes to projectile motion on page 53.

Tip: θ, ϕ and ψ are all Greek letters that can be used to represent different angles.

Tip: Be careful if your forces are acting in opposite directions — when you find the resultant components you'll end up taking away instead of adding. In these cases, it's always good to mark the direction you've chosen as positive on any diagrams you use.

Resolving a vector on a slope

You should always resolve vectors in the directions that make the most sense for the situation you're dealing with. If you've got an object on a slope, choose your directions along the slope and at right angles to it.

Example — **Maths Skills**

An apple with a weight of 1.5 N is at rest on a slope inclined at 29° to the horizontal, as shown in Figure 5. Find the component of its weight that acts along the slope.

This time, instead of resolving the vector into vertical and horizontal components, you're resolving it into components parallel and perpendicular to the slope (W_1 and W_2).

To find W_1, use $opp = \sin\theta \times hyp$

$$\Rightarrow W_1 = \sin 29° \times 1.5 = 0.7272...$$
$$= 0.73 \text{ N (to 2 s.f.)}$$

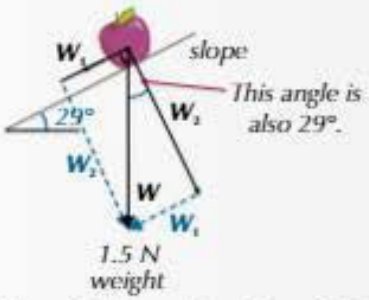


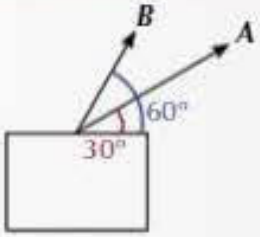
Figure 5: An apple with a weight of 1.5 N at rest on a slope.

Resolving vectors to find the resultant

Resolving vectors also gives you another way to find the resultant vector of two vectors that aren't at right angles to each other. Resolve the vectors into their horizontal and vertical components, and add up the vertical and horizontal components separately. Then you just need to combine the two to get the resultant vector. It can be a lot less fiddly than drawing accurate scale diagrams.

Example — **Maths Skills**

Two forces, A and B, are pulling on a block, shown to the right. A has a magnitude of 15 N and acts at 30° to the horizontal. B has a magnitude of 10 N and acts at 60° to the horizontal. Calculate the resultant force on the block to 2 significant figures.



For force A:

$A_x = A\cos\theta = 15\cos 30°$
$A_y = A\sin\theta = 15\sin 30°$

For force B:

$B_x = B\cos\phi = 10\cos 60°$
$B_y = B\sin\phi = 10\sin 60°$

The resultant horizontal component (F_x) is:
$F_x = A_x + B_x = 15\cos 30° + 10\cos 60° = 17.990...$ N

And the resultant vertical component (F_y) is:
$F_y = A_y + B_y = 15\sin 30° + 10\sin 60° = 16.160...$ N

Then using Pythagoras' theorem (page 42), the size of the resultant vector, R, is:

$$R = \sqrt{F_x^2 + F_y^2} = \sqrt{17.990...^2 + 16.160...^2} = 24.1827... \text{ N}$$

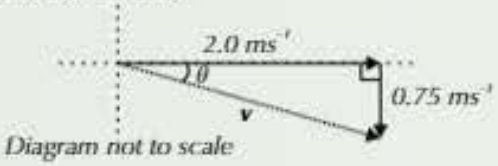
The angle between R and the horizontal is:

$$\psi = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{16.160...}{17.990...}\right) = 41.93°$$

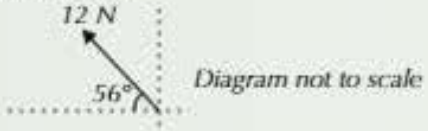
So the resultant force on the block is 24 N (to 2 s.f.) acting at 42° (to 2 s.f.) to the horizontal.

Practice Questions — Application

Q1 Without drawing a scale diagram, find the magnitude and direction of a paper plane's resultant velocity $\boldsymbol{v}$, from its horizontal and vertical components, shown below.



Q2 Without drawing a scale diagram, find the horizontal and vertical components of this force.



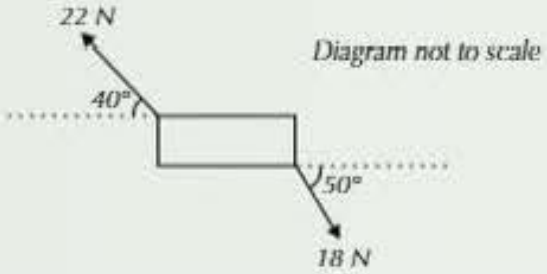
Q3 Use a scale diagram to find magnitude and direction of the resultant force produced by a 2.9 N force north and a 4.1 N force east.

Q4 A brick with a weight of 20.0 N is on a slope at 25° to the horizontal.

a) Find the component of the brick's weight that acts down the slope.

b) Find the size of the normal contact force exerted on the brick by the slope.

Q5 Two children are pulling on a toy. The forces exerted on the toy are shown below. Calculate the resultant force, including the angle at which it acts to the horizontal, on the toy.



Tip: The normal contact force due to an object's weight acts perpendicular to the surface. It is equal in size to the component of the weight applied to the surface, but acts in the opposite direction.

Tip: For question 5, it's easiest to choose upwards as the positive vertical direction and to the left to be the positive horizontal direction.

Practice Questions — Fact Recall

Q1 What's the difference between a scalar and a vector?

Q2 What name is given to a vector formed by adding vectors together?

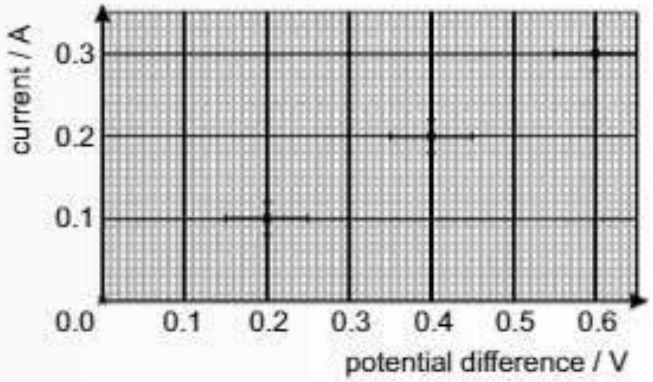
Section Summary

Make sure you know...

- That all physical quantities have a numerical value and a unit.
- The Système International base quantities and their units.
- The units derived from S.I. base units, including any units covered in this book.
- How to check the homogeneity of equations using S.I. base units.
- The prefixes for units and their meanings.
- How to estimate the sizes of physical quantities covered in this book.
- How to make predictions of how changes in, or incorrect recordings of, a variable will affect the results of an experiment.
- The meaning of the terms random error, systematic error and zero error.
- How the effect of errors on an experiment can be reduced.
- How errors affect the precision and accuracy of results.
- How to calculate and plot the absolute and percentage uncertainties in a result.
- That when adding or subtracting data that has uncertainties, the uncertainty of the result is found by adding together the absolute uncertainties of the data.
- That when multiplying or dividing data that has uncertainties, the uncertainty of the result is found by adding together the percentage uncertainties of the data.
- That when raising data that has an uncertainty to a power, the uncertainty of the result is found by multiplying the percentage uncertainty by the power.
- That the percentage difference is a way of measuring the accuracy of a result.
- How to calculate the percentage difference of a result.
- How to use error bars to represent the uncertainty in a measurement.
- That axes of graphs and headings of tables should be written in the form "quantity / unit".
- The meaning of the term 'worst line'.
- How to use worst lines to calculate the uncertainty in a result.
- The difference between scalar and vector quantities.
- Examples of both scalar and vector quantities.
- How to add and subtract vectors using a scale drawing.
- How to use trigonometry to find a resultant vector of two perpendicular vectors.
- How to resolve a vector into two components at right angles to each other.

Exam-style Questions

1 An *I-V* graph for a resistor is shown below.



What is the percentage uncertainty in the measurement of 0.4 V?

A 0.05%

B 0.1%

C 5%

D 12.5%

(1 mark)

2 A box of books is sliding down a frictionless slope inclined at 30.0° to the horizontal. The weight of the box of books is 15 ± 0.5 N.

(a) Calculate the size of the component of the box's weight that acts down the slope. Calculate the percentage uncertainty in this value.

(3 marks)

(b) More books are added to the box. A 23 N force is then applied to the box. This force acts parallel to the horizontal. It produces a 3.0 N resultant force up the slope. Calculate the new weight of the box.

(3 marks)

3 A student investigates the force constant of a spring, by measuring the extension of the spring for different stretching forces.

(a) The student realises that the mass balance he has been using has caused a zero error. State whether this is a type of random or systematic error, and describe the difference between systematic and random errors.

(2 marks)

(b) The spring was extended linearly by 3.0 ± 0.05 cm when a force of 60 ± 5 N was applied to it. Calculate the force constant of the spring and the uncertainty in this value. Use the equation: force = force constant × extension.

(4 marks)

Learning Objectives:

- Understand what is meant by the terms displacement, instantaneous speed, average speed, velocity and acceleration.
- Be able to use the equations of motion for constant acceleration in a straight line:
  $v = u + at$
  $s = \frac{1}{2}(u + v)t$
  $s = ut + \frac{1}{2}at^2$
  $v^2 = u^2 + 2as$

Specification References 3.1.1 and 3.1.2

Tip: Acceleration could mean a change in speed or direction or both — since velocity is speed in a given direction.

Figure 1: *All objects fall through a vacuum with the same acceleration (g). There's more on this on pages 50-52.*

Tip: These equations are sometimes called 'suvat equations'.

1. Constant Acceleration

There's a set of famous equations that you can use to work out an object's displacement, acceleration and starting and finishing velocity. These equations work for an object in uniform acceleration, and are really useful in physics.

Displacement, velocity and acceleration

You won't get far in physics without coming across **speed**, **displacement**, **velocity** and **acceleration**. Displacement, velocity and acceleration are all vector quantities (page 41), so the direction matters.

Speed — How fast something is moving, regardless of direction.

Displacement — How far an object's travelled from its starting point in a given direction.

Velocity — The rate of change of an object's displacement (its speed in a given direction).

Acceleration — The rate of change of an object's velocity.

During a journey, the **average speed** is just the total distance covered divided by the total time elapsed. The speed of an object at any given point in time is known as its **instantaneous speed**.

The constant acceleration equations

There are four main equations that you use to solve problems involving uniform (constant) acceleration. You need to be able to use them, but you don't have to know how they're derived — it's just shown here to help you learn them. The equations use 5 different letters:

s — displacement (in m) u — initial velocity (in ms^{-1})
v — final velocity (in ms^{-1}) a — acceleration (in ms^{-2}) t — time (in s)

- Acceleration is the rate of change of velocity. From this you get:

$$a = \frac{(v - u)}{t} \quad \text{so} \quad \boxed{v = u + at} \quad (1)$$

- Displacement = average velocity × time.
  If acceleration is constant, the average velocity is just the average of the initial and final velocities, so:

$$\boxed{s = \tfrac{1}{2}(u + v)t} \quad (2)$$

- Substitute the expression for v from equation 1 into equation 2 to give:

$$s = \tfrac{1}{2}(u + u + at)t = \tfrac{1}{2}(2ut + at^2) \Rightarrow \boxed{s = ut + \tfrac{1}{2}at^2} \quad (3)$$

- You can derive the fourth equation from equations 1 and 2:

Use equation 1 in the form: $a = \frac{(v-u)}{t}$

Multiply both sides by s, where: $s = \frac{1}{2}(u+v)t$

This gives: $as = \frac{(v-u)}{t} \times \frac{1}{2}(u+v)t$

The t's on the right cancel, so: $2as = (v-u)(v+u)$

$= v^2 - uv + uv - u^2$

so: $\boxed{v^2 = u^2 + 2as}$ ④

Example — **Maths Skills**

A mouse runs 10 m as it accelerates uniformly from 1.5 ms^{-1} to 3.5 ms^{-1}. Calculate the mouse's acceleration.

First of all, write out what you know:

$s = 10$ m
$u = 1.5$ ms^{-1}
$v = 3.5$ ms^{-1}
$a = ?$

Choose the equation with only one unknown quantity.
So you need to use: $v^2 = u^2 + 2as$

Substitute the values into the equation, then rearrange to find acceleration.

$$3.5^2 = 1.5^2 + (2 \times a \times 10) \Rightarrow a = \frac{12.25 - 2.25}{20} = 0.5 \text{ ms}^{-2}$$

Examples — **Maths Skills**

A car accelerates steadily from rest at a rate of 4.2 ms^{-2} for 6.0 seconds. Calculate its final speed.

As always, start by writing down what you know:

$u = 0$ ms^{-1}
$a = 4.2$ ms^{-2}
$t = 6.0$ s
$v = ?$

Then choose the right equation... $v = u + at$

$v = 0 + (4.2 \times 6.0) = 25.2 = 25$ ms^{-1} (to 2 s.f.)

Calculate the distance travelled in 6.0 seconds.

$u = 0$ ms^{-1}
$v = 25.2$ ms^{-1}
$a = 4.2$ ms^{-2}
$t = 6.0$ s
$s = ?$

You can use: $s = \frac{1}{2}(u+v)t$ ⇓ $s = \frac{1}{2}(0 + 25.2) \times 6.0 = 75.6$ m $= 76$ m (to 2 s.f.)

or: $s = ut + \frac{1}{2}at^2$ ⇓ $s = 0 + (\frac{1}{2} \times 4.2 \times 6.0^2) = 75.6$ m $= 76$ m (to 2 s.f.)

Tip: You can use either $s = \frac{1}{2}(u+v)t$ or $s = ut + \frac{1}{2}at^2$ to find the distance travelled, but it's safer to use $s = ut + \frac{1}{2}at^2$ because it doesn't rely on previously calculated values.

Practice Questions — Application

Q1 A cat runs with uniform acceleration from rest to 10 ms^{-1} in 20 s. What's its displacement after this time?

Q2 The brakes are applied to a train travelling at 25 ms^{-1} and it takes 18 s to stop with uniform deceleration. What is its deceleration?

Q3 An electric tram sets off from rest and travels 103 m in 9.2 seconds. Calculate its acceleration during this time, assuming it was uniform.

Tip: Deceleration is the same as negative acceleration.

Practice Questions — Fact Recall

Q1 What is a) the velocity of an object? b) the acceleration of an object?

Q2 What are the 4 'suvat equations' for constant acceleration?

Learning Objectives:

- Know that an object in free fall experiences an acceleration of g.
- Be able to use the equations of motion for constant acceleration in a straight line for bodies falling in a uniform gravitational field without air resistance.
- Know and be able to use techniques and procedures to determine the acceleration of free fall, such as using a trapdoor and electromagnet arrangement (PAG1).

Specification Reference 3.1.2

2. Acceleration Due To Gravity

If you drop a ball (or anything else heavier than air) from a height, it accelerates towards the ground due to gravity. If no other forces act on the ball (like air resistance) the ball is said to be in 'free fall'.

What is free fall?

Free fall is when there's gravity acting on an object and nothing else. It's defined as the motion of an object undergoing an acceleration of 'g'. You need to remember:

- Acceleration is a vector quantity — and 'g' acts vertically downwards.
- The magnitude of 'g' is usually taken as 9.81 ms^{-2}, though it varies slightly at different points on the Earth's surface.
- The only force acting on an object in free fall is its weight.
- Objects can have an initial velocity in any direction and still undergo free fall as long as the force providing the initial velocity is no longer acting.

'g' is also used for gravitational field strength, where $g = 9.81$ Nkg^{-1} (p.76). If you break down Nkg^{-1} into S.I. units, they are actually the same as ms^{-2}, so it's the same 'g'.

Free fall and the equations of motion

You need to be able to work out speeds, distances and times for objects moving vertically in a uniform gravitational field with an acceleration of g. As g is a constant acceleration you can use the equations of motion. But because g acts downwards, you need to be careful about directions. To make it clear, there's a sign convention: upwards is positive, downwards is negative.

Tip: Make sure you learn these conventions — remember that direction is important when using vectors.

- g is always downwards, so it's usually negative.
- t is always positive.
- u and v can be either positive or negative.
- s can be either positive or negative.

Tip: See pages 48 and 49 for the equations of motion.

Figure 1: *A ball bearing that is falling vertically is assumed to be in free fall. This is because they are small and quite streamlined, so any air resistance acting on them is assumed to be negligible. This makes them useful for free fall experiments (see the next page).*

Case 1: No initial velocity

This means an object is just falling — initial velocity $u = 0$. Acceleration $a = g = -9.81$ ms^{-2}. Hence the equations of motion become:

$$v = gt \qquad v^2 = 2gs$$
$$s = \frac{1}{2}gt^2 \qquad s = \frac{1}{2}vt$$

Case 2: An initial velocity upwards

This means it's projected up into the air. The equations of motion are just as normal, but with $a = g = -9.81$ ms^{-2}.

Case 3: An initial velocity downwards

This is like case 2 — the equations of motion are as normal with $a = g = -9.81$ ms^{-2}.

Examples — **Maths Skills**

A tile falls from a roof 25 m high.
Calculate its speed when it hits the ground.

As before, start by writing out what you know:

$s = 25$ m
$u = 0$ ms^{-1} (since the tile's stationary to start with)
$g = 9.81$ ms^{-2}
$v = ?$

We want to find v, we know s and g, and the tile has no initial velocity. So use $v^2 = 2gs$:

$v^2 = 2 \times 9.81 \times 25$
$v^2 = 490.5$
$\Rightarrow v = \sqrt{490.5} = 22.147... = 22$ ms^{-1} (to 2 s.f.)

Tip: Here, all of the values we're interested in act downwards, so we can define downwards as the positive direction, so all the vector quantities are positive.

Alex throws a stone upwards. She throws it with an initial velocity of 4.00 ms^{-1}. What is the maximum height reached by the stone?

You know $u = 4.00$ ms^{-1}, $v = 0$ ms^{-1} and $a = g = -9.81$ ms^{-2}
You need to find s.

Use $v^2 = u^2 + 2as$, $s = (v^2 - u^2) \div 2a$

$s = (0^2 - 4.00^2) \div (2 \times -9.81)$
$= 0.81549...$ m
$= 0.815$ m (to 3 s.f.)

Tip: Acceleration due to gravity acts against the motion of the stone, so it's negative here.

Determining *g* using free fall

PRACTICAL ACTIVITY GROUP 1

One method you can use to determine the acceleration due to gravity g using free fall involves dropping a ball bearing onto a trap door using an electromagnet. You can carry out the experiment by following these steps:

- Set up the equipment shown in Figure 2.

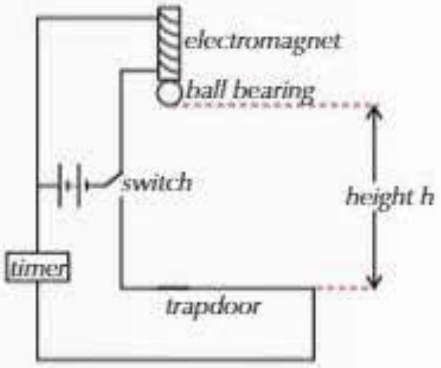


Figure 2: *Circuit diagram for an experiment to determine* ***g*** *using free fall.*

- Measure the height h from the bottom of the ball bearing to the trapdoor.
- Flick the switch to simultaneously start the timer and disconnect the electromagnet, releasing the ball bearing.
- The ball bearing falls, knocking the trapdoor down and breaking the circuit — which stops the timer. Record the time t shown on the timer.
- Repeat this experiment three times and average the time taken to fall.

Tip: Whether you use this experiment or another method, be sure you are working safely. Do a risk assessment before you start.

Tip: You could reduce the error in your measurement of h by using a set square to make sure your eye is level with the ruler.

Exam Tip
You could also measure g using light gates. Drop the ball between two light gates that are a known distance apart (h), and use the light gates to measure the velocity at each gate (u and v). You can then use the equation $v^2 = u^2 + 2gh$ (from the equation on page 49) to find g.

- To calculate g from the results, rearrange the equation $h = \frac{1}{2}gt^2$ (page 50) to get $g = 2h \div t^2$, where t is the time measured by the timer and h is the height that the ball bearing has fallen.
- Repeat this experiment for several different heights. Calculate a value of g for each height.
- Average the values of g for each of the heights to find your final value of g.

Identifying and reducing errors

The most significant source of random error in this experiment will be in the measurement of h. Using a ruler, you'll have an uncertainty of about ±1 mm.

Using a small and heavy ball bearing means you can assume air resistance is so small you can ignore it.

Using this kind of mechanism to automatically release and time the ball bearing's fall means you can measure times with a smaller uncertainty than if you tried to drop the ball and time the fall using a stopwatch.

If you were to do this by hand, you'd get a larger uncertainty due to human reaction times (it will take you a fraction of a second to press the buttons on the stopwatch). But there may still be a small systematic error if there is a delay in the switch or timing mechanism.

Tip: If you need to buff up on your practical skills, head over to pages 5-22.

Exam Tip
Unless you're given the value of g in an exam question, you should assume it has a value of 9.81 ms^{-2}.

Practice Questions — Application

Q1 A ball is dropped from the top of a cliff and takes 6.19 s to hit the ground. Assuming there's no air resistance, how tall is the cliff?

Q2 A spanner is dropped from a height of 6.83 m. Assuming there's no air resistance, how fast will it be going when it hits the ground?

Q3 Give two ways errors can be reduced when carrying out an experiment to determine the value of g by measuring the length of time an object takes to fall a set distance.

Practice Questions — Fact Recall

Q1 What's the only force present in free fall motion?

Q2 a) Describe an experiment you could do to obtain data that you could use to determine the value of g.

b) Explain how you can use your results to find g.

3. Projectile Motion

Objects such as a thrown ball or a bullet leaving a gun have projectile motion. They follow a curved path and experience free fall. The key thing here is to treat the horizontal and vertical parts of motion separately.

Learning Objectives:

- Understand the independence of the vertical and horizontal motion of a projectile.
- Understand and be able to perform calculations involving the two-dimensional motion of a projectile with constant velocity in one direction and constant acceleration in a perpendicular direction.

Specification Reference 3.1.3

Projectile motion

Any object given an initial velocity and then left to move freely under gravity is a projectile. In **projectile motion**, the horizontal and vertical components of the object's motion are completely independent. Projectiles follow a curved path because the horizontal velocity remains constant, while the vertical velocity is affected by the acceleration due to gravity, g.

Example — **Maths Skills**

A cannon ball is fired horizontally with a velocity of 120 ms^{-1} from 1.5 m above the ground. How long does it take to hit the ground, and what is its change in horizontal displacement in this time? Assume the cannon ball acts as a particle, the ground is horizontal and there is no air resistance.

Start with the vertical motion — it's constant acceleration under gravity:

You know $u = 0$ (no vertical velocity at first), $s = -1.5$ m and $a = g = -9.81$ ms^{-2}. You need to find t.

Use $s = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2s}{g}} = \sqrt{\frac{2 \times -1.5}{-9.81}} = 0.5530...$ s

So the ball hits the ground after 0.55 seconds (to 2 s.f.).

Then do the horizontal motion:

The horizontal motion isn't affected by gravity or other forces, so it moves at a constant speed. This means you can just use velocity $= \frac{\text{displacement}}{\text{time}}$.

Now $v_h = 120$ ms^{-1}, $t = 0.5530...$ s and $a = 0$. You need to find s_h.

$s_h = v_h t = 120 \times 0.5530... = 66.36... = 66$ m (to 2 s.f.)

Exam Tip
If you're doing AS or A Level Maths, you'll probably cover projectiles in mechanics.

Exam Tip
"Assume the [object] acts as a particle and there is no air resistance" means you don't need to worry about any forces other than the one caused by gravity (weight).

Tip: v_h is the horizontal velocity and s_h is the horizontal displacement.

Projectile motion at an angle

If something's projected at an angle (like, say, a javelin) you start off with both horizontal and vertical velocity. This can make solving problems trickier.

To solve this kind of problem, you need to use this method:

- Resolve the initial velocity into horizontal and vertical components.
- Use the vertical component to work out how long it's in the air and/or how high it goes.
- Use the horizontal component to work out how far it goes in the horizontal direction while it's in the air.

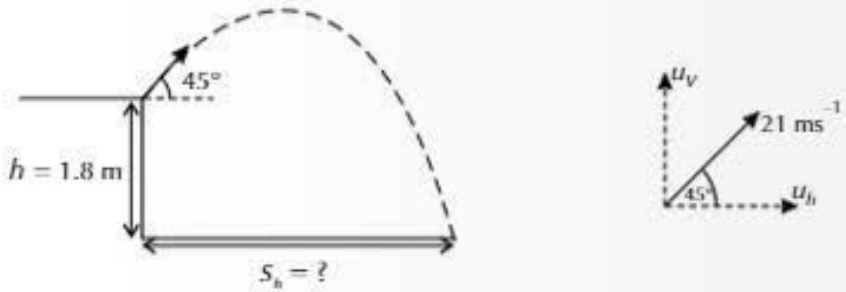
Tip: Resolving vectors was covered on pages 43-44.

Figure 1: *The only thing carrying a javelin forwards after it's been thrown is the initial horizontal velocity given to it by the thrower.*

Tip: v_v is negative because the javelin is travelling downwards when it lands.

Example — **Maths Skills**

A javelin is thrown with a velocity of 21 ms^{-1} at an angle of 45° to the ground from a height of 1.8 m. How far does the javelin travel? Assume the javelin acts as a particle, the ground is horizontal and there is no air resistance.



Start by resolving the velocity into horizontal and vertical components:

$u_h = \cos 45° \times 21 = 14.84...$ ms^{-1}
$u_v = \sin 45° \times 21 = 14.84...$ ms^{-1}

Then find how long it's in the air for — start by finding v_v. The javelin starts from a height of 1.8 m and finishes at ground level, so use $s = -1.8$ m:

$v_v^2 = u_v^2 + 2gs_v \Rightarrow v_v = \sqrt{14.84...^2 + 2 \times (-9.81) \times (-1.8)} = -15.99...$ ms^{-1}

Then find the time it stays in the air:

$$s_v = \frac{(u_v + v_v)}{2} \times t \Rightarrow t = \frac{s_v}{(u_v + v_v)} \times 2 = \frac{-1.8}{14.84... - 15.99...} \times 2 = 3.144...\text{ s}$$

Now you can work out how far it travels horizontally in this time:

$s_h = u_h t = 14.84... \times 3.144... = 46.68... = 47$ m (to 2 s.f.)

Figure 2: *A bullet acts as a projectile in free fall after leaving the gun (if you ignore air resistance).*

Practice Questions — Application

Q1 A gun fires a bullet at 502 ms^{-1} horizontally. If the gun was held 1.61 m above the ground, how far will the bullet travel? Assume there's no air resistance and the ground is horizontal.

Q2 A catapult hurls a rock from ground level at 25 ms^{-1}, 60.0° to the horizontal. Assuming there's no air resistance and that the ground is horizontal, calculate:

a) The amount of time the rock stays in the air.

b) How far away from the catapult the rock will land.

Q3 A golf ball is hit at 12.1 ms^{-1} at an angle of 31.5° above the horizontal from a podium 4.20 m above the surface of the ground. Calculate the maximum height above the ground reached by the ball. Assume there's no air resistance and the ground is horizontal.

Practice Questions — Fact Recall

Q1 What's free fall motion called when the object is given an initial velocity?

Q2 What should you do if you need to use the equations of uniform acceleration on an object that has an initial velocity at an angle to the horizontal, in order to find the horizontal distance travelled?

4. Displacement-Time Graphs

Displacement-time graphs show an object's position relative to its starting point over a period of time. They're useful because they can be used to describe an object's motion as well as find its velocity at a given point.

Learning Objectives:

- Be able to plot displacement-time graphs.
- Understand graphical representations of displacement, speed, velocity and acceleration on displacement-time graphs.
- Know that the gradient of an object's displacement-time graph gives the velocity of the object.

Specification Reference 3.1.1

Plotting displacement-time graphs

You need to be able to plot **displacement-time graphs** for moving objects. The suvat equations from page 48 can be used to work out values to plot. Displacement is plotted on the *y*-axis and time on the *x*-axis.

Example — **Maths Skills**

Plot a displacement-time graph for a panther who accelerates constantly from rest at 2.0 ms^{-2} for 5.0 seconds.

You want to find $\boldsymbol{s}$, and you know that:

$\boldsymbol{a} = 2.0$ ms^{-2}
$\boldsymbol{u} = 0$ ms^{-2}

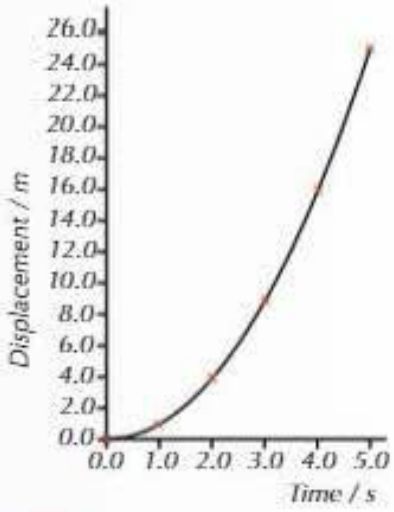
Use $\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$ to find values of t and $\boldsymbol{s}$ to plot on the graph.
If you substitute in $\boldsymbol{u}$ and $\boldsymbol{a}$, this simplifies to:

$\boldsymbol{s} = (0 \times t) + (\frac{1}{2} \times 2.0t^2)$
$= t^2$

Now pick values of t between 0 and 5.0 seconds and work out $\boldsymbol{s}$ at those points with $\boldsymbol{s} = t^2$:

t (s)	s (m)
0.0	0.0
1.0	1.0
2.0	4.0
3.0	9.0
4.0	16.0
5.0	25.0

...then plot the graph:



Tip: Try to use time intervals that make plotting the graph easier. In this example, using intervals of 1 second gives you 6 points to plot, which is enough for a neat curve.

Figure 1: *GPS and displacement-time graphs can be used to track a sea turtle's movement between mating seasons.*

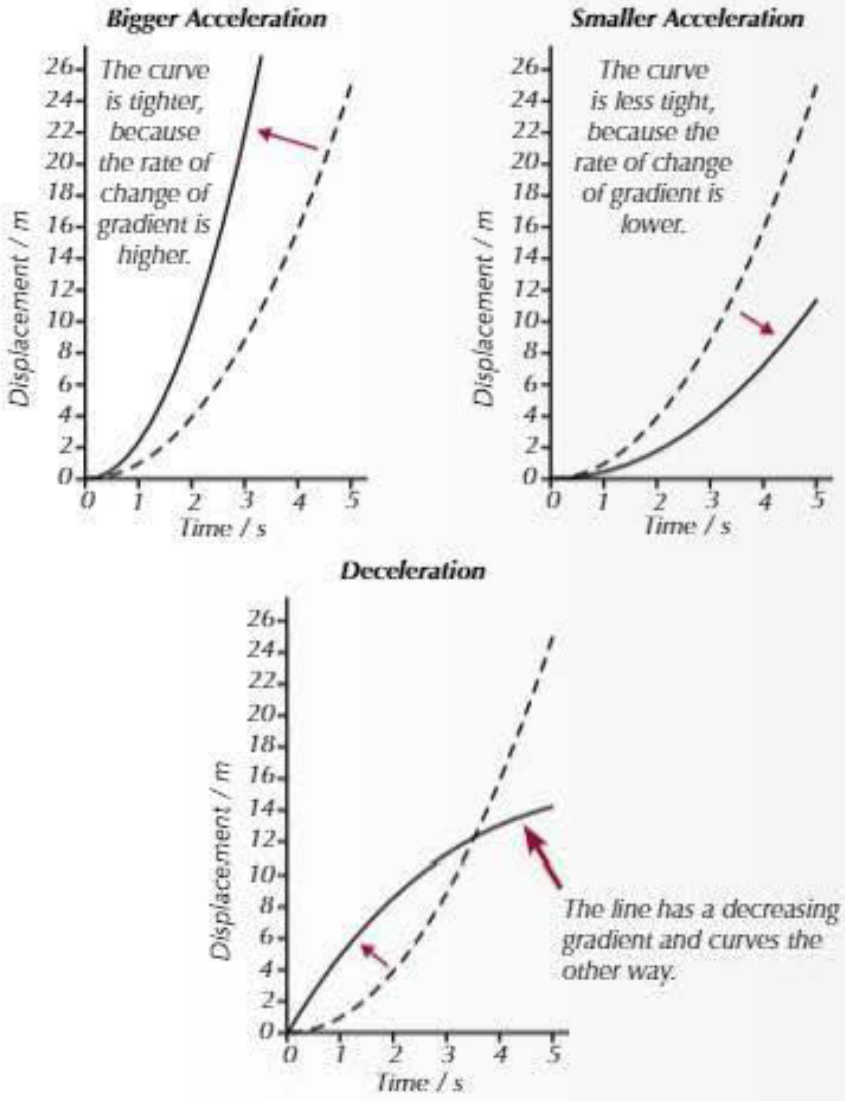
Acceleration on displacement-time graphs

The gradient of a displacement-time graph shows velocity. Acceleration is the rate of change of velocity (see page 48), so on a displacement-time graph, acceleration is the rate of change of the gradient.

A graph of displacement against time for an accelerating object always produces a curve. If the object's accelerating at a uniform rate, then the rate of change of the gradient will be constant. Acceleration is shown by a curve with an increasing gradient (like the one in the example above). Deceleration is shown by a curve with a decreasing gradient.

Changing the acceleration of the panther in the example on the previous page would change the gradient of the displacement-time graph like this:

Tip: You could use spreadsheet modelling to plot the *s*-*t* graph for the panther. See pages 481-482 for more.


Bigger Acceleration
The curve is tighter, because the rate of change of gradient is higher.
Displacement / m
Time / s
Smaller Acceleration
The curve is less tight, because the rate of change of gradient is lower.
Displacement / m
Time / s
Deceleration
The line has a decreasing gradient and curves the other way.
Displacement / m
Time / s

Tip: Note that in the case of deceleration, the panther must have been already moving at $t = 0$. Otherwise its displacement would be negative.

Figure 2: *The effect of changing the acceleration on the gradient of a displacement-time graph.*

Finding the velocity

When velocity is constant (but not zero), the displacement-time graph is a diagonal straight line. As you saw on the previous page, the gradient of a displacement-time graph shows velocity. This is because velocity is defined as:

$$\text{velocity} = \frac{\text{change in displacement}}{\text{time taken}} \quad \text{or:} \quad v = \frac{\Delta s}{\Delta t}$$

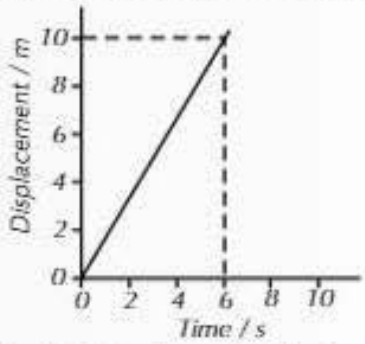
Tip: If a section of a displacement-time graph is horizontal (gradient = 0), the object's velocity is zero — it's not moving.

Tip: A negative gradient means the object's moving in a negative direction.

On the graph, this is $\frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x}$, i.e. the gradient.

Example — **Maths Skills**

The graph below shows a car's displacement over time. What's the car's velocity between 0 and 6 seconds?



To work out the velocity, find the change in displacement and the change in time during the period given in the question:

$$v = \frac{\Delta y}{\Delta x} = \frac{10 - 0}{6 - 0} = \frac{10}{6} = 1.666... = 1.7 \text{ ms}^{-1} \text{ (to 2 s.f.)}$$

Tip: Although it might seem pointless subtracting zero, the section of the graph you're working with won't always start at 0.

Velocity and curved displacement-time graphs

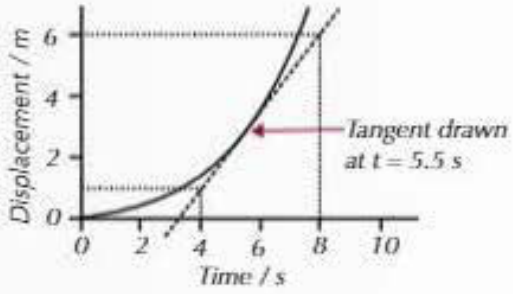
If the gradient isn't constant (i.e. if it's a curved line), it means the object is accelerating and the velocity is constantly changing.

An object's instantaneous velocity is just its velocity at a particular moment in time. To find the instantaneous velocity at a certain point, you need to draw a **tangent** to the curve at that point to find its gradient.

Figure 3: *A displacement-time graph of a runner during a race can be used to work out their velocity at any point in the race.*

Example — **Maths Skills**

The graph below shows the displacement of a ball rolling down a slope over time. Find its velocity at 5.5 s.



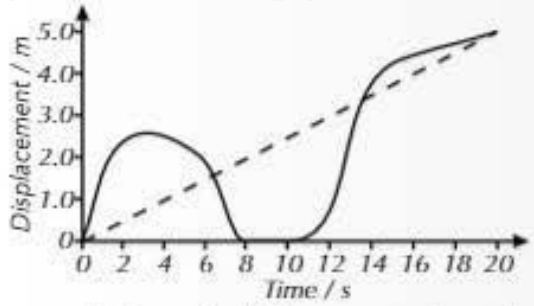
Start by drawing a tangent to the curve at 5.5 s (this has been done already). You can then draw horizontal and vertical lines from the tangent to the axes to find values for Δy and Δx.

$$v = \frac{\Delta y}{\Delta x} = \frac{6 - 1}{8 - 4} = \frac{5}{4} = 1.25 \text{ ms}^{-1}$$

The average velocity is just the total change in displacement of the object divided by the total time taken. You can ignore any variations in acceleration and velocity, you just need the time and total displacement.

Example — **Maths Skills**

The graph below shows the displacement of a model railway train along a straight track over time. Calculate the average velocity of the train during the time period shown on the graph.



average velocity = total change in displacement ÷ time taken

$= 5.0 \div 20 = 0.25\ ms^{-1}$

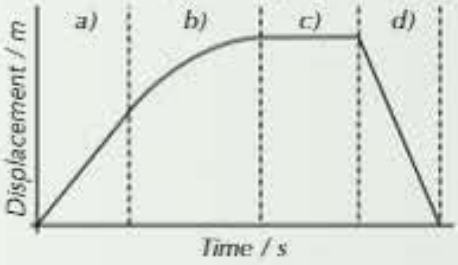
Practice Questions — Application

Q1 Use the data in the table below to plot a displacement-time graph.

t (s)	s (m)
2	6
2.5	7.5
3.5	10.5
5	15
7	21
8	24

Figure 4: *The graph in Q2 might describe a cyclist's displacement as they go up and down a hill.*

Q2 The graph below shows the displacement of a cyclist during a journey. Describe what's happening in parts a), b), c) and d).



Q3 A rocket accelerates at a constant rate from rest to 100.0 ms^{-1} in 5.0 s. Plot a displacement-time graph for the rocket during this time.

Tip: Start by finding the rocket's acceleration.

Q4 a) Use the data in the table below to plot a displacement-time graph.

t (s)	s (m)
0.0	0.0
1.0	1.5
2.0	6.0
3.0	13.5
4.0	24.0
5.0	37.5

b) Use your graph to find the velocity of the object at 3.0 s?

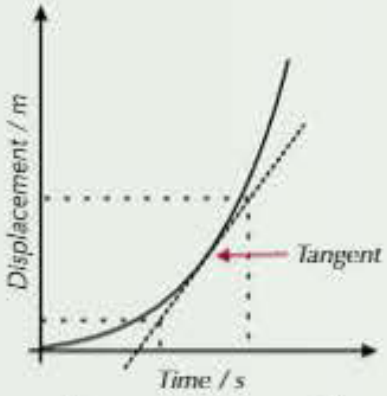
Practice Questions — Fact Recall

Q1 What kind of motion does a curved displacement-time graph show?

Q2 What kind of motion does a straight line on a displacement-time graph show?

Q3 What does the rate of change of gradient on a displacement-time graph represent?

Q4 What value can be calculated using the tangent on the distance-time graph below?



Q5 What is the difference between average and instantaneous velocity?

Q6 How would you calculate the average velocity of an object moving with non-uniform acceleration using a displacement-time graph?

Figure 5: *The displacement-time graph for a bus is likely to show a non-uniform acceleration as it makes frequent stops.*

Learning Objectives:

- Understand graphical representations of displacement, speed, velocity and acceleration on velocity-time graphs.
- Be able to plot velocity-time graphs.
- Know that the gradient of an object's velocity-time graph gives the acceleration of the object.
- Know that the area under an object's velocity-time graph gives the displacement of the object.

Specification Reference 3.1.1

5. Velocity-Time Graphs

Velocity-time graphs show, as the name suggests, an object's velocity over time. As with displacement-time graphs, their shape can be used to find out about an object's movement at different points in time.

Finding the acceleration

The gradient of a **velocity-time graph** tells you the acceleration, since:

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time taken}} \quad \text{or:} \quad a = \frac{\Delta v}{\Delta t}$$

Uniform acceleration is always a straight line. The steeper the gradient, the greater the acceleration.

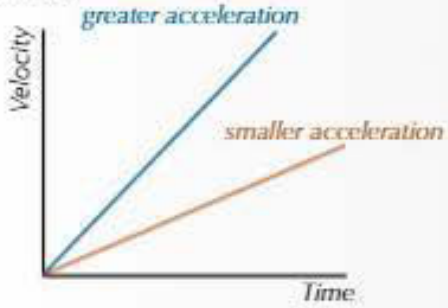


Figure 1: *A velocity-time graph showing the gradients of two different accelerations.*

Tip: The equation for a straight line is $y = mx + c$. You can rearrange the acceleration equation into the same form, getting $v = u + at$. So on a linear v-t graph, acceleration, a, is the gradient (m) and the initial speed, u, is the y-intercept (c).

Tip: In questions like this you could use the equations of motion to find the object's velocity at various times — in a similar way to the displacement-time graph example on page 55.

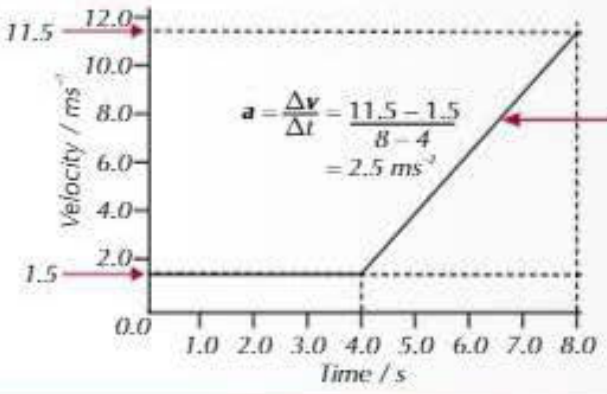
Example **Maths Skills**

A lion walks at 1.5 ms^{-1} for 4.0 s and then accelerates uniformly at a rate of 2.5 ms^{-2} for 4.0 s. Plot this information on a velocity-time graph.

Start by finding the lion's velocity at intervals between 0 and 8 s — it's 1.5 ms^{-1} for the first 4 seconds, then it increases by 2.5 ms^{-1} every second.

t (s)	v (ms^{-1})
0.0 – 4.0	1.5
5.0	4.0
6.0	6.5
7.0	9.0
8.0	11.5

Then just plot a graph with time on the x-axis and velocity on the y-axis:



The gradient is constant between 4 and 8 s and has a value of 2.5 ms^{-2}, representing the acceleration of the lion.

Finding displacement

A speed-time graph is very similar to a velocity-time graph. The big difference is that velocity-time graphs can have negative regions to show something travelling in the opposite direction.

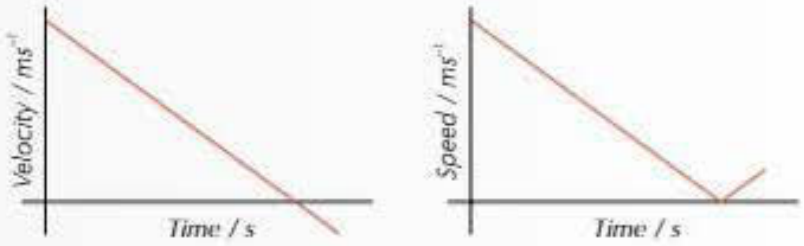


Figure 2: *The velocity-time and speed-time graphs for a ball being thrown up into the air.*

Tip: When the ball slows down and starts falling, its velocity will become negative but its speed will start increasing again.

The area under a velocity-time graph represents the displacement of an object, as displacement, **s**, of an object with uniform acceleration can be found using:

$$s = \tfrac{1}{2}(u + v)t = \text{average velocity} \times \text{time}$$

Tip: For velocity-time graphs, the areas under any negative parts of the graph count as 'negative areas', as they show the object moving the opposite way to whichever direction you took as being positive.

You can find the total distance travelled by an object using:

$$\text{distance travelled} = \text{average speed} \times \text{time}$$

Therefore the area underneath a speed-time graph is the distance travelled.

Tip: The examples below show the link between these equations and the area under a speed-time or velocity-time graph.

Examples — Maths Skills

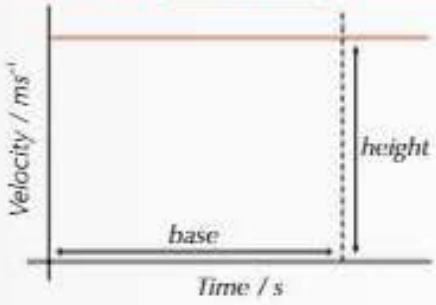


Figure 3: *The displacement for a rectangular velocity-time graph is just base × height.*

Average velocity $= \tfrac{1}{2}(u + v)$

But velocity is constant, so $u = v$.

So average velocity $= \dfrac{v + v}{2} = v$

$=$ final velocity

Area under graph

$=$ base × height

$=$ final velocity × time

$=$ average velocity × time

$=$ displacement

Tip: Remember, ***u*** is initial velocity and ***v*** is final velocity.

Tip: Here, displacement is the same as distance travelled because the velocity-time graph has no negative parts.

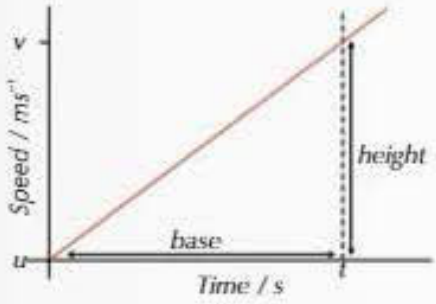


Figure 4: *The distance travelled for a triangular speed-time graph is ½ × base × height.*

Average speed $= \tfrac{1}{2}(u + v)$

But $u = 0$, so average speed $= \dfrac{v}{2}$

Area under graph

$= \tfrac{1}{2} \times$ base × height

$= \tfrac{1}{2} \times$ time × final speed

$= \tfrac{1}{2} \times t \times v$

$= \dfrac{v}{2} \times t$

$=$ average speed × time

$=$ distance travelled

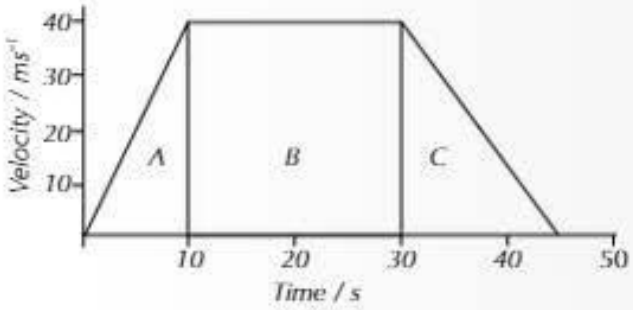
Tip: Remember, uniform acceleration and deceleration are shown by straight lines on a velocity-time or speed-time graph.

Tip: If the *v*-*t* graph is on a grid, you could also work out the area under the graph by counting how many squares make up the area. Multiplying the value of each square by the number of squares will give you the displacement.

Example — Maths Skills

A racing car accelerates uniformly from rest to 40.0 ms^{-1} in 10.0 s. It maintains this speed for a further 20.0 s before coming to rest by decelerating at a constant rate over the next 15.0 s. Draw a velocity-time graph for this journey and use it to calculate the total distance travelled by the racing car.

Start by drawing the graph and then splitting it up into sections:



Calculate the area of each section and add the three results together.

A: Area = ½ × base × height = ½ × 10.0 × 40.0 = 200 m

B: Area = base × height = 20.0 × 40.0 = 800 m

C: Area = ½ × base × height = ½ × 15.0 × 40.0 = 300 m

Total distance travelled = 200 + 800 + 300 = 1300 m

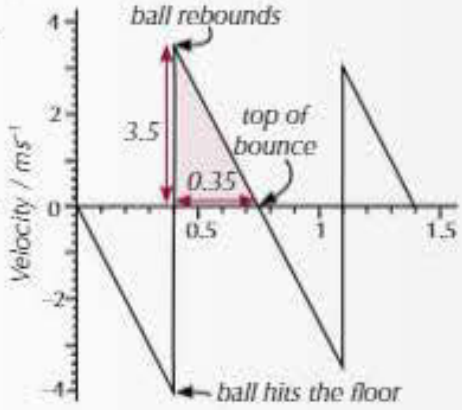
Figure 5: *When a ball bounces, it changes the direction in which it is travelling, so the sign of its velocity changes.*

Tip: The maximum velocity decreases with each bounce because some of the ball's kinetic energy is transferred into other forms when it hits the ground. This means the height of each bounce also decreases.

Tip: The straight, diagonal lines show that there is constant deceleration under gravity — see page 50.

Example — Maths Skills

A ball is dropped from table-height so it bounces vertically. It bounces twice before someone catches it. The ball's motion while it bounces is shown on the *v*-*t* graph below. Calculate how high the ball rebounds on the first bounce.



Before you try and calculate anything, make sure you understand what each part of the graph is telling you about the ball's motion.

1. When the ball is first dropped, the velocity of the ball is negative — so downwards is the negative direction.
2. The points where the ball hits the floor are shown by the vertical straight lines on the graph — the ball's speed remains roughly the same, but its direction (and velocity) changes the instant it hits the floor.
3. The points where the ball's velocity is zero show where the ball reaches the top of a bounce before starting to fall downwards.

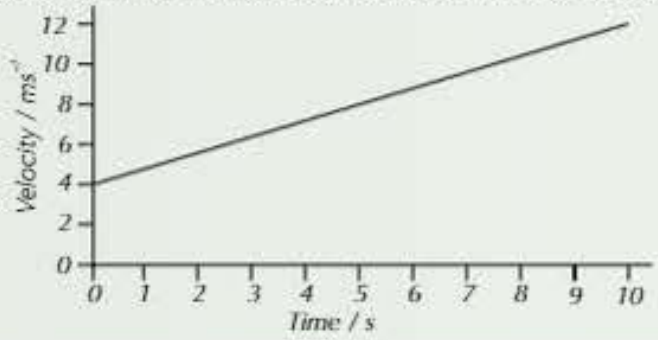
The height of the first bounce is the area under the graph between the time the ball first rebounds from the floor and the time it reaches the top of the bounce.

displacement = area under graph

= (3.5 × 0.35) ÷ 2 = 0.6125 = 0.61 m (to 2 s.f.)

Practice Question — Application

Q1 The graph below shows the velocity of a car over 10 seconds. Use the graph to find the displacement of the car in this time.



Tip: To find the area of a non-standard shape, try splitting it into two easier shapes (e.g. a rectangle and a triangle).

Non-uniform acceleration

If the acceleration is changing, the gradient of the velocity-time graph will also be changing — so you won't get a straight line. Increasing acceleration is shown by an increasing gradient — like in curve ① below. Decreasing acceleration is shown by a decreasing gradient — like in curve ② below.

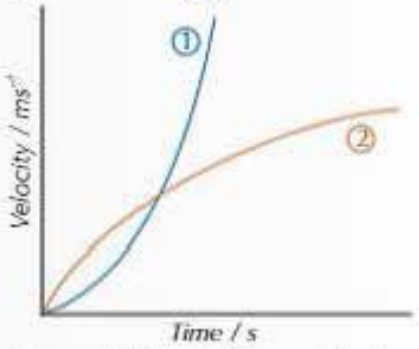


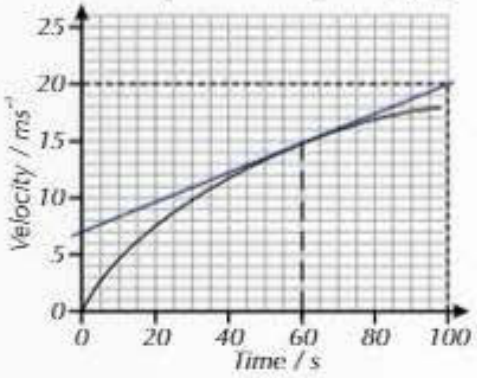
Figure 7: *Non-uniform acceleration on a velocity-time graph.*

Figure 6: *A speed-time graph of wind during a hurricane.*

For graphs like this, you can find the acceleration of an object at a particular point by drawing a tangent to the curve at that point and finding its gradient.

Example — **Maths Skills**

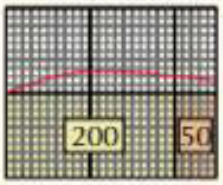
Find the acceleration shown by the velocity-time graph below at 60 seconds.



First draw a tangent to the curve at 60 seconds.

Then find the gradient of the tangent: gradient $= \frac{\Delta y}{\Delta x} = \frac{20 - 7}{100 - 0} = 0.13$ ms^{-2}

Exam Tip
The counting the squares method is useful, but it can be time consuming. If the graph you're given has lots of small squares, it might be quicker to split the area up into trapeziums and find the area of those instead. Otherwise, block out larger squares — e.g. in the graph below you know there are 100 small squares in a big square.



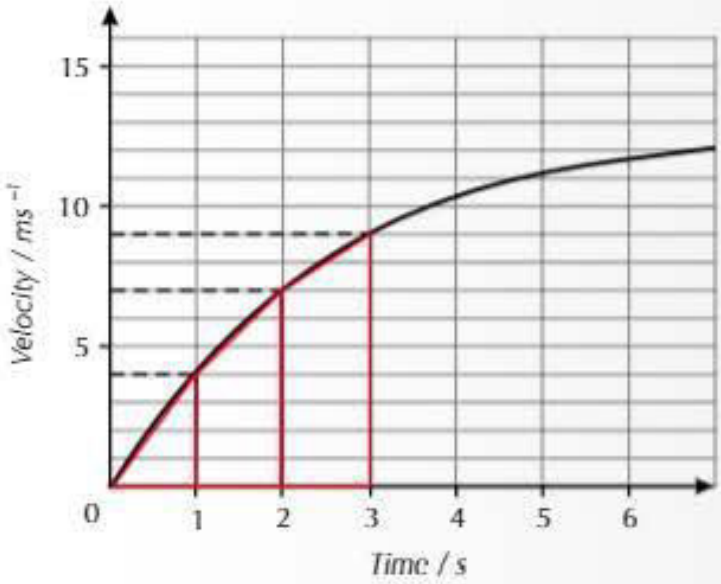
Displacement on non-linear velocity-time graphs

As our velocity-time graph is no longer a simple straight line, we have to use methods to estimate the area under the curve.

If the graph is on squared paper, an easy way to do this is just count the squares under the curve (see p.478). Another way is to split the curve up into trapeziums, calculate the area of each one and then add them all up.

Example — Maths Skills

A car decreases its acceleration as it approaches 13 ms^{-1}. Estimate its displacement between 0 and 3 seconds from the velocity-time graph.



Split the area under the curve up into trapeziums and a triangle.

0-1 s — estimate the area with a triangle.
The height of the triangle is 4.
The base of the triangle is 1.

$$A = \frac{1}{2}(1 \times 4) = 2\,\text{m}$$

1-2 s — estimate the area using a trapezium.

$$\text{Area} = \frac{1}{2}(a + b)h$$

a is the length of the first side, $a = 4$
b is the length of the second side, $b = 7$
h is the width of each strip, so $h = 1$

$$A = \frac{1}{2}(4 + 7) \times 1 = 5.5\,\text{m}$$

2-3 s — trapezium, $a = 7$, $b = 9$, $h = 1$

$$A = \frac{1}{2}(7 + 9) \times 1 = 8\,\text{m}$$

Now add the areas together — Total area = 2 + 5.5 + 8 = 15.5 m
The estimated overall displacement of the car is 15.5 m = **16 m (to 2 s.f.)**

Tip: If you did this by counting the squares (or rectangles in this case), each rectangle represents 1 m. There are 15.5 rectangles under the graph between 0 and 3 s, so you get 15.5 m, the same answer as the trapezium method.

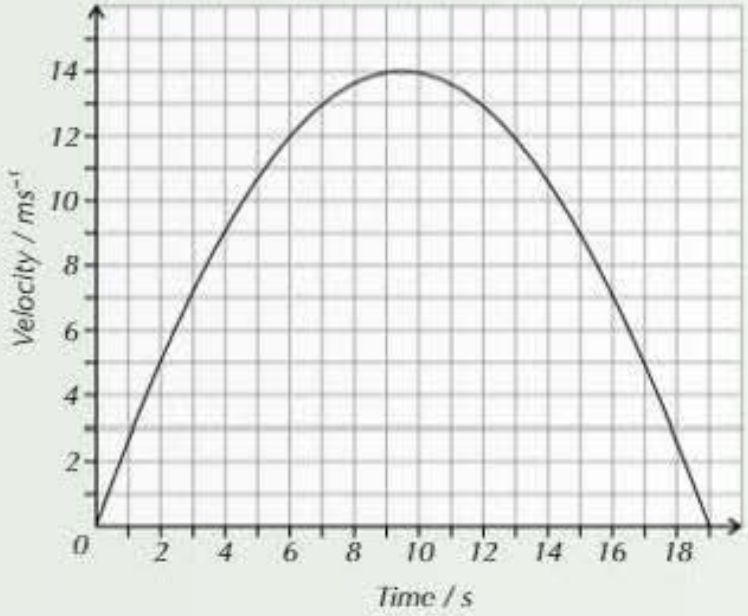
Practice Questions — Application

Q1 a) A cyclist goes down a hill with increasing acceleration. Use the table below to plot a velocity-time graph of the cyclist.

t (s)	v (ms^{-1})
0	0
1	1
2	4
3	9
4	16

b) Use the graph to find the acceleration at 2 seconds.

Q2 The acceleration of a car is being tested on a test-track. The graph below is the velocity-time graph for the car during the test.



Estimate the total distance travelled during the test.

Practice Questions — Fact Recall

Q1 What does the gradient of a velocity-time graph tell you?

Q2 How is uniform acceleration shown on a velocity-time graph?

Q3 What does the area under a velocity-time graph tell you?

Q4 How is non-uniform acceleration shown on a velocity-time graph?

Learning Objective:

- Know and understand techniques and procedures to investigate the motion and collisions of objects (PAG1).

Specification Reference 3.1.2

6. Investigating Motion

PRACTICAL ACTIVITY GROUP 1

If you want to do some experiments involving motion, you need to know the ins and outs of the apparatus you'll be using to make measurements. You need to make sure you know the best ways to measure the basic values.

Light gates

A light gate sends a beam of light from one side of the gate to a detector on the other side. When something passes through the gate, the beam of light is interrupted. The light gate (along with some computer software) measures how long the beam was interrupted for.

Light gates are often used to find the velocity or acceleration of an object. To find the velocity of an object, the length of the interrupting object should be input into the software. The software then uses velocity = length of object ÷ interruption time to calculate the velocity. To find the acceleration, the object needs to interrupt the beam twice, for example by attaching a piece of card in the shape of a 'U' to the top of the object. The software uses the difference in the velocity of the two sections of the card to calculate the acceleration (page 48).

Light gates are particularly useful because they give a much lower uncertainty than other methods of timing, like stopwatches. This is because the system doesn't rely on human reaction times and judgement.

***Figure 1**: The beam of light in a light gate being interrupted by a piece of card attached to a trolley.*

Tip: You could also do this experiment with an air track and a glider. The air track creates a cushion of air that the glider sits on (similar to the puck in air hockey), which reduces the friction in the experiment to almost nothing.

Tip: Remember, you need to carry out a risk assessment before you start any experiments.

Example

An experiment you can do using light gates involves a trolley rolling down a ramp.

For example, you can investigate how the distance a trolley has rolled affects its speed using the set-up shown in Figure 2.

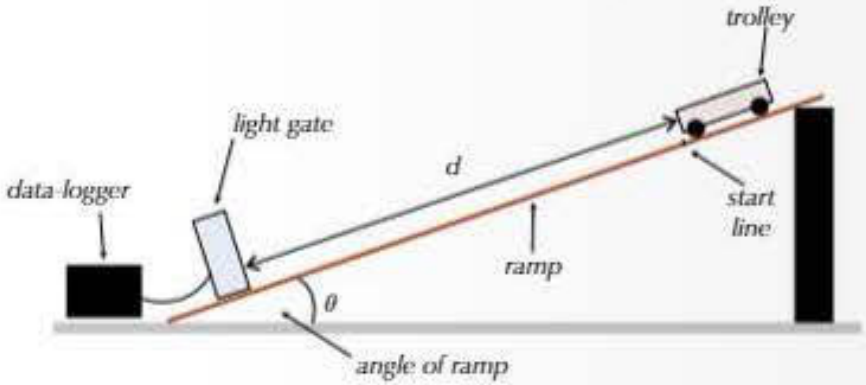


***Figure 2**: The experimental set-up for an investigation into the motion of a trolley as it travels down a ramp.*

- Measure the length of the trolley.
- Mark a start line on the ramp to make sure the trolley always starts from the same position.
- Measure the distance from the chosen start line to the light gate, d.

- Place the trolley on the ramp and line it up with the start line. Let go of it so its initial velocity, $\boldsymbol{u}$, is 0.
- The data logger will record the time taken for the trolley to pass through the light gate and calculate the velocity of the trolley as it passes through the gate.
- Change the starting position of the trolley, so d is varied.

Repeat this experiment for each distance 3 times and average the recorded velocities to reduce the error in your final result.

Ticker timers

Ticker timers are used to measure the speed of an object. A length of paper tape (ticker tape) is fed through the ticker timer and attached to the object whose speed you want to measure (see Figure 3). The ticker timer creates dots on the tape at regular time-intervals (e.g. every tenth of a second). As the object moves, a line of dots is created on the ticker tape. The further apart two consecutive dots are, the further the object has travelled in a set time, and the faster the object is moving.

You can calculate the speed of the object at a certain point by measuring the distance between two consecutive dots and dividing it by the time elapsed. You can find the acceleration by calculating the change in speed between two pairs of dots and then dividing this by the time elapsed between the two pairs of dots. The further apart the pairs of dots, the lower the percentage error in your calculation (page 16). If you know that the acceleration isn't constant, you can use three consecutive dots as your two pairs of dots. This will give the acceleration of the object at that point.

Ticker timers are generally a bit trickier to get your head around than light gates. They don't come with handy software that does the calculations for you, and it can be difficult to match parts of the ticker tape to specific sections of the object's journey.

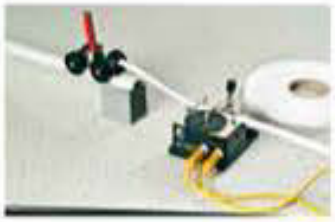


Figure 3: *A ticker timer being used to measure the speed of a trolley travelling down a ramp.*

Tip: For slow speeds, the dots may be very close together, so you may want to measure across e.g. five dots, and then calculate the time elapsed for five dots.

Video techniques

For a light gate to work, the object you're measuring the motion of has to pass completely through the light gate. So, if you want to measure the speed immediately before and after an event, such as in a collision, then light gates aren't suitable. In these situations, you can use video equipment to find speed.

Set up a video camera so it can record the event you're investigating. Make sure the video camera is filming the object from the side (perpendicular to the object's motion) and that there is a distance scale of some kind (e.g. a metre rule) within shot and alongside the path of the object.

Once you have filmed the event, you can use computer software to view your video frame by frame. Pick a point of reference on the metre stick and count how many frames it takes the object to pass that point.

Figure 4: *You should use a tripod, or a similar support, to keep your video camera level and still.*

Figure 5: *Images like this one are made by filming an action with a high frame-rate, and then combining the frames into a single picture.*

If you know how many frames per second the video is shot at (the frame rate of the video), you can calculate the time taken (t) for the whole object to pass that point. You can then use this time and the length (l) of the object to calculate its velocity using these equations:

$$\text{Time taken for the object to pass the point} = \text{Number of gaps between frames for an object to pass the point} \times \frac{1\text{ second}}{\text{Frame rate of camera}}$$

$$\text{velocity} = \frac{l}{t}$$

l = *length of object*

t = *time taken for object to pass a point*

This method is particularly useful when investigating collisions, such as how the velocity of an object before a collision affects its velocity after the collision.

Tip: Being able to do experiments involving collisions is particularly useful when it comes to investigating conservation of momentum (p.138).

Tip: You could also have a trolley colliding with another, stationary trolley. This is to investigate how the speed of the moving trolley affects the speed of both trolleys after the collision.

Example — Maths Skills

The set-up shown in Figure 6 is used to investigate the velocity of a trolley before and after it hits a wall. A video camera that records at 20 frames per second is used to record the experiment.

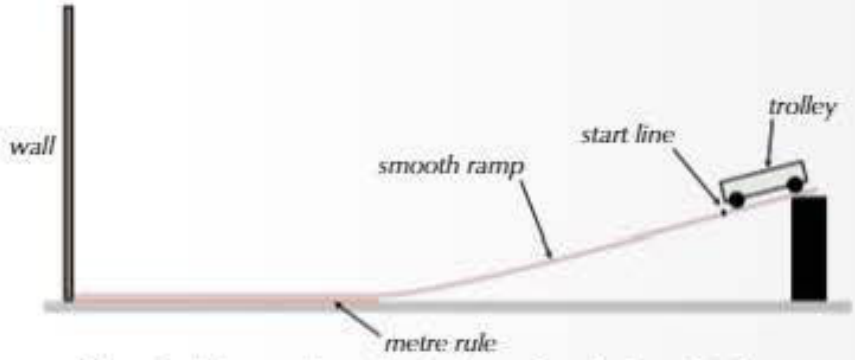


Figure 6: *The experimental set-up for an investigation into the motion of a trolley as it collides with a wall.*

Figure 7 shows three consecutive video frames of the trolley after it collides with the wall.
Calculate the velocity of the trolley from the results in Figure 7.

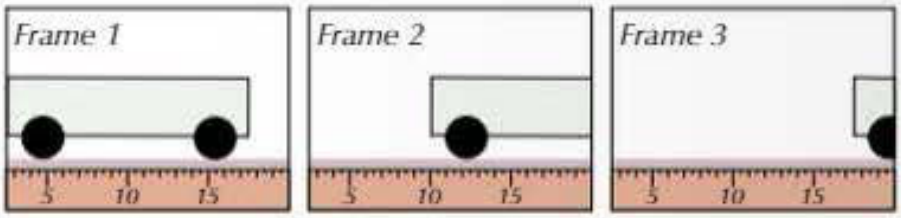


Figure 7: *Three consecutive frames of footage of the trolley moving along after it has hit the wall. The values on the metre rule are in centimetres.*

From Frame 1, you can see that the total length of the trolley is 15 cm.

To find the speed of the trolley, you need to find out how long it takes the trolley to pass a certain point.

For these frames, a good reference point is 17.5 cm.

It took three frames for the trolley to pass 17.5 cm, so there are 2 gaps between the frames.

$$\text{Time taken for the trolley to pass the point} = \text{Number of gaps between frames for the trolley to pass the point} \times \frac{1\,\text{second}}{\text{Frame rate of camera}}$$

$$= 2 \times \frac{1}{20} = 0.10\ \text{s}$$

Now calculate the velocity from the length of the trolley, l, and the time taken, t.
$l = 15\ \text{cm} = 0.15\ \text{m}$,
$t = 0.10\ \text{s}$

$$\text{velocity} = \frac{l}{t} = \frac{0.15}{0.10} = 1.5\ \text{ms}^{-1}$$

Tip: Remember to make sure all your variables are in the right units before you stick them into an equation.

ICT and data-loggers

A fairly standard piece of kit that is used for motion experiments in industry is an ultrasound position detector. This is a type of data-logger that automatically records the distance of an object from the sensor several times a second. If you attach one of these detectors to a computer with graph-drawing software, you can get real-time displacement-time and velocity-time graphs.

The main advantages of data-loggers over traditional methods are:

- The data is more accurate — you don't have to allow for human reaction times.
- Automatic systems have a much higher sampling rate than humans — ultrasound position detectors can take a reading ten times every second.
- You can see the data displayed in real time.

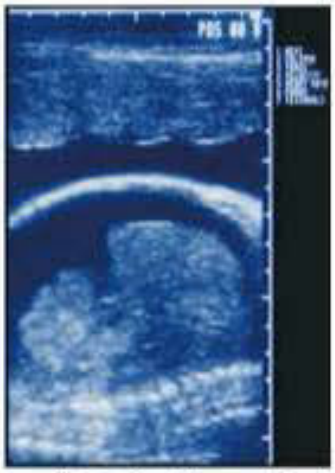


Figure 8: *Ultrasound position tracking is used to obtain medical images in real time.*

Practice Question — Application

Q1 A student is investigating the speed of a trolley using a ticker timer. The ticker timer makes 2 dots per second. The student cuts out the part of the ticker tape they are interested in, which is shown below.

Calculate the speed of the trolley between points A and B.

Practice Questions — Fact Recall

Q1 State and explain one advantage of using a light gate, instead of a stopwatch, to make time measurements.

Q2 A student wants to know how the velocity of a trolley changes after it hits a wall. Describe an experiment they could do to investigate this. Your answer should include a description of how measurements are taken and analysed.

Learning Objective:

- Know and be able to explain the terms: reaction time, thinking distance, braking distance and stopping distance.

Specification Reference 3.1.2

7. Stopping Distances

Stopping distances are important — awareness of them can make the difference between crashing and not. Stopping distance is split into thinking distance and braking distance, and different factors affect each.

Stopping distance

In an emergency (e.g. a hazard ahead in the road), a driver may perform an emergency stop. This is where maximum force is applied by the brakes in order to stop the car in the shortest possible distance. The longer it takes to perform an emergency stop, the higher the risk of crashing into whatever's in front.

The total **stopping distance** of a vehicle is the distance covered in the time between the driver first spotting a hazard and the vehicle coming to a complete stop. The total stopping distance is the sum of the thinking distance and the braking distance.

stopping distance = thinking distance + braking distance

- The **thinking distance** is the distance the vehicle travels during the driver's **reaction time** (the time between seeing a hazard and applying the brakes).
- The **braking distance** is the distance the vehicle travels after the brakes are applied until it comes to a complete stop, as a result of the braking force.

Many factors affect your total stopping distance — and you can break it down into thinking distance and braking distance to look at the factors that affect each of these.

Figure 1: *Tiredness can increase reaction times, so some roads have signs reminding motorists to take regular breaks and to not drive whilst tired.*

Thinking distance

You can calculate the thinking distance using:

thinking distance = speed × reaction time

So thinking distance is affected by two main factors:

1. How fast you're going — whatever your reaction time, the faster you're going, the further you'll go in that time.
2. How quick to respond you are, i.e. your reaction time — this can be affected by tiredness, illness, drugs, alcohol and a lack of concentration.

Braking distance

Braking distance depends on the braking force, friction between the tyres and the road, the mass and the speed. It is affected by:

1. How fast you're going — the faster you're going, the further you travel before you stop.
2. The mass of the car — the larger the mass of the car, the smaller the deceleration for a given braking force (p.79), and so the further it will travel before stopping.
3. How good your brakes are — all brakes must be checked and maintained regularly. Worn or faulty brakes won't be able to apply as much force as well-maintained brakes and could let you down catastrophically just when you need them the most, i.e. in an emergency.

4. How good the tyres are — tyres should have a minimum tread depth of 1.6 mm. In wet conditions, the tread pattern helps to stop water getting trapped between the tyres and the road — they provide a channel through which the water can 'escape'. With too little tread, the tyres may lose contact with the ground, causing the vehicle to slide.
5. How good the grip is — as well as the condition of the tyres, this depends on the weather conditions and the road surface. Water, ice, leaves, fuel spills, muck on the road etc. can greatly increase the braking distance. They can result in reduced friction between the tyres and the road — so you travel further before stopping and may skid. Often you only discover this when you try to brake hard.

Figure 2: *A good tyre tread depth helps reduce braking distance.*

Figure 3: *Fuel spills on roads can reduce grip and cause tyres to skid, increasing the braking distance.*

Typical stopping distances

The Highway Code contains typical stopping distances for a car travelling at different speeds (see Figure 4). As you've seen, lots of factors affect stopping distance, but this is a good indication of how stopping distance varies with speed.

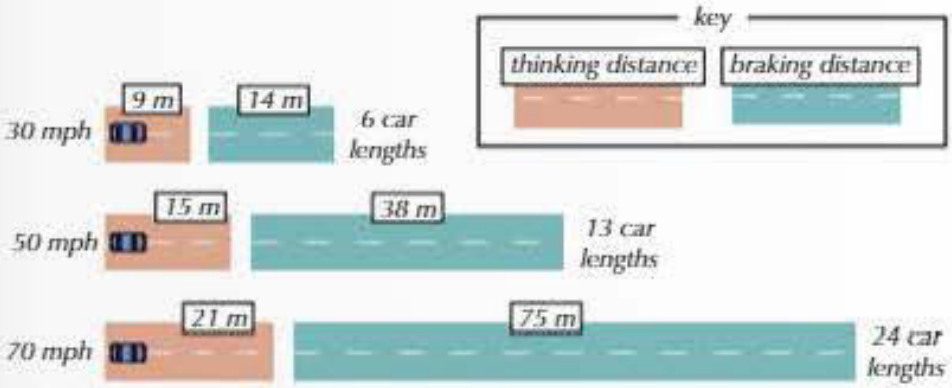


Figure 4: *Typical stopping distances taken from the Highway Code.*

Tip: Knowing about the stopping distances of vehicles helps the authorities to set speed limits and give advice to drivers that will help keep them and their passengers safe.

Practice Questions — Application

Q1 A driver in a car has a thinking distance of 15 m and a braking distance of 38 m. What's the stopping distance of the car?

Q2 Dahlia is driving her car at 25 ms^{-1} when she sees a hazard. She travels 14 m before she applies the brakes. Calculate her reaction time.

Practice Questions — Fact Recall

Q1 Define stopping distance, thinking distance and braking distance.

Q2 Other than speed, name three factors that affect braking distance.

Q3 Say whether each of the following would affect the thinking distance or the braking distance of a vehicle:

a) Ice on the road
b) Alcohol intake of the driver
c) Tiredness of the driver
d) Fuel spills

Section Summary

Make sure you know...

- That speed is how fast something is moving, regardless of its direction.
- That displacement is how far an object has travelled from its starting point in a given direction.
- That velocity is the rate of change of displacement and acceleration is the rate of change of velocity.
- That the average speed of an object is the total distance travelled by the object divided by the total time taken.
- That the instantaneous speed of an object is the speed of an object at a given point in time.
- The equations of motion for uniform acceleration and when to use them.
- What is meant by free fall and that all objects in free fall have the same acceleration due to gravity, ***g***.
- That ***g*** is directed vertically downwards and has a magnitude of 9.81 ms^{-2}.
- How to use the equations of motion for all objects in free fall.
- How to determine the value of ***g*** using a free fall method.
- That the vertical and horizontal components of projectile motion are independent.
- How to use the equations of uniform acceleration to solve problems that include projectile motion.
- How to plot a displacement-time graph.
- How to use a displacement-time graph to describe the motion of an object.
- How to calculate velocity from the gradient of a displacement-time graph.
- How to plot a velocity-time graph.
- How to use a velocity-time graph to describe the motion of an object.
- How to calculate acceleration from the gradient of a velocity-time graph.
- How to calculate displacement from the area under a velocity-time graph.
- How to estimate the area under a non-linear velocity-time graph.
- Some techniques you can use to investigate the motion and collisions of objects, including the use of light gates, data-loggers, ticker timers and video recordings and analysis.
- That the stopping distance of a vehicle is the total distance travelled between a driver seeing a hazard and the vehicle coming to a complete stop.
- That the stopping distance is the sum of the thinking distance and braking distance.
- That the reaction time of a driver is the time between the driver seeing a hazard and applying the brakes.
- That the thinking distance is the distance travelled during a driver's reaction time.
- That the braking distance is the distance travelled after the brakes are applied until the vehicle comes to a complete stop.
- How thinking distance depends upon the speed of the vehicle and the driver's reaction time and how it is affected by factors such as drug use and tiredness.
- How braking distance can be affected by factors such as the speed and mass of the vehicle, the weather, road conditions and the conditions of the brakes and tyres, and the implications for safety in an emergency.

Exam-style Questions

1 A student is investigating the value of ***g***. She drops a ball bearing so that it falls vertically through a light gate. The initial speed of the ball bearing is 0 ms^{-1}. Which of the following sets of measurements will allow her to calculate ***g***?

A Only the final speed of the ball bearing.

B Only the time taken for the ball bearing to reach the light gate.

C Only the distance between the ball bearing's starting point and the light gate, and time taken for the ball bearing to reach the light gate.

D Only the distance between the ball bearing's starting point and the light gate, and the mass of the ball bearing.

(1 mark)

2 A man is driving a car at a speed of 18.0 ms^{-1} when he makes an emergency stop. His reaction time during the emergency stop is 0.45 s. The car travels 24.9 m after he applies the brakes. What is the stopping distance of the vehicle?

A 8.1 m.

B 16.8 m.

C 24.9 m.

D 33.0 m.

(1 mark)

3 A tennis ball is dropped from a height and is in free fall. The speed of the ball when it reaches the ground is 3.80 ms^{-1}. What height was the tennis ball dropped from?

A 19.4 cm

B 73.6 cm

C 136 m

D 147 m

(1 mark)

4 A student throws a tennis ball at an angle of 60.0° to the horizontal, with an initial velocity of 2.50 ms^{-1}. How far does the ball travel horizontally in 3.00 s? You can assume that the ball does not reach the ground during this time and that air resistance is negligible.

A 6.50 m

B 1.25 m

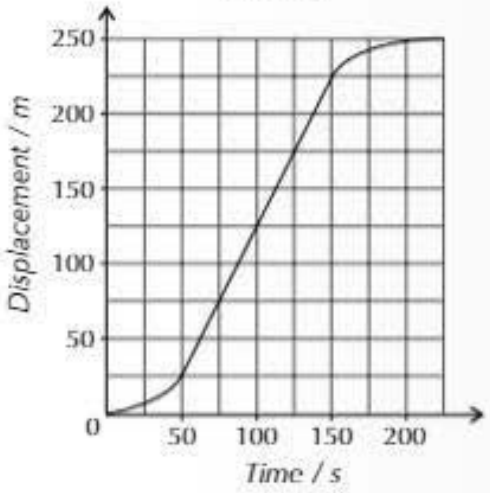
C 13.0 m

D 3.75 m

(1 mark)

5 A student wears a smart watch as she walks to the bus stop. Later, she uses the information to plot a displacement-time graph of her walk.
The graph is shown in **Fig. 5.1**.

Fig. 5.1



(a) Describe the student's motion through the course of her journey.

(2 marks)

(b) Calculate the student's average velocity for her entire journey.

(2 marks)

6 A student is investigating how the velocity of a trolley at the bottom of a ramp (***v***) changes when it is released from rest at different starting points on the ramp (***s***).
They use the set-up shown in **Fig. 6.1**. Their results are shown in **Fig. 6.2**.

Fig. 6.1

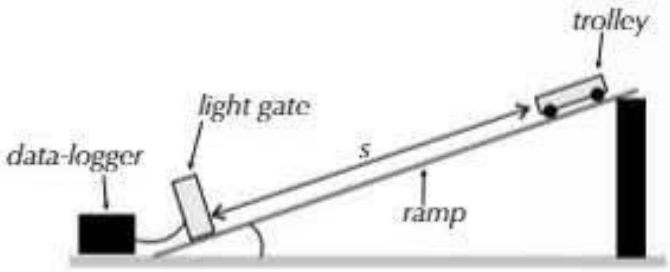


Fig. 6.2

distance travelled down the ramp / m	*final velocity / ms⁻¹*
0.20	1.22
0.40	1.73
0.60	2.12
0.80	2.45
1.00	2.74
1.20	3.00

(a) Plot a graph of final velocity squared against distance travelled down the ramp using the results in **Fig. 6.2**. Draw a line of best fit.

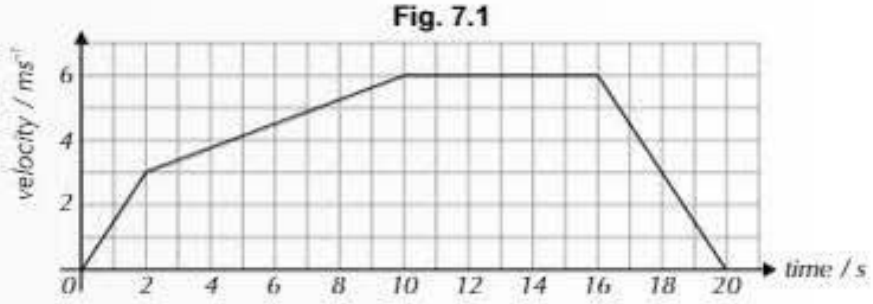
(2 marks)

(b) The student assumes that friction and air resistance on the trolley are negligible.
The acceleration is therefore constant acceleration due to gravity.
Use your graph to calculate the acceleration of the trolley down the ramp.

(3 marks)

7 **Fig. 7.1** shows the velocity-time graph for the motion of a cyclist during a 20 second time trial. The cyclist travels in a straight line during this time.

Fig. 7.1



(a) Find the cyclist's displacement at the end of the 20 seconds shown.

(2 marks)

(b) Find the acceleration of the cyclist during the last 4 s of the time trial.

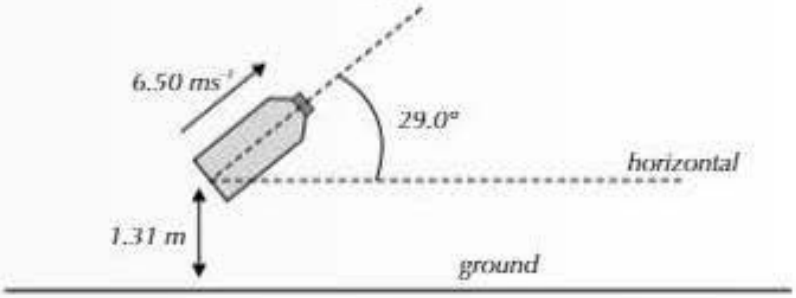
(2 marks)

(c) During a later bike ride, the cyclist allows herself to roll down a hill without pedalling. She starts at the top of the hill at rest. The road down the hill is 22.5 m long. The cyclist undergoes a constant acceleration of 3.40 ms^{-2} as she travels down the hill. Calculate how fast she will be going when she reaches the bottom of the hill. Assume air resistance and other resistive forces are negligible.

(2 marks)

The cyclist throws a water bottle with a velocity of 6.50 ms^{-1}, from a height of 1.31 m and at 29.0° above the horizontal, as shown in **Fig. 7.2**.

Fig. 7.2



(d) Calculate the vertical component of the velocity of the bottle when it hits the ground.

(2 marks)

(e) Calculate how long the bottle will take to hit the ground.

(2 marks)

Learning Objectives:

- Know what is meant by the weight of an object.
- Know that the newton (N) is the unit of force.
- Know and be able to use the equation $W = mg$.
- Understand what is meant by the centre of mass and centre of gravity of an object.
- Know how to experimentally determine the centre of gravity of an object.
- Know what is meant by the terms normal contact force, tension, friction and upthrust.
- Understand and be able to draw free-body diagrams.

Specification References 3.2.1 and 3.2.3

Tip: g is usually taken as $-9.81\ \text{Nkg}^{-1}$ because it acts downwards.

Figure 2: *The value of g decreases as you move away from a planet — which is important for satellite calculations.*

1. Mass, Weight and Force Basics

Before diving into how forces affect an object's motion, take some time to refresh your memory of the basics...

Mass and weight

The **mass** of an object is the amount of 'stuff' (or matter) in it. It's measured in kg. The greater an object's mass, the greater its resistance to a change in velocity (called its inertia). The mass of an object doesn't change if the strength of the gravitational field changes.

Weight is a force. It's measured in newtons (N), like all forces. Weight is the force experienced by a mass due to a gravitational field. The weight of an object does vary according to the size of the gravitational field acting on it.

Weight is given by the equation:

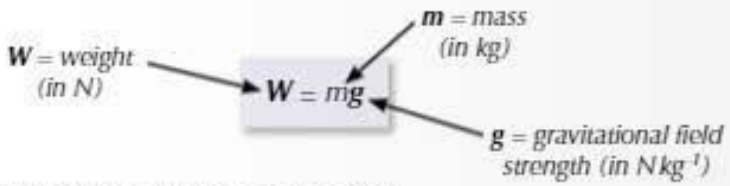


Figure 1 shows an example of how mass and weight vary for different values of g.

Name	Quantity	Earth (g = 9.81 Nkg^{-1})	Moon (g = 1.6 Nkg^{-1})
Mass	Mass (scalar)	150 kg	150 kg
Weight	Force (vector)	1471.5 N	240 N

Figure 1: *The mass and weight of an object on the Earth and on the Moon.*

Example **Maths Skills**

An astronaut has a mass of 85.0 kg. What would his weight be on Mars, where the value of g is 3.75 Nkg^{-1}?

$W = mg$

$= 85.0 \times 3.75 = 318.75 = 319$ N (to 3 s.f.)

Centre of mass

The **centre of mass** (or **centre of gravity**) of an object is the single point that you can consider its whole weight to act through (whatever its orientation). The object will always balance around this point, although in some cases the centre of mass falls outside the object. For a uniform regular solid, the centre of mass is at the centre of the object.

Figure 3: *The centres of mass of three objects.*

Finding the centre of gravity by experiment

You can find the centre of gravity of a flat object using a simple experiment:

- Hang the object freely from a point (e.g. one corner).
- Draw a vertical line downwards from the point of suspension — use a plumb bob to get your line exactly vertical.
- Hang the object from a different point, and draw another vertical line.
- The centre of gravity is where the two lines cross.

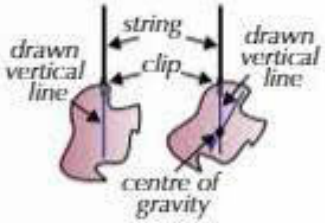


Figure 4: *The experimental set-up for finding the centre of gravity of a flat object.*

Tip: A plumb bob is just a piece of string or thread with a weight on the end of it, that hangs perfectly vertical.

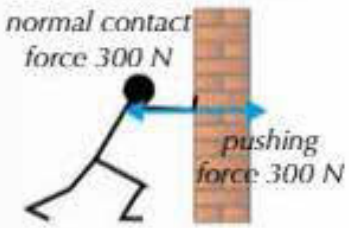
Types of forces

In addition to weight, you need to know some other common types of forces.

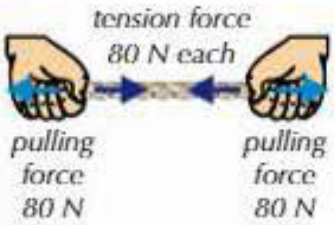
1. **Normal contact force** (or **reaction force**) — if an object exerts a force on a surface, the surface exerts an equal but opposite force on the object. The force acts normal (perpendicular) to the surface.
2. **Tension** — if a string is pulled tight, tension is the force pulling equally on the objects at either end of the string.
3. **Friction** — if an object is moving, it usually has a friction force acting on it in the opposite direction to motion.
4. **Upthrust** — if an object is submerged (or partially submerged) in a fluid, it experiences an upwards force called upthrust (see page 95).

Examples

- Normal contact force:
  The man pushes on the wall with a force of 300 N. The wall 'pushes back' with a normal contact force of 300 N.

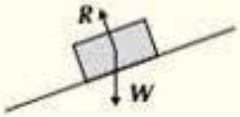


- Tension:
  A pulling force of 80 N is applied to each side of the rope. A tension force acts at each end of the rope to balance these forces.



- Friction:
  The car is moving at a constant speed. The 900 N driving force is balanced by a friction force of 900 N.

Tip: The normal contact force always acts at right angles to the surface. For example, an object on a slope will have a normal contact force (***R***) acting on it perpendicular to the slope. Its magnitude is equal to the component of its weight (***W***) that is perpendicular to the surface.



- Upthrust:
  The beach ball is floating in a pool of water. The weight of the beach ball acts downwards. It is balanced by a force of upthrust from the water.

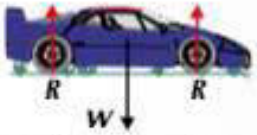
Free-body diagrams

A **free-body diagram** shows a single body on its own and the forces acting on it. The diagram should include all the forces that act on the body, but not the forces it exerts on the rest of the world.

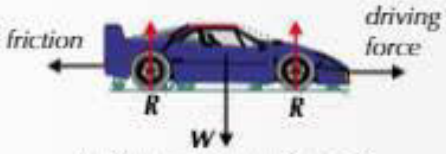
Remember, forces are vector quantities (p.41), so the arrows should show the direction of the forces, and be labelled with the size of the force.

Gravity pulls the satellite downwards towards Earth.

Gravity pulls man down, air resistance pushes man up.



Gravity pulls car down. Earth pushes car up (normal contact force, R).



Engine pushes car forward. Friction pulls car backwards. Gravity pulls car down. Earth pushes car up.

Figure 5: *Some free-body diagrams.*

Tip: All the forces in the diagrams in Figure 5 are coplanar — they're all in the same plane. You'll only need to deal with coplanar forces.

Practice Questions — Application

Q1 An object has a weight of 7.75 N on Pluto, where $g = 0.620\ \text{N kg}^{-1}$. Calculate the mass of the object.

Q2 A coconut with a weight of 11.4 N is falling from a tree. It experiences a 4.65 N force of air resistance. Draw a free-body diagram for the coconut.

Practice Questions — Fact Recall

Q1 Give the equation for weight.

Q2 What is the centre of mass of an object?

Q3 Where is the centre of gravity of a uniform, regular solid?

Q4 Give the name of the upwards force experienced by an object which is submerged (or partially submerged) in a fluid.

Q5 What is the name given to a diagram showing all of the forces acting on an object, but none of the forces it exerts on its surroundings?

2. Net Forces

If you have lots of forces acting on an object, it can be tricky to see what effect they will have on the object's motion, especially if the forces are acting at awkward angles to each other. Step forward net forces...

Learning Objectives:

- Know and be able to use the equation $F = ma$ (net force = mass × acceleration).
- Be able to solve problems involving one- and two-dimensional motion under a constant force.

Specification Reference 3.2.1

Net force and acceleration

The **net force** (or **resultant force**) on an object is the sum of all of the forces acting on the object, accounting for their relative directions. An object can only accelerate (change speed, change direction, or both — see page 48) if a non-zero net force is acting on it. In other words, a net force is needed for an object to accelerate.

The acceleration of an object is proportional to the net force acting on it. This can be written as the well-known equation:

The acceleration given by this equation is always in the same direction as the net force used to calculate it.

Tip: From this equation, 1 N = 1 kg ms^{-2}. This is the definition of a newton. $F = ma$ is a special case of Newton's Second Law (see page 135).

Example — **Maths Skills**

A 14 kg box is on the floor. A man pushes the box so that a resultant force of 1.2 N is acting on it. Calculate the initial acceleration of the box.

Rearrange $F = ma$ for acceleration:

$a = F \div m = 1.2 \div 14 = 0.08571... = 0.086$ ms^{-2} (to 2 s.f.)

Why do all objects fall at the same rate?

Consider two balls dropped at the same time, ball 1 being heavy and ball 2 being light. Then use the equation above to find their acceleration:

Tip: This only works if you ignore air resistance.

Example — **Maths Skills**

mass = m_1
resultant force = F_1
acceleration = a_1

By Newton's second law:

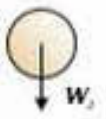
$$F_1 = m_1 a_1$$

Ignoring air resistance, the only force acting on the ball is weight, given by $W_1 = m_1 g$ (where g = gravitational field strength = 9.81 Nkg^{-1}).

So: $F_1 = m_1 a_1 = W_1 = m_1 g$

So: $m_1 a_1 = m_1 g$, then m_1 cancels out to give $a_1 = g$

mass = m_2
resultant force = F_2
acceleration = a_2



By Newton's second law:

$$F_2 = m_2 a_2$$

Ignoring air resistance, the only force acting on the ball is weight, given by $W_2 = m_2 g$ (where g = gravitational field strength = 9.81 Nkg^{-1}).

So: $F_2 = m_2 a_2 = W_2 = m_2 g$

So: $m_2 a_2 = m_2 g$, then m_2 cancels out to give $a_2 = g$

In other words, the acceleration is independent of the mass. It makes no difference whether the ball is heavy or light.

Figure 1: *In a vacuum with no air resistance, a feather will fall at the same rate as a heavy ball.*

Finding the net force

For one-dimensional problems, where all the forces are acting along the same line, you can find the net force by simply adding the forces together.

Tip: If the net force acting on an object acts along the line of motion of the object, the object will keep moving in the same direction and will either speed up or slow down (see p.135).

Example — **Maths Skills**

Figure 2 shows the forces acting on a firework rocket shortly after it is fired into the air. Calculate the net force acting on the firework.

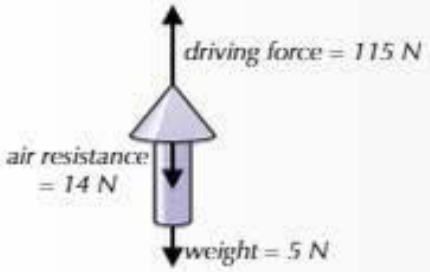


***Figure 2**: The forces acting on a firework rocket.*

Let up be positive, and down be negative.
Add all the forces together to find the net force.

Net force = 115 N + (–14 N) + (–5 N)
= 115 N – 19 N
= 96 N

So the net force is 96 N upwards.

For two-dimensional problems, with two forces acting at an angle to each other, you can find the net force by drawing a vector triangle. Draw the forces as vector arrows tip-to-tail, then draw in the third side to represent the net force (as shown on page 41).

You can then use trigonometry, such as Pythagoras' theorem (p.252) or the sine and cosine rules (p.488), to calculate the net force.

***Figure 3**: A block accelerating down a slope. It experiences a force of weight directly downwards, a normal contact force perpendicular to the slope, and a force of friction directed up the slope. This results in an overall net force down the slope.*

Example — **Maths Skills**

Two children are making a snowman. They roll a snowball along the ground. Child A applies a force of 5.0 N northwards, child B exerts a force of 2.0 N eastwards. You can assume no friction acts on the snowball. Find the net force on the snowball.

Draw the forces as vector arrows, touching tip-to-tail, then complete the triangle by drawing a net force arrow, $\boldsymbol{R}$.

By Pythagoras' theorem:

$\boldsymbol{R}^2 = 5.0^2 + 2.0^2$,

so $\boldsymbol{R} = \sqrt{29} = 5.4$ N (to 2 s.f.)

Use the tangent function to find the angle θ.

$\tan\theta = \frac{2.0}{5.0}$

so $\theta = \tan^{-1} 0.4$

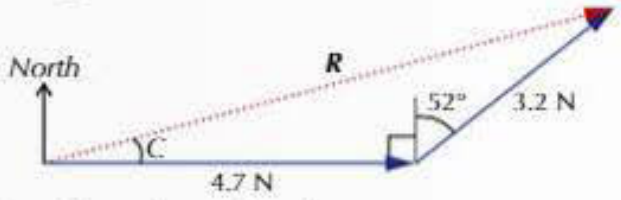
$= 22°$ (to 2 s.f.)

So the net force is 5.4 N at an angle of 22° from north.

Example — **Maths Skills**

Child A and child B are rolling a second snowball along the ground. Child A applies a force of 4.7 N eastwards, and child B exerts a force of 3.2 N at a bearing of 052°. You can assume no friction acts on the snowball. Find the net force on the snowball.

Draw the forces as vector arrows, touching tip-to-tail, then complete the triangle by drawing a net force arrow, ***R***.



To find the size of ***R***, use the cosine rule:

$a^2 = b^2 + c^2 - 2bc\cos(A)$

where $a = \boldsymbol{R}$, $b = 4.7$, $c = 3.2$, and $A = 52 + 90 = 142°$. So:

$\boldsymbol{R} = \sqrt{4.7^2 + 3.2^2 - (2 \times 4.7 \times 3.2 \times \cos(142°))} = 7.48554... = 7.5$ N (to 2 s.f.)

To find angle C, use the sine rule:

$$\frac{c}{\sin(C)} = \frac{a}{\sin(A)}$$

$$C = \sin^{-1}\left(\frac{\sin(142°)}{7.48554...} \times 3.2\right) = 15.25...°$$

To find the angle of ***R*** from north, subtract C from 90°.

Angle of ***R*** from north $= 90 - 15.25...$

$= 74.74... = 75°$ (to 2 s.f.)

So, the net force is 7.5 N at an angle of 75° from north.

Tip: You need to use the sine and cosine rules when the forces aren't perpendicular.

Tip: Remember, a bearing is just the number of degrees clockwise from north.

Tip: When using the sine and cosine rules, you need to make sure the sides and angles of your triangle are labelled correctly. See page 488 for a full explanation of how to do this.

Tip: You could also work this out by drawing a really accurate scale drawing (see p.41).

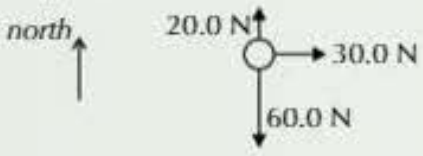
Practice Questions — Application

Q1 Calculate the net force acting on a 24.1 kg object that has an acceleration of 3.5 ms^{-2}.

Q2 A net force of 18 N is acting on a toy car. It has an acceleration of 29 ms^{-2}. What is the mass of the toy car?

Q3 Below is the free-body diagram of a cyclist cycling along a road while experiencing a cross-wind.

a) Calculate the net force on the cyclist. Give the direction as a clockwise angle from east.



b) The cyclist and bicycle have a total mass of 89.0 kg. Calculate the magnitude of their acceleration.

Practice Question — Fact Recall

Q1 What is meant by the net force acting on an object?

Q2 Define the variables and their units in the equation $\boldsymbol{F} = m\boldsymbol{a}$.

Learning Objectives:

- Understand how an object can be in equilibrium while under the action of forces.
- Know the conditions for equilibrium of three coplanar forces.
- Be able to use triangles of forces to solve problems involving equilibrium.

Specification Reference 3.2.3

Tip: An object in equilibrium (i.e. not accelerating) is either at rest or moving with a constant velocity (p.135).

Tip: You could do this for any number of coplanar forces. As long as the vector arrows, drawn to scale and tip-to-tail, form a closed shape, the object is in equilibrium.

Tip: Remember, the magnitude of a vector is its size (without the direction), see page 41.

Figure 3: *These rocks are in a state of equilibrium — all the forces on them must cancel out for them to be balanced. This is true for all balanced objects.*

3. Equilibrium

Time to find out how you can solve force problems for objects that are in equilibrium. Have a quick flick back to pages 41-45 — the vectors stuff will be pretty handy here.

Forces in equilibrium

If an object is in **equilibrium**, all the forces acting on it are balanced and cancel each other out. In other words, there's no net force on an object in equilibrium, so it isn't accelerating.

When only two forces act on an object, the object is in equilibrium if they're equal and opposite. If there are three forces acting on an object, there are two ways you can go about solving equilibrium problems:

Triangle of forces

Forces acting on an object in equilibrium form a closed loop when you draw them to scale and tip-to-tail. For the case of three coplanar forces, this will form a triangle of forces.

Figure 1: *Three balanced force vectors shown acting from a point and in a triangle of forces.*

Example **Maths Skills**

Figure 2 shows all the forces acting on a particle. Given that the particle is in equilibrium, find the magnitude of the missing force *P*.

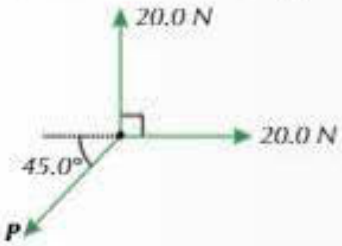
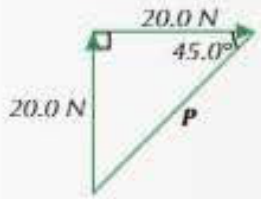


Figure 2: *Three balanced forces acting on a particle.*

The particle is in equilibrium, so the forces will form a triangle of forces when drawn to scale and tip-to-tail.



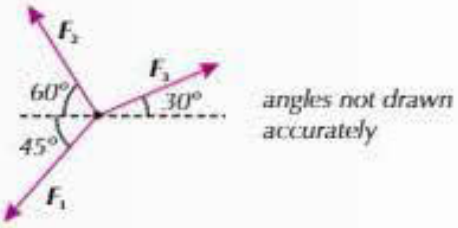
This means you can use trigonometry to find the magnitude of the force, *P*.

$$\cos 45.0° = \frac{20.0}{P}$$

$$P = \frac{20.0}{\cos 45.0°} = 28.2842... = 28.3 \text{ N (to 3 s.f.)}$$

Resolving forces in two perpendicular directions

You saw how to resolve vectors into perpendicular components on page 43, and it's no different for the forces acting on objects in equilibrium. If an object is in equilibrium, the sum of the components in each direction must be equal to zero. To find the components of each force in a direction, just use trigonometry:



↑ Vertical components	→ Horizontal components
$-\sin 45° \times F_1$	$-\cos 45° \times F_1$
$\sin 60° \times F_2$	$-\cos 60° \times F_2$
$\sin 30° \times F_3$	$\cos 30° \times F_3$

Figure 4: *Three forces acting at a point, and their horizontal and vertical components.*

Tip: When you're resolving forces, remember that the force being resolved is always the hypotenuse.

Tip: Remember, the direction matters with forces, so forces in the negative direction have a negative value. In this case, the negative directions are down and to the left.

Example — Maths Skills

Figure 5 shows all the forces on a particle in equilibrium. Find the magnitude of the missing force Q.

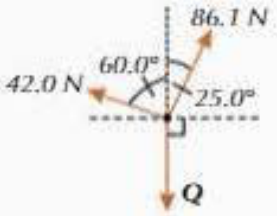


Figure 5: *The forces acting on a particle in equilibrium.*

The missing force Q acts vertically downwards. So for the particle to be in equilibrium, the magnitude of the force, Q, must equal the sum of the vertical components of the other two forces.

Vertical component of 42.0 N force $= 42.0 \cos 60.0°$

Vertical component of 86.1 N force $= 86.1 \cos 25.0°$

So, $Q = 42.0 \cos 60° + 86.1 \cos 25.0°$
$= 21.0 + 78.033...$
$= 99.033...$
$= 99.0$ N (to 3 s.f.)

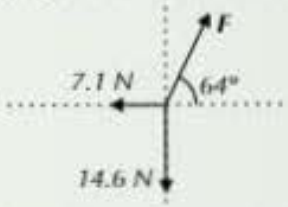
Tip: The horizontal component of a vertical vector is zero, and vice versa. Since you're told the object is in equilibrium, you don't need to worry about the horizontal components.

Tip: You could also use the cosine rule to answer this question (see page 488).

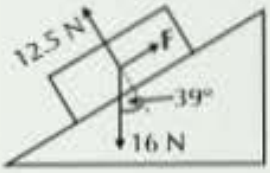
Practice Questions — Application

Q1 Show with a triangle of forces that an object is in equilibrium if only these forces are acting on it: 12 N acting east, 5 N north, 13 N at 23° anticlockwise from west.

Q2 The three forces shown below are in equilibrium.
Find the magnitude of the unknown force ***F***.



Q3 The diagram below shows the forces acting on a box on a slope.
The box is in equilibrium. Calculate the size of force ***F***.



Practice Questions — Fact Recall

Q1 What is the size of the net force acting on an object in equilibrium?

Q2 What is meant by 'resolving a force'?

4. Moments and Torques

A moment is the turning effect of a force around a point. Two moments of equal size acting parallel to each other are called a couple.

Moments and turning effects

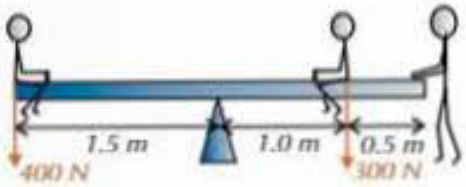
A **moment** is the turning effect of a force around a turning point. The moment of a force depends on the size of the force and how far the force is applied from the turning point (or **pivot**). It is defined as:

M = moment of a force (in Nm) → $M = Fx$ ← F = size of force (in N), x = perpendicular distance between the line of action of the force and the turning point (in m)

The **principle of moments** states that for a body to be in equilibrium, the sum of the clockwise moments about any point equals the sum of the anticlockwise moments about the same point. If the moments aren't balanced, the object will turn.

Example — **Maths Skills**

Two children sit on a seesaw as shown on the right. An adult balances the seesaw at one end. Find the size and direction of the force, F, that the adult needs to apply to do this.



Take moments about the turning point. In equilibrium,

$$\sum \text{anticlockwise moments} = \sum \text{clockwise moments}$$

$$400 \times 1.5 = (300 \times 1.0) + 1.5F$$

$$600 = 300 + 1.5F$$

$$F = \frac{600 - 300}{1.5} = 200 \text{ N downwards}$$

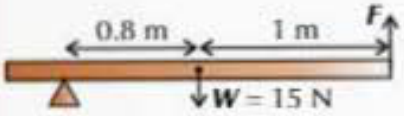
We know the force acts downwards because that's the direction required to produce a clockwise moment.

Centre of mass and moments

When calculating moments, you may also need to consider any turning effects due to the weight of the object that's turning. The line of action of the weight is drawn from the object's centre of mass (see page 76). If the centre of mass is directly over the pivot, there will be no resultant moment in either direction, so you can ignore the weight of the object in your moment calculations. However, if this isn't the case, then you do need to consider the weight.

Example — **Maths Skills**

A regular, symmetrical plank of wood with a weight of 15 N is in equilibrium, as shown below. Calculate the magnitude of the force, F.



$$\sum \text{anticlockwise moments} = \sum \text{clockwise moments}$$

$$15 \times 0.8 = F \times 1.8$$

$$F = \frac{15 \times 0.8}{1.8} = 6.66... = 7 \text{ N (to 1 s.f.)}$$

Learning Objectives:

- Be able to calculate the moment of a force about a point.
- Know the principle of moments and use it to solve problems involving moments.
- Know what is meant by a couple.
- Be able to calculate the torque of a couple.
- Understand how an object can be in equilibrium while under the action of torques.

Specification Reference 3.2.3

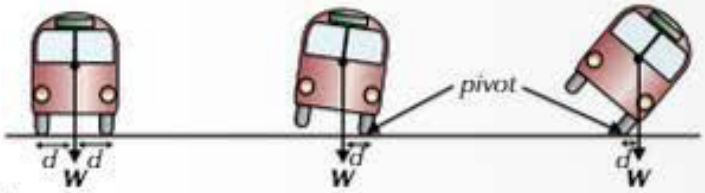
Tip: The perpendicular distance means the distance along a line that makes a right angle with the line of action of the force (i.e. the shortest possible distance between the pivot and the line in which the force acts).

Tip: The symbol Σ means "the sum of".

Tip: Remember, for regular, symmetrical objects, the centre of mass is at the centre of the object. So in the first example on this page it can be assumed that the centre of mass of the seesaw is above the pivot, and so the moment of its weight is zero.

An object will topple over if the line of action of its weight falls outside its base area. This is because a resultant moment occurs, whose turning force causes the object to fall.

Figure 2: *Cranes need a wide, heavy base to stop them toppling.*



No resultant moment. Bus is stable and stationary.

Net anticlockwise moment about pivot. Bus will rotate to the left and sit back upright.

Net clockwise moment about pivot. Bus will rotate to the right and fall over.

Figure 1: *Moments acting on a bus at different angles of tilt.*

The higher the centre of mass, and the smaller the base area, the less stable the object will be. An object will be very stable if it has a low centre of mass and a wide base area.

This is used a lot in design — for example, racing cars have a wide base area and a low centre of mass to prevent them from toppling over on fast corners.

Moments in levers

In a **lever**, an effort force acts against a load force by means of a rigid object rotating around a pivot. Levers are really useful in situations where you need a larger turning effect. Examples include spanners, wheelbarrows and scissors. They increase the distance from the pivot a force is applied, so you need less force to get the same moment. You can use the principle of moments to answer lever questions.

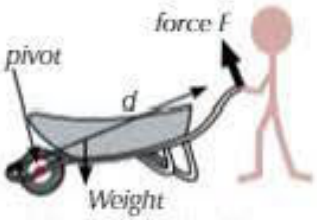


Figure 3: *A wheelbarrow is a type of lever used to reduce the force needed to lift a heavy load.*

Figure 5: *Spanners are a type of lever. They have long handles for larger turning effects.*

Example — **Maths Skills**

Find the size of the force exerted by the biceps in holding a bag of gold still. The bag of gold weighs 100 N and the forearm weighs 20 N.

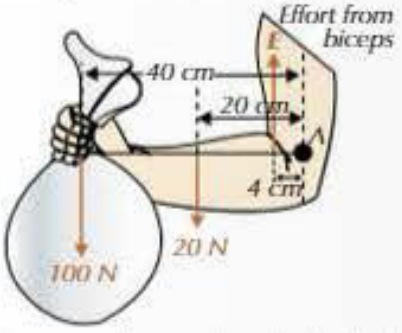


Figure 4: *A biceps muscle providing the clockwise moment needed to balance the anticlockwise moments.*

Take moments about *A*. In equilibrium:

$$\sum \text{anticlockwise moments} = \sum \text{clockwise moments}$$

$$(100 \times 0.4) + (20 \times 0.2) = 0.04F$$

$$40 + 4 = 0.04F$$

$$F = \frac{40 + 4}{0.04} = 1100 = 1000 \text{ N (to 1 s.f.)}$$

Forces on supports

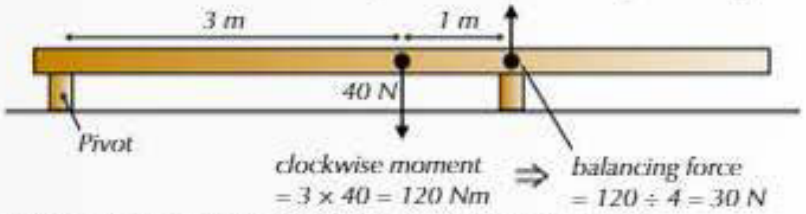
If an object is being held up by supports (e.g. chair legs, car tyres, etc.), the force acting on each support won't always be the same. The closer the object's centre of mass is to a support, the stronger the force on the support. It's all to do with the principle of moments — the anticlockwise and clockwise moments must be equal. So a support closer to the centre of mass will experience a larger force.

Exam Tip
Make sure you understand why different amounts of force can act on different supports, as you might be asked about it in the exam.

Example **Maths Skills**

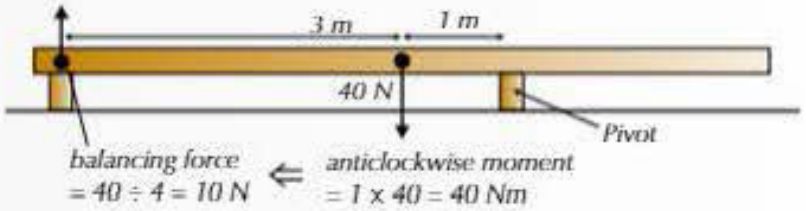
A plank with a weight of 40 N is resting on two supports 3 m and 1 m from the plank's centre of mass. Find the upwards force provided by each support.

Start by treating one of the supports as the pivot and finding how much force is needed to balance the moments provided by the weight of the plank:



Tip: The balancing force is found by rearranging $M = Fx$ to $F = M \div x$.

Then do the same, treating the other support as the pivot:



Tip: This is the force the supports experience too. It's Newton's third law — see p.136 for more on this.

So the support furthest from the centre of mass provides 10 N of force, while the support closest to the centre of mass provides 30 N of force.

Couples and torque

A **couple** is a pair of forces of equal size which act parallel to each other, but in opposite directions. A couple doesn't cause any resultant linear force, but does produce a turning effect. This type of turning effect is called a **torque**. The size of this torque depends on the size of the forces and the distance between them.

Tip: Couples are coplanar — they act in the same plane.

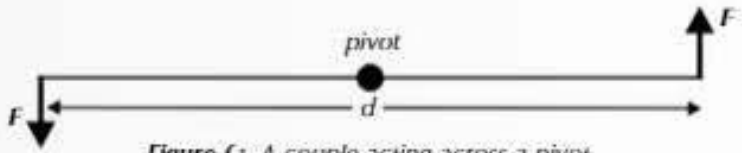


Figure 6: *A couple acting across a pivot.*

You can derive a formula for torque from the equation for moments, $M = Fx$ (p.85). In Figure 6, you can see that each force of the couple is providing a moment in the same direction, equal to:

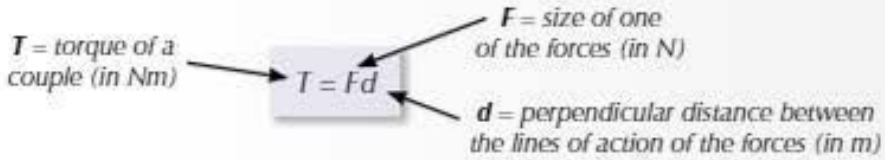
$$M = F \times \frac{d}{2}$$

Torque, T, is the sum of the moments provided by the couple, so:

$$T = (F \times \frac{d}{2}) + (F \times \frac{d}{2}) = Fd$$

Tip: Remember, the two forces in a couple have the same magnitude as each other.

This gives the equation:


T = torque of a couple (in Nm)
T = Fd
F = size of one of the forces (in N)
d = perpendicular distance between the lines of action of the forces (in m)

Tip: Remember... for couples you need to use the perpendicular distance between the forces, not the distance between the one force and the pivot.

Example **Maths Skills**

A cyclist turns a sharp right corner by applying equal but opposite forces of 25 N to the ends of the handlebars. The length of the handlebars is 0.60 m. Calculate the torque applied to the handlebars.

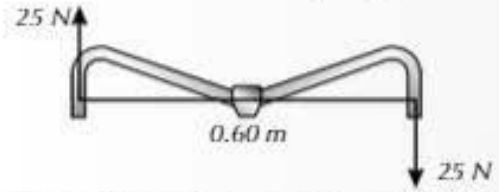


Figure 7: *Turning force being applied across bicycle handlebars.*

$T = F \times d$

$= 25 \times 0.60 = 15$ Nm

Like with the principle of moments, for an object to be in equilibrium, the clockwise torques have to equal the anticlockwise torques.

Example **Maths Skills**

A torque of 120 Nm is applied to turn a revolving door. Two people apply a couple to the door to stop it moving. They are each stood 1.2 m from the pivot. Calculate the size of the force, *F*, applied by each person.

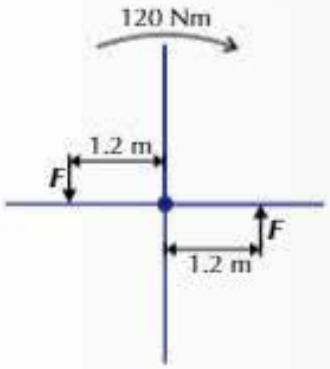


Figure 8: *An oil worker applying a torque to a tap on an oil pipeline.*

In order for the door to not be moving, it must be in equilibrium.

So the anticlockwise torque supplied by the people must equal the clockwise torque. So, anticlockwise torque = 120 Nm.

The distance between the two forces of the couple = 1.2 + 1.2 = 2.4 m

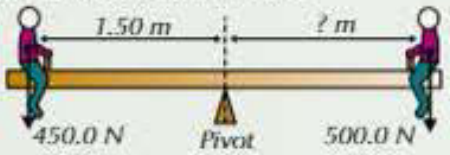
Rearrange the torque equation for force:

$$F = \frac{T}{d} = \frac{120}{2.4} = 50 \text{ N}$$

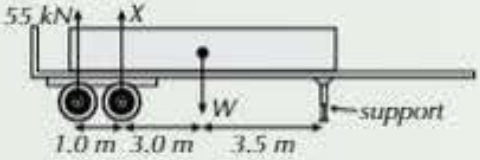
Practice Questions — Application

Q1 Find the moment provided by a force of 73.1 N acting at a perpendicular distance of 0.25 m from a pivot.

Q2 The diagram below shows two children on a seesaw. If the child on the left stays where they are, how far from the pivot should the child on the right sit to balance the seesaw?



Q3 The diagram below shows a loaded trailer of mass 24 000 kg.



a) Calculate the moment of the trailer's weight about the support.

b) Calculate the upward force X acting on the right-hand wheel.

Q4 An airtight chamber is kept shut by a release wheel with a diameter of 0.35 m. The release wheel won't open unless a torque of at least 50.0 Nm is applied across it. What's the minimum force that, when applied to both sides of the wheel, will open the chamber door?

Tip: Remember, for an object to be balanced, the total clockwise moments on the object must be equal to the total anticlockwise moments on the object.

Practice Questions — Fact Recall

Q1 What is a moment?

Q2 What name is given to a pair of forces of equal size which act parallel to each other, in opposite directions?

Learning Objectives:

- Know what is meant by friction.
- Know that drag is the frictional force experienced by an object travelling through a fluid.
- Know the factors that affect the drag acting on an object travelling through air.
- Know what is meant by terminal velocity.
- Understand the motion of an object falling in a uniform gravitational field in the presence of drag.
- Know and be able to carry out techniques and procedures to determine terminal velocity in fluids (PAG1).

Specification References 3.2.1 and 3.2.2

5. Drag and Terminal Velocity

When an object moves, it feels a resistive force called friction (or drag for a fluid) that opposes its motion. When the friction forces equal the driving forces, the object stops accelerating and reaches its terminal velocity.

Friction

Friction is a force that opposes motion. There are two main types of friction — contact friction and fluid friction.

Contact friction happens between solid surfaces (which is what we usually mean when we just use the word 'friction').

'Fluid' is a word that means either a liquid or a gas — something that can flow. Fluid friction is known as **drag**, or fluid resistance or air resistance. Three things affect fluid friction:

- The force depends on the **viscosity** (thickness) of the fluid.
- The force increases as the speed increases. For simple situations it's directly proportional (see page 13), but you don't need to worry about the mathematical relationship.
- The force depends on the size and shape of the object moving through it. The larger the area pushing against the fluid, the greater the resistance force.

There are three things you need to remember about frictional forces:

- They always act in the opposite direction to the motion of the object.
- They can never speed things up or start something moving.
- They convert kinetic energy into heat.

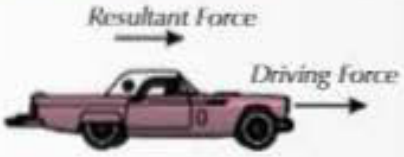
Terminal velocity

Terminal velocity happens when frictional forces equal the driving force. An object will reach a terminal velocity at some point if there's a driving force that stays the same all the time, and a frictional or drag force (or collection of forces) that increases with speed.

There are three main stages to reaching terminal velocity:

Example

The car accelerates from rest using a constant driving force.



As the speed increases, the resistance forces increase (because of things like turbulence). This reduces the resultant force on the car and hence reduces its acceleration.

Eventually the car reaches a speed at which the resistance forces are equal to the driving force. There is now no resultant force and no acceleration, so the car carries on at a constant speed.

Exam Tip

You don't need to know about turbulence or why resistive forces increase in the exam.

Motion graphs for terminal velocity

You need to be able to recognise and sketch the graphs for velocity against time and acceleration against time for the terminal velocity situation.

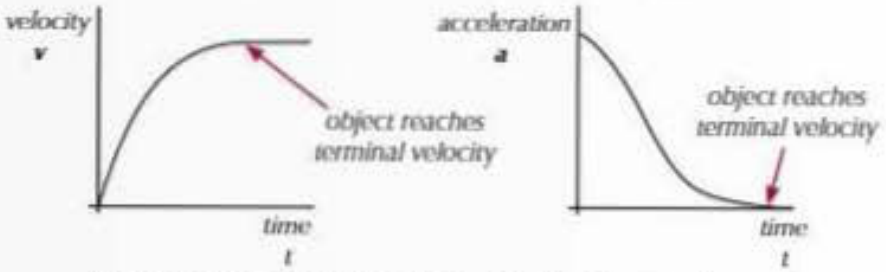


Figure 1: *The velocity-time and acceleration-time graphs for an object reaching terminal velocity.*

Exam Tip
These graphs might come up in the exam, so make sure you get lots of practice with them — try shutting the book and sketching them from memory.

Terminal velocity of a parachutist

When something starts falling through air in a uniform gravitational field, its weight causes it to accelerate. The weight of the object is a constant force acting downwards. Air resistance is a frictional force opposing this motion, which increases with speed. So before a parachutist opens the parachute, exactly the same thing happens as with the car:

Example

A skydiver leaves a plane and will accelerate until the air resistance equals his weight.

He will then be travelling at a terminal velocity. But the terminal velocity of a person in free fall is too great to land safely — so he needs to increase the upwards force of air resistance, to slow him down to a lower speed.

Before reaching the ground he will open his parachute, which immediately increases the air resistance so it is now bigger than his weight.

This slows him down until his speed has dropped enough for the air resistance to be equal to his weight again. This new terminal velocity is small enough to survive landing.

Figure 2: *A parachute increases the drag acting on a skydiver, so they hit the ground at a slower speed.*

Velocity-time graph for a parachutist

The velocity-time graph for this situation is a bit different, because you have a new terminal velocity being reached after the parachute is opened:

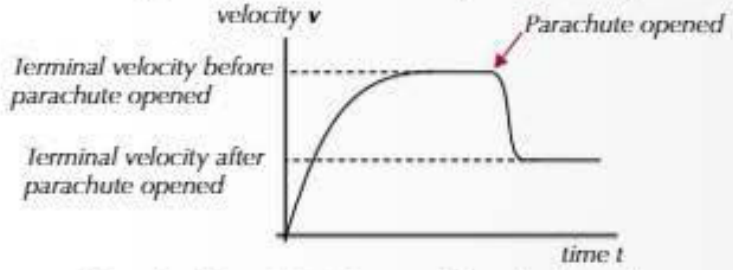


Figure 3: *The velocity-time graph for a skydiver who releases his parachute after reaching terminal velocity.*

Tip: If you need a reminder about velocity-time graphs, go to pages 60-65.

Investigating terminal velocity

PRACTICAL ACTIVITY GROUP 1

Things falling through any fluid will reach a terminal velocity (if it falls for long enough). You can calculate the terminal velocity of a ball bearing in a viscous (thick) liquid (e.g. wallpaper paste) by setting up an experiment like this:

- Put elastic bands around the tube of viscous liquid at fixed distances from the top of the tube (measured with a ruler). Then drop a ball bearing into the tube, and use a stopwatch to record the time at which it reaches each band.
- Repeat this a few times to reduce the effect of random errors on your results, using a strong magnet to remove the ball bearing from the tube.
- Calculate the times taken by the ball bearing to travel between consecutive elastic bands and calculate an average for each reading. Use the average times and the distance between bands to calculate the average velocity between each pair of elastic bands.
- You should find that the average velocity increases at first, then stays constant — this is the ball bearing's terminal velocity in the viscous liquid used.
- Use your average velocity data to plot a graph of velocity against time. Draw a smooth curve and use it to estimate the terminal velocity.

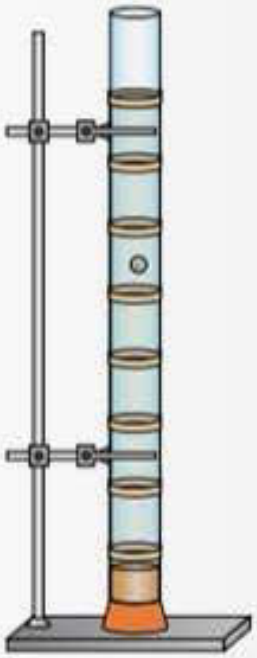


Figure 4: *A possible set-up for an investigation into terminal speed in a viscous liquid.*

Tip: If you do this experiment, be sure to think about how you'd do it safely — do a full risk assessment before you start, and be extra careful not to spill slippery liquids on the floor.

Tip: You might be asked to draw force diagrams as the ball bearing falls. Remember that the forces are balanced when the ball reaches terminal velocity.

Figure 5: *Crude oil being poured into a beaker. Crude oil has a higher viscosity than water, so an object falling through it would reach a lower terminal velocity than if it was falling through water.*

Factors that affect terminal velocity

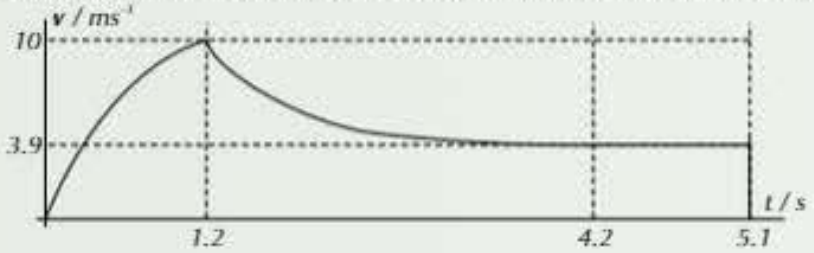
You can change parts of your experiment to see what effect they have on terminal velocity and the time taken to reach terminal velocity.
For example you could:

- Change the liquid — the terminal velocity will be lower in more viscous (thicker) liquids because the drag is greater. Try mixing water into wallpaper paste and see by how much the terminal velocity increases when the drag is lower.
- Change the size of the ball. What happens if the ball is larger? Or smaller?

- Change the shape of the thing you are dropping.
  The drag force will be greater on less streamlined shapes.
- Change the mass of the thing you are dropping, while keeping the size the same (this might be a bit tricky). You should find that heavier objects reach a faster terminal velocity because a greater drag force is needed to balance the extra weight. (Remember, objects with different masses only fall at the same rate if drag is ignored — p.79.)

Practice Questions — Application

Q1 A ball is dropped above a cylinder of water. It falls through the air and lands in the water after 1.2 seconds. It then reaches a terminal velocity after 4.2 seconds. Its motion is shown by the graph below.



a) Describe the ball's acceleration from when it's dropped to when it reaches the bottom of the cylinder (at 5.1 seconds).

b) Sketch a velocity-time graph for the same ball if it were dropped straight into the water (without falling through air first). Include the ball's terminal velocity in your sketch.

Q2 Explain how air resistance affects the maximum speed of a car.

Tip: Falling into water has a similar effect on the ball's velocity to opening a parachute.

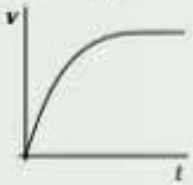
Practice Questions — Fact Recall

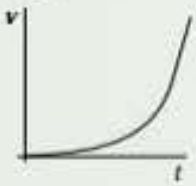
Q1 What is meant by friction and drag?

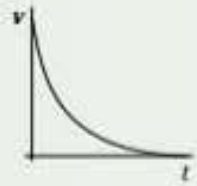
Q2 In what direction does a frictional force act?

Q3 What can you say about the frictional forces and the driving forces acting on an object when it reaches terminal velocity?

Q4 Which of these graphs shows the velocity-time graph for an object falling through air and reaching a terminal velocity?

a) 

b) 

c) 

Q5 How does a skydiver reduce his or her terminal velocity?

Q6 Sketch and label the equipment a student could use to investigate the terminal velocity of a ball bearing in a viscous fluid.

Learning Objectives:

- Know what is meant by the density of a material.
- Be able to use the formula $\rho = \frac{m}{V}$.
- Know what is meant by pressure and be able to use the equation $p = \frac{F}{A}$ for solids, liquids and gases.
- Be able to use the equation $p = h\rho g$ to calculate pressure.
- Understand what is meant by the upthrust on an object in a fluid.
- Know and understand Archimedes' principle.

Specification Reference 3.2.4

6. Density, Pressure and Upthrust

Density is a property that all materials have, and different materials have different densities. The greater the density of a fluid, the greater the pressure an object will experience when it is submerged within it.

What is density?

Density is a measure of the 'compactness' of a substance. It relates the mass of a substance to how much space it takes up. The density of a material is its mass per unit volume:

ρ = density in kg m^{-3} → $\rho = \frac{m}{V}$ ← m = mass in kg, V = volume in m^3

If you're given mass in g and volume in cm^3, you can work out the density of an object in g cm^{-3} (1 g cm^{-3} = 1000 kg m^{-3}). The density of an object depends on what it's made of. The density of a material doesn't vary with size or shape.

Example — **Maths Skills**

Aluminium has a density of 2.7 g cm^{-3}.
Calculate the volume of a 460 g solid object made out of aluminium.

Rearrange the formula $\rho = \frac{m}{V}$ to get $V = \frac{m}{\rho}$.

Substitute m = 460 g and ρ = 2.7 g cm^{-3} into the rearranged equation to find the volume of the object.

$$V = \frac{460}{2.7} = 170.37... = 170 \text{ cm}^3 \text{ (to 2 s.f.)}$$

What is pressure?

Pressure is the amount of force applied per unit area. It is measured in pascals (Pa), which are equivalent to newtons per square metre (N m^{-2}). You can calculate the pressure over an area for a solid, liquid or gas using:

p = pressure (in Pa) → $p = \frac{F}{A}$ ← F = force (in N), A = area (in m^2)

Example — **Maths Skills**

A triangular-based pyramid stands on a table. It has a mass of 2.5 kg, and its base has an area of 62 cm^2. Calculate the pressure (in pascals) exerted on the table by the pyramid.

Pressure is give by: $p = \frac{F}{A}$

First, calculate the force exerted by the pyramid.

The force from the pyramid is equal to its weight. So:

$F = mg = 2.5 \times 9.81 = 24.525$ N

Convert its base area to m^2: $A = 62 \div 10\,000 = 6.2 \times 10^{-3}$ m^2

So, $p = \frac{24.525}{6.2 \times 10^{-3}} = 3955.64...$ Pa = 4000 Pa (to 2 s.f.)

Tip: Make sure you know how to find the area of some basic shapes (e.g. rectangles, triangles, circles, etc). You may need to calculate the area of a surface to calculate the pressure.

Fluid pressure

When an object is inside a fluid, particles of the fluid collide with the object. These collisions exert a force on the surface of the object, and hence a pressure is exerted on the object by the fluid. This pressure depends on the density of the fluid, and how deep the object is within the fluid.

A denser fluid will have more particles in a given volume. This means there are more particles that are able to collide — which means more collisions and a larger total force exerted. So the pressure exerted by the fluid is higher.

As depth increases, the number of particles above that point increases. The weight of these particles adds to the pressure experienced at that point, so fluid pressure increases with depth.

The pressure acting on an object due to a fluid can be calculated from the depth of the object in the fluid (h), the density of the fluid (ρ) and the gravitational field strength (g).

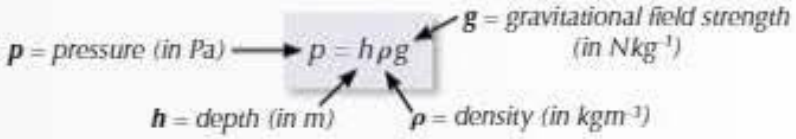


Figure 1: *Water emerges out of each spout at different speeds. The deeper the spout, the faster the water spurts out, due to the greater fluid pressure.*

Example — **Maths Skills**

An ice cube is partially submerged in a glass of water. The bottom face of the ice cube is at a depth of 3 cm. The density of water is 1 000 kg m^{-3}. Calculate the pressure acting on the bottom of the ice cube due to the water.

$h = 3$ cm $= 0.03$ m
$\rho = 1000$ kg m^{-3}
$g = 9.81$ N kg^{-1}

$p = h\rho g$
$= 0.03 \times 1000 \times 9.81$
$= 294.3$ Pa $= 300$ Pa (to 1 s.f.)

Upthrust

Upthrust is an upwards force that fluids exert on objects that are completely or partially submerged in the fluid. It's caused because the top and bottom of a submerged object are at different depths. Consider the cuboid in Figure 2.

- Since $p = h\rho g$, there is a difference in pressure, which causes an overall upwards force known as upthrust.
- The force acting on the top of the body due to the fluid is $F_1 = pA = h_1\rho gA$, where A is the surface area of a face of the cube.
- The force acting on the bottom of the body due to the fluid is $F_2 = pA = h_2\rho gA$
- So the net force (upthrust) acting on the body is:

$$\text{upthrust} = (h_2 - h_1)\rho gA$$

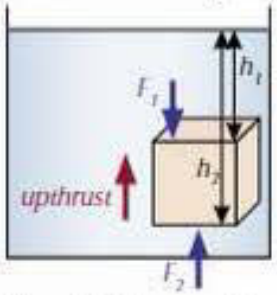


Figure 2: *Diagram of an object experiencing upthrust.*

Tip: Remember, when you're using the upthrust equation, you need to make sure your units are correct. Since you'll be using $g = 9.81$ N kg^{-1}, density needs to be in kg m^{-3}, distances need to be in m and areas need to be in m^2.

Figure 3: *Oil floats on water because it has a lower density. The bottle floats because the average density of the bottle and the air inside it combined is lower than the density of the water.*

Figure 4: *A submarine changes its weight in order to move up and down in the water.*

Archimedes' principle says that when a body is completely or partially immersed in a fluid, it experiences an upthrust equal to the weight of the fluid it has displaced. This is because:

$$(h_2 - h_1)\rho g A = V\rho g = mg = W$$

The average density of an object determines whether it floats or sinks. An object will float on a fluid if it has a lower density than the fluid. This is because the object will only displace a volume of water that weighs as much as the object does. This volume of water is lower than the volume of the object (because of the difference in their densities), so the object will only be partially immersed. At this point, the upthrust balances the weight of the object, and it floats.

If an object is more dense than the fluid, the water it displaces will always weigh less than the object itself. This means the upthrust will always be less than the weight of the object, and so the object will sink.

Example

Submarines make use of upthrust to dive underwater and return to the surface. To sink, large tanks are filled with water to increase the weight of the submarine so that it exceeds the upthrust. To rise to the surface, the tanks are filled with compressed air to reduce the weight so that it's less than the upthrust.

Tip: The volume of a sphere is $V = \frac{4}{3}\pi r^3$.

Tip: Remember, $1\ \text{g cm}^{-3} = 1000\ \text{kg m}^{-3}$.

Practice Questions — Application

Q1 Find the density of an object with a mass of 360 kg and a volume of 0.45 m^3.

Q2 The air pressure inside an inflated basketball is 159 000 Pa. The total force acting on the inner surface of the basketball is 26 500 N. Calculate the inner surface area of the basketball.

Q3 A solid aluminium cylinder has a volume of $9.1 \times 10^{-4}\ \text{m}^3$. Aluminium has a density of $2700\ \text{kg m}^{-3}$. What is the mass of the cylinder?

Q4 An orange has a mass of 0.15 kg and a radius of 4.0 cm. Show that it has a density of approximately $560\ \text{kg m}^{-3}$.

Q5 A uniform cylinder is submerged in water. The radius of the cylinder is 5.00 cm. The top of the cylinder is at a depth of 65.0 cm, and the bottom of the cylinder is at a depth of 76.0 cm. The density of water is $1.00\ \text{g cm}^{-3}$. Calculate the upthrust on the cylinder.

Practice Questions — Fact Recall

Q1 State the equation for density. Define any symbols that you use.

Q2 An object is submerged in water, and displaces an amount of water. What is the upthrust on the object equal to?

Q3 State how the densities of a solid object and fluid must differ for the object to float when it is placed in the fluid.

Section Summary

Make sure you know...

- That weight is the force experienced by an object with mass due to a gravitational field.
- That the unit of force is the newton, N.
- How to use the equation $\boldsymbol{W} = m\boldsymbol{g}$ to make calculations involving weight.
- That the centre of mass (or centre of gravity) of an object is the point where the weight of an object can be considered to act from.
- That for a uniform regular solid, its centre of gravity is at the centre.
- How to perform an experiment to determine the centre of gravity of a flat object.
- What is meant by the normal contact force, tension, friction and upthrust.
- That a free-body diagram shows all the forces acting on an object, but none of the forces it exerts on other objects.
- How to draw free-body diagrams.
- That the net force on an object is the sum of all the forces acting on the object.
- The equation $\boldsymbol{F} = m\boldsymbol{a}$ and how to use it in calculations involving net force and acceleration.
- How to calculate a net force acting on an object for one- and two-dimensional motion.
- That if all forces acting on an object cancel each other out, the object is in equilibrium.
- That if three coplanar forces are acting on an object in equilibrium, the forces can be drawn as a closed triangle of forces.
- How to resolve forces into perpendicular components.
- How to calculate the moment of a force about a point using the equation $M = Fx$.
- That the principle of moments states that for an object to be balanced, the sum of all clockwise moments acting on the object must equal the sum of all anticlockwise moments acting on it.
- That a couple is a pair of equal sized forces that applies a turning effect to an object.
- How to calculate the torque of a couple using the equation $T = Fd$.
- That for an object to be in equilibrium, the torques acting on it must balance.
- That drag is a form of friction which acts against the motion of an object moving through a fluid.
- That an object is travelling at its terminal velocity when the driving forces and frictional forces acting on it are balanced.
- How different factors affect the air resistance acting on an object, such as its speed, size and shape.
- How an object reaches terminal velocity, in terms of the forces acting on it.
- How to sketch and interpret velocity-time and acceleration-time graphs for an object reaching terminal velocity.
- That an object falling in a uniform gravitational field in the presence of drag will eventually reach a terminal velocity.
- How to perform an experiment to determine the terminal velocity of an object in a fluid.
- That density is a measure of the compactness of a substance.
- How to use the equation $\rho = \frac{m}{V}$ to make calculations involving density.
- That pressure is the force exerted on a surface per unit area.
- How to use the equation $p = \frac{F}{A}$ to find pressure in problems involving solids, liquids and gases.
- How to use the equation $p = h\rho g$ to find the pressure exerted on a surface due to the fluid it's in.
- How to calculate the upthrust exerted on an object by a fluid.
- That Archimedes' principle states that the upthrust on an object that is fully or partially submerged in a fluid is equal to the weight of the fluid displaced by the object.

Exam-style Questions

1 An object that weighs 29.43 N on Earth has a weight of 26.61 N on Venus.
What is the gravitational field strength on Venus?

A 9.04 $N kg^{-1}$

B 0.90 $N kg^{-1}$

C 8.87 $N kg^{-1}$

D 9.81 $N kg^{-1}$

(1 mark)

2 Which of the following statements about air resistance on a moving object is/are true?

1 Air resistance is independent of the size and shape of the object.

2 Air resistance increases as the velocity of the object increases.

3 Air resistance is smaller for objects with a lower mass.

A Only statement 2 is true.

B Only statements 1 and 2 are true.

C Only statements 1 and 3 are true.

D All of the statements are true.

(1 mark)

3 Which of the following statements regarding equilibrium is true?

A A body in equilibrium has no forces acting on it.

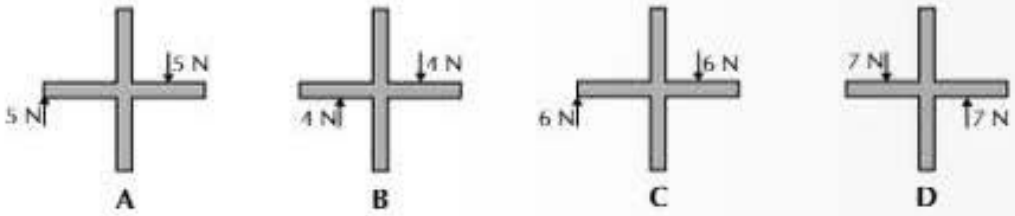
B A body in equilibrium cannot accelerate.

C A body cannot be in equilibrium if it is under the action of more than one moment.

D A body that is changing direction is in equilibrium.

(1 mark)

4 The diagrams below show couples of forces being applied to four identical taps.
The taps are drawn to scale. Which tap is experiencing the largest torque?



(1 mark)

5 A student is investigating the properties of an unknown liquid, X.

(a) The student wants to investigate the terminal velocity of a ball bearing as it travels through liquid X. Describe an experiment that the student could perform to investigate this.

(3 marks)

The student then pours 450 cm^3 of liquid X into a beaker, and weighs it. The mass of the full beaker is 862 g. The empty beaker has a mass of 250 g.

(b) Calculate the density of liquid X. Give your answer in $g\,cm^{-3}$.

(2 marks)

(c) The student places an apple into liquid X. The apple floats in the liquid. The bottom of the apple is at a depth of 4.0 cm below the surface. Calculate the pressure due to liquid X acting on the bottom of the apple.

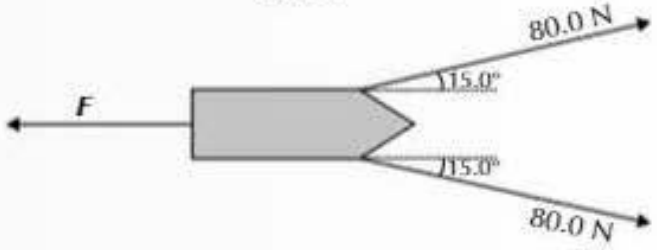
(2 marks)

(d) The apple has a mass of 185 g. Calculate the upthrust on the apple.

(2 marks)

6 A dinghy is at rest on water. It is then pulled through water by two constant forces, as shown in **Fig. 6.1**. A resistive force, ***F***, acts against the dinghy's motion.

Fig. 6.1



(a) After a period of time, the dinghy reaches terminal velocity. Describe how the dinghy reaches terminal velocity.

(3 marks)

(b) Sketch the velocity-time graph for the dinghy, starting from rest until it reaches terminal velocity.

(1 mark)

(c) Calculate the size of force ***F*** when the dinghy is at terminal velocity.

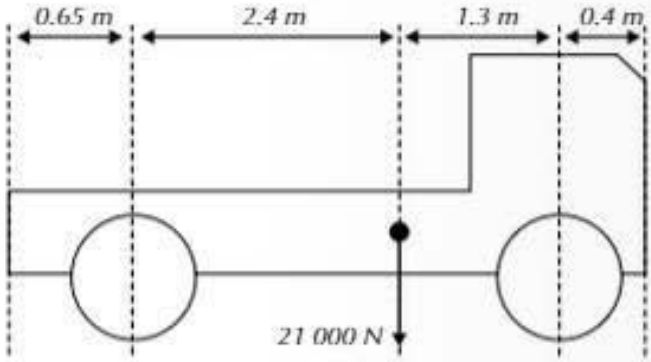
(2 marks)

(d) Once the dinghy is travelling at terminal velocity, the two 80.0 N forces are removed. The mass of the dinghy is 83.2 kg. Calculate the deceleration of the dinghy immediately after the 80.0 N forces are removed.

(2 marks)

7 A truck is used to carry rocks. The truck has a weight of 21 000 N and is 4.75 m long. When it's carrying no load the centre of mass is 1.3 m from the centre of the front wheels and 2.4 m from the centre of the rear wheels. The truck is shown in **Fig. 7.1**.

Fig. 7.1



(a) Define the moment of a force about a point or pivot.

(1 mark)

(b) State and explain which set of wheels, front or back, will exert a greater upwards force on the truck when the truck is carrying no load.

(2 marks)

(c) A rock is placed on the very edge of the back of the truck, and the front wheels remain on the ground but no longer feel any force from the truck. Calculate how much the rock weighs.

(2 marks)

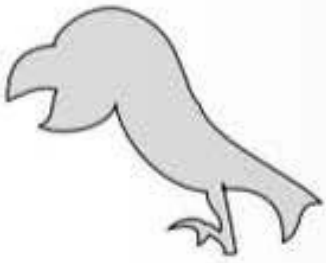
8 A student is investigating the centre of gravity of objects.

(a) Define the centre of gravity of an object.

(1 mark)

Fig. 8.1 shows a flat toy parrot.

Fig. 8.1



(b) Describe an experiment the student could perform to find the centre of gravity of the toy parrot.

(3 marks)

(c) Cranes are tall structures used to move heavy objects on construction sites. Describe the features the base of a crane must have and explain why these features are needed.

(3 marks)

1. Work and Power

You'll have met work and power at GCSE. In physics, they have specific meanings, and they can be calculated with a set of equations.

Learning Objectives:

- Understand the concept of work done by a force.
- Know that the transfer of energy is equal to work done.
- Be able to define the unit joule.
- Be able to calculate the work done by a force moving an object, including when the force is at an angle to the direction of motion, using $W = \boldsymbol{F}x\cos\theta$.
- Know that power is the rate of doing work.
- Be able to calculate power using $P = \frac{W}{t}$.
- Be able to define the unit watt.
- Be able to derive $P = \boldsymbol{F}v$ and know how to use it.

Specification References 3.3.1 and 3.3.3

What is work?

Work is done whenever energy is transferred — they are just two ways of saying the same thing. Here are some examples of work being done:

Activity	*Work done against*	*Final energy form*
Lifting up a box.	*Gravity*	*Gravitational potential energy*
Pushing a chair across level floor.	*Friction*	*Heat*
Pushing two magnetic north poles together.	*Magnetic Force*	*Magnetic energy*
Stretching a spring.	*Stiffness of spring*	*Elastic potential energy*

Figure 1: *A table showing examples of work being done and the final energy form for each activity.*

Usually you need a force to move something because you're having to overcome another force. The thing being moved has **kinetic energy** while it's moving, which is transferred to other forms of energy when the movement stops.

The word 'work' in physics means the amount of energy transferred from one form to another when a force causes a movement of some sort. It's measured in **joules** (J).

Calculating work done

When a car tows a caravan, it applies a force to the caravan to move it to where it's wanted. The force and the displacement of the caravan are in the same direction. To find out how much work has been done, you need to use the equation:

$$W = Fx$$

W = work done in J; F = force causing motion in N; x = displacement in m

- Work is the energy that's been changed from one form to another — it's not necessarily the total energy. E.g. moving a book from a low shelf to a higher one will increase its gravitational potential energy, but it had some gravitational potential energy to start with. Here, the work done would be the increase in gravitational potential energy, not the total gravitational potential energy.
- Remember the displacement needs to be measured in metres — if you have displacement in cm or km, you need to convert it to metres first.

Figure 2: *When you strike a match, work is done against friction to produce heat.*

Tip: Have a look at page 32 for more on unit prefixes.

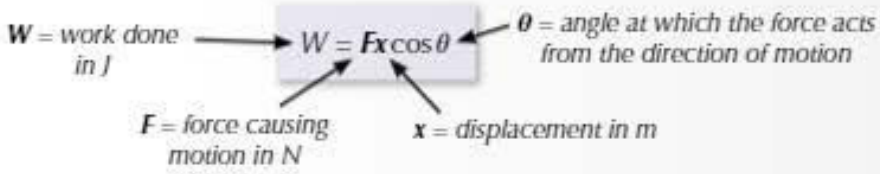
- The force will be a fixed value in any calculations, either because it's constant or because it's the average force.
- The equation assumes that the direction of the force is in the same direction and along the same line as the movement.
- The equation gives you the definition of the joule (symbol J): 'One joule is the work done when a force of 1 newton moves an object through a distance of 1 metre'.

Forces at an angle

Tip: Remember force is a vector, so you can resolve it into components (see page 43).

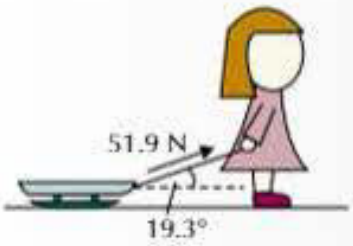
Sometimes the direction of movement of an object is different from the direction of the force acting on it. In this case you need to find the component of the force that acts in the direction of the movement.

Because you only need to worry about the component of the force in the direction of the motion, you only ever need to resolve it in that direction. In general for a force at an angle to the direction of motion, you can find the work with this equation:


W = work done in J
W = Fx cos θ
θ = angle at which the force acts from the direction of motion
F = force causing motion in N
x = displacement in m

Example — Maths Skills

A girl pulls a sledge with a force of 51.9 N for 100.0 m. The string she uses to pull the sledge is at an angle of 19.3° to the ground. Calculate the work she does in pulling the sledge over this distance.



To calculate the work done in a situation like the one above, you need to consider the horizontal and vertical components of the force. The only movement is in the horizontal direction. This means the vertical component of the force is not causing any motion (and hence not doing any work) — it's just balancing out some of the weight, meaning there's a smaller reaction force.

The horizontal component of the force is causing the motion, so to calculate the work done, this is the only force you need to consider. So resolving the force to find the horizontal component (F_x) we get:

$$F_x = 51.9 \times \cos 19.3° = 48.98... \text{ N}$$

$$\Rightarrow W = F_x \times x = 48.98... \times 100.0 = 4898.32... = 4900 \text{ J (to 3 s.f.)}$$

If the force and displacement are in the same direction, then $\theta = 0$, and $\cos\theta = 1$. This means $W = Fx$, which you saw on the previous page.

Power and work

Power means many things in everyday speech, but in physics it has a special meaning. Power is the rate of doing work — in other words it is the amount of energy transferred from one form to another per second.

You calculate the power from this equation:

$$P = \frac{W}{t}$$

P = *power in W*; ***W*** = *work done in J*; ***t*** = *time in s*

The **watt** (symbol W) is defined as a rate of energy transfer equal to 1 joule per second (Js^{-1}). Make sure you learn this definition.

Figure 3: *A flea's legs transfer a small amount of energy, but in a very short time — this makes them very powerful.*

Moving objects

Sometimes, for a moving object, it's easier to use another version of the power equation. It's derived like this:

- You know $P = \frac{W}{t}$.
- You also know $W = Fx$, which gives $P = \frac{Fx}{t}$.
- But $v = \frac{x}{t}$, which you can substitute into the above equation to give:

$$P = Fv$$

P = *power in W*; ***F*** = *force causing the motion in N*; ***v*** = *velocity in* ms^{-1}

Exam Tip
You'll be given this equation in your exam data and formulae booklet.

It's easier to use this if you're given the velocity in the question. Learn this equation as a shortcut to link power and speed.

Example — **Maths Skills**

A car is travelling at a speed of 10.0 ms^{-1} and is kept going against the frictional force by a driving force of 525 N in the direction of motion. Find the power supplied by the engine to keep the car running.

Use the shortcut $P = Fv$, which gives:

$$P = 525 \times 10.0 = 5250\ \text{W}$$

If the force and motion are in different directions, you can replace ***F*** with $F\cos\theta$ to get:

$$P = Fv\cos\theta$$

θ = *angle at which the force acts from the direction of motion*

Exam Tip
This equation doesn't appear in your data and formulae booklet. Make sure you know how to find the power of a moving object when the force isn't in the direction of the object's motion.

Tip: It's a good idea to draw a diagram of the set-up before you start calculating — it usually helps to clear things up a bit.

Practice Questions — Application

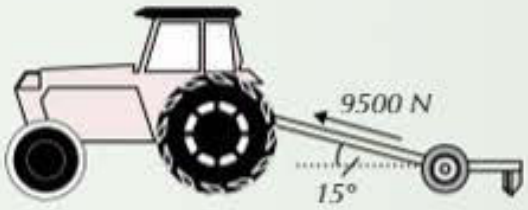
Q1 Ibrahim pushes a desk 2.81 m across a flat floor. If he pushes with a steady force of 203 N, how much work does he do?

Q2 A car operating at a power of 60.1 kW is travelling at a steady 34.7 ms^{-1}. How much force is being provided by the engine?

Q3 Koda pulls a desk 1.39 m across the floor by pulling a rope attached to it at 13.1° to the horizontal. If he pulls with a steady force of 371 N, how much work does he do?

Q4 A child is pulling a cart along by a string at 15.2° to the horizontal. The child is pulling with a constant force of 83.1 N and the cart is moving horizontally at 2.99 ms^{-1}. What is the power of the child?

Q5 The diagram below shows a tractor pulling a plough with a constant force.



It does 980 kJ of work pulling the plough the length of a field.

a) If it takes the tractor 22 seconds to pull the plough the length of the field, what is the tractor's power?

b) Calculate the length of the field.

Practice Questions — Fact Recall

Q1 What's transferred when work is done?

Q2 What is the definition of the joule?

Q3 Power is the rate of doing what?

Q4 What's the equation for finding the power of a moving object?

2. Kinetic Energy and Gravitational Potential Energy

Learning Objectives:

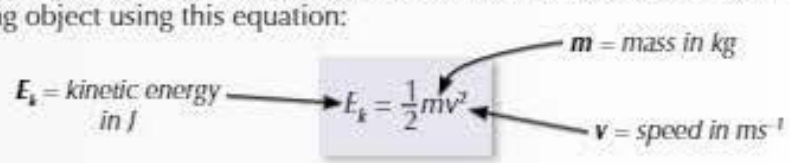
- Know and be able to derive $E_k = \frac{1}{2}mv^2$ and use it to calculate the kinetic energy of an object.
- Know and be able to derive $E_p = mgh$ and use it to calculate the gravitational potential energy of an object in a uniform gravitational field.

Specification Reference 3.3.2

You've probably come across kinetic energy and gravitational potential energy before, but don't assume you know it all. This time you could be asked to derive the equations in the exam.

Kinetic energy

Kinetic energy is the energy of anything moving. You can calculate it for a moving object using this equation:



Example **Maths Skills**

A car with a mass of 903 kg is travelling at a steady speed of 20.6 ms^{-1}. How much kinetic energy does it have?

Just put the numbers into the equation:

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 903 \times 20.6^2 = 191\,598.54 = 192 \text{ kJ (to 3 s.f.)}$$

Tip: A kilojoule (kJ) is equal to 1000 joules (J).

To derive this equation you need to think about an object undergoing a change in kinetic energy. When the kinetic energy of an object increases, work is done on the object. Similarly, when the object's kinetic energy decreases, work is done by the object. Work done is equal to energy transferred so the work done is equal to the change in kinetic energy of the object:

Tip: Remember work done, $W = \boldsymbol{Fx}$ (p.101).

$$E_k = \boldsymbol{Fx}$$

You also know that $\boldsymbol{F} = m\boldsymbol{a}$ (p.79) and $\boldsymbol{a} = \frac{\boldsymbol{v}-\boldsymbol{u}}{t}$ so:

$$\boldsymbol{F} = m\left(\frac{\boldsymbol{v}-\boldsymbol{u}}{t}\right)$$

And from page 48, $\boldsymbol{x}$ (displacement) can be written:

$$\boldsymbol{x} = \frac{(\boldsymbol{u}+\boldsymbol{v})}{2}t$$

Substituting these two into the E_k equation gives:

$$E_k = m \times \frac{(\boldsymbol{v}-\boldsymbol{u})}{t} \times \frac{(\boldsymbol{u}+\boldsymbol{v})}{2} \times t$$

Cancelling the 't's and expanding the brackets:

$$\begin{aligned} E_k &= \frac{1}{2}m(\boldsymbol{v}-\boldsymbol{u})(\boldsymbol{u}+\boldsymbol{v}) \\ &= \frac{1}{2}m(\boldsymbol{v}^2 + \boldsymbol{uv} - \boldsymbol{uv} - \boldsymbol{u}^2) \\ &= \frac{1}{2}m(\boldsymbol{v}^2 - \boldsymbol{u}^2) \\ &= \frac{1}{2}m\boldsymbol{v}^2 - \frac{1}{2}m\boldsymbol{u}^2 \end{aligned}$$

Exam Tip
Make sure you understand this derivation. It could bag you some easy marks in the exam.

The kinetic energy of an object is the kinetic energy compared to when the object is at rest, i.e. when $\boldsymbol{u} = 0$ and the object has no kinetic energy.

Substituting $\boldsymbol{u} = 0$ into the equation above gives you:

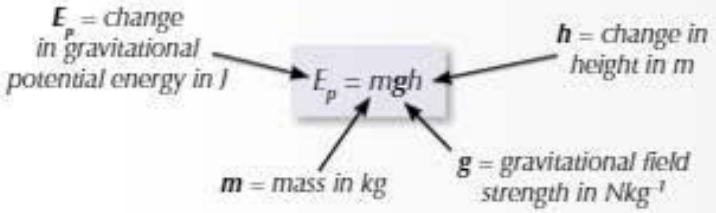
$$E_k = \frac{1}{2}mv^2$$

Tip: You only need the magnitude, not the direction, of $\boldsymbol{v}$ in this equation.

Gravitational potential energy

Gravitational potential energy is the energy something has due to its position in a gravitational field. The greater the height of the object, the greater its gravitational potential energy.

You normally want to find the change in an object's gravitational potential energy, E_p, which you can work out using the equation:


E_p = change in gravitational potential energy in J
$E_p = mgh$
h = change in height in m
m = mass in kg
g = gravitational field strength in Nkg^{-1}

To derive this equation, use the idea that the gravitational energy gained is equal to the work done in moving the object a distance h upwards:

$$E_p = W = F \times h$$

The force that work is done against is the force of gravity, which is equal to mg (see p.76). So:

$$E_p = F \times h = mgh$$

This equation only works when g is constant. This is known as a uniform field. You can assume g is constant close to the Earth's surface.

Tip: Remember, g is equal to 9.81 Nkg^{-1} on Earth, but can be very different elsewhere, see p.76. So an object could have a different gravitational potential energy on another planet even if it's at the same height above the surface.

Example — **Maths Skills**

A man with a mass of 82.5 kg climbs up a cliff and gains 28.5 kJ of gravitational potential energy. Calculate how high the man climbed.

You can rearrange $E_p = mgh$ to get $h = \frac{E_p}{mg}$

Then just put the numbers in:

$$h = \frac{28\,500}{82.5 \times 9.81} = 35.214... = 35.2 \text{ m (to 3 s.f.)}$$

Figure 1: *An astronaut can jump much higher on the moon than they could on Earth by doing same amount of work. This is because g is lower on the moon (p.76) so for a given E_p, h will be bigger (since $E_p = mgh$).*

Practice Questions — Application

Q1 A child with a mass of 32.4 kg jumps 0.285 m into the air. Calculate how much gravitational potential energy she has gained at the top of her jump.

Q2 A 770 kg helicopter descends from a height of 890 m above the ground to a height of 760 m. Calculate the gravitational potential energy that the helicopter loses during this descent.

Q3 A bus is travelling along a road at a speed of 16 ms^{-1}. Calculate the mass of the bus if it has 150 kJ of kinetic energy.

Practice Questions — Fact Recall

Q1 When does an object have kinetic energy?

Q2 When does an object gain gravitational potential energy?

3. Conservation of Energy

You'll no doubt have already met the idea that energy can be transferred from one type to another. Some energy transfers are useful, but others aren't.

Learning Objectives:

- Be able to recall the principle of conservation of energy.
- Understand what is meant by the efficiency of a mechanical system and be able to use the equation for efficiency.
- Be able to apply knowledge of energy in different forms to transfer and conservation problems, including exchanges between gravitational potential energy and kinetic energy.

Specification References 3.3.1, 3.3.2 and 3.3.3

The principle of conservation of energy

The **principle of conservation of energy** states that:

> Energy cannot be created or destroyed. Energy can be transferred from one form to another but the total amount of energy in a closed system will not change.

Whenever energy is converted from one form to another, some is always 'lost'. It's still there (i.e. it's not destroyed) — it's just not in a form you can use.

Example

Not all the energy input into a motor is converted to useful energy (e.g. kinetic energy) — it's not destroyed, but it is converted to less useful forms of energy.

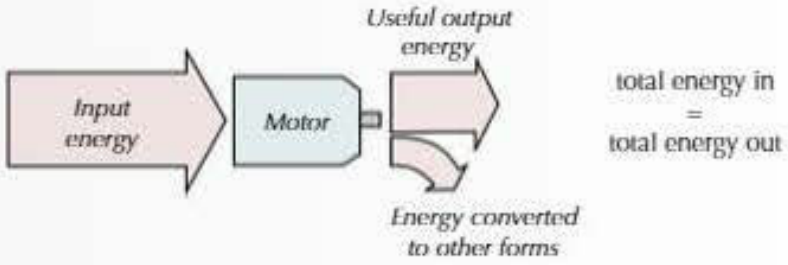


Figure 1: *The input energy is equal to the useful energy output + energy converted to other forms.*

Most often, energy is lost as heat — computers and TVs are always warm when they've been on for a while. No device, except possibly a heater, transfers all energy to useful energy — some energy is always lost as heat. (You want heaters to give out heat, but in other devices the heat loss isn't useful.) Often the heat that is lost is caused by friction.

Energy can be lost in other forms too, such as sound — the important thing is the lost energy isn't in a useful form and you can't get it back.

You can talk about how well energy is transferred in terms of **efficiency**. It's a measure of how much of the input energy is transferred usefully by a mechanical system. The more useful energy you get out of a machine for what you put into it, the more efficient it is. You can use the following equation to calculate efficiency as a percentage:

$$\text{efficiency} = \frac{\text{useful output energy}}{\text{total input energy}} \times 100$$

Exam Tip
Luckily you can normally assume that friction is zero in exam questions (unless stated otherwise).

Tip: To get efficiency as a decimal, just use the first part of the equation and don't multiply by 100.

Exam Tip
Some questions will be kind and give you the useful output energy — others will tell you how much is wasted. You just have to subtract the wasted energy from the total input energy to find the useful output energy.

Example — **Maths Skills**

A meat grinder has a useful energy output of 725 J when making minced meat. If its efficiency is 89.1%, work out the total input energy.

You know that: $\text{efficiency} = \frac{\text{useful output energy}}{\text{total input energy}} \times 100$

Which you can rearrange to:

$$\text{total input energy} = \text{useful output energy} \div \frac{\text{efficiency}}{100}$$

Then total input energy $= 725 \div \frac{89.1}{100} = 813.69... = 814$ J (to 3 s.f.)

Transfers between kinetic and gravitational potential energy

The principle of conservation of energy nearly always comes up when you're doing questions about changes between kinetic and gravitational potential energy.

Often when an object moves, energy is being transferred from one type to another. For example, kinetic energy can be transferred to gravitational potential energy by doing work, and vice versa.

Figure 2: *Rollercoasters transfer gravitational potential energy to kinetic energy and vice versa.*

Examples

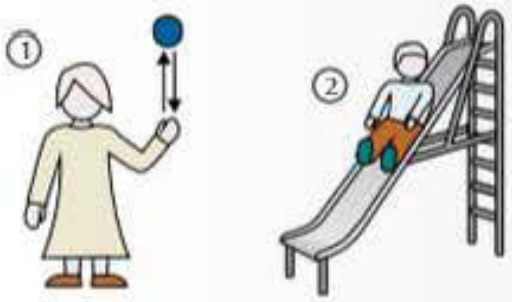


Figure 3: *Some examples of energy transfers between kinetic and potential energy.*

1. As Becky throws the ball upwards, kinetic energy is converted into gravitational potential energy. When it comes down again, that gravitational potential energy is converted back into kinetic energy.

2. As Dominic goes down the slide, gravitational potential energy is converted into kinetic energy.

Exam Tip
You could be given a graph showing the variation of gravitational potential energy (E_p) and kinetic energy (E_k) with distance or time for an object — remember, the sum of E_p and E_k should remain constant if there are no frictional forces.

In real life there are also frictional forces — in the example above, not all of Dominic's gravitational potential energy would be transferred to kinetic energy. Some would be lost due to frictional forces.

You're usually told to ignore friction in exam questions — this means that in the examples above you can assume that the sum of the kinetic and gravitational potential energy is constant. So for a falling object with no air resistance, the gain in kinetic energy is equal to the loss in gravitational potential energy:

$$\frac{1}{2}mv^2 = mgh$$

You need to be able to use this rule to solve problems. The classic example is the simple pendulum. In a simple pendulum, you assume that all the mass is in the bob at the end.

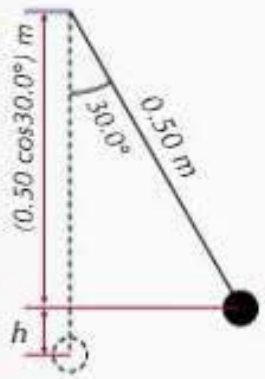
Example — **Maths Skills**

A simple pendulum has a mass of 700 g and a length of 50 cm (both correct to 2 significant figures).
It is pulled out to an angle of 30.0° from the vertical.

(a) Find the gravitational potential energy stored in the pendulum bob relative to its lowest point.

You can work out the increase in height, h, of the end of the pendulum using trigonometry.

Start by drawing a diagram:



Gravitational potential energy
$= mgh$
$= 0.70 \times 9.81 \times (0.50 - 0.50\cos 30.0°)$
$= 0.460...$
$= 0.46$ J (to 2 s.f.)

Tip: $0.50\cos 30.0°$ is the vertical side of the triangle formed when the pendulum swings, so the change in height is 0.50 (original height) – $0.50\cos 30.0°$ (new height).

Exam Tip
You could be asked to apply this stuff to just about anything in the exam. Rollercoasters are an exam favourite.

(b) The pendulum is released. Find the maximum speed of the pendulum bob as it passes through the vertical position.

To find the maximum speed, assume no air resistance, then use $mgh = \frac{1}{2}mv^2$.

So $\frac{1}{2}mv^2 = 0.460....$ Rearrange to find v:

$$v = \sqrt{\frac{2 \times 0.460...}{0.70}} = 1.1464... = 1.1 \text{ ms}^{-1} \text{ (to 2 s.f.)}$$

OR cancel the ms and rearrange to give:

$$v^2 = 2gh$$
$$= 2 \times 9.81 \times (0.50 - 0.50\cos 30.0°) = 1.3142...$$
$$\Rightarrow v = \sqrt{1.3142...} = 1.1464... = 1.1 \text{ ms}^{-1} \text{ (to 2 s.f.)}$$

Practice Questions — Application

Q1 Find the useful energy output of a 20.0% efficient engine with an energy input of 2900 J.

Q2 An apple falls from a tree and accelerates uniformly to a velocity of 7.8 ms^{-1} before it hits the ground. Assuming there is no air resistance, calculate the height the apple fell from.

Q3 a) How much gravitational potential energy does a falcon with a mass of 650 g lose when it dives 103 m?

b) As it dives, 95% of its gravitational potential energy is converted to kinetic energy. If it started stationary, how fast is it moving after the dive?

c) The falcon misses its prey and soars back up, converting 80.0% of its kinetic energy to gravitational potential energy. How high will it climb before needing to flap its wings?

Figure 4: *Birds of prey are highly streamlined so that only a small amount of energy is transferred to heat through drag.*

Practice Questions — Fact Recall

Q1 What's the principle of conservation of energy?

Q2 What is the efficiency of a mechanical system a measure of?

Section Summary

Make sure you know...

- That whenever work is done, energy is transferred.
- Some examples of work done and the transfers involved.
- That work is measured in joules.
- That one joule is the work done when a force of 1 newton moves an object through a distance of 1 metre.
- How to use $W = \boldsymbol{F}x\cos\theta$ to calculate the work done by a force moving an object, and recall that $W = \boldsymbol{F}x$ when the force is in the direction of motion ($\theta = 0°$).
- That power is the rate of doing work, and how to calculate it using $P = \frac{W}{t}$.
- That power is measured in watts.
- That one watt is defined as a rate of energy transfer equal to 1 joule per second.
- How to derive and use $P = \boldsymbol{F}v$ for calculations involving the force and velocity of moving objects.
- The equation $E_k = \frac{1}{2}mv^2$, how to derive it and how to use it to calculate the kinetic energy of an object.
- The equation $E_p = mgh$, how to derive it and how to use it to calculate the gravitational potential energy of an object in a uniform gravitational field.
- That the principle of conservation of energy says that energy cannot be created or destroyed. Energy can be transferred from one form to another but the total amount of energy in a closed system will not change.
- What is meant by the efficiency of a mechanical system.
- How to calculate efficiency using: $\text{efficiency} = \frac{\text{useful output energy}}{\text{total input energy}} \times 100$.
- How energy is commonly wasted.
- That for a falling object with no air resistance, the gain in kinetic energy is equal to the loss in potential energy.
- How to apply energy conservation to transfer and conservation problems.

Exam-style Questions

1 A mobile phone is connected to a power supply to charge.
Of the 565 J of energy supplied to the phone, 45.2 J is wasted in the form of heat.
What is the efficiency of this process?

A 8.00%

B 12.5%

C 92.0%

D 98.1%

(1 mark)

2 A farmer wants to move some farming equipment down the road.

(a) He lifts the equipment into a cart.
State what the farmer is doing work against as he lifts the equipment.

(1 mark)

The farmer uses a horse to pull the cart along a straight road.
Fig 2.1 shows the horse pulling the cart with a constant force.

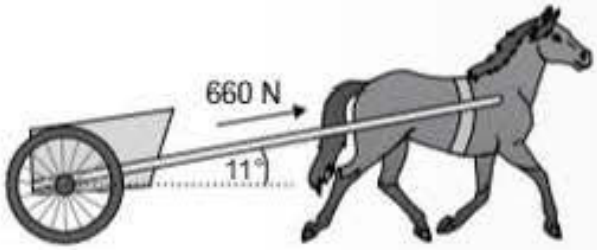


Fig 2.1

(b) Calculate the amount of work done by the horse if it pulls the cart 95 m.

(2 marks)

3 A cyclist pedals along a road.

(a) When the cyclist is travelling at 6.5 ms^{-1}, the brakes are applied and the bike comes to a stop after 3.0 s. If the cyclist and bike together have a mass of 74.8 kg, calculate their combined kinetic energy just before the brakes are applied.

(2 marks)

(b) Calculate the power of the brakes.

(2 marks)

(c) Later on, the cyclist allows himself to roll down a hill from rest without pedalling.
The top of the hill is 22.5 m higher than the bottom of the hill.
Calculate how fast he will be going when he reaches the bottom of the hill.
Assume air resistance and other resistive forces are negligible.

(2 marks)

1. Hooke's Law

Applying a force to a material can stretch it. When some materials are stretched, they follow Hooke's law — but only up to a certain point.

Learning Objectives:

- Know the terms extension and compression.
- Know what is meant by tensile and compressive deformation.
- Know Hooke's law, and be able to use the formula $F = kx$.
- Know the meaning of a force constant, k, of a spring or wire.
- Be able to interpret force-extension (or compression) graphs.
- Know techniques and procedures used to investigate force–extension characteristics for arrangements which may include springs, rubber bands or polythene strips (PAG2).

Specification Reference 3.4.1

Forces and deformation

When more than one force is applied to an object, **deformation** (a change in the object's shape or size) will occur. More than one force is needed to cause deformation — without a secondary supportive force, the work done by a force on an object will simply move the object.

There are two general types of deformation. **Tensile deformation** is caused by applying balanced outward forces, which leads to extension (stretching). **Compressive deformation** is caused by applying balanced inward forces, which leads to compression (squashing) of the object.

If a light metal wire of original length L is supported at the top and then has a weight attached to the bottom, it stretches. The weight pulls down with force F. Once the wire has stopped stretching, the forces will be in equilibrium and there will be an equal and opposite reaction force at the support.

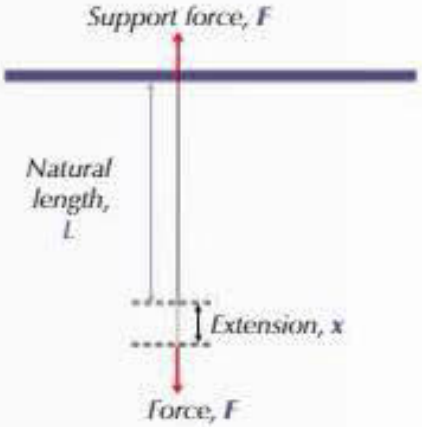


Figure 1: *A supported metal wire extending by x when a weight is attached.*

Tip: The metal wire is light — this means you can ignore the force acting downwards due to the weight of the wire.

What is Hooke's law?

Robert Hooke discovered that the extension, x, of some objects (e.g. most wires and springs) is proportional to the load or force applied, F.
He wrote this as:

k is the **force constant** (or the **stiffness** of an object). It is the force needed to extend an object that obeys Hooke's law by 1 m. It depends on the material that the object is made from, as well as its length and shape.

Exam Tip
If you're given a question like this in the exam, make sure the extension is in m before putting it into the equation.

Example — **Maths Skills**

A force of 6.0 N is applied to a metal wire, which extends by 0.040 m. Calculate the force constant of the wire.

Rearrange the formula $F = kx$ to get $k = \frac{F}{x}$.

Then substitute $F = 6.0$ N and $x = 0.040$ m into the rearranged equation to find the force constant of the wire.

$$k = \frac{6.0}{0.040} = 150\,\text{Nm}^{-1}$$

Tip: If two things are proportional, it means that if one increases, the other increases by the same proportion.

Hooke's law and springs

A metal spring also changes length when you apply a pair of opposite forces. The extension or compression of a spring is proportional to the force applied — so Hooke's law applies.

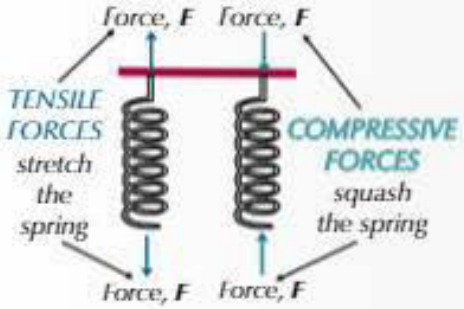


Figure 2: *Metal springs with tensile and compressive forces acting on them.*

Hooke's law works just as well for **compressive forces** as **tensile forces**. For a spring, k has the same value whether the forces are tensile or compressive (though this isn't true for all materials).

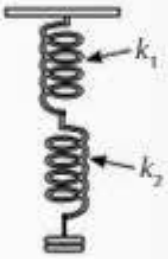
Springs in series and parallel

If a force is applied to more than one spring (or wire), you can combine the force constants of the individual objects to find the overall force constant of the system. You can then treat the system as one spring with force constant k. How you combine the force constants depends on how the springs are arranged.

In series, the reciprocal of the combined force constant is equal to the sum of the reciprocals of the individual force constants:

Tip: Don't get these equations confused with the series and parallel resistance equations on page 176.

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$



In parallel, the combined force constant is just the sum of the force constants of the individual springs:

$$k = k_1 + k_2$$

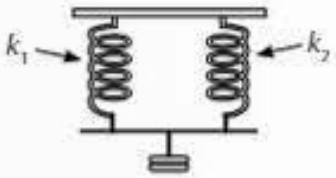
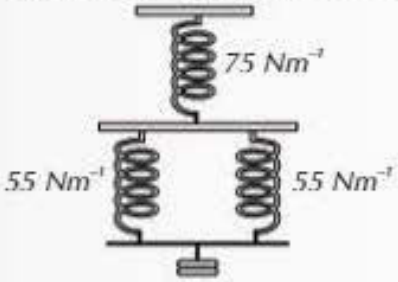


Figure 3: *Springs were used in parallel in old train suspension systems, providing a large combined spring constant.*

Example — **Maths Skills**

Calculate the combined force constant for the springs shown below.



The two springs in parallel at the bottom of the spring have a combined force constant of:

$k_{bottom} = k_1 + k_2 = 55 + 55 = 110 \text{ Nm}^{-1}$

Rearrange $\frac{1}{k} = \frac{1}{k_{top}} + \frac{1}{k_{bottom}}$ to get:

$$k = 1 \div \left(\frac{1}{k_{top}} + \frac{1}{k_{bottom}}\right) = 1 \div \left(\frac{1}{75} + \frac{1}{110}\right)$$

$= 44.5945... = 45 \text{ Nm}^{-1}$ (to 2 s.f.)

The limit of proportionality

There's a limit to the force you can apply for Hooke's law to stay true. Figure 4 shows force against extension for a typical metal wire or spring.

The first part of the graph shows Hooke's law being obeyed — there's a linear relationship between force and extension and it goes straight through the origin. The gradient of the straight line is the force constant, k.

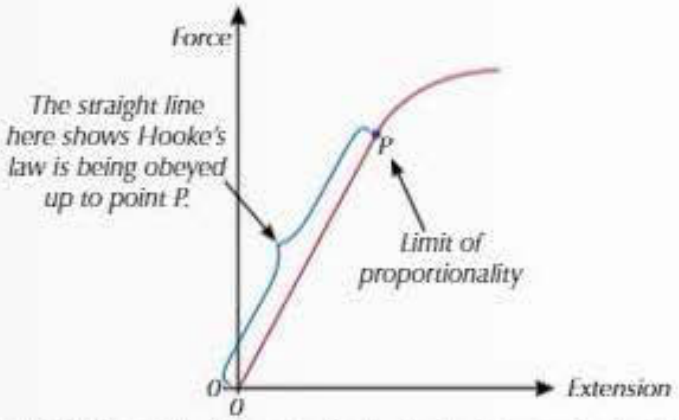


Figure 4: *The graph of force against extension for a typical metal wire.*

Metals generally obey Hooke's law up to the **limit of proportionality**, marked P on the graph. The limit of proportionality is the point beyond which the force is no longer proportional to extension. Beyond P, the graph starts to curve. The limit of proportionality is also known as **Hooke's law limit**. Hooke's law doesn't just apply to metal springs and wires — most other materials obey it up to a point. But be careful — there are some materials, like rubber, only obey Hooke's law for really small extensions.

Exam Tip
You may be asked to give two features of a graph which show that the material under test obeys Hooke's law. Just remember, the graph of a material which obeys Hooke's law will start with a straight line through the origin.

Tip: You get a similar graph for compression too.

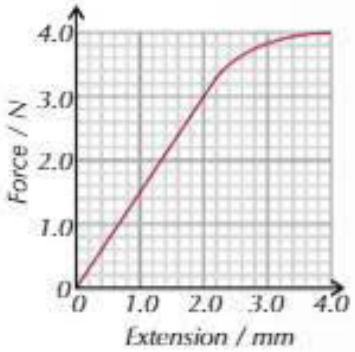
Tip: There's more on force-extension graphs on pages 118-120.

Tip: The limit of proportionality can also be described as the point beyond which the force-extension graph is no longer linear (straight).

Tip: Force-extension graphs are specific for the tested object and depend on its dimensions — different wire lengths and diameters of the same metal will produce different force-extension graphs.

Example — **Maths Skills**

Below is a force-extension graph for a spring. Calculate the force constant, k, in Nm^{-1} for the spring.



k is the gradient of the graph up to the limit of proportionality.

For this graph, this is shortly after the point where the load is 3.0 N and the extension is 2.0 mm.

Convert 2.0 mm into m, to get 0.0020 m.

Then $k = \frac{F}{x} = \frac{3.0}{0.0020} = 1500\ Nm^{-1}$.

Investigating extension

PRACTICAL ACTIVITY GROUP 2

Figure 5 shows the experimental set-up you could use in the lab to investigate how the extension of an object varies with the force applied to it.

Tip: Do a risk assessment before you start. When carrying out this experiment, make sure you're stood up so you can move out of the way quickly if the weights fall, and use goggles in case the spring snaps.

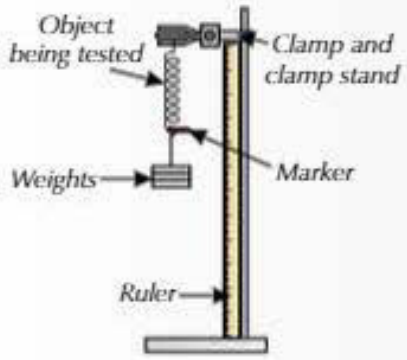


Figure 5: *An experimental set-up used to investigate how extension varies with force.*

Tip: You may want to conduct a pilot experiment beforehand to help you work out the sizes of masses to use.

Tip: If you have unknown masses, rather than known weights, the object can be suspended by a newton meter — this will let you calculate the additional force being applied each time a mass is added.

- Measure the natural length of the spring (when no load is applied) with a millimetre ruler clamped to the stand. Make sure you take the reading at eye level and add a marker (e.g. a thin strip of tape) to the bottom of the spring to make the reading more accurate.
- Add a weight to the spring and allow it to come to rest. Record the weight and measure the new length of the spring. The extension is the change in length.
- Repeat this process until you have enough measurements (no fewer than 6).
- Plot a force-extension graph of your results. Where you can draw a straight line of best fit, the object obey's Hooke's law, and the gradient is equal to k (from $F = kx$). If you've loaded the object beyond its limit of proportionality, the graph will start to curve.
- You could try measuring and comparing the spring constants and force-extension characteristics of different materials, such as springs, rubber bands and polythene strips.

Practice Questions — Application

Q1 A metal spring has a spring constant of 1250 Nm^{-1}. When a force is applied, the spring extends proportionally by 1.60 cm. Calculate the force applied.

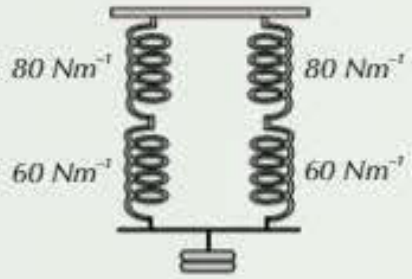
Q2 A spring is compressed proportionally 0.80 mm by a force of 20 N (correct to 2 s.f.). Calculate the spring constant of the spring in Nm^{-1}.

Q3 The original length of a metal wire is 20.0 cm. A force of 55.0 N is applied and the wire extends to a new length of 22.0 cm. Assume that the limit of proportionality has not been reached.

a) Calculate the extension of the wire in metres.

b) Calculate the spring constant of the length of wire.

Q4 Calculate the combined force constant for the springs shown in the system below.



Tip: Remember, Hooke's law applies for compression as well as tension. Huzzah.

Practice Questions — Fact Recall

Q1 Write down the formula for Hooke's law, defining all the symbols you use.

Q2 What is meant by the limit of proportionality for a material?

Q3 A spring suspended from a clamp stretches when a weight is attached to the bottom. Describe the measurements you would need to take and the calculations you would have to perform to find the extension of the spring.

Exam Tip
You'll be given the formula for Hooke's law in your data and formulae booklet, so you don't need to know it by heart.

2. Elastic and Plastic Deformation

It won't surprise you to hear that different objects behave in different ways when a force is applied to them. Force-extension graphs are a useful way of showing these behaviours.

Learning Objectives:

- Apply knowledge and understanding of force–extension (or compression) graphs.
- Know what is meant by elastic and plastic deformations of materials.
- Know that for a force-extension (or compression) graph, work done is the area under the graph.
- Be able to calculate elastic potential energy using $E = \frac{1}{2}Fx$ and $E = \frac{1}{2}kx^2$.

Specification Reference 3.4.2

The elastic limit

Figure 1 shows a graph of force against extension for a typical metal wire or spring. At the limit of proportionality, the graph starts to curve. The point marked *E* on the graph is called the **elastic limit**. If you increase the force past the elastic limit, the material will be permanently stretched. This means when all the force is removed, the material will be longer than at the start.

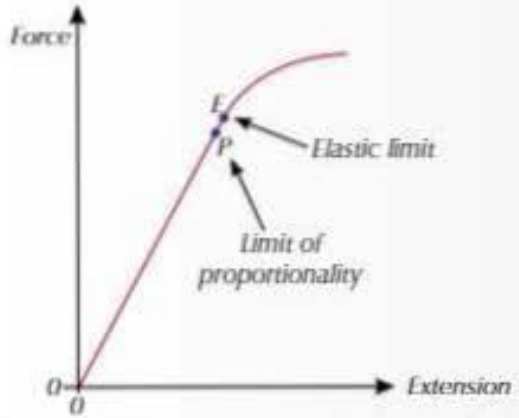


Figure 1: *The graph of force against extension for a typical metal wire.*

Figure 2: *Guitar strings deform elastically after being plucked.*

Figure 3: *Bullets made from metals often behave plastically and permanently deform on impact.*

Types of deformation

Elastic deformation

Up to their elastic limit, materials behave elastically. This means the material returns to its original shape once the forces are removed. Elastic materials are able to be stretched or compressed without permanently deforming.

Plastic deformation

A material stretched past its elastic limit shows **plastic deformation**. The material does not return to its original shape when the forces on it are removed. Plastic materials are permanently deformed by forces. When we say plastic here, we just mean a material that undergoes plastic deformation. It has nothing to do with the materials called 'plastic', that things like water bottles are made of.

Work done in deformations

When a material is stretched or compressed, work has to be done to deform the material. Force-extension graphs are a useful way of showing how much energy has been stored by the deformed material.

Force-extension graphs

On a force-extension graph, the work done is given by the area under the graph. Before the elastic limit, all the work done in deforming a material is stored as potential energy in the material. This stored energy is called **elastic potential energy**.

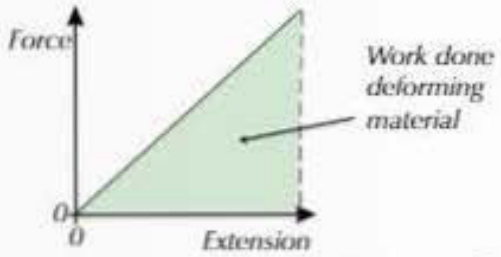
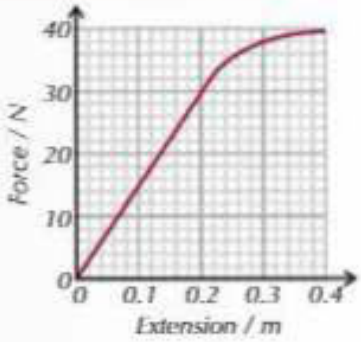


Figure 4: *The area under a force-extension graph for a stretched material is equal to the work done stretching it.*

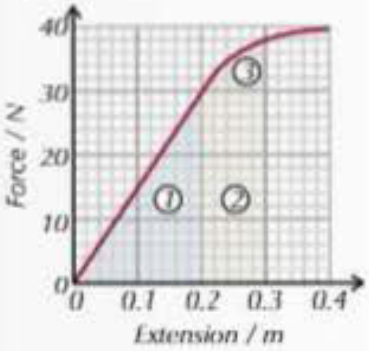
Tip: Remember, work done = energy transferred (page 101).

Example — **Maths Skills**

Shown to the right is a force-extension graph for a metal spring. Find the work done stretching the spring to an extension of 0.3 m.



To find the work done, you need to find the area under the graph shown highlighted below:



You can approximate the area using triangles and rectangles:

Area 1: ½ × 30 N × 0.2 m = 3 J

Area 2: 30 N × 0.1 m = 3 J

Area 3: ½ × 8 N × 0.1 m = 0.4 J

Total work done ≈ 3 J + 3 J + 0.4 J
= 6.4 J

You can also approximate the area using the squares on the grid:

There are (approximately) 6.5 big squares, each worth: 10 N × 0.1 m = 1 J
So the total work done is ≈ 6.5 × 1 J = 6.5 J.

Exam Tip
The examiners will accept a range of answers for questions like this, as it's tricky to get the area under a curve exactly right. But that doesn't mean you can be slapdash when working areas out — still do it carefully.

Tip: You could also count the number of small squares. Oh joy.

Calculating energy stored

Provided a material obeys Hooke's law, the potential energy stored inside it can be calculated quite easily using a formula. This formula can be derived using a force-extension graph and work done.

The energy stored by the stretched material is equal to the work done on the material in stretching it.

Work done is equal to force × displacement (see page 101). But on a force-extension graph, the force acting on the material is not constant. Therefore you need to work out the average force acting on the material, from zero to F, which is $\frac{1}{2}F$:

$$\text{work done} = \tfrac{1}{2}Fx$$

On the graph, this is just the area underneath the straight line from the origin to the extension (x). So the area under the graph represents the work done or the energy stored. And so the elastic potential energy, E, is:

$$E = \tfrac{1}{2}Fx$$

Tip: This is just the same as working out the area of a triangle: (½ × base × height).

Exam tip
In the exam, they might ask you to explain how the formula $E = \frac{1}{2}Fx$ can be derived using a force-extension graph.

Because Hooke's law is being obeyed, $F = kx$, which means F can be replaced in the equation to give:

$$E = \frac{1}{2}kx^2$$

Work done to permanently deform an object

If the material is stretched beyond the elastic limit, some work is done permanently deforming the material. This will not be stored as elastic potential energy.

Force-extension graphs can show when work has been done to permanently deform an object. For example, you can plot a force-extension graph of what happens when force has been added and then removed from an object. If the unloading line and the loading line don't start and end at the same points, the material has deformed plastically and work has been done extending the object permanently.

Tip: Remember that up to point *P*, the gradient of the graph gives the force constant of the wire.

Figure 5 is a force-extension graph for a metal wire that has been stretched beyond its elastic limit (*E*).

Tip: If you apply a big enough force to fracture the object you are investigating, you can't draw the unloading line. The loading line should just stop at the point the fracture occurred (see pages 128 and 130).

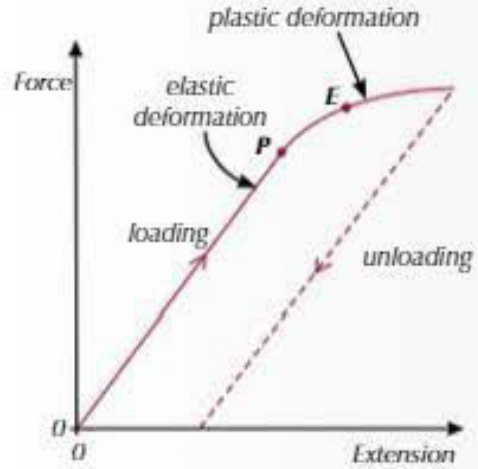


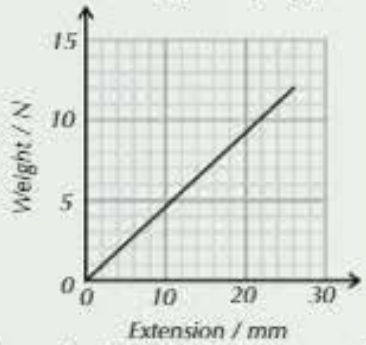
Figure 5: *A force-extension graph for a metal wire.*

When the load is removed, the extension decreases. The unloading line is parallel to the loading line because the force constant k is still the same. Because the wire was stretched beyond its elastic limit (*E*) and deformed plastically, it has been permanently stretched. This means the unloading line doesn't go through the origin. The area between the two lines is the work done to permanently deform the wire.

Practice Questions — Application

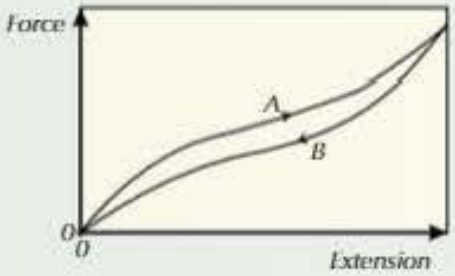
Q1 Calculate the elastic potential energy stored in a spring when a force of 30 N has produced an elastic extension of 1.2 cm. You may assume that the limit of proportionality has not been reached.

Q2 A student hangs weights from the end of a spring and records its extension. Her results are shown in the graph below. Calculate the total work done in stretching the spring during this experiment.



Q3 Sketch and label a typical force-extension graph for a metal wire as a force is applied to stretch the wire up to and beyond its elastic limit, then gradually removed.

Q4 Below is a force-extension graph for an investigation into how extension varies with force for a rubber band. The band was loaded with weights, shown by curve A. The band was then unloaded, shown by curve B.



Is the rubber band elastic or plastic? Explain your answer.

Tip: Watch out for the units in Q1 — to calculate the elastic potential energy, the force has to be in newtons and the extension has to be in metres.

Practice Questions — Fact Recall

Q1 What is meant by the elastic limit of a material?

Q2 What does it mean if a material is deforming elastically?

Q3 What does it mean if a material is deforming plastically?

Q4 What is shown by the area between the loading and unloading lines on a force-extension graph of a material undergoing plastic deformation?

Learning Objective:

- Be able to define stress, strain and ultimate tensile strength.

Specification Reference 3.4.2

3. Stress and Strain

Two samples of the same material with different dimensions will stretch different amounts under the same force. Stress and strain are measurements that take into account the size of the sample, so a stress-strain graph is the same for any sample of a particular material.

Tensile stress and tensile strain

A material subjected to a pair of opposite forces may deform, i.e. change shape.

Tensile stress, σ, is defined as the force applied, F, divided by the cross-sectional area, A:

$$\sigma = \frac{F}{A}$$

The units of stress are Nm^{-2} or pascals, Pa.

A stress causes a strain. **Tensile strain**, ε, is defined as the change in length, i.e. the extension, divided by the original length of the material:

$$\varepsilon = \frac{x}{L}$$

Figure 2: *A material having its tensile stress tested by a machine.*

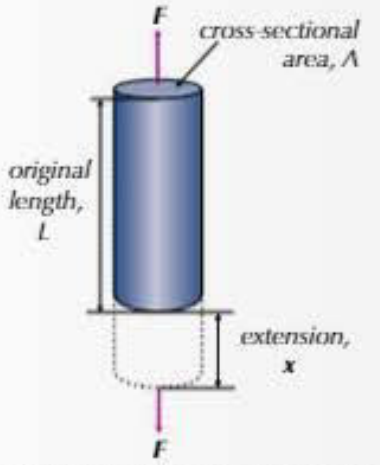


Figure 1: *A pair of opposite tensile forces acting on an object.*

Strain has no units because it's a ratio — it's usually just written as a number.

It doesn't matter whether the forces producing the stress and strain are tensile or compressive — the same equations apply. The only difference is that you tend to think of tensile forces as positive, and compressive forces as negative.

Ultimate tensile strength

As a greater and greater tensile force is applied to a material, the stress on it increases.

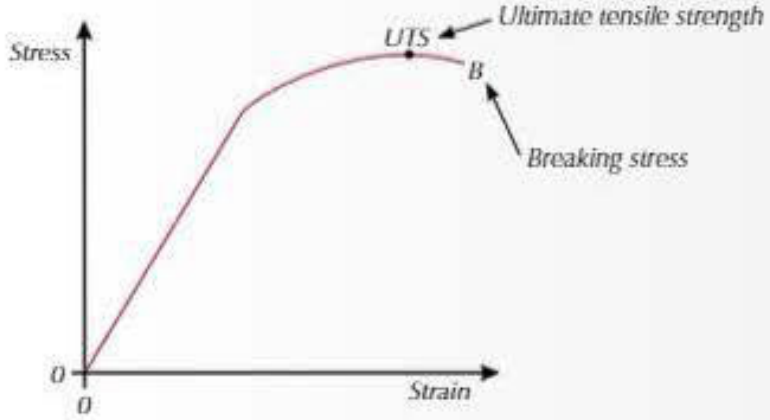


Figure 3: *A stress-strain graph showing the ultimate tensile strength and breaking stress of a material.*

Tip: The stress-strain graphs for brittle materials don't look like this — see page 128. Page 130 has more on how to interpret stress-strain graphs like this one.

Tip: If an object has an uneven cross-section, the stress and strain will be different in different parts of the object — but it will always be somewhere on the stress-strain graph.

The effect of the stress is to start to pull atoms in the material apart from one another. Eventually the stress becomes so great that atoms separate completely, and the material breaks. This is shown by point B on the graph in Figure 3. The stress an object experiences when this occurs is called the **breaking stress**.

The point marked UTS on the graph in Figure 3 is called the **ultimate tensile strength**. This is the maximum stress that the material can withstand before it will break. Once you go past the UTS, the material will break no matter what you do. Engineers have to consider the UTS and breaking stress of materials when designing a structure.

Figure 4: *Copper deforms plastically before breaking under sufficient stress.*

Example — **Maths Skills**

A rope has a cross-sectional area of 8.0×10^{-3} m². A tensile force is applied to the rope and slowly increased. The rope breaks after a force of 1.8×10^{5} N is applied. Calculate the UTS of the material.

$$\sigma = \frac{F}{A} = \frac{1.8 \times 10^{5}}{8.0 \times 10^{-3}} = 2.25 \times 10^{7}$$
$$= 2.3 \times 10^{7} \text{ Pa (or Nm}^{-2}\text{) (to 2 s.f.)}$$

You could also write 2.3×10^{7} Pa as 23 MPa.

Tip: 1 MPa is the same as 1×10^{6} Pa.

Practice Questions — Application

Q1 A force of 50.0 N is applied to wire with a cross-sectional area of 3.1×10^{-6} m². Calculate the stress on the wire.

Q2 A rope has an original length of 12.0 cm. After a force is applied, its length becomes 12.3 cm. Calculate the strain on the rope.

Q3 A wire with a cross-sectional area of 1.2×10^{-7} m² has an ultimate tensile stress of 3.8×10^{8} Pa. Calculate the maximum force you could apply to the wire before it breaks.

Practice Questions — Fact Recall

Q1 What is meant by:

a) tensile stress,

b) tensile strain,

c) breaking stress,

d) ultimate tensile strength?

Learning Objectives:

- Know that Young modulus = $\frac{\text{tensile stress}}{\text{tensile strain}}$, and be able to use the equation $E = \frac{\sigma}{\varepsilon}$.
- Be able to describe techniques and procedures used to determine the Young modulus for a metal (PAG2).

Specification Reference 3.4.2

4. The Young Modulus

The Young modulus is a measure of how stiff a material is. It is really useful for comparing the stiffness of different materials, for example if you're trying to find out the best material for making a particular product.

Calculating the Young modulus

When you apply a load to stretch a material, it experiences a tensile stress and a tensile strain (see page 122). Up to a point called the limit of proportionality, the stress and strain of a material are proportional to each other. So below this limit, for a particular material, stress divided by strain is a constant. This constant is called the **Young modulus**, E.

$$\text{Young modulus} = \frac{\text{tensile stress}}{\text{tensile strain}} \qquad E = \frac{\sigma}{\varepsilon} = \frac{F \div A}{x \div L} = \frac{FL}{Ax}$$

Where F = force in N, A = cross-sectional area in m², L = initial length in m and x = extension in m. The units of the Young modulus are the same as stress (Nm⁻² or Pa), since strain has no units.

Tip: The Young modulus is used by engineers to make sure their materials can withstand sufficient forces.

The Young modulus experiment

PRACTICAL ACTIVITY GROUP 2

Figure 1 shows an experiment you could use to find out the Young modulus of a metal:

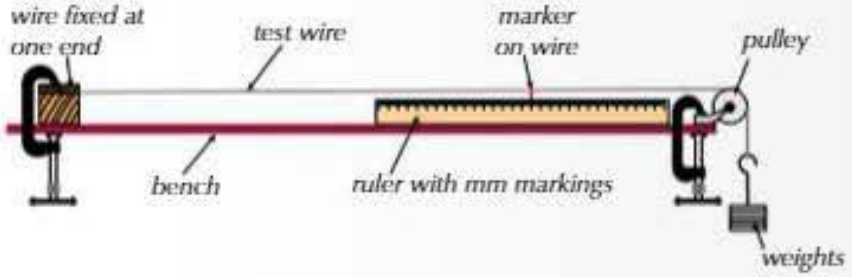


Figure 1: *Experimental set-up for determining the Young modulus of a test metal.*

Tip: Do this experiment standing up so you can get out of the way quickly if the weights fall. And wear safety goggles in case the wire snaps. You should also do a full risk assessment before starting work.

Tip: You should do a pilot experiment in which you plot a force-extension graph for an identical piece of test wire to find its limit of proportionality. That way you can make sure you get nowhere near it in this experiment.

Tip: To reduce random errors you should use a thin marker on the wire.

Tip: Extensions can be very small, so you could use a travelling microscope to measure them more precisely than with a ruler.

- The test wire should be thin, and as long as possible. The longer and thinner the wire, the more it extends for the same force — this reduces the uncertainty in your measurements.
- First you need to find the cross-sectional area of the wire. Use a micrometer to measure the diameter of the wire in several places and take an average of your measurements. By assuming that the cross-section is circular, you can use the formula for the area of a circle: Area = $\pi \times \text{radius}^2$.
- Clamp the wire to the bench (see Figure 1 above) so you can hang weights off one end of it. Start with the smallest weight necessary to straighten the wire. (Don't include this weight in your final calculations.)
- Record the starting position of the marker. Measure the distance between the fixed end of the wire and the marker — this is your unstretched length. Make sure you position yourself so that the marker and ruler are at eye level when measuring the position of the marker.

- Then if you increase the weight, the wire stretches and the marker moves.
- Increase the weight in steps (e.g. 1 N intervals), recording the marker reading each time — the extension is the difference between this reading and the starting position of the marker. Use a mass meter or a set of digital scales to accurately find the weight you add at each step.
- You can use your results from this experiment to calculate the stress and strain of the wire and plot a stress-strain curve.

Tip: If you unload the wire, you can re-measure the extension for each weight to make sure you haven't gone past the wire's elastic limit.

Finding the Young modulus using a graph

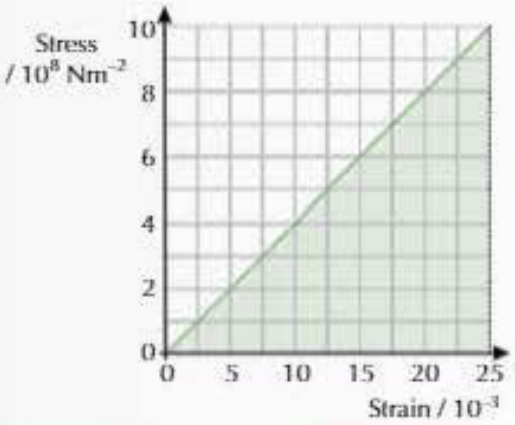
You can plot a graph of stress against strain from your results — see Figure 3. The gradient of the graph gives the Young modulus, E.

$$E = \frac{\text{stress}}{\text{strain}} = \text{gradient}$$

Tip: You can also measure the Young modulus in the lab using Searle's apparatus. This is a bit more accurate, but it's harder to do and the equipment's more complicated.

Example — **Maths Skills**

The stress-strain graph below is for a thin metal wire. Find the Young modulus of the wire from the graph.



E = stress ÷ strain = gradient

$$\text{Gradient} = \frac{\Delta\text{stress}}{\Delta\text{strain}} = \frac{10 \times 10^8}{25 \times 10^{-3}} = \mathbf{4 \times 10^{10}\ Nm^{-2}}$$

Tip: Remember, when using the gradient to work out the Young modulus, you can only use it up to the limit of proportionality (p.115). After then, the stress and strain are no longer proportional.

Up to the elastic limit, the area under the graph gives the elastic potential energy stored per unit volume, i.e. the energy stored per 1 m^3 of wire.

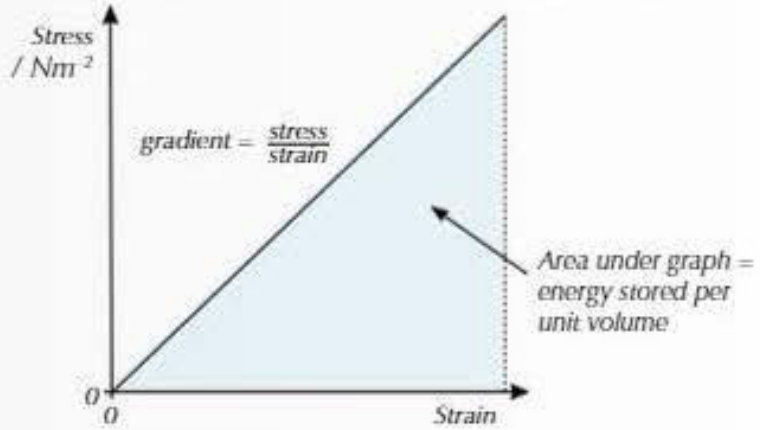


Figure 3: *A stress-strain graph showing how to calculate the gradient and energy stored per unit volume.*

Figure 2: *Steel has a high Young modulus, which means under huge stress there's only a small strain. This makes it a useful building material for things like bridges.*

The stress-strain graph is a straight line provided that Hooke's law is obeyed, so up to the limit of proportionality you can also calculate the elastic potential energy per unit volume as:

$$\text{energy per unit volume} = \tfrac{1}{2}\sigma\varepsilon$$

Example — **Maths Skills**

A 1.2 metre long wire with a diameter of 2.0 cm stores 5.2 J when it is stretched. If the stress on the wire is 5.9×10^7 Nm^{-2}, calculate the strain on the wire. Assume the wire has not been stretched past its limit of proportionality.

First calculate the energy per unit volume:

$$\text{energy per unit volume} = \frac{5.2}{1.2 \times \pi \times (0.020 \div 2)^2} = 1.3793... \times 10^4\ \text{Jm}^{-3}$$

Rearranging the formula above:

$$\varepsilon = \frac{2 \times \text{energy per unit volume}}{\sigma} = \frac{2 \times 1.3793... \times 10^4}{5.9 \times 10^7}$$

$$= 0.0004675... = \mathbf{4.7 \times 10^{-4}\ (to\ 2\ s.f.)}$$

Tip: Don't forget to convert any lengths to m and areas to m^2 when working out the Young modulus.

Practice Questions — Application

Q1 a) A copper wire has an original length of 1.0 m and a diameter of 1.1 mm. What is the cross-sectional area of the wire?

b) A force of 23 N is applied to the wire and it extends by 0.20 mm. Find the Young modulus of copper.

Q2 A nylon wire with a cross-sectional area of 8.0×10^{-7} m^2 has a Young modulus of 3.5×10^8 Pa. A force of 100 N is applied to the wire.

a) Calculate the stress on the wire.

b) Calculate the strain on the wire.

Q3 A block with an initial length of 254 cm is compressed by 0.11 cm. The stress on the block is 4 200 000 m^{-2}. Assuming the block has not been deformed past its limit of proportionality, calculate the energy stored per unit volume by the block.

Practice Questions — Fact Recall

Q1 What are the units of the Young modulus?

Q2 Name four things you would need to measure when carrying out an experiment to find the Young modulus of a wire.

Q3 Give one safety precaution you should take when carrying out an experiment to find the Young modulus of a wire.

Q4 What is the gradient of the linear part of a stress-strain graph equal to?

5. Stress-Strain Graphs

Because materials have different properties, their stress-strain graphs look different too — you need to know the graphs for ductile, brittle and polymeric materials.

Learning Objective:

- Be able to interpret stress-strain graphs for typical ductile, brittle and polymeric materials.

Specification Reference 3.4.2

Ductile materials

The diagram shows a stress-strain graph for a typical **ductile** material, such as copper. You can change the shape of ductile materials by drawing them into wires or other shapes. The important thing is that they keep their strength when they're deformed like this.

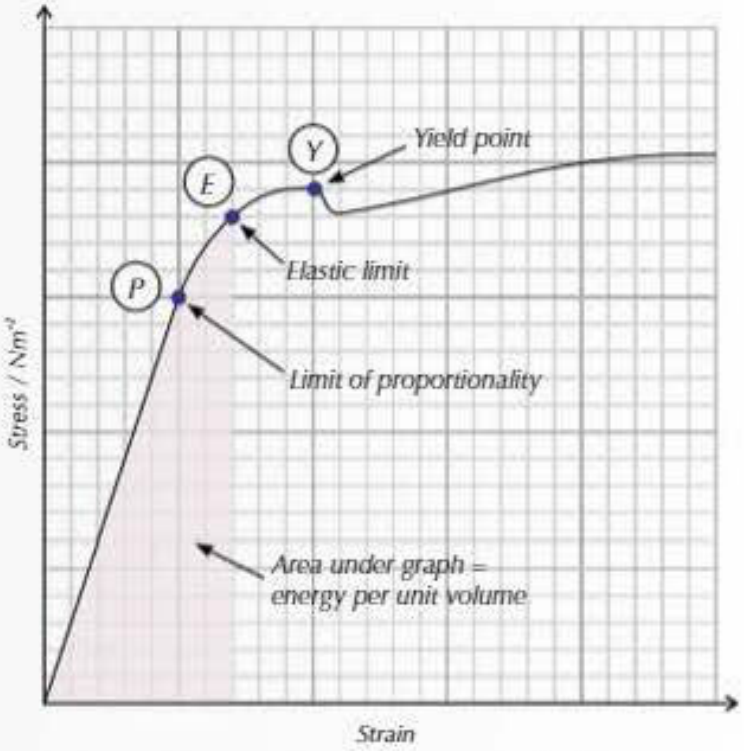


Figure 1: *A stress-strain graph for a typical ductile material.*

Before point *P*, the graph is a straight line through the origin. This shows that the material is obeying Hooke's law (page 113). The gradient of the line is constant — it's the Young modulus (see pages 124-125).

Point *P* is the limit of proportionality — after this, the graph starts to bend. At this point, the material stops obeying Hooke's law, but would still return to its original size and shape if the stress was removed.

Point *E* is the elastic limit — at this point the material starts to behave plastically. From point *E* onwards, the material would no longer return to its original size and shape once the stress was removed.

Point *Y* is the **yield point** — here the material suddenly starts to stretch without any extra load. The yield point (or yield stress) is the stress at which a large amount of plastic deformation takes place with a constant or reduced load.

The shaded area under the graph gives the energy stored in the material per unit volume (see page 125) up to the elastic limit.

Tip: Remember, these are different to force-extension graphs. Force-extension graphs are specific to the tested object and are affected by its dimensions. Stress-strain graphs describe the general behaviour of a material, because stress and strain are independent of the dimensions.

Tip: You can plot graphs like this from experiments like the one on p.124. From these graphs you can see if a material is behaving elastically or plastically.

Tip: Plastic deformation is useful if you don't want a material to return to its original shape, e.g. drawing copper into wires or gold into foil (see Figure 2).

Figure 2: *Rolls of gold foil being made at a factory.*

Figure 3: *Ceramics are brittle materials.*

Brittle materials

Brittle materials don't deform plastically, but break when the stress on them reaches a certain point. Glass, ceramics and polystyrene are all brittle materials.

Below is a stress-strain graph typical of a brittle material. The stress-strain graph for a brittle material doesn't curve.

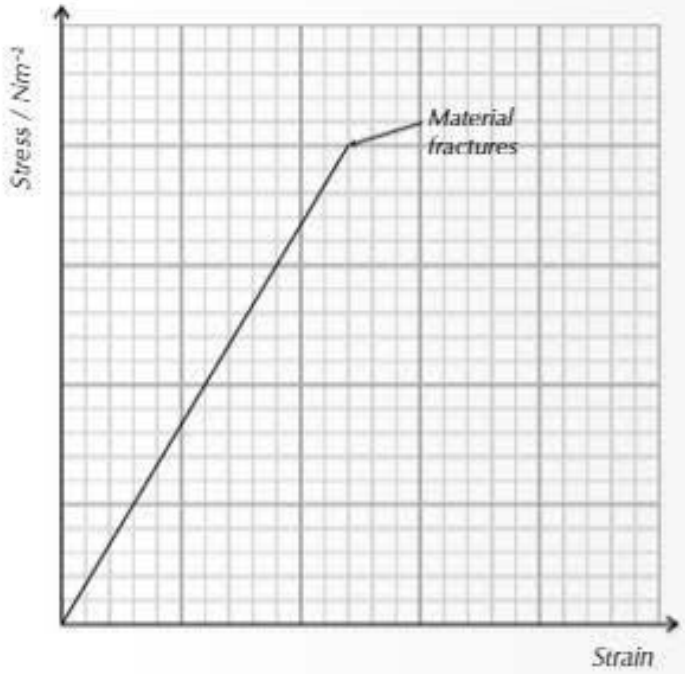


Figure 4: *A typical stress-strain graph for a brittle material.*

Tip: Notice how the straight line just stops. Very dramatic.

Tip: It's not only brittle materials that fracture. Some materials will deform plastically up to a point, then fracture if the force is too great (see page 130).

The graph starts with a straight line through the origin. So brittle materials also obey Hooke's law. However, when the stress reaches a certain point, the material snaps — it doesn't deform plastically.

Brittle materials can be quite weak if they have cracks. When stress is applied to a brittle material any tiny cracks at the material's surface get bigger and bigger until the material breaks completely. This is called brittle fracture.

Polymeric materials

The molecules that make up **polymeric** (or polymer) materials are arranged in long chains. They have a range of properties, so different polymers have different stress-strain graphs.

Polythene

Tip: You might also see polythene referred to as polyethylene.

Polythene is a common plastic, and is generally used in packaging. It is a ductile material that behaves plastically — applying a stress to it stretches it into a new shape.

Figure 6 shows the stress-strain graph for polythene.

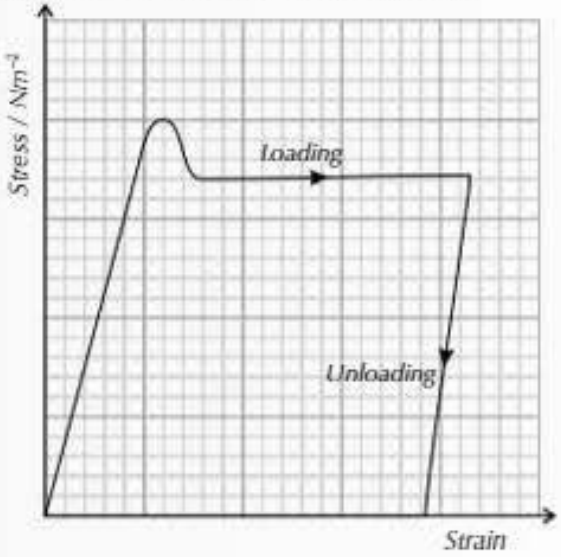


Figure 6: *A typical stress-strain graph for polythene.*

Figure 5: *Polythene is often used to make bottles and containers as it's more resistant to breakage than brittle alternatives like glass.*

The graphs starts with a straight line through the origin, as polythene initially obeys Hooke's Law. When the load is increased past its elastic limit, the polythene begins to deform plastically. The graph doesn't return to the origin so the polythene is permanently deformed into a new shape.

Rubber

Rubber is another polymeric material but it behaves differently to polythene. Figure 8 shows a typical stress-strain graph for rubber.

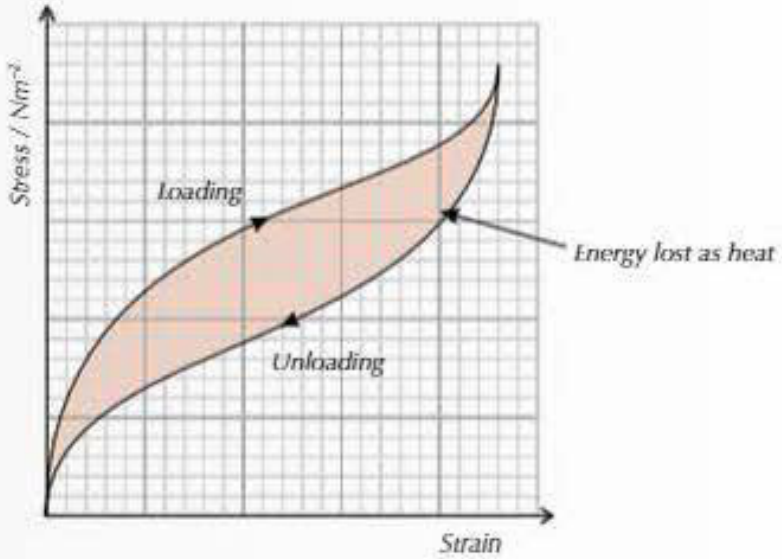


Figure 8: *A typical stress-strain graph for rubber.*

Figure 7: *Rubber is used in the manufacture of tyres. Some of the energy transferred to the tyres by bumps in the road is transferred to thermal energy, making the ride less bouncy.*

Rubber returns to its original length when the load is removed — it behaves elastically.

The loading and unloading curves for rubber are different. The energy released when the rubber is unloaded is less than the work done to stretch the rubber. This is because some of the elastic potential energy stored in the stretched rubber is converted to heat.

The amount of energy converted to heat per unit volume is given by the area between the loading and unloading curves.

Tip: Try repeatedly stretching and releasing a rubber band. You should feel it get hotter.

Material properties

It's useful to be able to describe how materials behave under stress so that materials with different properties can be easily compared in areas such as construction and manufacturing. For example, a material that can withstand high stress before deforming may be desirable when building the foundations for a house, but you wouldn't use it for stuffing soft furnishings.

Exam Tip
You might be asked about why a given material is appropriate, or inappropriate, for a specific purpose. Think carefully about the properties of the material and how it might behave under the forces required.

You've already seen that if the stress on a material becomes too great, the material can break (p.122). The **strength** of a material is a measure of its ability to withstand stress without breaking. The stronger the material, the higher its ultimate tensile strength. Materials with a very low ultimate tensile strength are described as being weak.

Stiffness is a measure of how much a material can resist deformation in response to stress. Stiff materials are difficult to stretch or compress, so they have a large Young's modulus. For a given stress, a stiff material will have a lower strain (i.e. a smaller extension) than a less stiff material.

A stiff material doesn't have to be strong (and vice versa). Some stiff materials, such as polystyrene, break under a low stress, and some strong materials aren't very stiff.

Figure 9 shows the stress-strain curves for materials of different strengths and stiffnesses.

Exam Tip
When a line on a stress-strain graph just stops, you can assume the material has reached its breaking stress and fractured (unless the question says otherwise).

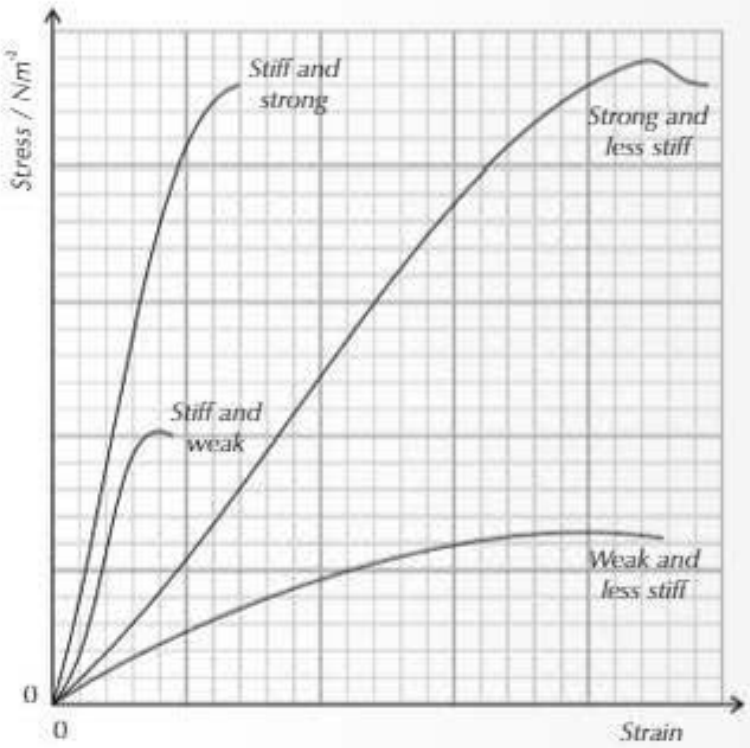
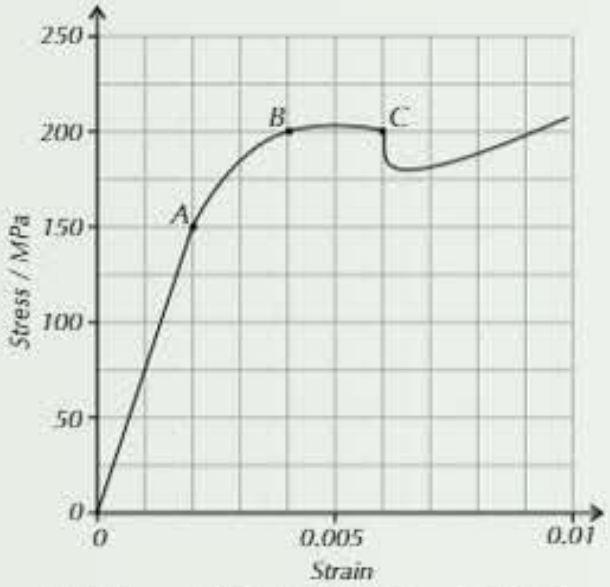


Figure 9: *Stress-strain curves for different materials.*

Stiff materials have a steep gradient on a stress-strain graph, as they show very little strain under stress, while less stiff materials have a shallower gradient. Stress-strain curves for strong materials end at a higher stress than weak materials.

Practice Questions — Application

Q1 The diagram below shows a stress-strain graph for a material with three important points marked on it.



a) After point B the material starts to deform plastically. Give the name of point B.

b) What law does the material obey between the origin and point A?

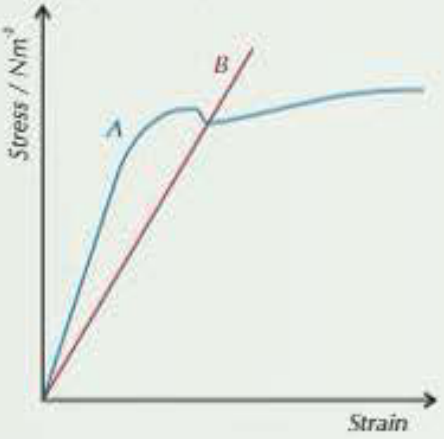
c) Which point on the graph marks the yield point of the material?

d) Find the Young modulus of the material.

e) Calculate the elastic potential energy stored per unit volume in the material up to the limit of proportionality.

Q2 The graph to the right is a stress-strain graph for two different materials.

Which line, A or B, shows a brittle material? Explain your answer.



Tip: Think about the features of both lines.

Practice Questions — Fact Recall

Q1 What is the yield point of a material?

Q2 Give one example of a polymeric material.

Q3 Do brittle materials obey Hooke's law? Explain your answer.

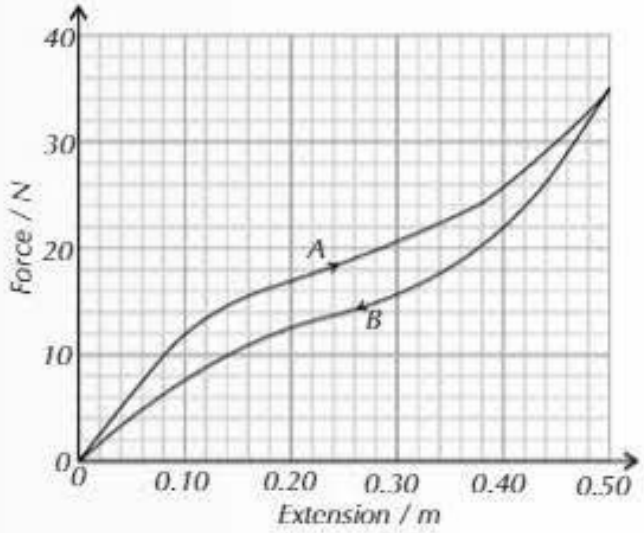
Section Summary

Make sure you know...

- What is meant by the terms extension and compression.
- What is meant by the terms tensile deformation and compressive deformation.
- That Hooke's law states that the extension of an object is proportional to the load or force applied to it.
- How to use the formula for Hooke's law, $\boldsymbol{F} = k\boldsymbol{x}$, where k is the force constant of an object.
- That the force constant, k, is the force needed to extend an object that obeys Hooke's Law by 1 m, and it has units of Nm^{-1}.
- How to combine the force constants of individual objects in series or parallel to find the overall force constant of the system.
- That the limit of proportionality of an object is the maximum force that can be applied to an object before it no longer obeys Hooke's Law.
- How to investigate the force-extension characteristics for objects (or arrangements of objects), which may include springs, rubber bands and polythene strips.
- How to plot and interpret force-extension (or compression) graphs for springs and wires.
- That the elastic limit of an object is the maximum force that can be applied to an object before it permanently changes shape.
- That if a material deforms elastically, it will return to its original shape when the force applied to it is removed, but if it deforms plastically, it will permanently change shape.
- That the area under a force-extension (or compression) graph is equal to the work done.
- That all the work done stretching a material elastically is stored as elastic potential energy.
- How to calculate elastic potential energy using $E = \frac{1}{2}\boldsymbol{F}\boldsymbol{x}$ and $E = \frac{1}{2}k\boldsymbol{x}^2$.
- That the tensile stress of a material, σ, is measured in Nm^{-2} or Pa and is defined as the force applied to the material, $\boldsymbol{F}$, divided by its cross-sectional area, A.
- That the tensile strain of a material, ε, has no units and is defined as the extension of the material, $\boldsymbol{x}$, divided by its original length, L.
- That breaking stress is the stress experienced by a material at the point it breaks.
- That the ultimate tensile strength of a material is the maximum stress a material can experience before it will break.
- That the Young modulus of a material is the ratio of stress to strain, and is characteristic of that material.
- How to carry out an investigation to find the Young modulus of a material.
- How to calculate the Young modulus for materials from their stress-strain graphs.
- That, up to the elastic limit, the area under a stress-strain graph is equal to the material's elastic potential energy per unit volume.
- That the energy per unit volume for a deformed object can be calculated up to its limit of proportionality using the formula: energy per unit volume = $\frac{1}{2}\sigma\varepsilon$
- That a ductile material is a material which maintains its strength when it is deformed, and how to interpret the stress-strain graph of a typical ductile material.
- That a brittle material is a material which breaks before it can undergo plastic deformation, and how to interpret the stress-strain graph of a typical brittle material.
- How to interpret typical stress-strain graphs of polymeric materials such as polythene and rubber.
- That the strength of a material is a measure of how large a stress it can withstand before breaking.
- That the stiffness of a material is a measure of how resistant it is to deformation while under stress.

Exam-style Questions

1 The diagram below shows a force-extension graph for a rubber cord. Curve *A* shows loading and curve *B* shows unloading of the cord.



The rubber cord has a cross-sectional area of 5.0×10^{-6} m^2 and had an initial length of 0.80 m.

What is the Young modulus for the material at an extension of 0.080 m? Assume that curve *A* is linear for small loads.

A 1.2×10^5 Pa

B 2.0×10^5 Pa

C 1.2×10^7 Pa

D 2.0×10^7 Pa

(1 mark)

2 A spring is used in a pen as part of the mechanism which opens and closes it. The spring is compressed elastically by 5.0 mm when a user pushes it down with a force of 1.1 N. Calculate the elastic potential energy stored by the spring when it is compressed by 18 mm.

A 0.036 J

B 7.1 J

C 36 J

D 0.071 J

(1 mark)

3 A piano manufacturer wants to find the Young modulus of a sample of piano wire.

(a)* Describe an experimental method that the piano manufacturer could use to find the Young modulus of the piano wire.
Your answer should include:

- a labelled diagram
- a discussion of ways to reduce the effect of errors on the results

(6 marks)

(b) **Fig. 3.1** shows the stress and strain on the piano wire as it was stretched.

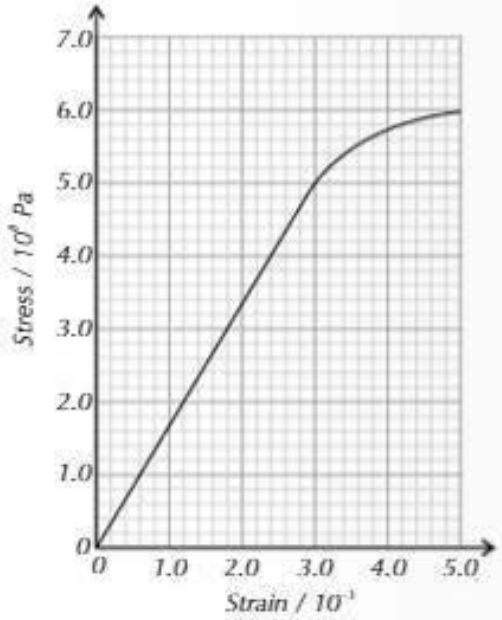


Fig. 3.1

Use the graph to calculate the Young modulus of the wire.
Give your answer in an appropriate unit.

(2 marks)

(c) If the wire has an initial length of 1.6 m and a cross-sectional area of 1.0×10^{-7} m², calculate how much elastic potential energy is stored by the wire at a stress of 3.0×10^{8} Pa.

(3 marks)

4 A chandelier is used as part of the scenery for a stage show.

(a) The chandelier is suspended from the ceiling above the stage by a 2.0 m steel cable. The tension in the cable will be 2.0 kN and it must not extend more than 0.20 mm. The Young modulus for steel is 2.10×10^{11} Pa.
Calculate the minimum cross-sectional area the cable should have, in m².

(2 marks)

(b) As part of the show, the chandelier falls to the floor and shatters.
The chandelier is made from a brittle material that will break under a low stress.
The Young modulus for the material is 6.0×10^{10} Nm⁻², and it has a breaking stress of 3.0×10^{7} Nm⁻². Draw the stress-strain graph for this material.

(3 marks)

* The quality of your response will be assessed in this question.

1. Newton's Laws of Motion

Newton's laws of motion describe the relationship between the forces acting on an object and its motion. You might have already met these ideas at GCSE — they're really important in mechanics, so they crop up a lot in physics.

Learning Objectives:
- Know Newton's three laws of motion.
- Know that net force is equal to the rate of change of momentum, $F = \frac{\Delta p}{\Delta t}$.

Specification Reference 3.5.1

Newton's 1st law of motion

Newton's 1st law of motion states that:

"The velocity of an object will not change unless a resultant force acts on it."

This means a body will stay still or move in a straight line at a constant speed, unless there's a resultant force acting on it. If the forces acting on a body aren't balanced, the overall resultant force will make the body accelerate. This could be a change in direction, speed, or both (see Newton's 2nd law below).

Example

An apple sitting on a table won't go anywhere because the forces on it are balanced.

reaction (*R*) (force of table pushing apple up) = **weight** (*mg*) (force of gravity pulling apple down)

Figure 1: *Sir Isaac Newton, the British physicist who devised the three laws of motion still used in modern mechanics.*

Newton's 2nd law of motion

Newton's 2nd law of motion says that the rate of change of momentum of an object is equal to the net force which acts on the object.
This can be written as the equation:

F = *net force (in N)* → $F = \frac{\Delta p}{\Delta t}$ ← $\frac{\Delta p}{\Delta t}$ = *rate of change of momentum (in kg ms^{-1})*

Try to remember:
- The net force is the vector sum of all the forces (page 79).
- The net force is always measured in newtons.
- The mass is always measured in kilograms.
- The change in momentum is always in the same direction as the net force and is measured in kg ms^{-1}.

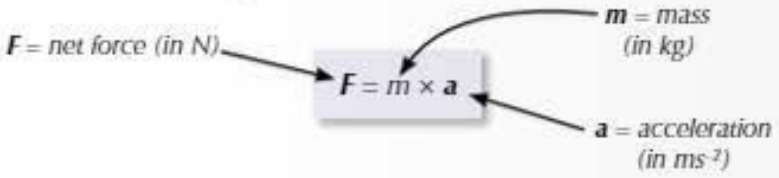
Tip: There's more on this equation on p.142.

Exam Tip
This equation crops up all over the place in physics, so make sure you know how to use it.

Tip: Momentum (p) is covered on page 138.

Tip: Momentum depends on mass and velocity (p.138). As long as v isn't close to the speed of light, an object's mass is constant so momentum only depends on velocity.

In the special case where mass is constant, Newton's 2nd Law can be written as the well-known equation:



This equation says that the more force you have acting on a certain mass, the more acceleration you get. It also says that for a given force, the more mass you have, the less acceleration you get.

But, if the mass of the object is changing, such as if it is accelerating at close to the speed of light, then you can't use $F = ma$. (You don't need to know why the mass changes in this case.) Newton's 2nd law still applies, it's just that the 'rate of change of momentum' bit refers to a change in mass and velocity.

Example **Maths Skills**

Zola is playing hockey. The ball is coming towards her at a speed of 4.6 ms^{-1}. She hits it so that it travels in the opposite direction at a speed of 10.2 ms^{-1}. Her stick is in contact with the ball for 0.84 seconds. The ball has a mass of 161 g.
Calculate the average force exerted on the ball during this time.

Tip: The ball reverses direction, so you need to give one of the velocities a negative value. Choose whichever makes the maths easier.

You'll need to use $F = ma$, so first calculate a:

$$a = \frac{\Delta v}{\Delta t} = \frac{10.2 - (-4.6)}{0.84} = \frac{14.8}{0.84} = 17.619... \text{ ms}^{-2}$$

$m = 161 \text{ g} = 0.161 \text{ kg}$, so:

$F = 0.161 \times 17.619... = 2.836... = 2.84 \text{ N}$ (to 3 s.f.)

Newton's 3rd law of motion

There are a few different ways of stating **Newton's 3rd law**, but the clearest way is:

"If object A exerts a force on object B, then object B exerts an equal but opposite force on object A."

You'll also hear the law as "every action has an equal and opposite reaction". But this can wrongly sound like the forces are both applied to the same object. (If that were the case, you'd get a resultant force of zero and nothing would ever move anywhere.)

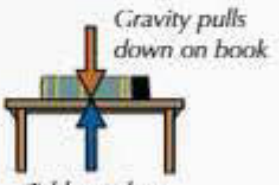
The two forces actually represent the same interaction, just seen from two different perspectives:

- If you push against a wall, the wall will push back against you, just as hard. As soon as you stop pushing, so does the wall.
- If you pull a cart, whatever force you exert on the rope, the rope exerts the exact opposite pull on you.
- When you go swimming, you push back against the water with your arms and legs, and the water pushes you forwards with an equal-sized force.

Figure 2: *A swimmer moves forwards because the water pushes back against them, as they push against the water.*

Newton's 3rd law applies in all situations and to all types of force. But the pairs of forces are always the same type, e.g. both gravitational or both electrical. Sometimes it looks like Newton's 3rd law is being applied, but it's not.

Example



Both forces are acting on the book, and they're not of the same type. These are two separate interactions. The forces are equal and opposite, resulting in zero acceleration, so this is an example of Newton's First Law.

Tip: In this example, the resultant force is zero and the acceleration is zero, which is an example of Newton's Second Law.

Practice Questions — Application

Q1 Draw a diagram showing what forces are acting on a ball when it's at rest on the ground.

Q2 A rocket burns through its fuel stores to accelerate upwards. Explain why you cannot use $F = ma$ to calculate the net force on the rocket in this case.

Q3 Why does a bird lift into the air when it flaps its wings?

Q4 A car is pulling a caravan behind it at a constant speed. The additional resistive force acting on the system due to the caravan at this speed is 780 N. The link between the car and the caravan breaks and they separate. The car immediately begins to accelerate at 0.41 ms^{-2}. Calculate the mass of the car.

Q5 Two ice skaters of mass 55.0 kg and 60.0 kg push against each other. The heavier ice skater accelerates away at 2.3 ms^{-2}. What will the magnitude of the lighter ice skater's acceleration be?

Exam Tip
Using ice skaters is a common way of saying there's no friction involved — the only force is from where they push against each other.

Practice Question — Fact Recall

Q1 State Newton's three laws of motion, and briefly explain what they mean.

Learning Objectives:

- Be able to calculate linear momentum; $p = mv$.
- Understand the vector nature of momentum.
- Know the principle of conservation of momentum.
- Be able to solve collision problems in one dimension and in two dimensions.
- Understand the meaning of a perfectly elastic collision and an inelastic collision.

Specification References 3.5.1 and 3.5.2

Tip: The other kind of momentum is 'angular momentum', but you don't need to know about that for now.

Tip: See page 94 for more on density.

Tip: In the example the positive direction was taken to be the direction in which the balloon was thrown. If you took the opposite direction as the positive, the balloon would have a negative momentum.

Figure 1: *Linear momentum is conserved in a Newton's cradle. When the first ball strikes and stops, the last ball moves off with an equal momentum.*

2. Momentum

Momentum is how much 'oomph' an object has, and the direction in which the 'oomph' acts. When objects collide, their overall momentum is conserved.

What is momentum?

The **linear momentum** of an object depends on two things — its mass and velocity. The product of these two values is the momentum of the object.

p = *linear momentum in* $kg\,ms^{-1}$ → $p = m \times v$ ← v = *velocity in* ms^{-1}

m = *mass in kg*

The word 'linear' means that the momentum acts in a straight line. You'll often see it just called momentum. Momentum and velocity are vectors (see page 41), so you need to remember to think about direction when doing calculations.

Example — **Maths Skills**

A water balloon of volume 4.2×10^{-3} m^3 is thrown at a speed of 8.5 ms^{-1}. If water has density 1.0×10^3 $kg\,m^{-3}$ and the rubber balloon itself has mass 12 g, calculate the balloon's total momentum.

Before you can calculate the balloon's total momentum, you need to work out the mass of the water.

Rearranging $\rho = \frac{m}{V}$:

$m = \rho V = 1.0 \times 10^3 \times 4.2 \times 10^{-3} = 4.2$ kg

Linear momentum = mass × velocity

= (mass of water + mass of balloon) × velocity

$= (4.2 + 0.012) \times 8.5 = 35.802 = 36$ $kg\,ms^{-1}$ (to 2 s.f.)

The principle of conservation of momentum

Assuming no external forces act, linear momentum is always conserved. This means the total linear momentum of two objects before they collide equals the total linear momentum after the collision. This can be used to work out the velocity of objects after a collision.

Example — **Maths Skills**

A skater of mass 75 kg and velocity 4.0 ms^{-1} collides with a stationary skater of mass 55 kg. The two skaters join together and move off in the same direction. Calculate their velocity v after impact.

Figure 2: *The skaters, before and after the collision.*

Momentum before = Momentum after

$(75 \times 4.0) + (55 \times 0) = 130v$

$300 = 130v$

So $v = 2.307...$

$= 2.3$ ms^{-1} (to 2 s.f.)

The same principle can be applied in situations that don't involve a collision, like explosions. For example, if you fire an air rifle, the forward momentum gained by the pellet is equal in magnitude to the backward momentum of the rifle, and you feel the rifle recoiling into your shoulder.

Example — **Maths Skills**

A bullet of mass 0.0050 kg is shot from a rifle at a speed of 220 ms^{-1}. The rifle has a mass of 4.0 kg. Calculate the velocity at which the rifle recoils.

Figure 3: *The rifle and bullet after the explosion.*

Momentum before explosion = Momentum after explosion

$$0 = (0.0050 \times 220) + (4.0 \times v)$$
$$0 = 1.1 + 4.0v$$
$$v = -0.275 = -0.28 \text{ ms}^{-1} \text{ (to 2 s.f.)}$$

Tip: There was no momentum before the explosion, so after the explosion both the rifle and the bullet must have the same magnitude of momentum, but in opposite directions. The rifle has a much smaller final velocity than the bullet because it's got a much greater mass.

In reality, collisions and explosions usually happen in more than one dimension. In two-dimensional collisions, momentum is conserved in both dimensions. You can solve two-dimensional collision problems by resolving vectors (see page 43).

Example — **Maths Skills**

Ball A collides with stationary ball B, as shown in Figure 4. After the collision, the two balls move off in different directions. Ball A has a mass of 40 g. Calculate the mass, *m*, of ball B.

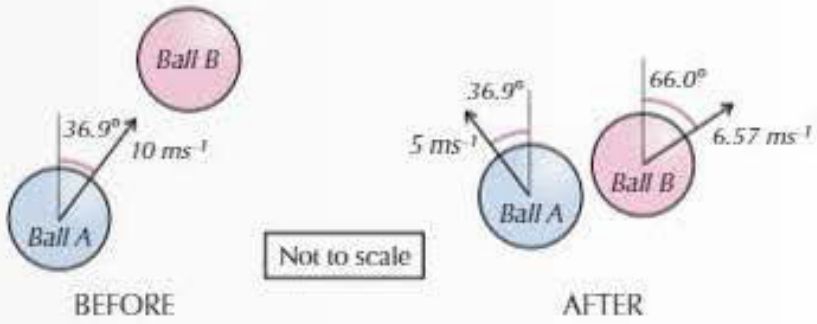


Figure 4: *Balls A and B before and after the collision.*

Exam Tip
Two-dimensional problems like this will only be in your exam if you're taking the full A-level course.

You can work this out using conservation of momentum in just the horizontal direction (or just the vertical). Start by picking the positive direction — let's say right is positive.

horizontal momentum before the collision
= horizontal momentum after the collision

$$(0.04 \times 10 \sin 36.9°) + (m \times 0) = -(0.04 \times 5 \sin 36.9°) + (m \times 6.57 \sin 66.0°)$$

$$\text{so, } m = \frac{0.04 \times (10 \sin 36.9° + 5 \sin 36.9°)}{6.57 \sin 66.0°} = 0.0600... \text{ kg}$$
$$= 60 \text{ g (to 2 s.f.)}$$

Tip: The horizontal component of the momentum of ball A is negative after the collision — it's to the left.

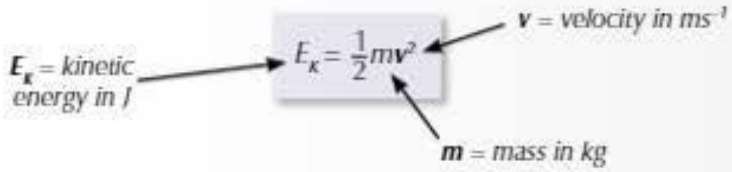
Tip: You could check your answer by doing the same calculation for the vertical direction.

Elastic and inelastic collisions

An **elastic collision** is one where momentum is conserved and **kinetic energy** is conserved — i.e. no energy is dissipated as heat, sound, etc. Kinetic energy is the energy that an object has due to its motion. You saw on page 105 that the equation for kinetic energy is:

Tip: Elastic collisions are sometimes called 'perfectly elastic'.

Tip: Collisions between gas particles are elastic, otherwise air would get colder and colder until there was no motion left.



Example — Maths Skills

A bowling ball of mass 5.00 kg is travelling at a velocity of 5.00 ms^{-1} when it collides with a stationary bowling ball of mass 3.00 kg. The velocity of the lighter ball after the collision is 6.25 ms^{-1} in the same direction as the heavier ball before the collision. Show that the collision is elastic.

Figure 5: *The balls, before and after the collision.*

Momentum is conserved, so momentum before = momentum after.

$$(5.00 \times 5.00) + (3.00 \times 0) = (5.00 \times \boldsymbol{v}) + (3.00 \times 6.25)$$
$$25 = 5\boldsymbol{v} + 18.75$$
$$\boldsymbol{v} = 1.25 \text{ ms}^{-1}$$

Check to see if kinetic energy is conserved:

Kinetic energy before = $(\frac{1}{2} \times 5.00 \times 5.00^2) + (\frac{1}{2} \times 3.00 \times 0^2) = 62.5$ J

Kinetic energy after = $(\frac{1}{2} \times 5.00 \times 1.25^2) + (\frac{1}{2} \times 3.00 \times 6.25^2) = 62.5$ J

So total kinetic energy is conserved in the collision — therefore this is an elastic collision.

Exam Tip
Before you start a momentum calculation, always draw a quick sketch of the relevant objects, before and after the collision — then it's much easier to figure out what's going on.

Figure 6: *A dropped ball loses energy to heat and sound when it collides inelastically with the floor, and rebounds to a lower height.*

If a collision is **inelastic**, it means that some of the kinetic energy is converted into other forms during the collision. Linear momentum is always conserved in inelastic collisions though.

Example — **Maths Skills**

A toy lorry (mass 2.00 kg) travelling at 3.00 ms^{-1} crashes into a toy car (mass 0.800 kg), travelling in the same direction at 2.00 ms^{-1}. The velocity of the lorry after the collision is 2.60 ms^{-1} in the same direction. Calculate the new velocity of the car and the total kinetic energy before and after the collision.

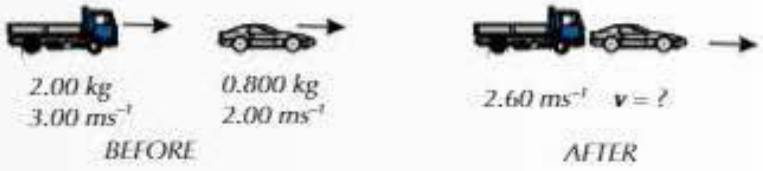


Figure 8: *The toy lorry and car, before and after the collision.*

Momentum is conserved, so:
Momentum before collision = Momentum after collision

$$(2.00 \times 3.00) + (0.800 \times 2.00) = (2.00 \times 2.60) + (0.800 \times \boldsymbol{v})$$

$$7.60 = 5.20 + 0.800\boldsymbol{v}$$

$$2.40 = 0.800\boldsymbol{v}$$

$$\boldsymbol{v} = 2.40 \div 0.800 = 3.00 \text{ ms}^{-1}$$

Kinetic energy before = E_k of lorry + E_k of car
$= \frac{1}{2}m\boldsymbol{v}^2$ (lorry) + $\frac{1}{2}m\boldsymbol{v}^2$ (car)
$= \frac{1}{2}(2.00 \times 3.00^2) + \frac{1}{2}(0.800 \times 2.00^2)$
$= 10.6$ J

Kinetic energy after = $\frac{1}{2}(2.00 \times 2.60^2) + \frac{1}{2}(0.800 \times 3.00^2)$
$= 10.36 = 10.4$ J (to 3 s.f.)

The difference in the two values is the amount of kinetic energy dissipated as heat, sound, or in damaging the vehicles — so this is an inelastic collision.

Figure 7: *In the real world, most collisions are at least slightly inelastic. These billiard balls will lose energy as sound when they collide.*

Practice Questions — Application

Q1 A man sitting in a stationary boat throws a 1.0 kg rock horizontally out of the boat at 10 ms^{-1}. If the total mass of the man and boat is 125 kg, how fast will the boat move in the opposite direction to the rock?

Q2 An ice hockey puck of mass 165 g has a velocity 2.25 ms^{-1} (in the positive direction) when it collides with an identical puck moving at 4.75 ms^{-1} in the opposite (negative) direction. If the first puck has a final velocity of −4.25 ms^{-1}, calculate the final velocity of the second puck. Is this an elastic or inelastic collision?

Q3 The nozzle of a fire hose has cross-sectional area 5.6×10^{-4} m^2 and shoots water at a rate of 8.4×10^{-3} m^{3}s^{-1}. Find the momentum of the water leaving the hose, if water has a density of 1000 kgm^{-3} (to 2 s.f.).

Tip: If you're struggling with Q3, first find the mass of water that leaves the hose every second, then find how fast the water leaves the nozzle.

Practice Questions — Fact Recall

Q1 What is the equation used to calculate linear momentum?

Q2 What is the principle of conservation of momentum?

Q3 What is the difference between a perfectly elastic and an inelastic collision?

Learning Objectives:

- Know what is meant by the impulse of a force and be able to calculate it using: impulse = $F\Delta t$.
- Know that impulse is equal to the area under a force-time graph.

Specification Reference 3.5.1

3. Impulse and Vehicle Safety

Impulse is just a word for the change in momentum of a body. You can work it out if you know the average force acting on a body and the time it acts over.

Calculating impulse

Newton's Second Law can be rearranged to give:

$$F\Delta t = \Delta p$$

The **impulse** of a force is defined as the product of average force and time:

$$\text{impulse} = F\Delta t$$

So, the impulse on a body is equal to the change in momentum of that body.

$F\Delta t$ = *impulse in Ns* → $F\Delta t = \Delta p$ ← Δp = *change in momentum in kg ms*$^{-1}$

Tip: Impulse is only talking about the change of momentum of one of the objects, whilst conservation of momentum applies to the whole system. So the impulse of an object can change although momentum is conserved.

Example — **Maths Skills**

A stationary golf ball of mass 45 g is hit with a club. The club provides an average force of 1200 N on the ball, causing the ball to travel at a speed of 25 ms^{-1}. Calculate the time the club was in contact with the ball for.

First calculate the change in momentum of the ball:

$\Delta p = \Delta(mv) = m\Delta v = 0.045 \times (25 - 0) = 1.125$ kg ms^{-1}

Rearranging $F\Delta t = \Delta p$:

$$\Delta t = \frac{\Delta p}{F} = \frac{1.125}{1200} = 0.0009375 = 0.94 \text{ ms (to 2 s.f.)}$$

Impulse is the area under a force-time graph — this is really handy for solving problems where the force changes.

Tip: For improved accuracy, you could use a spreadsheet to plot your F–t graph and calculate the area underneath it.

Example — **Maths Skills**

The graph shows the resultant force acting on a toy car. If the car is initially at rest, calculate the impulse given to the car over the first 3 seconds.

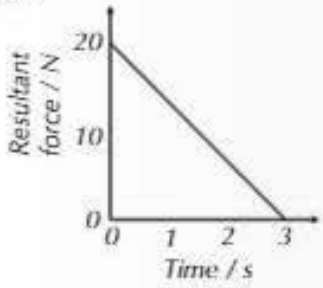


Figure 1: *Graph of resultant force against time.*

Impulse is the area under the graph, so you need to find the area under the graph between 0 and 3 s.

Area under graph = $\frac{1}{2} \times 3 \times 20 = 30$ Ns

For more complicated force-time graphs, you might need to estimate the area under the graph by counting squares or approximating curves as straight lines (see page 478).

Impact forces

Since force is equal to rate of change of momentum — remember $F = \frac{\Delta p}{\Delta t}$, the force of an impact can be reduced by increasing the time of the impact. Similarly, the force can be increased by reducing the impact time.

Examples

- The less time your foot is in contact with a football when kicking it, the more force you will kick it with (assuming the change in momentum is the same).
- Packaging for fragile objects is designed to reduce the magnitude of any impact forces during transport or storage. The packaging takes longer to deform on impact, and so absorbs the shock and protects the packaged object.

Figure 2: *Egg boxes are designed to reduce the impact on the eggs if they're dropped. The packaging crumples, increasing the impact time.*

Tip: The same is true in many other sports — for example, the shorter the impact time between a bat and ball, the greater the force.

Vehicle design

When a car crashes, there is a big change in speed — possibly from 70 mph (about 110 km h^{-1}) to zero. This means there is a big change in momentum, and potentially large forces, which can injure passengers. Modern vehicle safety features are designed using the fact that the force of an impact can be reduced by increasing the time of the impact.

Examples

- **Crumple zones**
  These are areas at the front and rear of a car that are designed to crumple on impact. They absorb some of the car's kinetic energy when they deform, which would otherwise be transferred to the passengers and whatever the car had collided with. They also increase the time taken for the car to slow down, which reduces the forces acting on passengers and whatever the car has hit.
- **Seat belts**
  Seat belts are designed to stretch slightly. They protect wearers in a crash in a number of ways. They hold the wearer in place in the car, stopping them from being thrown from their seats. They also absorb some of the wearer's kinetic energy by stretching and increase the time that the wearer comes to a stop over, Δt (again, by stretching).
- **Air bags**
  Air bags are 'cushions' in the dashboard and elsewhere that inflate very quickly on collision. They protect the passengers by making passengers slow down more gradually (increasing Δt) and by stopping passengers from hitting the dashboard, steering wheel, etc. during a crash.

Tip: Understanding the forces involved in road collisions has saved many lives — it's estimated that the three-point seat belt alone has saved over a million lives worldwide in the last 50 years.

Figure 3: *Cars have specially designed 'crumple zones' that extend the time of a collision. This reduces the force experienced by the car and occupants.*

Tip: There's more about material deformation over on page 118.

Example — **Maths Skills**

Jian's car bumps into the back of a stationary bus. The car was travelling at 2 ms^{-1} and comes to a stop in 0.20 s. Jian was wearing his seatbelt and takes 0.80 s to stop. Jian's mass is 75 kg.

a) Calculate the average force acting on Jian during the accident.

$F = \frac{\Delta p}{\Delta t}$ So, for Jian: $F = \frac{(75 \times 2) - (75 \times 0)}{0.80} = 187.5$
$= 190$ N (to 2 s.f.)

b) Calculate the average force that would have acted on Jian if he had stopped as quickly as the car.

Again, $F = \frac{\Delta p}{\Delta t}$ but $\Delta t = 0.20$ s. So: $F = \frac{(75 \times 2) - (75 \times 0)}{0.20}$
$= 750$ N

Figure 4: *Crash test dummies are used in simulations to model the effects of a collision on a human body.*

Early cars didn't have many safety features in the event of a crash. But as cars became faster and the number of cars on the roads increased, car manufacturers started to think about how to make cars safer. This led to the development of the three-point seat belt, crumple zones and air bags back in the 1950s.

At the same time, engineers were developing better ways of understanding the forces acting on passengers in a collision. They developed increasingly sophisticated crash test dummies that could measure all the forces that different passengers experience in different kinds of collisions. Crash tests also allowed car manufacturers to test the effectiveness of certain safety features, so they could see how much they reduced the likelihood of injury.

Wearing a seat belt is now mandatory (everyone has to do it), and features such as crumple zones and air bags have become standard features of modern cars (along with many more sophisticated safety features).

The risks of car safety features

Figure 5: *A young girl using a booster seat to reduce the risks posed by her seatbelt.*

Seat belts can cause bruising during a crash. They can also be dangerous for small children — if the top part of the belt lies across the child's neck this can cause injury in a crash, and if the child is too small for a seat belt it may not secure them properly. In the UK, children must use booster seats or cushions to reduce these risks.

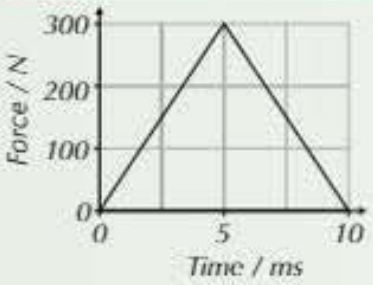
Air bags are designed for use with seat belts and can be dangerous if you're not wearing one. Air bags inflate very rapidly, with a lot of force. If a passenger seat belt isn't secured properly then the passenger can keep moving forwards quickly as the car slows down. They could hit the air bag as it is inflating with a force big enough to cause injury.

Air bags are also dangerous when using rear-facing child seats — the air bag inflates behind the child and can throw the child seat towards the car seat with some force. It is now illegal to use a rear-facing child seat in the UK in a seat fitted with an air bag.

Most of these risks are caused by not using a car's safety features properly. If you use a seat belt and air bags as you are meant to, you are far safer in a car than you would be without them.

Practice Questions — Application

Q1 The figure on the right shows a graph of force against time for a tennis ball during contact with a racket. Calculate the magnitude of the impulse acting on the ball.



Q2 Until the 1950s, cars were manufactured to be uniformly rigid as it was generally thought that this was safer. Explain why this is incorrect.

Tip: Rigid materials are stiff and difficult to deform.

Q3 A crash test is conducted on a new car model to test the effectiveness of its safety features. A crash test dummy is placed in the driver's seat and the car is driven into a concrete wall.

a) A seat belt was fastened over the dummy before the test. Explain how the seat belt reduces the forces acting on the dummy during the crash.

b) Explain why airbags can be dangerous for passengers when used without a seat belt.

Q4 A landing aeroplane of mass 18 000 kg touches down on a runway at a velocity of 125 $km\,h^{-1}$. The brakes are applied and a resultant horizontal force of 62 000 N acts on the aeroplane. Find the time it takes for the aeroplane to come to a stop.

Tip: The mass before and after is the same, so the velocity is the only thing that changes.

Practice Questions — Fact Recall

Q1 What is impulse?

Q2 Describe how airbags protect passengers in the event of a crash.

Q3 Describe the risks associated with seat belts.

Q4 Explain how crumple zones work.

Section Summary

Make sure you know...

- That the velocity of an object will not change unless a resultant force acts on it (Newton's First Law).
- That net force is equal to the rate of change of momentum (Newton's Second Law).
- How to calculate net force, $\boldsymbol{F}$, from change in momentum, $\Delta\boldsymbol{p}$, and change in time, Δt, using $\boldsymbol{F} = \frac{\Delta \boldsymbol{p}}{\Delta t}$.
- That $\boldsymbol{F} = m\boldsymbol{a}$ is a special case of Newton's Second Law, and can only be applied when the body's mass is constant.
- That if object A exerts a force on object B, then object B exerts an equal but opposite force on object A (Newton's Third Law).
- That the forces exerted by objects A and B in Newton's Third Law are always of the same type.
- That linear momentum is the product of mass and velocity.
- That momentum is a vector quantity and so has a magnitude and direction.
- That momentum is always conserved.
- How to use the principle of conservation of momentum to solve one-dimensional and two-dimensional collision problems.
- That kinetic energy is conserved in elastic collisions but not inelastic collisions.
- That the impulse of a force is defined as the product of average force and time.
- That impulse is equal to the area under a force-time graph.
- How to estimate the area under non-linear force-time graphs.
- What crumple zones are, and how they reduce the forces acting on passengers in a collision.
- How seat belts protect wearers and the risks associated with them.
- How airbags protect passengers and the risks associated with them.
- That the introduction of mandatory safety features in cars resulted from research into reducing the forces involved in collisions, and therefore reducing the risk of injury.

Exam-style Questions

1 A ball is shot from a cannon. The ball has a much smaller mass than the cannon. Which combination of the following statements is true?

1. Kinetic energy is conserved when the ball is launched.
2. The momentum of the cannon is equal in magnitude to that of the ball.
3. The cannon moves backwards at the same speed that the ball moves forwards.

A All three statements are true.
B Only 1 and 2 are true.
C Only 2 is true.
D Only 1 and 3 are true.

(1 mark)

2 Which pair of forces below are an example of Newton's Third Law?

A When the end of a spring is pulled downwards, the spring extends downwards.
B When a car is being towed by a lorry, the lorry pulls forwards on the car and friction pulls backwards on the car.
C When a cat jumps, the earth pulls down on the cat and the cat pulls up on the earth.
D When a child moves down a slide, the earth's gravity pulls the child down and the slide pushes up on the child.

(1 mark)

3 A ball of mass 0.25 kg is travelling with velocity 1.2 ms^{-1}. It collides with a second ball that is travelling towards it at 0.3 ms^{-1}. The mass of the second ball is half that of the first ball. After the collision, the two balls move together with the same velocity. How fast are they moving?

A 0.45 ms^{-1}
B 0.70 ms^{-1}
C 0.75 ms^{-1}
D 0.90 ms^{-1}

(1 mark)

4 A 1.00×10^3 kg car is participating in a race. During a long stretch of straight track, it applies a force of 1.70×10^3 N to accelerate from 26.5 ms^{-1} to 31.0 ms^{-1} in 2.50 s.

(a) State Newton's Second Law.

(1 mark)

(b) During the race, the back bumper, with mass 50.0 kg, falls off the car. When it reaches the straight stretch of track again, the same force again applied again over 2.50 s to increase its speed from 26.5 ms^{-1}. Calculate the speed the car reaches.

(2 marks)

5 At a fairground, a dodgem car of total mass 325 kg travelling at 2.40 ms^{-1} collides loudly with the wall of the dodgem arena, and comes to a complete stop. The wall is fixed and does not move.

(a) Is this an elastic or inelastic collision? Explain your answer.

(2 marks)

(b) Calculate the magnitude of the impulse that acts on the dodgem.

(2 marks)

(c) Calculate what the impulse would have been if the dodgem had been travelling at double the speed.

(1 mark)

(d) Two dodgems of equal mass collide in the arena. **Fig. 5.1a** and **Fig. 5.1b** show the dodgems before and after the collision respectively.

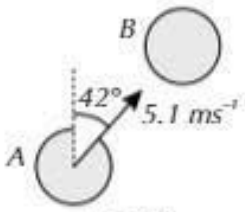


Fig 5.1a

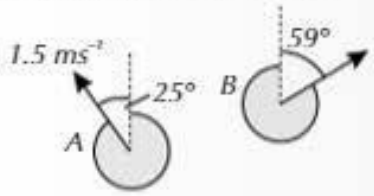


Fig 5.1b

Calculate the velocity of dodgem B after the collision.

(2 marks)

6 A force is applied to a stationary trolley to push it across a horizontal floor. **Fig. 6.1** shows the force applied to the trolley against time.

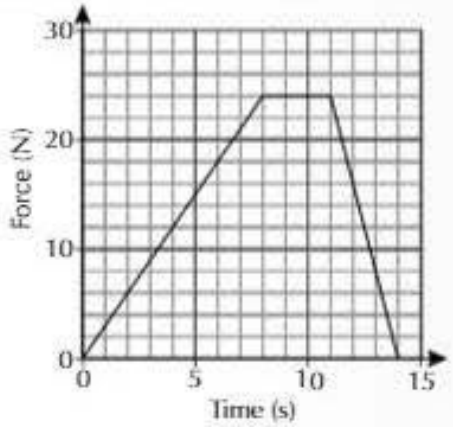


Fig 6.1

(a) Calculate the magnitude of the impulse that acts on the trolley.

(2 marks)

(b) The trolley reaches a velocity of 2.2 ms^{-1} after the force is applied. Assuming friction is negligible, calculate the mass of the trolley.

(2 marks)

(c) Explain why the trolley continues to move at a constant velocity after the force has been removed.

(1 mark)

1. Circuit Diagrams

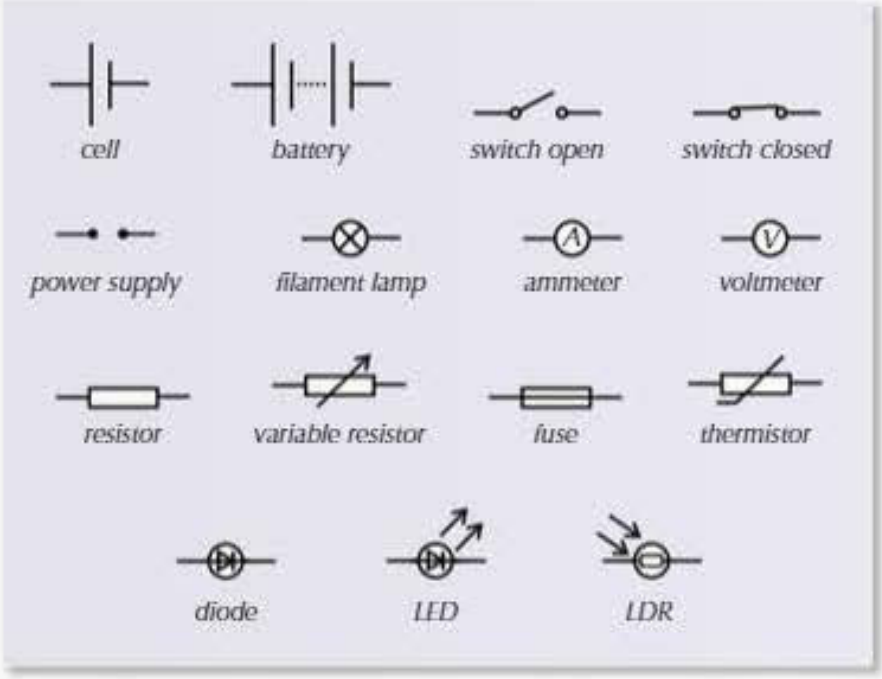
Module 4: Section 1 is all about electricity. Before we start going into all the detail, it's important that you can understand circuit diagrams.

Learning Objectives:
- Understand circuit symbols.
- Be able to draw circuit diagrams using circuit symbols.

Specification Reference 4.2.1

Circuit symbols

In physics, we use circuit symbols to represent different electrical components. Here are some of the basic ones you should recognise...



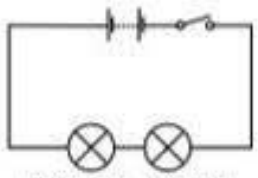


Figure 1: *A simple circuit diagram.*

Figure 2: *An electronic circuit. The red dome-shaped components to the right of the picture are LEDs.*

Drawing circuits

The lines between components in a circuit diagram represent wires and show how the components are connected together. You can connect components in series (see Figure 3) or in parallel (Figure 4).

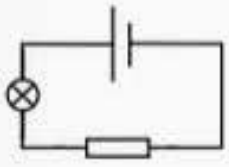


Figure 3: *A simple series circuit.*

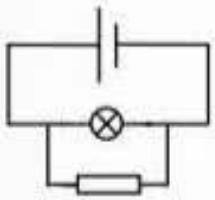


Figure 4: *A simple parallel circuit.*

Exam Tip
You could be asked to draw a circuit diagram in the exam — so make sure you get loads of practice at it before then.

Tip: A circuit needs to be complete for a current to flow through it.

Tip: Series and parallel circuits will be covered in more detail on p.172 and p.176.

Practice Questions — Application

Q1 Draw the circuit symbols for a diode, a fuse and an ammeter.

Q2 Draw a circuit diagram showing a battery, resistor and LDR in series.

Learning Objectives:

- Know the definition of electric current as rate of flow of charge.
- Be able to calculate current using $I = \frac{\Delta Q}{\Delta t}$.
- Know the coulomb as the unit of charge.
- Understand conventional current and electron flow.
- Know that the elementary charge e equals 1.60×10^{-19} C.
- Know that the net charge on a particle or an object is quantised and a multiple of e.
- Understand current as the movement of electrons in metals and movement of ions in electrolytes.
- Understand the distinction between conductors, semiconductors and insulators in terms of n.
- Know what is meant by the mean drift velocity of charge carriers.
- Be able to use $I = Anev$, where n is the number density of charge carriers.

Specification References 4.1.1 and 4.1.2

Figure 2: *Ideas about electric charge have been around for hundreds of years. Philosophers in Ancient Greece noticed attraction between pieces of amber and fabric, which we now call static electricity.*

2. Current

After all that stuff at GCSE about electricity, you wouldn't think there was much left to learn. Well, unfortunately you'd be wrong...

What is current?

The current in a wire is like water flowing in a pipe. The amount of water that flows depends on the flow rate and the time. It's the same with electricity — **current** is the rate of flow of charge.

I = current in amperes (A) → $I = \frac{\Delta Q}{\Delta t}$ ← Q = charge in coulombs (C)

t = time in seconds (s)

The **coulomb** is the unit of charge. One coulomb (C) is defined as the amount of charge that passes in 1 second when the current is 1 ampere.

Example — **Maths Skills**

A component has a current of 0.430 A passing through it. Calculate the charge passing through the component in exactly 3 minutes.

First convert the time taken from minutes to seconds:

3 minutes × 60 seconds per minute = 180 seconds.

Then substitute the values for current and time taken into the charge formula given above:

$\Delta Q = I \times \Delta t = 0.430 \times 180 = 77.4$ C

You can measure the current flowing through a part of a circuit using an **ammeter**. You always need to attach an ammeter in series (so that the current through the ammeter is the same as the current through the component — see page 175).

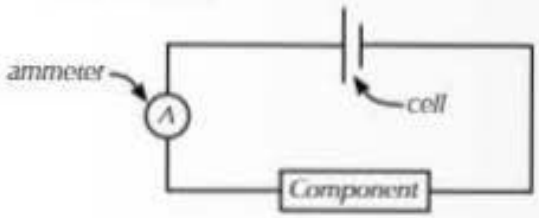


Figure 1: *An ammeter connected in series with a component.*

In Figure 1, you would label the current as flowing from the positive terminal of the cell (the longer line in the cell symbol), through the ammeter and then through the component to the negative terminal (the shorter line in the cell symbol). This is **conventional current** — it is said to flow from positive to negative. However, in most cases the actual movement of particles is in the opposite direction. Electrons, the usual charge carrier in circuits (see next page), are attracted towards the positive terminal, so the flow of electrons is from negative to positive.

This difference occurred because ideas about electricity and current were around long before the electron was actually discovered. Since the direction of flow makes no difference to the effect the current has in a circuit, and current can flow in either direction depending on the charge carriers involved, the idea of conventional current stuck around as the default.

Charge carriers

Charge carriers are electrically charged particles that are free to move in an object. The type of particle and the number of charge carriers in a given volume depend on the material.

Electrons and ions

In most objects (and all metals), the charge carriers are electrons. Each electron carries the same amount of charge, $-e$, where e is the **elementary charge**:

$$e = 1.60 \times 10^{-19}\,\text{C}$$

The elementary charge is the smallest quantity of charge than can be found on its own — the net charge on any object will always be a multiple of e. This means that we can say that charge is quantised.

In other materials, mainly liquids and gases, **ions** are the charge carriers. Ionic crystals like sodium chloride are insulators. Once molten though, the liquid conducts. Positive and negative ions are the charge carriers. The same is true in an ionic solution like copper sulfate solution. A substance containing ions that conducts electricity like this is called an **electrolyte**.

Tip: Protons have an equal but opposite charge to electrons — i.e. each proton has a charge of $+e$.

Tip: In a metal, the charge carriers are free electrons — the ones from the outer shell of each atom.

Conductors and insulators

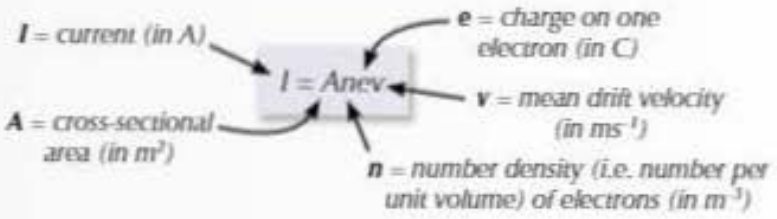
In metals and other good conductors, there are lots of charge carriers per unit volume (n). This means that a current can easily flow through them. **Semiconductors** have fewer charge carriers (p.159) and perfect **insulators** have no charge carriers, so they don't allow a current to flow. In real life, insulators just have a very small n.

Tip: Gases are insulators, but if you apply a high enough p.d. across them, electrons get ripped out of atoms, giving you a spark (ions along a path).

Mean drift velocity

When current flows through a wire, you might imagine the electrons all moving uniformly in the same direction. In fact, they move randomly in all directions, but tend to drift towards the positive terminal of the circuit.

The current is proportional to the average speed of this drifting — known as the **mean drift velocity**. You can use the continuity equation below to determine the size of a current if you know the mean drift velocity.


I = current (in A)
e = charge on one electron (in C)
I = Anev
v = mean drift velocity (in ms⁻¹)
A = cross-sectional area (in m²)
n = number density (i.e. number per unit volume) of electrons (in m⁻³)

So the current through a wire can be increased by:

- Increasing the number of electrons — more charge is carried past a point in a given time. You can do this by either increasing the number density of electrons or increasing the cross-sectional area of the wire.
- Increasing the mean drift velocity of the electrons — it takes less time for a given amount of charge to be transferred. (You can achieve this by increasing the potential difference across the wire — see page 153.)

Tip: The mean drift velocity is much less than the electrons' actual speed (which is about 10^6 ms^{-1}).

Tip: If you're using different charge carriers, just replace e with the charge on each carrier, and let n be the number density of charge carriers.

Tip: Semiconductors can change their number density of electrons (page 159).

Deriving the continuity equation

First, imagine a section of wire that has a length of L and a cross-sectional area of A (see Figure 3).

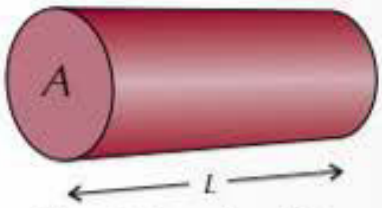


Figure 3: *A section of wire.*

The number of charge carriers in this section is equal to the number of charge carriers per unit volume × the volume of the section, or:

$$\text{no. charge carriers} = ALn$$

The total charge carried by these charge carriers is equal to the number of charge carriers × the charge on each charge carrier, or:

$$Q = ALne$$

To find the time taken for this amount of charge to pass through the section of the conductor, you have to use speed = distance ÷ time.

$$\text{time} = \text{length of section} \div \text{mean drift velocity, or } t = \frac{L}{v}$$

Then substitute both of these into the equation for current from page 150.

$$I = \frac{\Delta Q}{\Delta t} = \frac{ALne}{\left(\frac{L}{v}\right)} = Anev$$

Practice Questions — Application

Q1 A particle has a charge of 4e.
What is the charge of the particle in coulombs?

Q2 A cell has a charge of 91 C passing through it every 32 seconds.
Calculate the current passing through the cell.

Q3 A copper wire has a cross-sectional area of 0.50×10^{-6} m^2.
The number density of electrons for copper is 8.5×10^{28} m^{-3}.
A current of 12 A flows through the wire. Calculate the mean drift velocity of the free electrons in the wire.

Practice Questions — Fact Recall

Q1 Define the term current.

Q2 Describe the direction of the flow of conventional current.

Q3 What is a charge carrier?

Q4 State the value of the elementary charge, e.

Q5 What is the number density of charge carriers for a perfect insulator?

Q6 Give one way of increasing the current through a wire.

3. Potential Difference

So you've learnt all about current, but you can't get a current to flow without a source of potential difference. Potential difference is all about the work done and energy transferred to get a charge carrier around a circuit.

Learning Objectives:

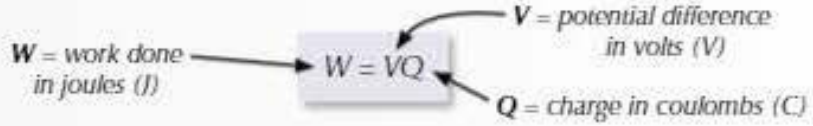
- Understand potential difference (p.d.).
- Know that the unit of potential difference is the volt.
- Be able to calculate energy transferred using $W = VQ$.
- Understand the energy transfer $eV = \frac{1}{2}mv^2$ for electrons and other charged particles.

Specification Reference 4.2.2

What is potential difference?

To make electric charge flow through a circuit, you need to transfer energy to the charge — this energy is supplied by the power source, e.g. a battery. When a charge flows through the power source it is 'raised' through a potential and energy is transferred to the charge.

When energy is transferred, we say that work is done — so the power source does work to move the charge around the circuit. The **potential difference** (p.d.), or voltage, between two points is defined as the work done to move a unit charge between those points.



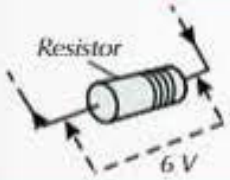
The potential difference across a component is 1 volt when 1 coulomb of charge does 1 joule of work to pass through the component.

$$1\,\mathbf{V} = 1\,\mathbf{J C^{-1}}$$

Tip: When a charge flows through a component, it transfers energy to the component (it does work).

Tip: Back to the 'water analogy' again... the p.d. is like the pressure that's forcing water along the pipe.

Example — **Maths Skills**



Here you do 6 J of work moving each coulomb of charge through the resistor, so the p.d. across it is 6 V.

$$V = \frac{W}{Q} = \frac{6}{1} = 6\,\text{V}$$

The energy gets converted to heat.

You can measure the potential difference across an electrical component by using a **voltmeter**. You always need to attach a voltmeter in parallel (see Figure 2).

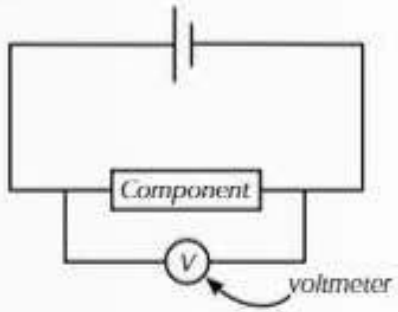


Figure 2: *A voltmeter connected in parallel with a component.*

Figure 1: *A voltmeter connected across a filament bulb.*

Tip: The maximum value that a voltmeter or an ammeter can measure is called the full scale deflection.

Kinetic energy of charges

When a charged particle is accelerated by a potential difference, it gains kinetic energy equal to the amount of work done on the particle. The larger the potential difference, the more energy the particle gains and so the faster it travels. You can work out the speed of the particle by using the equation from the last page and $E_k = \frac{1}{2}mv^2$ from page 105.

$$\text{Work done} = \text{Kinetic energy gained}$$

$$VQ = \tfrac{1}{2}mv^2$$

For electrons, you can write this using the elemental charge, e — see p.151:

Tip: You'll come across this equation again in quantum physics on page 233.

Tip: This equation gives you the speed of a single electron. Don't get it confused with the mean drift velocity from p.151.

Tip: An electron's mass is 9.11×10^{-31} kg (see page 572).

Example — **Maths Skills**

An electron is accelerated from rest by a potential difference of 12 V. Calculate the final speed of the electron.

$eV = \frac{1}{2}mv^2$

$v^2 = 2eV \div m$

$v = \sqrt{2eV \div m} = \sqrt{(2 \times 1.60 \times 10^{-19} \times 12) \div (9.11 \times 10^{-31})}$

$= 2.053... \times 10^6 = 2.1 \times 10^6$ ms^{-1} (to 2 s.f.)

Practice Questions — Application

Q1 In a circuit, it takes 114 J to move 56.0 C of charge through a filament lamp. Calculate the potential difference across the lamp.

Q2 An alpha particle has a mass of 6.64×10^{-27} kg and a charge of +2e. It is accelerated from rest to a speed of 2.20×10^4 ms^{-1}. Calculate the size of the potential difference that accelerated the alpha particle.

Practice Questions — Fact Recall

Q1 Define the potential difference between two points.

Q2 How should a voltmeter be connected in a circuit to measure the potential difference across a component?

Q3 State the equation that gives the kinetic energy of an electron after it is accelerated from rest through a potential difference. Define all symbols used.

4. Resistance and Resistivity

This topic covers everything you need to know about resistance. It's Georg Simon Ohm you've got to blame for these pages — this German physicist developed Ohm's law and got the unit for resistance named after him.

Learning Objectives:

- Know that resistance is defined by $R = \frac{V}{I}$.
- Know that resistance is measured in ohms.
- Know Ohm's law.
- Know what is meant by the resistivity of a material.
- Be able to use the equation $R = \frac{\rho l}{A}$.
- Understand the variation of resistivity of metals and semiconductors with temperature.
- Know techniques and procedures used to determine the resistivity of a metal (PAG3).

Specification References 4.2.3 and 4.2.4

What is resistance?

If you put a potential difference (p.d.) across an electrical component, a current will flow. How much current you get for a particular potential difference depends on the **resistance** of the component.

You can think of a component's resistance as a measure of how difficult it is to get a current to flow through it. Resistance is measured in **ohms** (Ω). A component has a resistance of 1 Ω if a potential difference of 1 V makes a current of 1 A flow through it. This equation defines resistance:

$$R = \frac{V}{I}$$

R = resistance in ohms (Ω)

V = potential difference in volts (V)

I = current in amperes (A)

This equation is commonly known as **Ohm's law**.

Example — **Maths Skills**

A fixed resistor has a resistance of 4.43 Ω and the potential difference across it is 12.0 V. Calculate the current flowing through the fixed resistor.

You just need to rearrange the resistance equation and plug the numbers in.

$I = V \div R$

$= 12.0 \div 4.43$

$= 2.7088... = 2.71$ A (to 3 s.f.)

Figure 1: *Georg Simon Ohm — the physicist who developed Ohm's law.*

Resistivity

If you think about a nice, simple electrical component, like a length of wire, its resistance depends on:

- **Length (*l*)**. The longer the wire, the more difficult it is to make a current flow through it. The resistance is proportional to the length of the wire.
- **Area (*A*)**. The thicker the wire, the easier it will be for the electrons to pass along it.
- **Resistivity (ρ)**. This is a property of the material the component is made from. It is a measure of how much a particular material resists current flow. It's defined as the resistance of a 1 m length with a 1 m^2 cross-sectional area. It is measured in ohm-metres (Ωm).

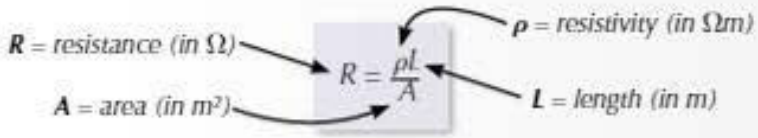
Resistivity depends on the structure of the material as well as on environmental factors such as temperature and light intensity.

For metals, the resistivity increases as the temperature increases. This is because charge is carried through metals by free electrons in a lattice of positive ions. Heating the metal causes the ions to vibrate more, which makes the electrons collide with the lattice more often. This causes energy to be lost, usually as heat, and increases the overall resistance of the component.

For semiconductors, increasing the temperature causes more charge carriers to be released (p.159), so the resistivity decreases.

Exam Tip
You'll be given this equation in the exam, but not Ohm's law, so make sure you learn it.

You can calculate the resistance of a component using the equation:



Tip: Don't confuse resistance and resistivity. Resistance is a property of an object and it depends on the material and dimensions of the object. Resistivity is a property of a material.

Examples — Maths Skills

A piece of metal has a length of 0.50 cm, a square cross-sectional area with a width of 11 cm, and a resistance of 1.22×10^{-8} Ω. Find the resistivity of the metal.

Convert all lengths into metres.

The cross-sectional area (A) of the metal = 0.11 m × 0.11 m = 0.0121 m²

Length = L = 0.0050 m

Rearrange the resistance equation for the resistivity of the metal:

$$\rho = \frac{RA}{L} = \frac{1.22 \times 10^{-8} \times 0.0121}{0.0050} = 2.9524 \times 10^{-8} = 3.0 \times 10^{-8}\ \Omega\text{m (to 2 s.f.)}$$

Exam Tip
For resistivity calculations, don't forget that you need to have the length in m and the cross-sectional area in m² — you'll lose marks if you don't use the correct units.

The heating element in a toaster is a bare nichrome wire with a radius of 0.2 mm and a length of 0.8 m. Find the resistance of the wire. (Resistivity of nichrome = 1.10×10^{-6} Ωm.)

First of all, you need to find the cross-sectional area of the wire (A) in m². Assuming the wire to be cylindrical, its cross-sectional area will be the area of the circle, πr^2.

Wire radius = 0.2 mm = 2×10^{-4} m, so $A = \pi(2 \times 10^{-4})^2 = 1.256... \times 10^{-7}$ m²

Plug the numbers into the equation $R = \frac{\rho L}{A}$ to get:

$R = (1.10 \times 10^{-6} \times 0.8) \div 1.256... \times 10^{-7} = 7.0028...\ \Omega = 7.0\ \Omega$ (to 2 s.f.)

Finding the resistivity of a metal

PRACTICAL ACTIVITY GROUP 3

In this experiment, you'll be finding the resistance of a test wire made from the metal you want to know the resistivity of. You can then use this resistance and the dimensions of the wire to calculate the resistivity of the metal using the resistance equation above.

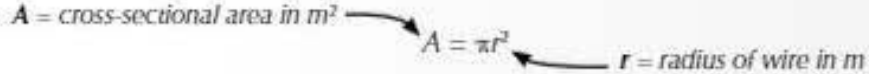
Before you start, you need to do a risk assessment to make sure you are aware of all the safety issues and risks.

Figure 2: *A micrometer, used to measure very small distances. Micrometers are covered in more detail on page 17.*

Calculating cross-sectional area

The first thing you want to calculate is the cross-sectional area of the test wire. Measure the diameter of the wire in at least three different places along the wire using a micrometer.

Find the mean diameter and halve it to find the mean radius of the wire. You can assume that the cross-section of the wire is a circle, so you can calculate the cross-sectional area using this formula:



Measuring resistivity

Set up the experiment as shown in Figure 3, clamping the wire to the ruler so it is aligned with the zero reading on the ruler's scale.

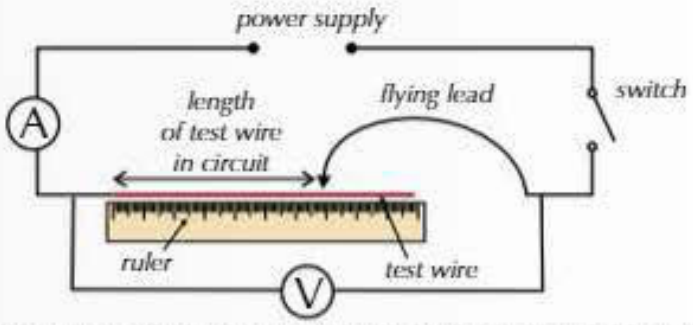


Figure 3: *The experimental set-up for determining resistivity of a metal.*

1. Attach the flying lead to the test wire and measure the length of the test wire connected in the circuit.
2. Close the switch and measure the current through the circuit and the potential difference across the test wire. Open the switch again once you've taken your measurements and use these values to calculate the resistance of the wire.
3. Repeat step 2 at least one more time for this length of wire and calculate the mean resistance.
4. Reposition the flying lead and repeat steps 1 to 3 to get an average resistance for each of several different lengths of test wire.
5. Plot a graph of average resistance (in Ω) against length (in m) using your results. Your graph and line of best fit should look similar to Figure 4.

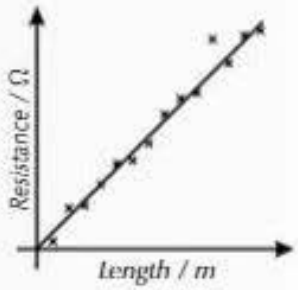


Figure 4: *A typical graph and line of best fit of resistance against length for a uniform wire.*

Since this graph is a straight line through the origin, the gradient of the graph is equal to $\frac{R}{L}$. By rearranging the equation for resistivity (see page 156), you will find that $\frac{R}{L} = \frac{\rho}{A}$.

This means that to find the resistivity of the wire metal in Ωm, you just need to multiply the gradient of the graph by the cross-sectional area of the wire (in m^2).

Exam Tip
You can assume ammeters and voltmeters are ideal unless you're told otherwise — see page 162.

Tip: A flying lead is just a wire with a crocodile clip at the end to allow connection to any point along the test wire.

Tip: If the wire heats up, its resistance (and resistivity) will increase. You can minimise how much it heats up by keeping the current low and using a switch to make sure current only flows through the wire for short amounts of time while you're taking your measurements.

Tip: Don't forget — if you get an anomalous result when you calculate a resistance, you should discount it from calculations for the average resistance. You should also discount any anomalous values when drawing a line of best fit.

Tip: The main sources of random errors in this experiment are likely to be the temperature of the wire changing and measuring the length of the wire. How to deal with errors and uncertainties in practicals is covered in Module 1 (on pages 16 and 17) and Module 2 (p.36–40).

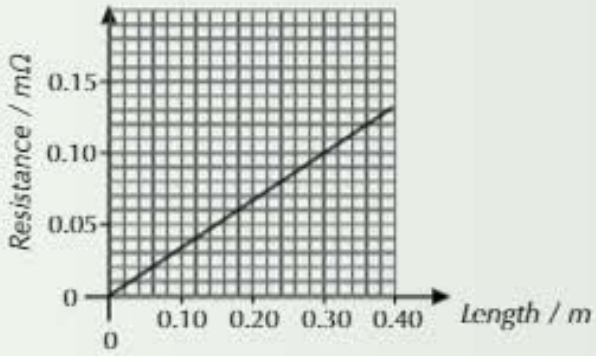
Practice Questions — Application

Q1 A thermistor has a resistance of 8.62 Ω. Calculate the potential difference across the thermistor when there is a current of 2.10 A flowing through it.

Q2 A fuse has a current of 1.21 A flowing through it. Calculate the resistance of the fuse when the potential difference across it is 13.4 V.

Q3 Copper has a resistivity of 1.72×10^{-8} Ωm. Calculate the resistance of a 40 cm long piece of copper wire that has a cross-sectional area of 2.8×10^{-5} m².

Q4 The graph below shows the resistance against length for a test wire of an unknown material. The radius of the test wire was 4.0 mm. Calculate the resistivity of the material the wire was made from.



Exam Tip
Remember to look out for prefixes on units. If your results seem too large or too small, check that you haven't missed one. There's more about prefixes on page 32.

Practice Questions — Fact Recall

Q1 What is meant by resistance?

Q2 State the equation used to calculate the cross-sectional area of a wire, stating any assumptions you make.

Q3 Describe an experiment you could do to find the resistivity of a metal.

Q4 Explain why it is important to keep the temperature constant when doing an experiment to find the resistivity of a metal. Suggest how you could keep the temperature of the material constant.

5. Types of Conductor

So you've come across Ohm's law, but not every electrical component follows it. Unfortunately, you have to know the difference between ohmic and non-ohmic components, including some fancy ones like LDRs and thermistors.

Learning Objectives:

- Understand the variation of the resistivity of semiconductors with temperature.
- Know what is meant by a negative temperature coefficient (NTC) thermistor and understand the variation of its resistance with temperature.
- Know the term light-dependent resistor (LDR).
- Understand the variation of resistance of an LDR with light intensity.
- Know the techniques and procedures used to investigate the electrical characteristics for a range of ohmic and non-ohmic components (PAG3).

Specification References 4.2.3 and 4.2.4

Ohmic conductors

Conductors that obey Ohm's law (mostly metals) are called **ohmic conductors**. Ohm's law states that, provided the physical conditions (such as temperature) remain constant, the current through an ohmic conductor is directly proportional to the potential difference across it.

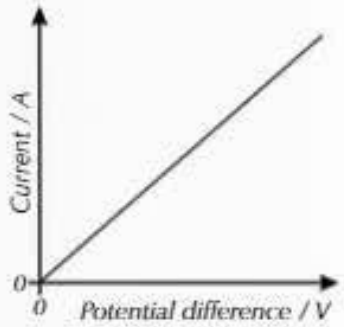


Figure 1: *I-V graph for an ohmic conductor.*

If you plot a graph of current against potential difference (page 162), you get a straight-line graph — doubling the p.d. doubles the current. What this means is that the resistance is constant — the gradient is always a fixed value.

Often factors such as light level or temperature will have a significant effect on resistance, so you need to remember that Ohm's law is only true for ohmic conductors under constant physical conditions.

Conductors that don't follow Ohm's law are called **non-ohmic conductors**. If you plot a graph of current against potential difference for these conductors, you don't get a straight line. There's more on this on pages 163-164.

Tip: The gradient of an *I-V* graph for an ohmic conductor (like the one shown in Figure 1) is $\frac{1}{R}$.

Semiconductors

Semiconductors are a group of materials that aren't as good at conducting electricity as metals, because they have far fewer charge carriers (i.e. electrons) available.

However, if energy is supplied to a semiconductor, e.g. by an increase in temperature or light intensity, more charge carriers can be released and the resistivity of the material decreases. This means that some can make excellent sensors for detecting changes in their environment (p.180).

Three common semiconductor components are thermistors, diodes and light-dependent resistors (LDRs).

Figure 2: *Semiconductors are used in loads of electrical appliances, like microwaves.*

Diodes

You've probably come across **diodes** before, in the form of LED lights. Diodes are designed to only let current flow in one direction, and only when a potential difference above a particular threshold voltage is applied across the diode. There's more about diodes over on page 164.

Thermistors

Figure 3: *Thermistor circuit symbol.*

A **thermistor** is a component with a resistance that depends on its temperature. You only need to know about NTC thermistors — NTC stands for 'Negative Temperature Coefficient'. This means that the resistance decreases as the temperature goes up, see Figure 4.

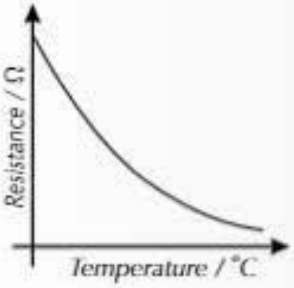


Figure 4: *A resistance vs temperature graph for an NTC thermistor.*

Warming the thermistor gives more electrons enough energy to escape from their atoms. This means that there are more charge carriers available, so the resistance is lower. This sensitivity to temperature makes them really good temperature sensors.

Figure 5: *A thermistor.*

Investigating the resistance of a thermistor

PRACTICAL ACTIVITY GROUP 3

You can use the circuit shown in Figure 6 to investigate how a thermistor's resistance varies with temperature.

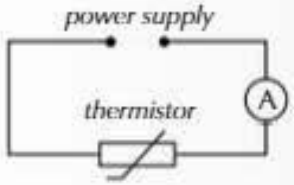


Figure 6: *Circuit used to investigate the resistance of a thermistor.*

Tip: Make sure you do a risk assessment before you start any practical. In this practical it's particularly important to make sure that the thermistor and any other components that will be in contact with the water bath are waterproof.

Tip: Having the thermistor in hot water, rather than trying to heat it directly, helps prevent the temperature from fluctuating while you're taking measurements, and affecting your results.

You can measure the temperature of a thermistor more easily by putting it in water (provided the thermistor is waterproof). Place the thermistor in a beaker and pour enough boiling water into the beaker to cover the thermistor. Turn on the power supply, and record its potential difference — this should be kept constant throughout the experiment. Measure and record the temperature of the water using a digital thermometer and the current through the circuit using the ammeter.

Continue to record the current and temperature for every 5 °C drop in temperature. Use your recorded values for potential difference and current to calculate the resistance of the thermistor at each temperature. Then plot a resistance-temperature graph. For an NTC thermistor, you should find as the temperature decreases, the resistance increases (and so the current will decrease) — your graph should look like the one in Figure 4.

Light-dependent resistors

A **light-dependent resistor** (LDR) is a component with a resistance that depends on the intensity of the light that is incident on it. The greater the intensity, the lower the resistance of the LDR.

The explanation for why this occurs is similar to the thermistor. Light provides the energy to release charge carriers and decrease the resistivity of the material. A higher intensity means that more photons are hitting the LDR each second, so more charge carriers are released each second.

The graph of resistance against light intensity for an LDR is shown in Figure 8.

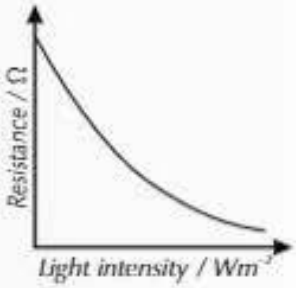


Figure 8: *A resistance vs light intensity graph for an LDR.*

Figure 7: *The circuit symbol for an LDR.*

Investigating the resistance of an LDR

PRACTICAL ACTIVITY GROUP 3

You can use the circuit in Figure 6 to investigate the resistance of an LDR — just replace the thermistor with an LDR.

Switch on the power supply, and record its potential difference — this should stay constant. Measure the current through the LDR, and then cover up part of the LDR's surface with a piece of thick paper. This will decrease the light intensity on the LDR. Gradually cover more of the surface with the paper, taking new current readings each time you do. Record the area of the uncovered surface for each current measurement. You can use the current and p.d. data to calculate the resistance of the LDR for different uncovered areas (i.e. light intensities). Then plot a graph of uncovered area against resistance. Your results should match the relationship shown in Figure 8.

Tip: The intensity of light is a measure of the energy transferred to a surface by light per second, per unit area. So the more area light hits, the greater the intensity of light on the surface. See page 192 for more on this.

Practice Questions — Fact Recall

Q1 What happens to the resistance of an ohmic conductor if you double the potential difference across it? Give a reason for your answer.

Q2 What does the resistance of a thermistor depend on?

Q3 Sketch a graph of resistance against light intensity for an LDR.

Learning Objectives:

- Know the techniques and procedures used to investigate the electrical characteristics for a range of ohmic and non-ohmic components (PAG3).
- Know and understand *I–V* characteristics of a resistor, filament lamp, thermistor, diode and light-emitting diode (LED).

Specification Reference 4.2.3

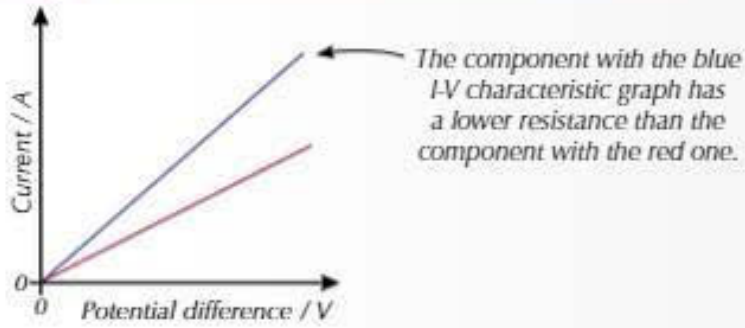
6. *I-V* Characteristics

You've had a glimpse of the I-V characteristic for an ohmic conductor — now it's time to meet a few more...

What are *I-V* characteristics?

The term '***I-V* characteristic**' is just a fancy way of saying a graph which shows how the current (*I*) flowing through a component changes as the potential difference (*V*) across it is increased. The shallower the gradient of a *I-V* characteristic, the greater the resistance of the component.

Example


Current / A
0
0
Potential difference / V
The component with the blue I-V characteristic graph has a lower resistance than the component with the red one.

A curve shows that the resistance is changing (see next page).

Tip: As always, do a risk assessment before you start any experiment.

Figure 2: *A variable resistor.*

Tip: You could record your results in a spreadsheet and use computer software to plot your *I-V* characteristics instead of drawing them by hand.

Exam Tip
In the exams, you can assume all voltmeters and ammeters are ideal unless the question tells you otherwise.

Finding the *I-V* characteristic of a component

You can use the circuit in Figure 1 to find the *I-V* characteristic for a component.

PRACTICAL ACTIVITY GROUP 3

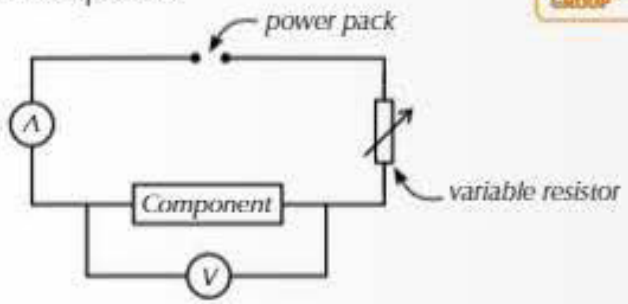


Figure 1: *A circuit that could be used to find the I-V characteristic for a component.*

By changing the resistance of the variable resistor, you can change the voltage across the component you're investigating and the current through it. You can plot an *I-V* characteristic for the component by recording a range of current and voltage values and plotting an *I-V* graph using them.

Ideal voltmeters and ammeters

You can normally assume that any voltmeters and ammeters used in an experiment are ideal. Voltmeters are assumed to have an infinite resistance (so no current flows through them) and ammeters are assumed to have no resistance (and so will have no potential difference across them).

Ohmic conductors

As you saw on page 159, the current through an ohmic conductor under constant physical conditions is directly proportional to the voltage. So their *I-V* characteristic is a straight line as their resistance doesn't change (see Figure 3). Fixed resistors and LDRs in constant conditions are both ohmic conductors.

Tip: Most metallic conductors are ohmic.

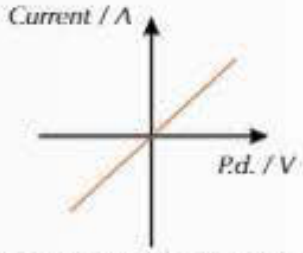


Figure 3: *The characteristic I-V graph for an ohmic conductor.*

Filament lamps

The *I-V* characteristic for a filament lamp is a curve that is steep for small voltages but gets shallower as the voltage rises (see Figure 5).

Figure 4: *Filament lamp circuit symbol.*

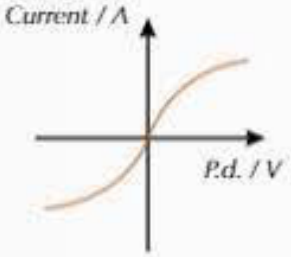


Figure 5: *The characteristic I-V graph for a filament lamp.*

Figure 6: *A filament lamp contains a thin coil of metal wire inside it.*

The filament is a thin coil of metal wire, so you might think it should have the same characteristic graph as a metallic conductor. But, a filament is designed to heat up and glow. An increase in current means an increase in temperature, and an increase in resistance, which causes the current to decrease again. This is why the *I-V* graph for the filament lamp levels off at high currents.

Most components with resistance heat up as the current increases, but at much higher currents than a filament lamp. This means there is usually a limit to the amount of current that can flow through a component.

Thermistors

The *I-V* characteristic for a thermistor gets steeper as the potential difference across it increases.

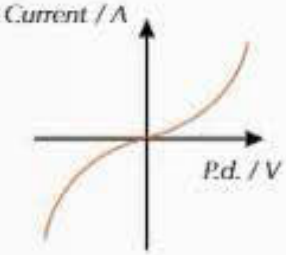


Figure 7: *The characteristic I-V graph for a thermistor.*

The explanation for this is similar to that for the filament lamp. As a current flows through the thermistor, the thermistor heats up. This causes more charge-carriers to be released (p.151), so the resistance of the thermistor decreases and the *I-V* characteristic becomes steeper.

Diodes

Figure 8: *Diode circuit symbol. Current flows through the diode in the direction of the arrow.*

Diodes (including light-emitting diodes (LEDs)) are made from semiconductors (see page 159) and are designed to let current flow in one direction only. Forward bias is the direction in which the current is allowed to flow. Most diodes require a voltage of about 0.6 V in the forward direction before they will conduct — this is called the threshold voltage. In reverse bias, the resistance of the diode is very high and the current that flows is very tiny.

Tip: You can use the threshold voltages of LEDs to determine Planck's constant — there's more on this on page 234.

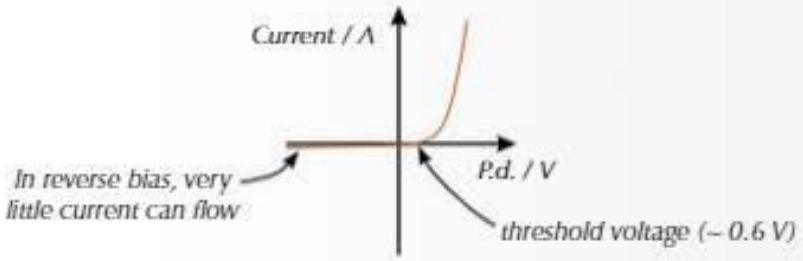
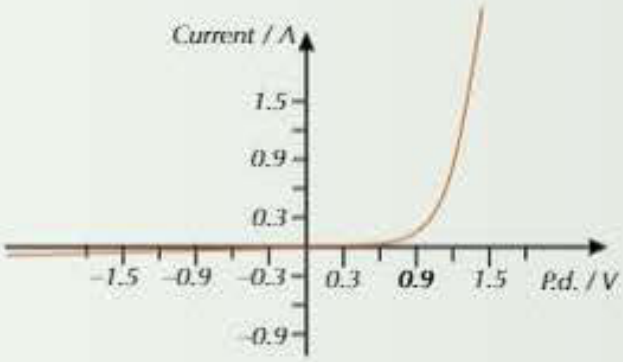


Figure 9: *The characteristic I-V graph for a diode.*

Practice Question — Application

Q1 The graph below shows the *I-V* characteristic for a component.



a) What type of component has this *I-V* characteristic?

b) Describe how the current and voltage vary in the forward bias.

c) Describe the resistance of the component in the reverse direction.

Practice Questions — Fact Recall

Q1 What resistances do you normally assume a voltmeter and an ammeter to have?

Q2 Sketch the *I-V* characteristic of:
a) an ohmic conductor under constant physical conditions,
b) a filament lamp.

Q3 Is a filament lamp an ohmic conductor?
How can you tell from its *I-V* characteristic?

7. Power and Electrical Energy

You should remember all about work and power from GCSE. Well, here they are again because you still need to know about them. Remember — the faster a device does work (transfers energy), the more powerful it is.

Learning Objectives:

- Be able to use the equations $P = VI$, $P = I^2R$ and $P = \frac{V^2}{R}$.
- Be able to calculate energy transferred using $W = VIt$.

Specification Reference 4.2.5

Power

Power (P) is defined as the rate of doing work. It's measured in watts (W), where 1 watt is equivalent to 1 joule of work done per second.

$$P = \frac{W}{t}$$

P = *power in watts (W)*; ***W*** = *work done in joules (J)*; ***t*** = *time in seconds (s)*

From this definition, you can derive an equation to find the power of electrical devices.

- From the definition of potential difference, you know that $W = VQ$.
- From the definition of current, you know that $Q = It$.
- Substitute these into the equation for power:

$$P = \frac{VQ}{t} = \frac{VIt}{t}$$

- And you end up with:

$$P = VI$$

P = *power in watts (W)*; ***V*** = *potential difference in volts (V)*; ***I*** = *current in amperes (A)*

Tip: If your mind needs refreshing about current and potential difference, flick back to pages 150-154.

Example — **Maths Skills**

A 3.0 A current flows through a component when a 12.0 V potential difference is applied across it. Calculate the power of the component.

$P = VI$

$= 12.0 \times 3.0 = 36$ W

Figure 1: *The label on an electrical appliance tells you its power rating — the rate at which it transfers energy.*

Equations for calculating power

As well as the two equations above, you can derive two more equations for calculating power in electrical circuits...

You know from rearranging Ohm's Law that:

$$V = IR$$

V = *potential difference in volts (V)*; ***I*** = *current in amperes (A)*; ***R*** = *resistance in ohms (Ω)*

By substituting this rearranged Ohm's Law into the above equation for power, you get:

$$P = I^2R$$

Exam Tip

The good news is you'll be given most of the boxed equations in the exam, so you don't have to memorise them all — hurrah.

You can also rearrange Ohm's Law to make I the subject:

$$I = \frac{V}{R}$$

And then substitute this into the equation for power to give:

$$P = \frac{V^2}{R}$$

Obviously, which equation you should use depends on what quantities you know.

Exam Tip
Always double-check that you've rearranged equations correctly in the exam — it'd be sad to lose marks for making such an easy mistake.

Examples — **Maths Skills**

A 24 W car headlamp is connected to a 12 V car battery. Assume the wires connecting the lamp to the battery have negligible resistance.

a) How much work is done by the lamp in 2.0 hours?

b) Find the total resistance of the lamp.

a) Number of seconds in 2.0 hours = $2 \times 60 \times 60 = 7200$ s
Rearrange the equation $P = W \div t$:
$W = P \times t = 24 \times 7200 = 172\,800$ J = 170 kJ (to 2 s.f.)

b) Rearrange the equation $P = \frac{V^2}{R}$, $R = \frac{V^2}{P} = \frac{12^2}{24} = \frac{144}{24} = 6.0\ \Omega$

A machine does 750 J of work every second.

a) What is the power rating of the machine?

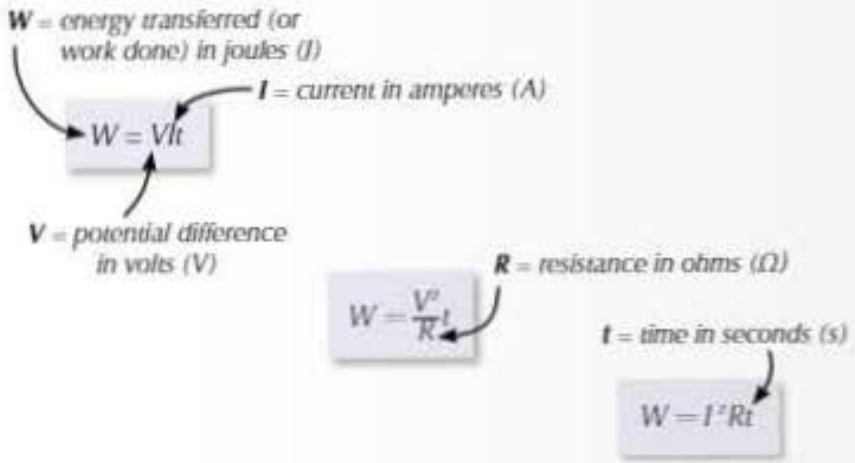
b) All of the machine's components are connected in series, with a total resistance of 30 Ω. What current flows through the machine's wires?

a) Power = $W \div t = 750 \div 1 = 750$ W

b) Rearrange the equation $P = I^2R$, $I = \sqrt{\frac{P}{R}} = \sqrt{\frac{750}{30}} = \sqrt{25} = 5$ A

Energy

Sometimes it's the electrical energy transferred by the device that you're interested in — this is equal to the work done. Just substitute $P = W \div t$ into the power equations, rearrange, and tah dah — you get these equations for the electrical energy transferred by a device:


W = energy transferred (or work done) in joules (J)
I = current in amperes (A)
W = VIt
V = potential difference in volts (V)
R = resistance in ohms (Ω)
W = V²/R t
t = time in seconds (s)
W = I²Rt

Example — **Maths Skills**

It takes 4.5 minutes for a kettle to boil the water inside it. A current of 4.0 A flows through the kettle's heating element once it is connected to the mains (230 V). How much energy does the kettle's heating element transfer to the water in the time it takes to boil?

Time the kettle takes to boil in seconds = $4.5 \times 60 = 270$ s.

Use $W = VIt$

$= 230 \times 4.0 \times 270$

$= 248\,400$ J

$= 250$ kJ (to 2 s.f.)

Tip: Electrical energy can also be given in kWh (kilowatt hours, see the next page). 1 kWh equals 3.6 MJ, so to convert kWh to J, multiply by 3.6×10^6.

Exam Tip

When you're doing these questions, make sure the time you use is always in seconds, or you'll get the wrong answer.

Practice Questions — Application

Q1 A battery provides 3400 J of energy per second. What is the power of the battery?

Q2 A car starter motor requires 12.5 kJ of energy to flow through it in 2.00 seconds to start the engine.

a) Calculate the power necessary to start the engine.

b) The car battery supplies 8.00 V to the starter motor. Calculate the current required to start the engine.

Q3 A motor has a power rating of 5.2 kW. The potential difference across the motor is 230 V. Calculate the total resistance of the motor.

Q4 A lamp has a potential difference of 230 V across it and a current of 1.2 A flowing through it. Calculate the energy transferred to the lamp in 45 seconds.

Q5 A circuit in an electric car converts 1250 J of electrical energy into heat every second. The resistance in that circuit of the car is 54.2 Ω. Calculate the current through that circuit.

Practice Questions — Fact Recall

Q1 Define the term power.

Q2 Write down three equations you could use to calculate electrical power.

Q3 Give an equation for the energy transferred by a device in terms of current, potential difference and time.

8. Domestic Electricity

Whilst you probably don't have to worry about real electricity bills just yet, you unfortunately need to know about them for your exam...

Learning Objectives:

- Know the kilowatt-hour (kWh) as a unit of energy.
- Be able to calculate the cost of energy.

Specification Reference 4.2.5

Electricity companies

Unless you live somewhere incredibly remote, your house will be connected to the national grid — a network of cables that transfers electricity from power stations to consumers. Of course, this isn't free — you have to pay your electricity supplier. Electricity suppliers charge customers based on the number of 'units' of electricity that have been used.

Another name for a unit is a kilowatt-hour (kWh), where 1 kWh = 3.6 million joules. Kilowatt-hours are used because a typical household uses loads of electricity each month, so it'd be impractical to try to measure this in joules. If you know the power of an appliance and the length of time it's used for, you can calculate the work it does in kWh.

Figure 1: *Electricity meters record the units of electricity used by a household.*

Tip: Remember, 1 kWh equals 3.6×10^6 J.

Example — **Maths Skills**

A 1500 W hairdryer is on for 10 minutes.
How much energy does it use in J and kWh?

In joules: $W = Pt = 1500 \times 10 \times 60 = 900\,000$ J

In kWh: 10 minutes $= 10 \div 60 = 0.166...$ hours
1500 watts $= 1500 \div 1000 = 1.5$ kW
$W = Pt = 1.5 \times 0.166... = 0.25$ kWh

Cost of electricity

To work out the cost of electricity you need to know how much you've used (in units) and the price of each unit. Then it's a simple matter of multiplying these two numbers together:

Cost = Number of units × Price per unit

Figure 2: *Real electricity bills can have multiple charges and discounts applied to them, but in your exams, you'll only be expected to know the basic equation on the right.*

Example — **Maths Skills**

How much does it cost to use an 800 W microwave oven for 6 minutes? Electricity costs 16.1p per unit.

6 minutes $= 6 \div 60 = 0.1$ hours
800 watts $= 800 \div 1000 = 0.8$ kW
$W = Pt = 0.8 \times 0.1 = 0.08$ kWh
Cost $= 0.08 \times 16.1 = 1.288 = 1$p (to 1 s.f.)

Of course, most people don't make a separate payment for their electricity each time they use a microwave — electricity providers bill them at regular intervals instead.

Electricity bills can look like they're written in a strange code — but luckily for you, the examples you'll see are easy to understand. Real ones aren't really that bad either — you just need to know where to look to find the important information. Take a look at the example on the next page.

Example

Charges for this period

	Previous	Latest	Total
Electricity used	29 125	29 605	480
Unit charge			10.25p
Total for this period			**£49.20**

- The electricity used is given in units (kWh) on an electricity bill.
- The 'previous' value is the reading from the electricity meter in the customer's house from the last time they were billed.
  The 'latest' value is what the electricity meter currently says.
- The total is the difference between the previous and latest values — i.e. the total electricity used in this time.
- The unit charge is the price of one unit of electricity.
- The total cost is found by multiplying the number of units used by the price per unit — 480 × 10.25 = 4920 p or £49.20.

Reducing electricity use

Reducing the amount of electrical energy you use can save you money and it can help the environment too.

Environmental impacts of generating electricity

Burning fossil fuels (coal, oil and gas) releases carbon dioxide into the atmosphere, which adds to the greenhouse effect and contributes to global warming. Burning coal and oil also releases sulfur dioxide. This can cause acid rain, which damages plants and buildings.

Nuclear power stations produce radioactive waste, which can be dangerous to the environment and human health if it isn't disposed of safely. Even renewable energy sources like biofuels have an impact because lots of land is needed to produce them.

Energy saving devices

Energy saving (fluorescent) light bulbs are more expensive than traditional filament bulbs, but are about four times more efficient. They also last for about ten years (as opposed to a year for filament bulbs) so reduce the number of bulbs needing to be disposed of and replaced. LED light bulbs are even more efficient than energy saving bulbs, and last even longer, but they cost more.

Domestic appliances (e.g. washing machines and ovens) sold in the UK all have an energy rating. This helps customers choose the appliance that's most efficient to run, which can reduce their environmental impact.

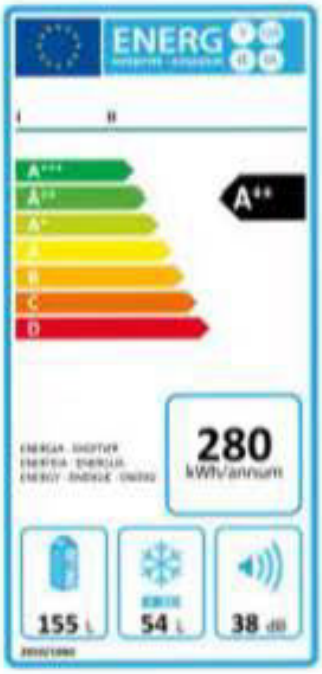


Figure 3: *Energy efficiency labelling, used for domestic appliances within the EU.*

Tip: Remember, efficiency is the useful output energy ÷ the total input energy (page 107).

Practice Questions — Application

Q1 A device transfers 72 000 J of energy.
Calculate the energy transferred in kWh.

Q2 A household's electricity bill for one month is £28.24.
Electricity is supplied to the house at 12.2p per unit. Calculate how many units of electricity the household used during the month.

Learning Objectives:

- Know what is meant by internal resistance.
- Know what is meant by the electromotive force (e.m.f.) of a source such as a cell or a power supply.
- Be able to calculate energy transferred using $W = \varepsilon Q$.
- Understand the distinction between e.m.f. and p.d. in terms of energy transfer.
- Know the meanings of the terms terminal p.d. and 'lost volts'.
- Be able to use the equations $\varepsilon = I(R + r)$ and $\varepsilon = V + Ir$.
- Be able to analyse circuits with more than one source of e.m.f.
- Understand techniques and procedures used to determine the internal resistance of a chemical cell or other source of e.m.f. (PAG4).

Specification References 4.2.2, 4.3.1 and 4.3.2

9. E.m.f. and Internal Resistance

There's resistance in almost all wires and components — including inside batteries and cells. This makes some resistance calculations a little more tricky. But don't worry — the next few pages will help you with those questions.

What is internal resistance?

Resistance comes from electrons colliding with atoms and losing energy. In a battery, chemical energy is used to make electrons move. As they move, they collide with atoms inside the battery — so batteries must have resistance. This is called **internal resistance**. Internal resistance is what makes batteries and cells warm up when they're used.

Load resistance is the total resistance of all the components in the external circuit. You might see it called 'external resistance'.

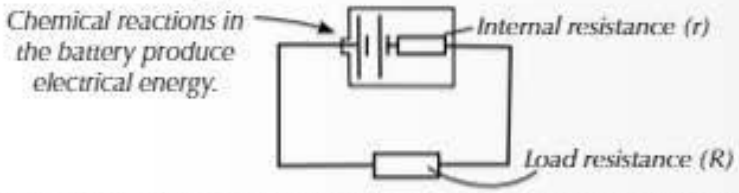
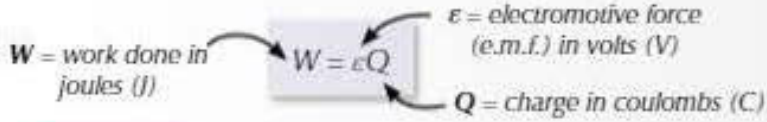


Figure 1: *A circuit diagram showing the internal and external resistances in a circuit.*

What is e.m.f.?

The total amount of work the battery does on each coulomb of charge is called its **electromotive force** or **e.m.f.** (ε). Be careful — e.m.f. isn't actually a force. It's measured in volts.



Example — **Maths Skills**

A battery has an e.m.f. of 10.2 V. Calculate the work done by the battery when 0.650 C of charge passes through it.

$W = \varepsilon Q$

$= 10.2 \times 0.650 = 6.63$ J

Tip: In general, you can assume the connecting wires in a circuit have no resistance — and it's no different for internal resistance problems. But in practice, they do have a small resistance.

The potential difference (p.d.) across the load resistance (R) is the work done when one coulomb of charge flows through the load resistance. This potential difference is called the **terminal p.d.** (V). If there was no internal resistance, the terminal p.d. would be the same as the e.m.f. However, in real power supplies, there's always some energy lost overcoming the internal resistance. The energy wasted per coulomb overcoming the internal resistance is called the **lost volts** (v).

Conservation of energy tells us for any electrical circuit:

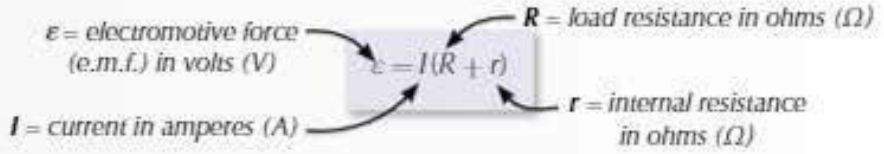
energy per coulomb supplied by the source = energy per coulomb transferred in load resistance + energy per coulomb wasted in internal resistance

Tip: There's more on the conservation of energy on page 107.

Calculations using e.m.f.

Examiners can ask you to do calculations with e.m.f. and internal resistance in lots of different ways. You've got to be ready for whatever they throw at you. Here are some of the equations you might need and how to derive them...

You'll get this equation for e.m.f. in the data and formulae booklet in the exam:


ε = electromotive force (e.m.f.) in volts (V)
I = current in amperes (A)
ε = I(R + r)
R = load resistance in ohms (Ω)
r = internal resistance in ohms (Ω)

Expanding the brackets of this equation gives: $\varepsilon = IR + Ir$

Then using the equation $V = IR$, you can substitute V and v for IR and Ir...

Rearranging the equation gives the equation for terminal p.d...

$$V = \varepsilon - v$$

And re-substituting v for Ir gives...

$$\varepsilon = V + Ir$$

These are all basically the same equation, just written differently. Which equation you should use depends on what information you've got, and what you need to calculate.

Tip: $V = IR$ is just the definition of resistance rearranged (see p.155).

Exam Tip
You'll only be given $\varepsilon = I(R + r)$ and $\varepsilon = V + Ir$ in the exam, so make sure you can work out the other versions of these equations.

Example — **Maths Skills**

A battery has an e.m.f. of 0.15 V and an internal resistance of 0.50 Ω. Calculate the terminal p.d. when the current flowing through the battery is 6.0 mA.

The values for the e.m.f., the internal resistance and the current have been given in the question — so rearrange the equation $\varepsilon = V + Ir$ into $V = \varepsilon - Ir$ to calculate the terminal p.d. 6.0 mA = 0.0060 A.

$V = \varepsilon - Ir = 0.15 - (0.0060 \times 0.50) = 0.147 = 0.15$ V (to 2 s.f.)

As mentioned on the previous page, some power (or energy) is lost as heat when overcoming the internal resistance in a power supply. You can calculate the energy dissipated (lost) per second due to the internal resistance of a power supply using $P = I^2R$ (see page 165).

Example — **Maths Skills**

A cell has an internal resistance of 0.35 Ω and current flowing through it of 0.60 A. Calculate the energy dissipated due to the internal resistance of the cell every second.

$P = I^2r = (0.60)^2 \times 0.35 = 0.126 = 0.13$ W (to 2 s.f.)

So the energy dissipated every second is 0.13 J (to 2 s.f.)

Exam Tip
You'll be given $P = I^2R$ in your exam data and formulae booklet.

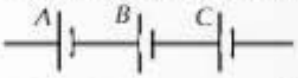
E.m.f. in series and parallel

For cells in series in a circuit, you can calculate the total e.m.f. of their combination by adding their individual e.m.f.s. This makes sense if you think about it, because each charge goes through each of the cells and so gains e.m.f. (electrical energy) from each one.

$$\varepsilon_{total} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \ldots$$

Example — **Maths Skills**

Three cells of negligible internal resistance are connected in series, as shown below. Cells *A*, *B* and *C* have an e.m.f. of 1.5 V, 1.0 V and 3.0 V respectively. Find the total e.m.f. of the combination of cells.



Total e.m.f. $= \varepsilon_A + \varepsilon_B + \varepsilon_C = 1.5 + 1.0 + 3.0 = 5.5\ \text{V}$

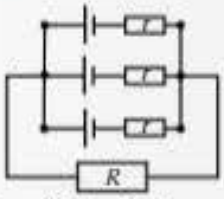
Figure 2: *The total e.m.f. of two identical batteries in parallel will follow the same rule as for cells — see right.*

For identical cells in parallel in a circuit, the total e.m.f. of the combination of cells is the same size as the e.m.f. of each of the individual cells. This is because the amount of charge flowing in the circuit doesn't increase by adding cells in parallel, but the number of paths the charges can take does. The current will split equally between identical cells. The charge only gains e.m.f. from the cells it travels through — so the overall e.m.f. in the circuit doesn't increase.

$$\varepsilon_{total} = \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \ldots$$

Example — **Maths Skills**

Three identical cells, each with an e.m.f. of 2.0 V and an internal resistance of 0.20 Ω, are connected in parallel in the circuit shown to the right.
A current of 0.90 A is flowing through resistor *R*.
Calculate the total p.d. across the cells.



In a parallel circuit, the total p.d. supplied by any number of identical e.m.f. sources connected in parallel is equal to the p.d. supplied by one.

$V_{total} = V_1 = V_2 = V_3$

So we can find the total potential difference from the potential difference of one cell.

Rearranging $\varepsilon = V + Ir$, we get $V = \varepsilon - Ir$

Since the current flowing through the circuit is split equally between each of the three cells, the current through one cell is $I \div 3$.

$I \div 3 = 0.90 \div 3 = 0.30\ \text{A}$

So, $V_{total} = 2.0 - (0.30 \times 0.20)$
$= 2.0 - 0.06$
$= 1.94\ \text{V}$
$= 1.9\ \text{V}$ (to 2 s.f.)

Measuring internal resistance and e.m.f.

PRACTICAL ACTIVITY GROUP 4

You can measure the internal resistance and e.m.f. of a cell or battery using the circuit in Figure 3.

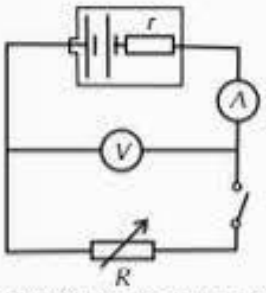


Figure 3: *The circuit needed to work out the internal resistance of a power source.*

1. Set the variable resistor (the load resistance) to its highest resistance.
2. Close the switch and record the current (I) through and potential difference (V) across the circuit.
3. Open the switch and close it again to get another set of current and potential difference readings for this load resistance. Then repeat to get a third set of values. Calculate the mean current and potential difference for this resistance from your results.
4. Decrease the resistance of the variable resistor by a small amount.
5. Repeat steps 2, 3 and 4 until you have a set of mean currents and potential differences for 10 different load resistances (over the widest possible range).
6. Plot a V-I graph for your mean data and draw a line of best fit — you get a straight-line graph (see Figure 4).
7. Make sure all other variables are kept constant when carrying out the experiment, including external factors like temperature (which affects the resistivity of materials, see page 155).

Tip: Make sure you have carried out a risk assessment before you start the experiment.

Tip: Remember that ammeters are always connected in series and voltmeters are always connected in parallel (see pages 150 and 153).

Tip: Remember not to include any anomalous measurements when calculating mean values.

Analysing the results

You can rearrange the equation $\varepsilon = V + Ir$ (from page 171) to get $V = -rI + \varepsilon$. The equation of a straight line is $y = mx + c$, where m = gradient and c = y-intercept. Since ε and r are constants, $V = -rI + \varepsilon$ is just the equation of a straight line.

You can just read ε and r from the graph — the intercept on the vertical axis is ε and the gradient is $-r$. As the graph is a straight line, you can find the gradient of the graph by dividing the change in y (p.d.) by the change in x (current).

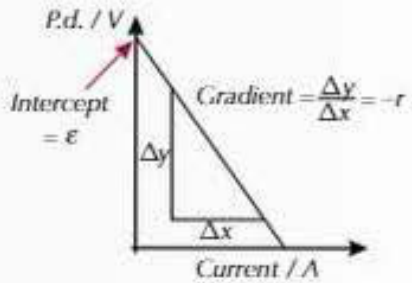


Figure 4: *Using a V-I graph to calculate e.m.f. and internal resistance.*

Tip: You can measure the approximate e.m.f. of a power source by connecting a high-resistance voltmeter across its terminals. Since the resistance of the voltmeter is so high, a very small current will flow, so the lost volts due to the internal resistance will be negligible. But the value you measure will still be very slightly less than the true e.m.f.

Tip: For all these questions assume that the resistance due to the wires in the circuit is negligible.

Exam Tip
In the exam, the examiners could write 'find the terminal p.d. of a battery' or 'find the p.d. across the terminals of a battery'. They mean exactly the same thing.

Tip: Remember, voltage and potential difference (p.d.) are the same thing.

Practice Questions — Application

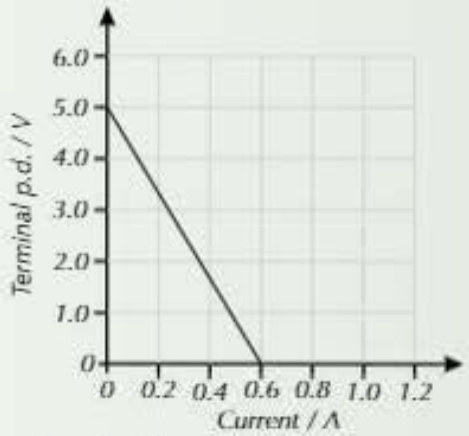
Q1 A power source has an e.m.f. of 2.50 V. The terminal potential difference is 2.24 V. Calculate the lost volts (v) for this power source.

Q2 A battery has a terminal potential difference of 4.68 V and an internal resistance of 0.89 Ω when 0.63 A of current is flowing through it. Calculate the e.m.f. of the battery.

Q3 The power source in a circuit has an e.m.f. of 15.0 V. The internal resistance of the power source is 8.28×10^{-3} Ω. Calculate the load resistance of the circuit when the current flowing through the power source is 26.1 A.

Q4 A power source has a current of 1.2 A flowing through it and an internal resistance of 0.50 Ω. Calculate the energy dissipated in the internal resistance of the power source each second.

Q5 A student varies the load resistance in a circuit to produce the *V-I* graph shown below.



a) Use the graph to find the e.m.f. (ε) of the power supply.

b) Use the graph to find the internal resistance of the power supply.

c) Describe how the graph would be different if the student swapped the power supply for one with the same e.m.f. but half the internal resistance.

Practice Questions — Fact Recall

Q1 What is the load resistance in a circuit?

Q2 What units is electromotive force measured in?

Q3 Explain what lost volts are.

Q4 What do the gradient and vertical intercept on a *V-I* graph for a power supply show?

10. Conservation of Energy and Charge in Circuits

Learning Objectives:

- Understand Kirchhoff's first and second laws as applied to electrical circuits.
- Understand Kirchhoff's second law in terms of the conservation of energy.
- Be able to calculate the resistance of two or more resistors in series, using $R = R_1 + R_2 + R_3 + ...$
- Be able to calculate the resistance of two or more resistors in parallel, using $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$
- Be able to analyse circuits with components in both series and parallel.

Specification Reference 4.3.1

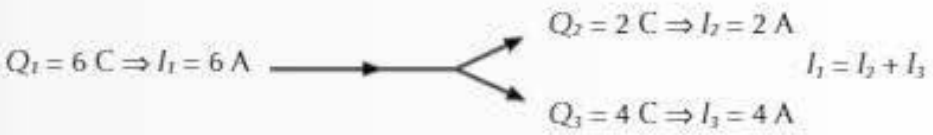
Conservation of energy is another topic you'll probably have met at GCSE. Questions on this stuff are almost guaranteed to be in the exam so I've put in a lot of time and effort to make sure this topic's as easy to grasp as possible...

Conservation of charge

As charge flows through a circuit, it doesn't get used up or lost. This means that whatever charge flows into a junction will flow out again. Since current is rate of flow of charge, it follows that whatever current flows into a junction is the same as the current flowing out of it.

Example

If a charge of 6 C flows into a junction...



... a charge of 6 C must flow out of it as well.

Gustav Kirchhoff was a German scientist who developed a set of laws for the current through and potential difference across different components in a circuit. Conservation of current is known as his first law.

Kirchhoff's first law

> The total current entering a junction = the total current leaving it.

Figure 1: *Gustav Kirchhoff, the German scientist that formulated Kirchhoff's laws.*

Conservation of energy

Energy is conserved. You already know that. In electrical circuits, energy is transferred round the circuit. Energy transferred to a charge is e.m.f. (see page 170), and energy transferred from a charge is potential difference (p.d.). In a closed loop, these two quantities must be equal if energy is conserved (which it is).

Kirchhoff's second law:

> The total e.m.f. around a series circuit = the sum of the p.d.s across each component.

This is Kirchhoff's second law in symbols:

$$\varepsilon = \Sigma IR$$

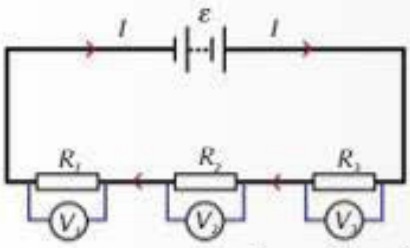
This symbol means 'sum of'.

Figure 2: *Fairy lights are the classic example of a series circuit.*

Applying Kirchhoff's laws

A typical exam question will give you a circuit with bits of information missing, leaving you to fill in the gaps. Not the most fun... but on the plus side you get to ignore any internal resistance stuff (unless the question tells you otherwise)... hurrah. You need to remember the following rules:

Series circuits



- There will be the same current at all points of the circuit (since there are no junctions).
- The e.m.f. is split between the components (by Kirchhoff's 2nd law), so:

$$\varepsilon = V_1 + V_2 + V_3$$

- The voltage splits proportionally to the resistance, as $V = IR$.

Tip: Remember that even in series circuits, voltmeters are placed in parallel with components.

Example

If you had a 1 Ω resistor and a 3 Ω resistor, you'd get 1/4 of the p.d. across the 1 Ω resistor and 3/4 across the 3 Ω.

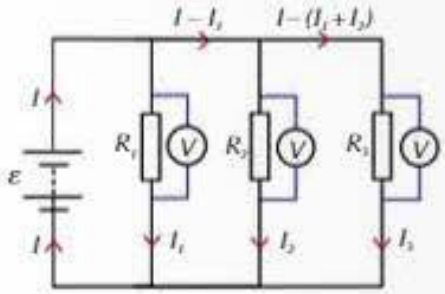
- $V = IR$, so if I is constant:

$$IR = IR_1 + IR_2 + IR_3 \text{ where } R \text{ is the total resistance}$$

- Cancelling the Is gives:

$$R = R_1 + R_2 + R_3$$

Parallel circuits



Tip: Don't worry if you don't quite understand what's going on here... it'll all become clear when you have a look at the example on the next page.

- The current is split at each junction, so:

$$I = I_1 + I_2 + I_3$$

- There is the same p.d. across all components as the p.d. across each branch of the circuit is equal to the e.m.f. So by substituting Ohm's Law rearranged for current, $I = \frac{V}{R}$, into the above equation we get:

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

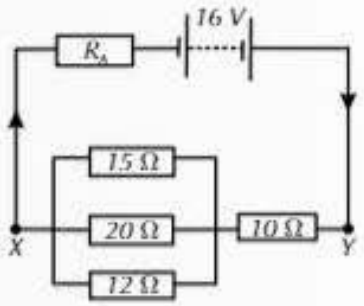
- Cancelling the Vs gives:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Tip: Remember, R in both series and parallel circuits represents the total (or effective) resistance of the circuit.

Example — **Maths Skills**

A battery of e.m.f. 16 V and negligible internal resistance is connected in a circuit as shown. All values are given to two significant figures.



Exam Tip
If an exam question tells you that something's got a negligible internal resistance, you can completely ignore it in your calculations.

a) Show that the group of resistors between X and Y could be replaced by a single resistor of resistance 15 Ω.

You can find the combined resistance of the 15 Ω, 20 Ω and 12 Ω resistors using:

$1/R = 1/R_1 + 1/R_2 + 1/R_3 = 1/15 + 1/20 + 1/12 = 1/5 \Rightarrow R = 5\ \Omega$

So overall resistance between X and Y can be found by:

$R = R_1 + R_2 = 5 + 10 = 15\ \Omega$

b) If $R_A = 20\ \Omega$:

i) calculate the potential difference (p.d.) across R_A.

Careful — there are a few steps here. You need the p.d. across R_A, but you don't know the current through it. So start there:
Total resistance in circuit = 20 + 15 = 35 Ω, so the current through R_A can be found using $I = V_{total} \div R = \frac{16}{35}$ A

Then you can use $V = IR_A$ to find the p.d. across R_A:

$V = \frac{16}{35} \times 20 = 9.1428... = 9.1$ V (to 2 s.f.)

Exam Tip
If you get a question like this in the exam and you don't know where to start, write down all the information you do know and work out anything you can work out. You might spot how to do the question whilst you're playing around with the numbers.

ii) calculate the current in the 15 Ω resistor.

You know the current flowing into the group of three resistors and out of it, but not through the individual branches. But you know that their combined resistance is 5 Ω (from part a)) so you can work out the p.d. across the group:

$V = IR = \frac{16}{35} \times 5 = \frac{16}{7}$ V

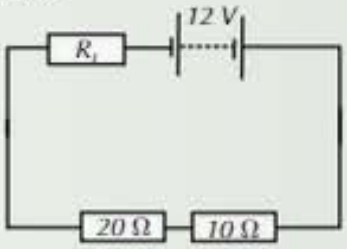
The p.d. across the whole group is the same as the p.d. across each individual resistor, so you can use this to find the current through the 15 Ω resistor:

$I = V \div R = \frac{16}{7} \div 15 = 0.15238... = 0.15$ A (to 2 s.f.)

Tip: Keeping numbers as fractions in your working can help avoid rounding errors creeping into your answers.

Practice Questions — Application

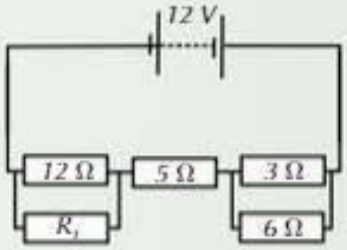
Q1 A battery of negligible resistance is connected in a circuit as shown to the right.



a) The total resistance of the circuit is 40 Ω. What is the resistance of the resistor marked R_1?

b) The current flowing through the battery is 0.4 A. What is the current flowing through the 10 Ω resistor?

Q2 The battery in the circuit below has a negligible internal resistance. The total resistance in the circuit is 10 Ω.



a) Calculate the resistance of R_1.

b) Calculate the potential difference across the 5 Ω resistor.

c) Calculate the current through the 6 Ω resistor.

Tip: Remember to calculate the resistance of resistors in parallel differently to the way you calculate resistors in series.

Practice Questions — Fact Recall

Q1 State Kirchhoff's first law.

Q2 State Kirchhoff's second law.

Q3 How are the resistors R_1 and R_2 connected if their total resistance is equal to $R_1 + R_2$?

Q4 Which type of circuit is $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ true for?

11. The Potential Divider

Potential dividers can be used to supply a varying potential difference. If only that was all you needed to know about them, but I'm afraid there's a bit more...

Learning Objectives:

- Understand a potential divider circuit.
- Be able to use potential divider equations, such as $\frac{V_1}{V_2} = \frac{R_1}{R_2}$ and $V_{out} = \frac{R_2}{R_1 + R_2} V_{in}$.
- Understand potential divider circuits with variable components, e.g. LDRs and thermistors.
- Know techniques and procedures used to investigate potential divider circuits which may include a sensor such as a thermistor or an LDR (PAG4).

Specification Reference 4.3.3

What is a potential divider?

At its simplest, a **potential divider** is a circuit with a voltage source and a couple of resistors in series. The potential difference across the voltage source (e.g. a battery) is split across the resistors in the ratio of the resistances. You can write this as:

V_1 = *p.d. across resistor 1 (in V)*

V_2 = *p.d. across resistor 2 (in V)*

$$\frac{V_1}{V_2} = \frac{R_1}{R_2}$$

R_1 = *resistance of resistor 1 (in Ω)*

R_2 = *resistance of resistor 2 (in Ω)*

So you can choose resistances to supply a potential difference, V_{out}, between zero and the potential difference across the power supply.

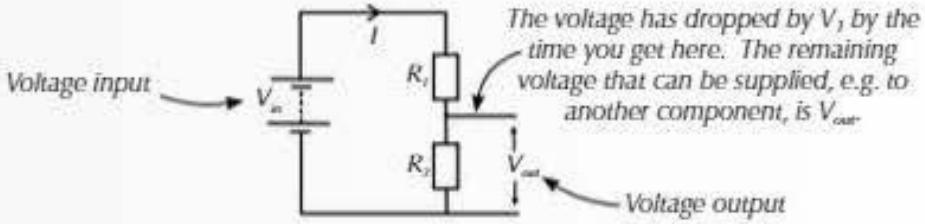

Voltage input
V_{in}
I
R_1
R_2
V_{out}
The voltage has dropped by V_1 by the time you get here. The remaining voltage that can be supplied, e.g. to another component, is V_{out}.
Voltage output

Figure 1: *A simple potential divider made up of two fixed resistors.*

You can find an equation relating V_{in} and V_{out} using $V = IR$ (see p.155). The total resistance in the circuit is $R = R_1 + R_2$, as the resistors are in series (see page 176). The total voltage across the resistors is V_{in} and the current through them is I, so $V_{in} = I(R_1 + R_2)$. Which when rearranged gives:

$$I = \frac{V_{in}}{R_1 + R_2}$$

You can also write I in terms of V_{out}, as $V_{out} = V_2$, and the resistance across R_2.

$$V_{out} = IR_2 \Rightarrow I = \frac{V_{out}}{R_2}$$

Substituting I into the previous equation and rearranging gives:

$$V_{out} = \frac{R_2}{R_1 + R_2} V_{in}$$

Tip: Make sure when you're using this equation that you plug in R_2 as the resistor that you're measuring the voltage over — otherwise you'll get completely the wrong answer.

Example — **Maths Skills**

For the circuit in Figure 1, $V_{in} = 9$ V and $V_{out} = 6$ V. Suggest one set of possible values for R_1 and R_2.

First find what fraction V_{out} is of V_{in}:

$$\frac{V_{out}}{V_{in}} = \frac{6}{9} = \frac{2}{3} \Rightarrow V_{out} = \frac{2}{3} V_{in}$$

$$V_{out} = \frac{R_2}{R_1 + R_2} V_{in}, \text{ so } \frac{R_2}{R_1 + R_2} = \frac{2}{3}$$

This multiplies out to give $3R_2 = 2R_1 + 2R_2 \Rightarrow R_2 = 2R_1$

So you could have, say, $R_2 = 200\ \Omega$ and $R_1 = 100\ \Omega$.

Tip: Voltmeters have a very high resistance. You can calibrate a voltmeter by connecting it across R_2 and setting it so that its reading matches the value of V_{out} given by the equation.

This equation will work as long as something with relatively high resistance is connected across R_2. But if you connect something with a relatively small resistance, then you start to run into problems. The equivalent resistance of the component and R_2 will be notably less than R_2, and so will cause a significant change in the current in the circuit. This will mean the equation no longer works properly, and will not give you an accurate answer for V_{out}.

Light and temperature sensors

PRACTICAL ACTIVITY GROUP 4

A light-dependent resistor (see p.160) has a very high resistance in the dark, but a lower resistance in the light. An NTC thermistor (page 160) has a high resistance at low temperatures, but a much lower resistance at high temperatures (it varies in the opposite way to a normal resistor and to a much greater extent). By using one of these components in a potential divider, your V_{out} can vary with light or heat, so it works as a light or heat sensor.

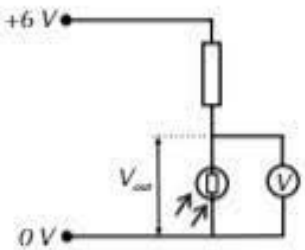


Figure 2: *A sensor used to detect changes in light levels — as it gets darker, V_{out} increases.*

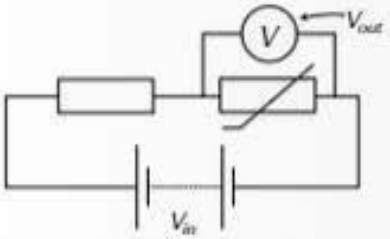


Figure 3: *A heat sensor circuit.*

Tip: Do a risk assessment before you start this experiment. Keep the rest of the circuit as far away from the Bunsen burner and the beaker of water as possible and make sure you waterproof the thermistor, e.g. by wrapping it in polythene.

Figure 3 shows the circuit you could use to create a heat sensor. Place the thermistor in a beaker of ice water. Measure the initial temperature of the water and record the voltage across the thermistor.

Heat the beaker gently using a Bunsen burner (make sure the water is well-stirred), and record the temperature and V_{out} at regular intervals over a suitable range (e.g. at 5 °C intervals over a range of 0-100 °C).

Plot a graph of voltage against temperature from your results. You should see that as the temperature of the water increased, V_{out} decreased. This is because as the NTC thermistor got hotter, its resistance decreased so it got a smaller 'share' of the total potential difference.

This kind of circuit could form the basis of a digital thermometer (the graph of voltage against temperature would be the thermometer's calibration curve), or could form part of the circuit for a thermostat in a central heating system.

Potentiometers

A potentiometer has a variable resistor replacing R_1 and R_2 of the potential divider (see Figure 5), but it uses the same idea. You move a slider or turn a knob to adjust the relative sizes of R_1 and R_2, which is useful when you want to change a voltage continuously, like in the volume control of a stereo.

Figure 4: *A rotary potentiometer. The relative sizes of R_1 and R_2 are controlled by the position of the dial.*

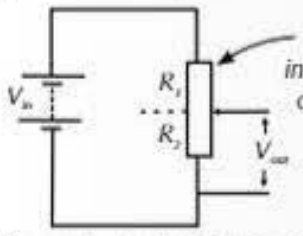
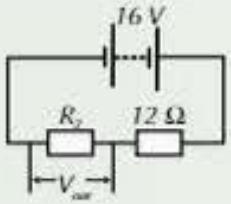


Figure 5: *A potentiometer.*

Practice Questions — Application

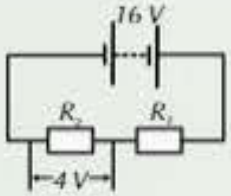
Q1 The circuit below shows a simple potential divider.



a) What would the output voltage be if $R_2 = 3.0\ \Omega$?

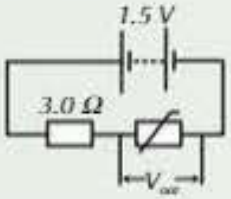
b) What would the resistance of R_2 have to be for V_{out} to be 5.0 V?

Q2 The potential divider below has the values of the resistors missed off.



Give one set of possible values for resistors R_1 and R_2.

Q3 The circuit diagram below shows part of a temperature sensor for a greenhouse.



When the greenhouse gets too hot this temperature sensor sets off an alarm.

a) The resistance of the thermistor when the alarm first starts to go off is 1.5 Ω. Calculate the voltage output of the circuit at this point.

b) The gardener starts to grow tropical plants in his greenhouse. He wants the alarm to go off when the voltage output drops to 0.30 V. Calculate the new resistance of the thermistor when the alarm first starts to go off.

Practice Questions — Fact Recall

Q1 What is a potential divider?

Q2 Write down the equation you would use to work out the voltage output of a potential divider. Define all the symbols you use.

Q3 How can you make a light sensor using a potential divider?

Q4 What is a potentiometer? Give an example of when it could be used.

Section Summary

Make sure you know...

- The circuit symbols for various components and how to draw circuit diagrams using these symbols.
- The definition of electric current as rate of flow of charge, and how to calculate its size using $I = \frac{\Delta Q}{\Delta t}$.
- That the coulomb is the unit of charge.

cont...

- The direction of flow of conventional current is opposite to the direction of actual electron flow.
- That the net charge on an object is quantised and a multiple of e, where e equals 1.6×10^{-19} C.
- That current is the movement of electrons in metals and movement of ions in electrolytes.
- How number density of charge carriers, n, differs between conductors, semiconductors and insulators.
- That the mean drift velocity of charge carriers is a measure of the average speed of the charge carriers.
- Know how to use the equation $I = Anev$ to calculate the current through a conductor with cross-sectional area, A, charge carrier number density, n, and mean drift velocity, v.
- That the potential difference (p.d.) between two points is the work done to move a unit charge between those two points, and is measured in volts.
- How to calculate the work done, or energy transferred, using $W = VQ$.
- That, for electrons and other charged particles being accelerated from rest by a potential difference V, $eV = \frac{1}{2}mv^2$, where e is the charge of the particle, m is the particle's mass, and v is its velocity.
- That resistance is defined by Ohm's law, $R = \frac{V}{I}$, and it is measured in ohms.
- That the resistivity of a material is a measure of how much that particular material resists current flow.
- How to use the equation $R = \frac{\rho l}{A}$ to calculate the resistance of a conductor.
- How the resistivity of metals and semiconductors varies with temperature.
- The techniques and procedures used to determine the resistivity of a metal.
- That a diode is a semiconductor device which will only let current flow through it in one direction, and that current will only flow if the p.d. across the diode is greater than the threshold voltage.
- That a negative temperature coefficient (NTC) thermistor is a semiconductor device whose resistance decreases with increasing temperature.
- That an LDR is a semiconductor device whose resistance decreases with increasing light intensity.
- How to carry out an experiment to investigate the electrical characteristics for a range of ohmic and non-ohmic components.
- The I-V characteristics of a resistor, filament lamp, thermistor, diode and a light-emitting diode (LED).
- How to use the power equations, $P = VI$, $P = I^2R$ and $P = \frac{V^2}{R}$.
- How to calculate energy transferred by an electrical component using $W = VIt$.
- That the kilowatt-hour (kWh) is a unit of energy and it can be used to calculate the cost of energy.
- What is meant by the internal resistance and electromotive force (e.m.f.) of a power supply.
- How to calculate the energy transferred by a source of e.m.f. using $W = \varepsilon Q$.
- What is meant by 'terminal potential difference' and the distinction between e.m.f. and p.d. in terms of energy transfer.
- That the 'lost volts' of an e.m.f. source is the energy wasted per coulomb due to internal resistance.
- How to use the equations $\varepsilon = I(R + r)$ and $\varepsilon = V + Ir$ and how to deal with sources of e.m.f. in series and parallel.
- The method used to determine the internal resistance of a chemical cell or other source of e.m.f.
- Kirchhoff's first and second laws as applied to electrical circuits, including understanding Kirchhoff's second law in terms of the conservation of energy.
- How to analyse circuits with components in both series and parallel.
- How to calculate the equivalent resistance of two or more resistors in series and in parallel.
- What is meant by a potential divider circuit and how to use the equations $\frac{V_1}{V_2} = \frac{R_1}{R_2}$ and $V_{out} = \frac{R_2}{R_1 + R_2} V_{in}$
- How to investigate potential divider circuits, which may include a sensor such as a thermistor or an LDR.

Exam-style Questions

1 Calculate the resistance of a component that has a power rating of 11 W if the potential difference being supplied is 230 V.

A $2.5 \times 10^3\ \Omega$

B $4.8 \times 10^3\ \Omega$

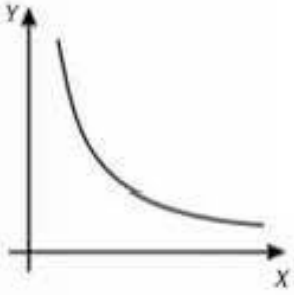
C $21\ \Omega$

D $5.8 \times 10^5\ \Omega$

(1 mark)

2 A wire made from a material with a constant resistivity ρ is placed in a complete circuit. An experiment is carried out in which X is the independent variable, and Y is the dependent variable. Assume all other variables remain constant.
Which of the following X and Y quantities would produce the graph shown below?
Wire length = L, radius of the wire = r and resistance of a wire = R.

	X	Y
A	L	R
B	R	L
C	r	R
D	L	r



(1 mark)

3 In one minute, an electric heater connected to a 160 V power supply transfers 120 kJ of heat energy. What is the current flowing through the electric heater?

A 8.00×10^{-2} A

B 1.25×10^{-2} A

C 750 A

D 12.5 A

(1 mark)

4 Which of the following statements is true?

A A diode is an example of an ohmic conductor.

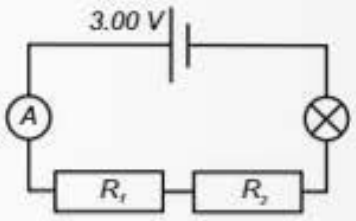
B The resistance of a diode is dependent on the light intensity of its surroundings.

C Diodes are designed to let current flow in one direction only.

D A diode is an example of an insulator.

(1 mark)

5 The circuit on the right has a cell connected in series to an ammeter, two resistors and a filament bulb.



The current flowing through the cell is 0.724 A.

(a) Calculate the power of the cell.

(2 marks)

(b) Calculate the charge passing through the cell in exactly 5 minutes.

(2 marks)

(c) It takes 56.5 J of energy to move the charge through resistor R_1 in exactly 5 minutes. Calculate the potential difference across R_1.

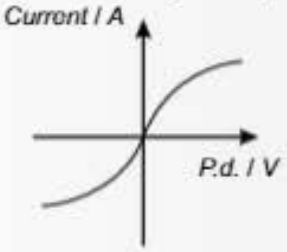
(2 marks)

(d) The ammeter and the cell have negligible resistances. The filament lamp has a resistance of 2.00 Ω. Calculate the potential difference across resistor R_2.

(2 marks)

(e) The *I-V* graph for the filament lamp is shown on the right. State whether the filament lamp is an ohmic conductor. Explain your answer.

(1 mark)



6 All batteries and power supplies have an internal resistance.

(a) Explain why a battery has an internal resistance.

(1 mark)

(b) Describe an experiment that could be carried out to accurately determine the internal resistance of a battery. Include a labelled circuit diagram in your answer.

(5 marks)

In an experiment to find the internal resistance of a battery, a student gathered the following data:

Terminal voltage (V)	Current (A)
1.00	2.79
3.00	2.36
4.00	1.18
6.00	1.71
8.00	1.29
9.00	1.07

(c) Draw a *V-I* graph of the data shown in the table.

(2 marks)

(d) Use the graph to find the e.m.f. of the battery.

(1 mark)

(e) Use the graph to determine the internal resistance of the battery.

(2 marks)

7 The circuit shown in **Fig 7.1** contains a battery with an e.m.f. of 12 V with a negligible internal resistance.
Component *A* has a resistance that varies with temperature.

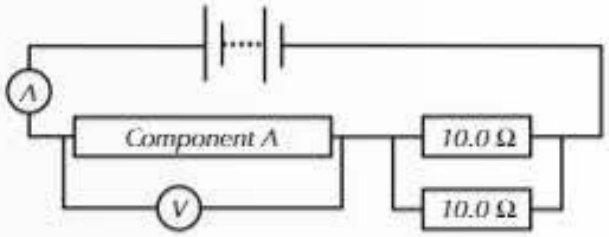


Fig 7.1

At 298 K the resistance of component *A* is 10.0 Ω.

(a) Calculate the total resistance in the circuit at 298 K.

(2 marks)

(b) Calculate the potential difference across component *A* at 298 K.

(2 marks)

Component *A* is cooled to 96 K. The resistance of the component drops to 3.21 mΩ.

(c) Calculate the current flowing through component *A* at 96 K.

(2 marks)

(d) Component *A* is a wire with a cross-sectional area of 3.05×10^{-6} m^2 and a length of 3.00 cm. Calculate the resistivity of the wire's material at 96 K.

(2 marks)

8 This question is about potential dividers.

(a) Draw a circuit to show the set-up of a potential divider using the following components: two cells (connected in series), a thermistor, a resistor and wires. The circuit must be set up so that the output voltage is the same as the potential difference across the resistor.

(2 marks)

Use the circuit drawn in part (a) to answer the following questions.

(b) The two cells connected in series provide an e.m.f. of 22 V. The resistor has a resistance of 6.0 Ω. At a temperature of 12°C the resistance of the thermistor is 9.2 Ω. Calculate the output voltage. Assume the cells have negligible internal resistance.

(1 mark)

(c) Calculate the energy dissipated by the resistor when the potential divider is used for 12 minutes.

(2 marks)

(d) The temperature of the circuit increases to 21°C. The output voltage is now 13 V. Calculate the resistance of the thermistor at 21°C.

(2 marks)

Learning Objectives:

- Understand the meaning of the term 'progressive waves'.
- Know the meaning of the terms displacement, amplitude, wavelength, period, frequency and phase difference in relation to waves.
- Understand what transverse waves and longitudinal waves are.
- Understand and be able to draw graphical representations of transverse and longitudinal waves.

Specification Reference 4.4.1

1. Progressive Waves

Waves are just vibrations. They transfer energy, but once they've passed through a region, everything goes back to normal... like they were never there.

What is a progressive wave?

A **progressive wave** (moving wave) carries energy from one place to another without transferring any material. Imagine a buoy bobbing up and down on a water wave — the buoy doesn't move from its location except to move up and down as the wave passes.

A wave is caused by something making particles or fields (e.g. electric or magnetic fields) oscillate (or vibrate) at a source. These oscillations pass through the medium (or field) as the wave travels, carrying energy with them in the direction of travel. A wave transfers this energy away from its source — so the source of the wave loses energy. You can tell waves carry energy from looking at the effects different waves can have. For example:

- Electromagnetic waves cause things to heat up.
- X-rays and gamma rays transfer so much energy that they knock electrons out of their orbits, causing ionisation.
- Loud sounds cause large oscillations of air particles which can make things vibrate.
- Sea waves can be used to generate electricity.

Wave basics

There are lots of quantities that you can use to describe a wave.

- **Displacement**, x, measured in metres
  — how far a point on the wave has moved from its undisturbed position. Going back to the buoy example above, the displacement would be how high the buoy is above sea level, or how low it is below sea level.
- **Amplitude**, A, measured in metres
  — the maximum magnitude of the displacement, i.e. the distance from the undisturbed position to the crest, or trough (see Figure 1). The amplitude of a bobbing buoy would be the distance from the undisturbed position (sea level) to the highest point it reaches above sea level, or the lowest point it reaches below sea level.

Tip: Make sure you learn all these wave properties, their symbols and the units they're given in. You could be asked to find them from a graph or a diagram of a wave.

Tip: Amplitude can either be measured at a crest or a trough. It doesn't have direction, only magnitude.

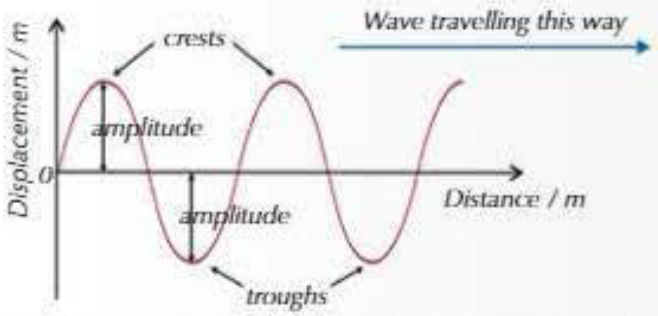


Figure 1: *Diagram showing the displacement and amplitude of a wave.*

- **Wavelength**, λ, measured in metres
  — the length of one whole wave oscillation or wave cycle, e.g. the distance between two crests (or troughs) of a wave.

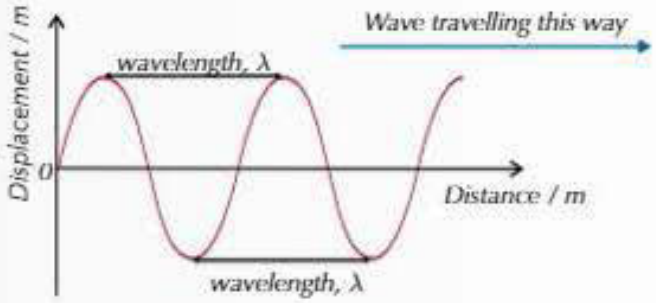
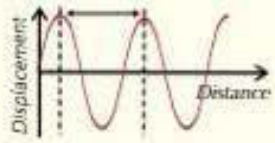


Figure 2: *Diagram to show the wavelength of a wave.*

Tip: One oscillation or wave cycle is a 'section' of a wave from crest to crest, or trough to trough.



- **Period**, T, measured in seconds
  — the time taken for one whole wave cycle, e.g. the time it takes a buoy to go from its highest point, back to its highest point again.
- **Frequency**, f, measured in hertz
  — the number of whole wave cycles (oscillations) per second passing a given point. Or the number of whole wave cycles (oscillations) given out from a source per second. For a buoy, it's the number of times it reaches its highest point per second.

Tip: Waves with different frequencies and wavelengths can have very different properties (see page 195).

- **Phase**
  — a measurement of the position of a certain point along the wave cycle.
- **Phase difference**
  — the amount by which one wave lags behind another wave, or the difference in phase between two points on a single wave.

Tip: The phase difference between two waves is the difference between a point on one wave, and the next identical point on the second wave.

Phase and phase difference can be measured in angles (in degrees or radians) or in fractions of a cycle (see page 208).

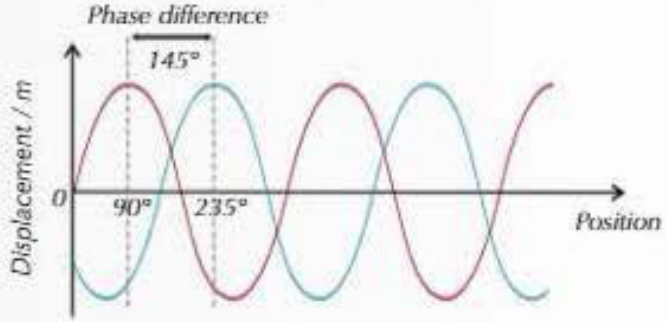


Figure 3: *Diagram to show the phase difference between waves.*

Types of wave

Waves can be either **transverse waves** or **longitudinal waves**.

Transverse waves

In transverse waves, the oscillation of the particles or field (i.e. the displacement) is at right angles to the direction of energy propagation (the direction the wave is travelling in). All electromagnetic waves (see page 194) are transverse. They travel as oscillating magnetic and electric fields — with oscillations perpendicular to the direction of energy transfer. Other examples of transverse waves are ripples on water, waves on strings, and some types of earthquake shock wave (S-waves).

Figures 4 and 5 show how a transverse wave can be demonstrated using a long spring.

Figure 5: *A photograph of a transverse wave produced on a spring.*

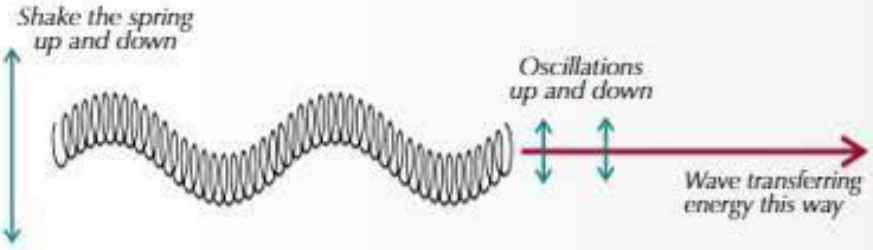


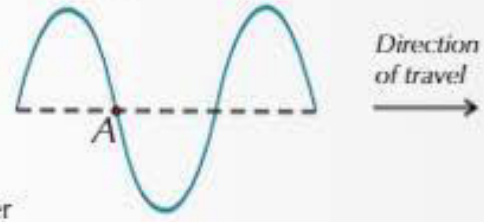
Figure 4: *A transverse wave on a spring.*

You can work out what direction a point on a transverse wave is moving in when given a snapshot of the wave.

Tip: This can be pretty confusing at first. If you're struggling, try drawing the wave moved slightly further along in the direction of travel. Look at how it lines up with the point you're given. If the wave is now above the point, it's moving upwards. If it's below the point, it's moving downwards.

Example

Look at the snapshot of the wave below.
Which direction is point A on the wave moving in?



1. Look at which direction the wave is travelling in — here the wave is moving from left to right.
2. The displacement of the wave just to the left of point A is greater than point A's. So as the wave travels along, point A will need to move upwards to have that displacement. (If the displacement to the left was less than point A's, point A would need to move down.)

There are two main ways of drawing transverse waves, as shown in Figure 6. They can be shown as graphs of displacement against distance along the path of the wave, or as graphs of displacement against time for a point as the wave passes.

Tip: When you use an oscilloscope (p.190) to look at a wave, it produces a displacement-time graph.

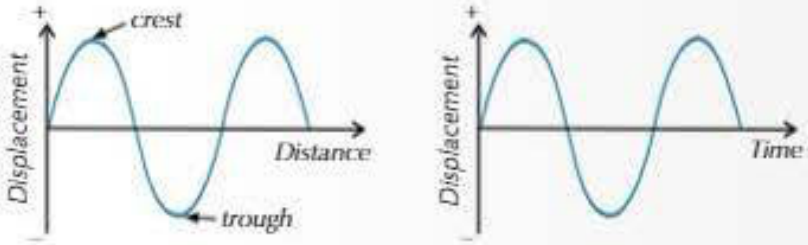


Figure 6: *Two ways of representing transverse waves graphically — as a displacement-distance graph (on the left) or as a displacement-time graph (on the right).*

Tip: Displacement is a vector so it has direction and magnitude (see page 41) — if a point has moved below its undisturbed position, it will have a negative displacement.

Both sorts of graph often give the same shape, so make sure you check out the label on the *x*-axis. On a displacement-distance graph, the separation between two crests (a wave cycle) gives the wavelength of the wave. On a displacement-time graph, this gives the period of the wave.

Displacements upwards from the centre line are given a + sign. Displacements downwards are given a – sign.

Longitudinal waves

In longitudinal waves the oscillations of the particles or field are in the direction of energy propagation. The most common example of a longitudinal wave is sound. A sound wave consists of alternate compressions and rarefactions of the medium it's travelling through. That's why sound can't go through a vacuum. Some earthquake shock waves (P-waves) are longitudinal.

Figures 7 and 8 show how a longitudinal wave can be demonstrated using a long spring.

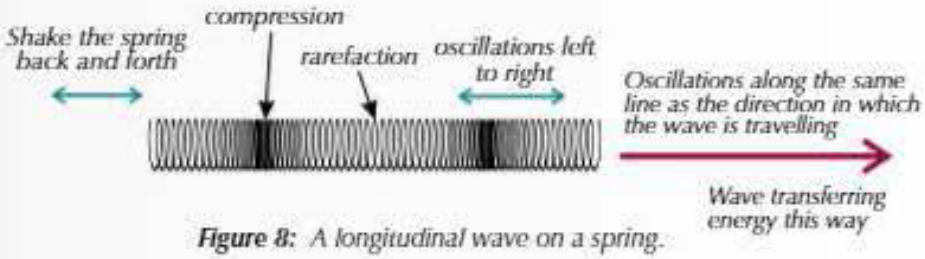

Shake the spring back and forth
compression
rarefaction
oscillations left to right
Oscillations along the same line as the direction in which the wave is travelling
Wave transferring energy this way

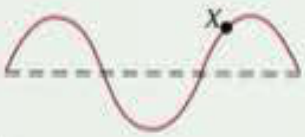
Figure 8: *A longitudinal wave on a spring.*

Figure 7: *A photograph of a longitudinal wave produced on a spring.*

It's hard to represent longitudinal waves graphically. You can plot them on displacement-distance or displacement-time graphs, just like transverse waves. They'll give you graphs that look exactly like the graphs for transverse waves in Figure 6. This can be confusing, as, unlike transverse waves, these graphs look nothing like the waves themselves. You'll usually see longitudinal waves plotted as displacement-time graphs.

Practice Questions — Application

Q1 The diagram on the right shows a snap shot of a transverse wave. The point marked *X* is moving upwards. What is the direction of travel of the wave?

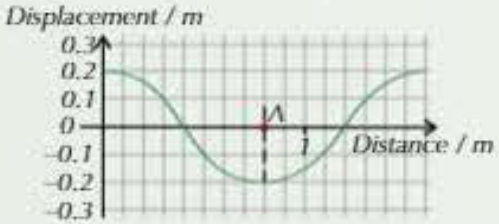


Q2 For the wave shown, find:

a) the displacement at A,

b) the amplitude,

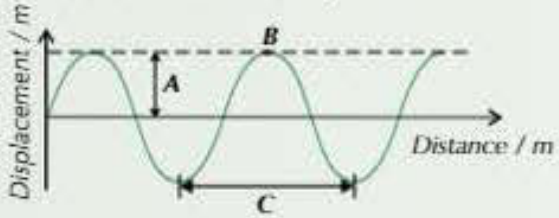
c) the wavelength.



Practice Questions — Fact Recall

Q1 How does a wave transfer energy through a region?

Q2 On the diagram below, what's shown by *A*, *B* and *C*?



Q3 Describe what is meant by the phase difference of two waves.

Q4 Describe the difference between longitudinal and transverse waves in terms of their oscillations.

Q5 Sketch a displacement-time graph for a longitudinal wave.

Learning Objectives:

- Be able to use the equation $f = \frac{1}{T}$.
- Know techniques and procedures involved in using an oscilloscope to determine frequency (PAG5).
- Be able to use the equation for the speed of a wave, $v = f\lambda$.
- Be able to use the equation for the intensity of a progressive wave, $I = \frac{P}{A}$.
- Know that, for a wave, intensity $\propto$ (amplitude)^2

Specification Reference 4.4.1

2. Frequency, Speed and Intensity

Now it's time for a bit of maths. You'll have probably come across the frequency and wave speed equations before now, but the new and exciting world of intensity is also coming up in this topic...

Frequency and period

Frequency and period are linked to each other. The number of whole wave cycles (oscillations) per second is 1 ÷ (time taken for one oscillation). So, by definition, the frequency is the inverse of the period:

$$f = \frac{1}{T}$$

f = *frequency in Hz (= s^{-1})*

T = *period in s*

Example — **Maths Skills**

A wave has a period of 0.25 seconds.
How many oscillations will pass a given point each second?

The number of oscillations passing a point per second is the frequency, so

$$f = \frac{1}{T} = \frac{1}{0.25} = 4.0 \text{ Hz}$$

So 4.0 oscillations pass each second.

Frequency and period on an oscilloscope

PRACTICAL ACTIVITY GROUP 5

A cathode ray **oscilloscope** (CRO) produces a function of voltage over time from a signal generator. The displayed function is called a trace.

The type of trace you get on an oscilloscope depends on the source it is connected to. If you plug an AC (alternating) supply into an oscilloscope, you get a trace that goes up and down in a regular pattern — some of the time it's positive and some of the time it's negative. A microphone converts sound waves into electrical signals which can be seen on an oscilloscope.

An example of an oscilloscope display is shown in Figure 2.

Figure 1: *An oscilloscope displaying the sound wave produced by a tuning fork.*

Tip: As always, remember to do a risk assessment before working in the lab.

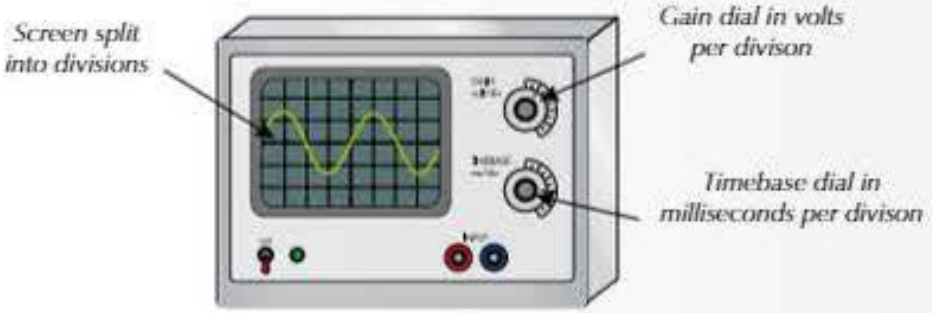


Figure 2: *A diagram of an oscilloscope, showing its display along with its gain and timebase dials.*

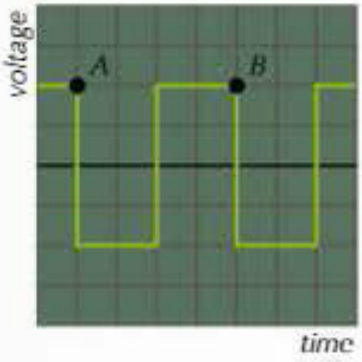
The screen is split into squares called divisions. The vertical axis is in volts. The volts per division shown on this axis is controlled by the gain dial. The horizontal axis is in seconds — also called the timebase. The seconds per division shown on this axis is controlled by the timebase dial. You can alter the gain and timebase to make it easy to read off measurements.

You can use oscilloscope traces to calculate wave properties, like frequency.

Figure 3: *An example of an oscilloscope being used to measure a wave.*

Example — **Maths Skills**

Find the frequency shown by the oscilloscope trace on the right. The timebase is set to 4.0 ms / div.



First calculate the period of the trace.
Point A to B is one cycle — it is 4 squares wide.

This means for one cycle, it takes
$4 \times 4.0 \text{ ms} = 16 \text{ ms}$.

So, the period $T = 16$ ms.

Now, use $f = \frac{1}{T}$ to find the frequency:

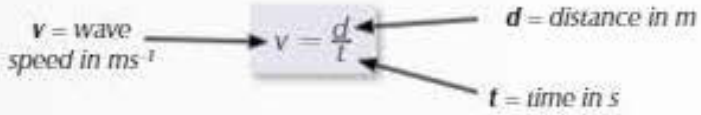
$$f = \frac{1}{16 \times 10^{-3}} = 62.5 \text{ Hz} = 63 \text{ Hz (to 2 s.f.)}$$

Tip: Make sure you carefully read the timebase and gain on an oscilloscope before you use it to do any calculations.

Exam Tip
If you can remember 1 Hz = 1 s^{-1}, it'll help you get your units right. Don't write s^{-1} in an exam though — Hz is the standard unit for frequency (see p.31 for more on units).

Wave speed

You can use speed = distance ÷ time to find wave speed.



You can derive the wave speed equation by imagining how long it takes for the crest of a wave to move across a distance of one wavelength.

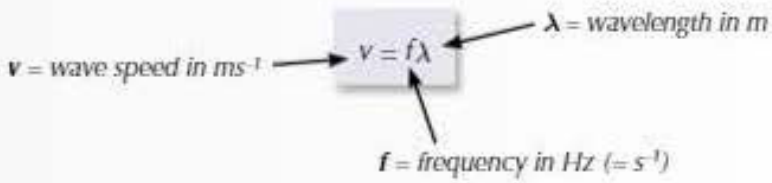
- The distance travelled is the wavelength, λ.
- The time taken to travel one wavelength is the period of the wave, which is equal to $\frac{1}{f}$.

Now substitute these values into the equation above to get the speed of a wave in terms of wavelength and frequency:

$$\text{wave speed } (v) = \frac{\text{distance } (\lambda)}{\text{time } \left(\frac{1}{f}\right)}$$

Dividing something by $\frac{1}{f}$ is the same as multiplying it by f.

So you get the wave speed equation:



Exam Tip
You don't need to know this derivation, but this should help you understand where the equation comes from, and why it works.

Tip: Remember, you're not measuring how fast a physical point (like one molecule of rope) moves. You're measuring how fast a point on the wave pattern moves.

Example — **Maths Skills**

Below is a diagram of a water wave.
Calculate the speed of the wave, if the frequency is 5 Hz.

First find λ by calculating the distance, e.g., between the first two peaks:

$\lambda = 2.5 - 0.5 = 2$ m

Then substitute λ and f into $v = f\lambda$ to find the speed of the wave:

$v = f\lambda = 5 \times 2 = 10$ ms^{-1}

Tip: When a wave changes speed, its wavelength changes but its frequency remains constant (see p.199).

Intensity

When you talk about "brightness" for light or "loudness" for sound, what you really mean is how much light or sound energy hits your eyes or your ears per second. The scientific measure of this is **intensity**. Intensity is the rate of flow of energy per unit area at right angles to the direction of travel of the wave. It's measured in Wm^{-2}.

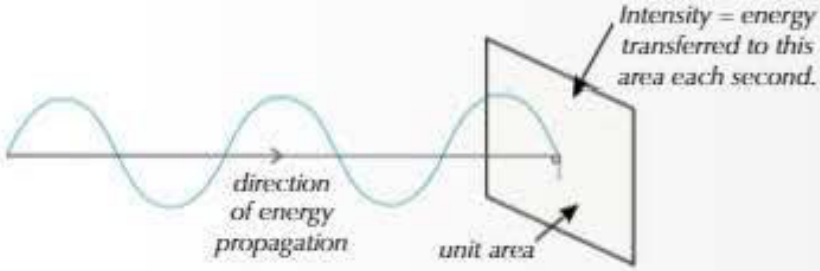


Figure 4: *A diagram demonstrating the intensity of a wave on a unit area.*

Tip: Remember, power is a measure of energy transferred per second.

You can calculate intensity using the equation:

$$I = \frac{P}{A}$$

I = *intensity in* Wm^{-2}; ***P*** = *power in W*; ***A*** = *area (at right angles to the wave's motion) in* m^2

Example — **Maths Skills**

A 28.0 W sound wave is incident at right angles to the surface of a square metal plate. The plate has sides of length 1.20 m. Calculate the intensity of the sound wave on the metal plate.

Surface area of the plate, $A = 1.2 \times 1.2$
$= 1.44$ m^2

$I = \frac{P}{A}$

$= \frac{28.0}{1.44} = 19.44... = 19.4$ Wm^{-2} (to 3 s.f.)

The intensity of a wave is also proportional to its amplitude squared. You don't need to know why this is the case, but you do need to learn the proportionality relation:

$$\text{Intensity} \propto (\text{Amplitude})^2$$

Tip: If two variables are proportional like this, then you can say that one variable is equal to the other variable multiplied by a constant. E.g. Intensity = constant × (amplitude)^2. This is helpful for finding missing values.

Example — **Maths Skills**

A wave has an intensity of 18 Wm^{-2}, and an amplitude of 1.5 m. The wave's intensity is reduced to 2 Wm^{-2}. Calculate the new amplitude of the wave.

We know that intensity $\propto$ (amplitude)^2.

So, $\dfrac{\text{intensity}}{(\text{amplitude})^2} = \text{constant}$

This means that $\dfrac{\text{final intensity}}{(\text{final amplitude})^2} = \dfrac{\text{initial intensity}}{(\text{initial amplitude})^2}$

$$\Rightarrow \text{final amplitude} = \sqrt{\frac{\text{final intensity}}{\text{initial intensity}}} \times \text{initial amplitude}$$

$$= \sqrt{\frac{2}{18}} \times 1.5 = \frac{1}{3} \times 1.5 = 0.5\,\text{m}$$

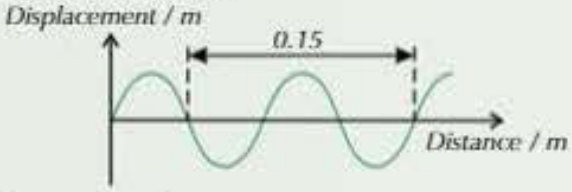
Practice Questions — Application

Q1 A wave hits a 0.2 m^2 panel at a right angle to the panel's surface. The intensity of the wave on the panel is 5 Wm^{-2}.

a) Calculate the power of the wave.

b) The amplitude of the wave is then doubled. Calculate the new intensity of the wave.

Q2 For the wave shown below:



a) Find its wavelength.

b) Find how long it takes the wave to travel 1.0 m, given that the frequency is 30.0 Hz.

Practice Questions — Fact Recall

Q1 How would you calculate the frequency of a wave, given its period?

Q2 What are the two axes of an oscilloscope display?

Q3 Describe how you would find the frequency of a wave shown on an oscilloscope from the oscilloscope display.

Q4 What is the equation that links wave speed, frequency and wavelength?

Q5 Define the intensity of a progressive wave.

Q6 What is the proportionality relation between intensity and amplitude?

3. Electromagnetic Waves

Perhaps the most important family of waves in physics is the electromagnetic spectrum. You need to know all the different types of EM waves, their properties, and how these properties change depending on wavelength.

Learning Objectives:

- Know the properties of electromagnetic waves.
- Know the electromagnetic spectrum.
- Know the orders of magnitude of wavelengths of the principal EM radiations from radio waves to gamma rays.

Specification Reference 4.4.2

What are electromagnetic waves?

Electromagnetic (EM) waves are a group of transverse waves. They consist of vibrating electric and magnetic fields, not matter, which means they are able to travel through a vacuum. These electric and magnetic fields are always at right angles to each other and to the direction of travel.

All electromagnetic waves have some properties in common:

- They all travel at the same speed in a vacuum — 3.00×10^8 ms^{-1} (to 3 s.f.).
- Like all waves, they can be refracted, reflected, diffracted and can undergo interference.
- They can be polarised, because they are transverse.
- They all obey the wave equation, $v = f\lambda$ (like all waves).

EM waves form a continuous series of waves, called the **electromagnetic spectrum**. The electromagnetic spectrum is split into seven categories based on wavelength. From the longest to the shortest wavelength, these are: radio waves, microwaves, infrared (IR), visible light, ultraviolet (UV), X-rays and gamma rays (see Figure 1).

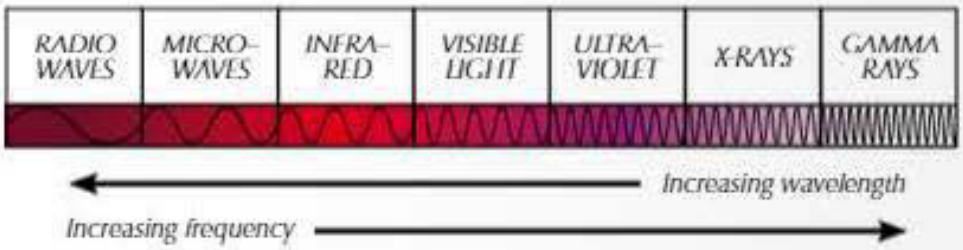


Figure 1: *The electromagnetic spectrum.*

Tip: The speed of EM waves in a vacuum is often referred to as c. It's an important number in quantum physics (see pages 232-244) and will be given to you in the data and formulae booklet.

Figure 2: *The visible light portion of the EM spectrum. Just like the colours above, each category of wave in the EM spectrum flows smoothly into the next one.*

Like all progressive waves, progressive EM waves carry energy. As you go along the spectrum, the frequency increases and the waves carry more energy. This is because the energy of an EM wave is proportional to its frequency (from $E = hf$, see p.232). So gamma rays have the highest energy, while radio waves have the lowest.

Towards the high frequency end of the UV range, waves become energetic enough to cause **ionisation**. This is where they transfer so much energy to an atomic electron, that the electron is removed from the atom. If this occurs in the human body it can kill cells or cause cancer. So, in general, the higher the frequency, the more dangerous the EM wave is.

Due to their wavelengths, different electromagnetic waves also behave differently in a given situation (e.g. some EM waves can diffract around hills, whilst others cannot). This means that electromagnetic waves have a range of uses. Figure 3 on the next page describes some of the most common uses.

Tip: Diffraction is where waves spread out, and it depends on the wavelength of the wave compared to the gap or obstacle causing the diffraction (p.211).

Type	Approximate Wavelength /m	Penetration	Uses
Radio waves	10^{-1} — 10^{6}	Pass through matter.	Radio transmissions.
Microwaves	10^{-3} — 10^{-1}	Mostly pass through matter, but cause some heating.	Radar. Microwave cooking. TV transmissions.
Infrared (IR)	7×10^{-7} — 10^{-3}	Mostly absorbed by matter, causing it to heat up.	Heat detectors. Night vision cameras. Remote controls. Optical fibres.
Visible light	4×10^{-7} — 7×10^{-7}	Absorbed by matter, causing some heating.	Human sight. Optical fibres.
Ultraviolet (UV)	10^{-8} — 4×10^{-7}	Absorbed by matter. Cause some ionisation.	Sunbeds. Security marks that show up under UV.
X-rays	10^{-13} — 10^{-8}	Mostly pass through matter, but cause ionisation as they pass.	To see damage to bones and teeth. Airport security scanners. To kill cancer cells.
Gamma rays	10^{-16} — 10^{-10}	Mostly pass through matter, but cause ionisation as they pass.	Irradiation of food. Sterilisation of medical instruments. To kill cancer cells.

Figure 3: *A table showing the different types of EM wave, their wavelength ranges, penetration properties, and examples of their uses.*

Figure 4: *An infrared camera being used to monitor the skin temperature of people arriving at an airport. High skin temperature can be a sign of illnesses such as flu.*

Figure 5: *Banknotes contain a fluorescent ink pattern, making them harder to forge. It only shows up under UV light of the correct frequency.*

Practice Question — Application

Q1 A radio wave with wavelength of 2.40×10^{5} m is travelling through a vacuum. Calculate the frequency of the radio wave.

Tip: Remember, $v = f\lambda$ (see page 191).

Practice Questions — Fact Recall

Q1 Are electromagnetic waves transverse or longitudinal waves?

Q2 Which type of wave in the EM spectrum has the highest energy?

Q3 State the wavelength range of the following parts of the electromagnetic spectrum:

a) microwaves b) ultraviolet light c) X-rays

Q4 State the three parts of the electromagnetic spectrum that are mostly absorbed by matter.

Learning Objectives:

- Understand what is meant by the term 'plane polarised waves'.
- Understand the polarisation of electromagnetic waves, e.g. light and microwaves.
- Know techniques and procedures used to observe polarising effects using light (PAG5).
- Know techniques and procedures used to observe polarising effects using microwaves (PAG5).

Specification References 4.4.1 and 4.4.2

4. Polarisation

When we draw waves, we usually show them vibrating in a single plane. But when real waves are produced, they're usually oscillating in a lot of different directions. This is where polarisation comes in...

Polarised waves

If you shake a rope to make a wave you can move your hand up and down or side to side or in a mixture of directions — it still makes a transverse wave. But if you try to pass waves in a rope through a vertical fence, the wave will only get through if the vibrations are vertical. The fence filters out vibrations in other directions. The result is a **plane polarised wave** (see Figure 1).

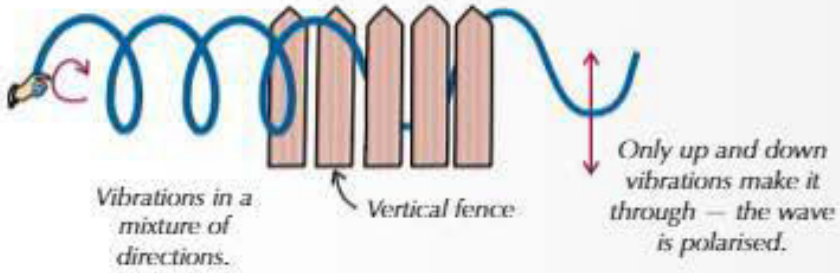


Figure 1: *The polarisation of a transverse wave through a fence.*

Tip: A plane is a flat surface in 3D space.

The plane in which a plane polarised wave vibrates is called the plane of polarisation — e.g. the rope wave was polarised in the vertical plane by the fence. Polarisation can only happen for transverse waves. The fact that you can polarise EM waves is one proof that they're transverse waves.

Figure 2: *Two polarising filters at right angles to each other block all light.*

Polarising light

PRACTICAL ACTIVITY GROUP 5

Ordinary light waves are a mixture of different directions of vibration. A **polarising filter** can be used to polarise light (and other waves). It only transmits vibrations in one direction. If you have two polarising filters at right angles to each other, then no light will get through — see Figure 2.

You can observe polarisation by shining unpolarised white light through two polarising filters. Start by aligning the transmission axes of two polarising filters so they are both vertical. Shine unpolarised light on the first filter. Keep the position of the first filter fixed and rotate the second one.

Tip: The transmission axis of a polarising filter shows the only direction of vibrations which can pass through the filter.

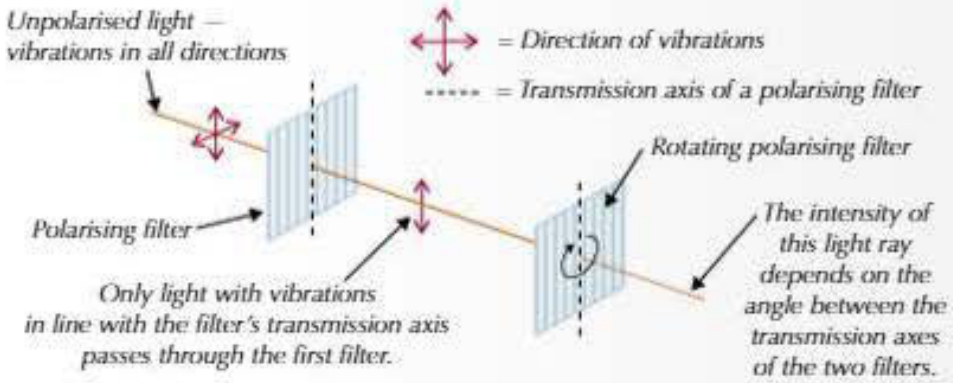

Unpolarised light — vibrations in all directions
= Direction of vibrations
= Transmission axis of a polarising filter
Rotating polarising filter
Polarising filter
Only light with vibrations in line with the filter's transmission axis passes through the first filter.
The intensity of this light ray depends on the angle between the transmission axes of the two filters.

Figure 3: *An unpolarised light wave passing through two polarising filters.*

Light that passes through the first filter in Figure 3 will always be vertically polarised. When the transmission axes of the two filters are aligned, all of the light that passes through the first filter also passes through the second. As you rotate the second filter, the amount of light that passes through the second filter varies.

As the second filter is rotated, less light will get through it as the vertical component of the second filter's transmission axis decreases. This means the intensity of the light getting through the second filter will gradually decrease. When the two transmission axes are at 45° to each other, the intensity will be half that getting through the first filter. When they're at right angles to each other no light will pass through — intensity is 0.

As you continue turning, the intensity should then begin to increase once again. When the two axes realign (after a 180° rotation), all the light will be able to pass through the second filter again. This is shown in Figure 4.

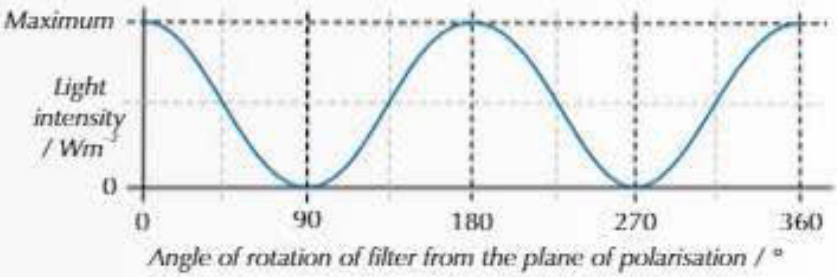


Figure 4: *The effect of rotating a polarising filter on the intensity of polarised light passing through it.*

As shown in Figure 4, at 90° and 270°, the light intensity is zero. This is because the transmission axis of the second filter will be perpendicular to the first filter, so no light will be able to get through. At 180° and 360°, the light intensity is at a maximum. This is because the transmission axes of both filters will be aligned, so all of the light passing through the first filter will be able to pass through the second.

Uses of polarising filters

You come across polarising filters more often than you might think.

Examples

- 3D films use polarised light to create depth — the filters in each lens are at right angles to each other so each eye gets a slightly different picture.
- Polaroid sunglasses also use polarising filters — light reflected off some surfaces is partially polarised so the sunglasses block this out to help prevent glare.

Tip: Just like vectors, you can think of the transmission axis of the rotating filter as having a vertical and horizontal component. The larger the vertical component, the more vertically polarised light will pass through the filter.

Tip: Light intensity is just the brightness of the light. See more about it on page 192.

Figure 5: *The light reflecting off water is partially polarised, so a polarising filter can let you see underwater shapes more clearly.*

Polarising microwaves

PRACTICAL ACTIVITY GROUP 5

Polarising filters don't work on microwaves — their wavelength is too long. Instead, metal grilles (squares full of metal wires which are all aligned, see Figure 6) are used to polarise them.

You can investigate the polarisation of microwaves using a microwave transmitter and a microwave receiver linked to a voltmeter. Place a metal grille between the microwave transmitter and receiver as shown in Figure 7. (Handily, microwave transmitters transmit vertically polarised microwaves, so you only need one metal grille.)

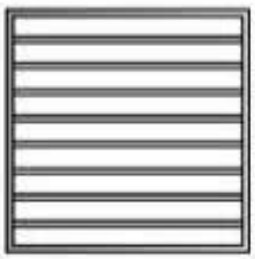


Figure 6: *A diagram of a metal grille, used to polarise microwave radiation.*

Tip: As always, make sure you carry out a risk assessment before you do either of the polarisation experiments. Make sure all of your electrical equipment is safely connected before you turn it on — microwave transmitters operate at very high voltages.

Figure 8: *A microwave transmitter.*

Tip: Don't get confused by metal grilles. The transmission axis for microwaves is perpendicular to the direction of the wires — which is the opposite of what you'd expect.

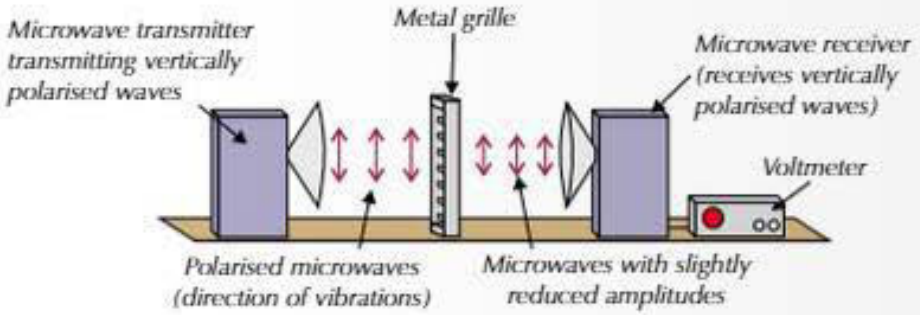

Microwave transmitter transmitting vertically polarised waves
Metal grille
Microwave receiver (receives vertically polarised waves)
Voltmeter
Polarised microwaves (direction of vibrations)
Microwaves with slightly reduced amplitudes

Figure 7: *A diagram of the experimental set-up for investigating the polarisation of microwaves by a metal grille.*

When microwaves meet a metal grille, some of their energy is absorbed by the grille. This is because the vibrating electric field of the microwave excites electrons in the metal. When the electrons de-excite, they re-emit this energy in all directions as microwaves. Only some of the emitted microwaves have oscillations in the vertical direction.

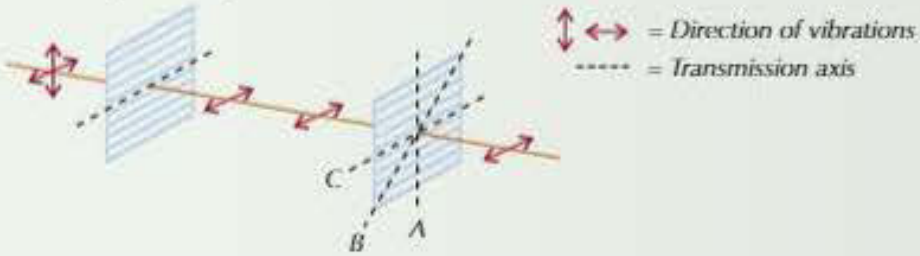
Microwave receivers only detect vertically polarised microwaves, so even if a re-emitted microwave travels towards the receiver, it might not be picked up. So whenever microwaves are absorbed by a metal grille, only a small fraction of their energy reaches the receiver.

Microwaves are absorbed most when the grille wires are aligned with the vibrations, and least when they're perpendicular to the vibrations. So a metal grille lets through most vibrations when the oscillations of the microwaves and the grille wires are at right angles to each other.

When using the setup in Figure 7, the voltmeter will measure the intensity of the radiation reaching the receiver. The intensity of microwaves passing through the grille is at a maximum when the direction of the vibration of the microwaves and the wires on the grille are at right angles to each other. This is because the minimum amount of energy is being absorbed. As you rotate the grille, the intensity of polarised microwaves able to pass through the grille decreases, so the reading on the voltmeter decreases. When the wires of the metal grille are aligned with the direction of vibration of the polarised waves, little to no signal will be shown on the voltmeter because most of the energy is being absorbed and re-emitted in all directions.

Practice Question — Application

Q1 The diagram below shows an unpolarised wave passing through two polarising filters.



a) Is this wave transverse or longitudinal? Explain your answer.

b) Which line, A, B or C, represents the transmission axis of the second polarising filter? Explain your answer.

c) Explain what would happen if the second polarising filter was rotated by 90°.

5. Reflection and Refraction

Reflection and refraction are two important wave effects that can change the direction a wave is travelling in. They happen when a wave hits a boundary between two different materials.

Learning Objectives:

- Understand reflection and refraction for all waves.
- Know and understand techniques and procedures used to demonstrate reflection and refraction using a ripple tank.

Specification Reference 4.4.1

What is reflection?

Reflection is when a wave is bounced back when it hits a boundary, as shown in the ray diagram in Figure 1.

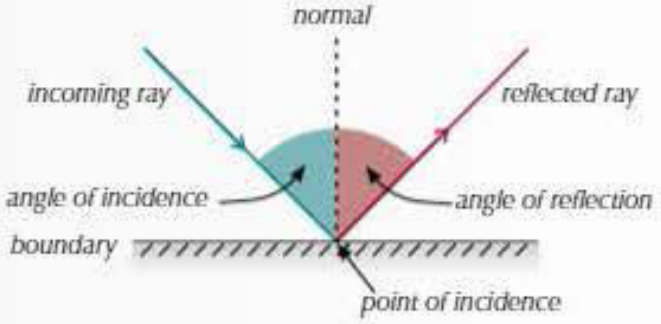


Figure 1: *A ray diagram to show the reflection of a wave at a boundary.*

A reflected wave always bounces off a boundary at the same angle to the normal as it hit the boundary. This is the law of reflection:

angle of incidence = angle of reflection

Tip: A ray shows the path taken by a wave.

Figure 2: *A ray of light being reflected by a flat mirror.*

What is refraction?

Refraction is the way a wave changes direction as it enters a different medium at an angle to the boundary, as shown by the ray diagram in Figure 4. The change in direction is a result of the wave slowing down or speeding up.

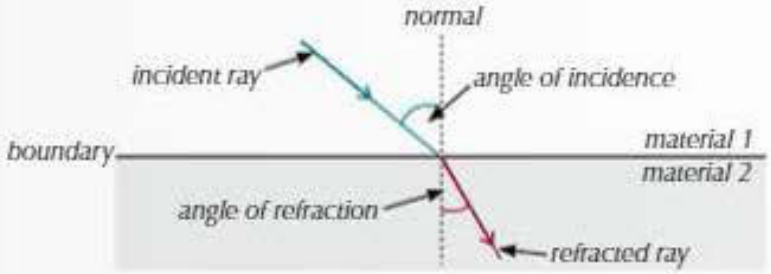


Figure 4: *A ray diagram showing the refraction of a wave as it moves from one material into another.*

Figure 3: *The pencil in the glass of water looks bent because of the refraction of light as it passes between the water and the air.*

Tip: For light, and other EM waves, we can talk about the 'optical density' of materials. Light will slow down when it passes into a more optically dense medium, and speed up when it passes into one that is less optically dense (p.202).

You can tell if the wave is speeding up or slowing down by the way it bends towards or away from the normal.

- If the ray bends towards the normal — it is slowing down.
- If the ray bends away from the normal — the wave is speeding up.

The frequency of a wave remains constant, no matter what the wave is travelling through. As $v = f\lambda$, this means that when a wave has changed speed, its wavelength must also have changed. If a wave speeds up in a medium, its wavelength increases and if it slows down, its wavelength decreases.

Tip: A ripple tank is a large, shallow tank of water which can be used to observe wave effects using water.

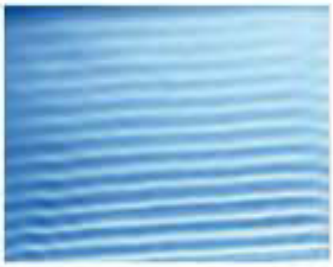


Figure 5: *A series of straight, parallel wave fronts, produced in a ripple tank.*

Tip: Close to the barrier, it might be difficult to make out the waves. This is because the incoming waves will be interfering with the reflected waves — there's more on this on pages 207-209.

Tip: Take care when working with ripple tanks. Make sure that the electrical cables powering the paddle don't come into contact with the water. And make sure you clear up any water spillages quickly, to prevent any damage to equipment, or accidents like people slipping on a wet floor.

Using ripple tanks to show wave effects

You can show the reflection and refraction of water waves using a ripple tank.

Demonstrating reflection

Set up the ripple tank so an oscillating paddle (connected to a signal generator and vibration generator) is creating regular waves with straight, parallel wave fronts, as shown in Figure 6. Place a barrier in the tank at an angle to the wave fronts.

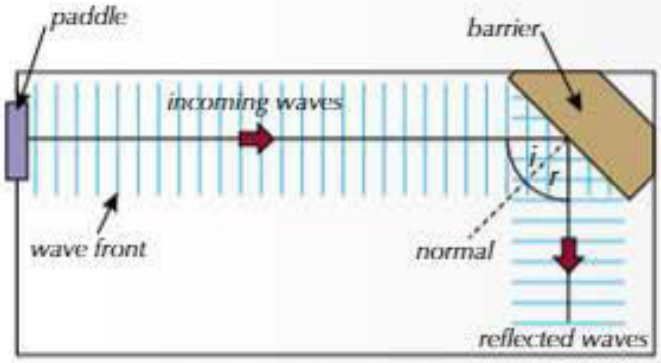


Figure 6: *A diagram of a ripple tank being used to demonstrate the reflection of water waves.*

The angle the incoming waves make with the normal to the barrier is the angle of incidence, *i*. The angle between the direction of the reflected waves and the normal to the barrier is the angle of reflection, *r*.

You should see the waves reflecting off the barrier and travelling in a different direction to the way they arrived. You can change the angle of incidence to see that the angle of reflection changes by the same amount. They are always equal to each other.

Demonstrating refraction

Put a block of plastic into the ripple tank, so that part of the tank contains a different depth of water than the rest of the tank. The point where the tank suddenly changes depth is the 'boundary'. Water waves travel faster in deeper water, so when they cross the boundary they should refract.

Set up the oscillating paddle so it is creating regular waves with straight, parallel wave fronts at an angle to the boundary, as shown in Figure 7.

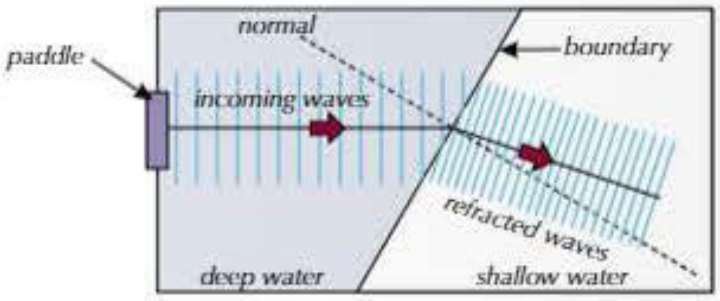


Figure 7: *A diagram of a ripple tank being used to demonstrate the refraction of water waves.*

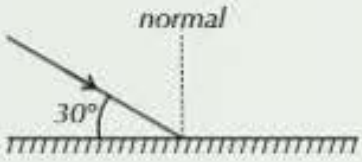
When the wave moves from the deeper water into the shallower water, it will slow down and refract. You should see the wave bend towards the normal to the boundary. You should also see that the wave fronts get closer together, which shows that the wavelength has decreased (due to $v = f\lambda$).

If you set up the experiment with the waves starting in the shallow end, you should see the opposite — the waves will bend away from the normal as they cross the boundary, and the wave fronts get further apart when they pass into the deeper water.

Tip: The change in depth of the ripple tank probably won't be enough to have a really strong effect. So you'll have to look closely to see the change in direction and wavelength.

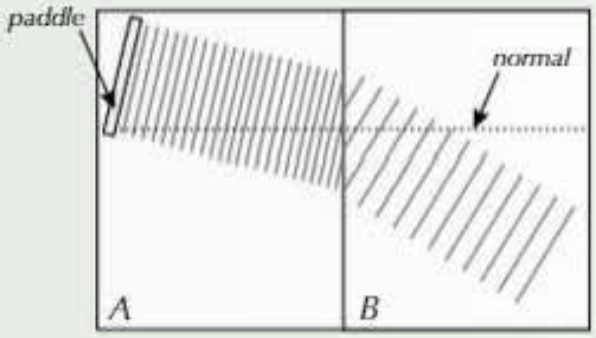
Practice Questions — Application

Q1 A light ray is incident on a mirror as shown below. Calculate the angle of reflection for the light ray.



Tip: Remember, in calculations of reflection and refraction, the angles aren't measured from the surface or boundary, but from the normal to the surface/ boundary.

Q2 A student is using a ripple tank to investigate wave effects. The ripple tank has two sections, A and B, which are at different depths. A snapshot of the student's experiment is shown below.



Which side of the tank, A or B, is deeper? Explain your answer.

Practice Questions — Fact Recall

Q1 State the law of reflection.

Q2 A wave hits a boundary between two materials at an angle to the normal. It slows down as it crosses the boundary.

a) State how its direction of motion changes.

b) State what happens to the wave's wavelength and frequency.

Learning Objectives:

- Know and understand the refraction of light.
- Know what is meant by the 'refractive index' of a material.
- Be able to calculate the refractive index of a material using $n = \frac{c}{v}$.
- Be able to use $n \sin\theta$ = constant to solve problems involving a ray crossing a boundary, where θ is the angle to the normal.
- Know and understand techniques and procedures used to investigate the refraction of light using ray boxes and transparent rectangular blocks (PAG6).
- Know what is meant by the term 'critical angle'.
- Be able to use the equation $\sin C = \frac{1}{n}$.
- Know what is meant by the total internal reflection of light.
- Know and understand techniques and procedures used to investigate total internal reflection of light using ray boxes and semi-circular blocks (PAG6).

Specification Reference 4.4.2

Tip: Remember, light bends towards the normal when it slows down (see page 199).

6. Refractive Index and Total Internal Reflection

You saw in the previous topic that waves changes speed and direction if they pass into a different medium. For light, you can work out the new direction if you know the refractive index of the materials.

The refractive index of a material

Light goes fastest in a vacuum. It travels slower in other materials, because it interacts with the particles in them. The more optically dense a material is, the more light slows down when it enters it. The **optical density** of a material is measured by its refractive index — the higher a material's optical density is, the higher its refractive index.

The **refractive index** of a material, n, is the ratio between the speed of light in a vacuum, c, and the speed of light in that material, v.

$$n = \frac{c}{v}$$

$c = 3.00 \times 10^8$ ms^{-1}

n = refractive index of the material

v = speed of light in material in ms^{-1}

The refractive index of a material is a property of that material only. The speed of light in air is only a tiny bit smaller than c. So you can assume the refractive index of air, $n_{air} = 1$.

Example — **Maths Skills**

The refractive index of a plastic block is 1.47.
What's the speed of light in the plastic?

First rearrange the refractive index equation to make v the subject:

$$n = \frac{c}{v} \Rightarrow v = \frac{c}{n}$$

$$v = \frac{c}{n} = \frac{3.00 \times 10^8}{1.47} = 2.04... \times 10^8 = 2.04 \times 10^8 \text{ ms}^{-1} \text{ (to 3 s.f.)}$$

Snell's law

If light is passing through a boundary between two materials, you can use Snell's law to calculate unknown angles or refractive indices.

The angle that incoming light makes to the normal, θ_i, is called the **angle of incidence**. The angle the refracted ray makes with the normal, θ_r, is the **angle of refraction**.

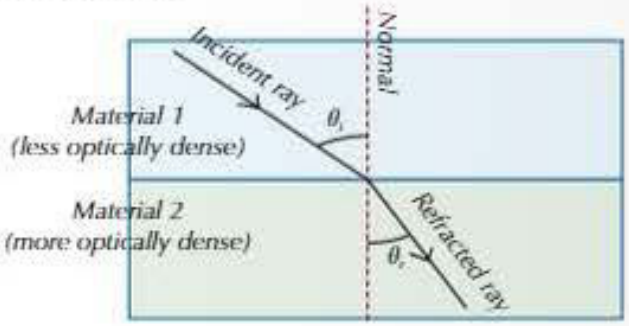
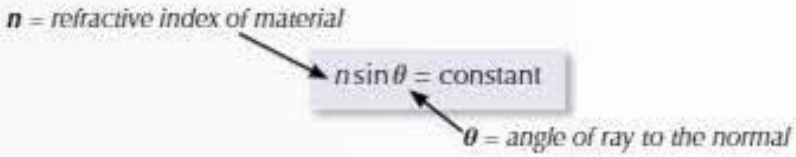
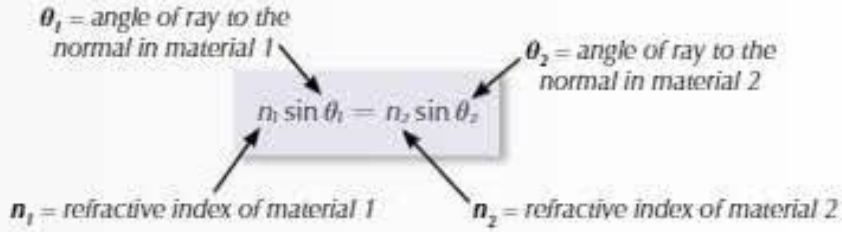


Figure 1: *Refraction of light passing into a more optically dense material.*

When a light ray passes across a boundary between two materials:



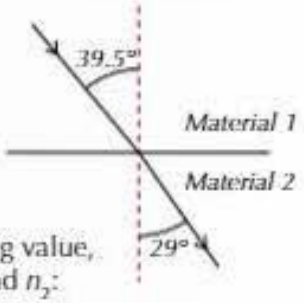
This means that for a light ray at a boundary between two materials, $n \sin\theta$ must be the same on either side. This can be written nicely as Snell's law:


θ1 = angle of ray to the normal in material 1
θ2 = angle of ray to the normal in material 2
n1 sin θ1 = n2 sin θ2
n1 = refractive index of material 1
n2 = refractive index of material 2

Exam Tip
You'll only get given the '$n \sin\theta$ = constant' equation in the exam, so make sure you remember what that equation actually tells you — that $n_1 \sin\theta_1 = n_2 \sin\theta_2$ at any boundary.

Example — **Maths Skills**

Calculate the refractive index of material 2 in the diagram, given that the refractive index is 1.1 for material 1.



First look at what information you've been given:

$n_1 = 1.1$, $n_2 = ?$, $\theta_1 = 39.5°$, $\theta_2 = 29°$

You can use the law of refraction to find the missing value, but you'll have to rearrange the equation first to find n_2:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \Rightarrow n_2 = \frac{n_1 \sin\theta_1}{\sin\theta_2}$$

$$\Rightarrow n_2 = \frac{1.1 \sin 39.5°}{\sin 29°} = 1.44... = 1.4 \text{ (to 2 s.f.)}$$

Tip: You can check your answer seems right by looking at how the direction changes. Here $\theta_1 > \theta_2$ so the ray has bent towards the normal. This shows the light ray has slowed down (p.199) so it has travelled into an optically denser material. This means that $n_1 < n_2$, which your answer shows.

Investigating refractive indices

PRACTICAL ACTIVITY GROUP 6

You can find the refractive index of a (transparent) material using a rectangular block of that material and a ray box.

- Place the rectangular block on a piece of paper and draw around it.
- Use the ray box to shine a beam of light into the glass block. Dim any other lights so you can see the path of the light beam through the block clearly.
- Trace the path of the incoming and outgoing beams of light either side of the block.
- Remove the block and join up the two paths you've drawn with a straight line. This shows the path the light beam took through the glass block. You should be able to see from your drawing how the path of the ray bent when entering and leaving the block.
- Measure the angles of incidence (θ_i) and refraction (θ_r) where the light enters the block (θ_{i1} and θ_{r1}) and exits the block (θ_{i2} and θ_{r2}) — see Figure 2.

Tip: Remember to carry out a risk assessment before starting any experiment.

Tip: You can also use a device called a refractometer to accurately measure the refractive index of a material. The machine shines a beam of light at the sample. You then view the refracted beam through a microscope and measure its angle of refraction.

Tip: Remember to draw a normal at the point where the ray enters and leaves the block.

Figure 3: *A green laser beam being refracted as it passes through a transparent rectangular block.*

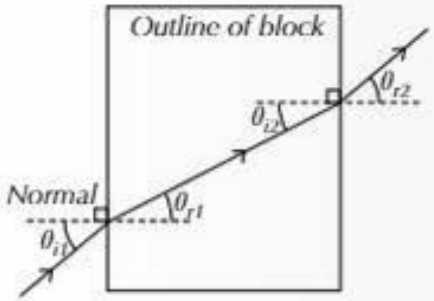


Figure 2: *An example of the type of drawing obtained from carrying out an experiment into the refraction of light through a transparent block.*

You can calculate the refractive index of the block using Snell's Law. Since $n_{air} = 1$, Snell's Law can be simplified to $\sin\theta_{i1} = n\sin\theta_{r1}$ at the entry point and $n\sin\theta_{i2} = \sin\theta_{r2}$ at the exit point. Plug in your measurements to calculate n for both points, and then find the average of the two to get your final value of n.

The critical angle of a boundary

When light goes from a more optically dense material into a less optically dense material (e.g. glass to air), it can get refracted along the boundary.

Shine a ray of light at a boundary going from refractive index n_1 to n_2, where $n_1 > n_2$, then gradually increase the angle of incidence. The light is refracted away from the normal, so as you increase the angle of incidence in material 1 (θ_1), the angle of refraction in material 2 (θ_2) gets closer and closer to 90°. Eventually θ_1 reaches a **critical angle** (C) for which $\theta_2 = 90°$. The light is refracted along the boundary — see Figure 4.

This can happen for any boundary where the light is passing from a more optically dense material (n_1, higher refractive index) into a less optically dense material (n_2, lower refractive index).

Figure 5: *Light enters the prism from underneath and then strikes the prism-air boundary at around the critical angle. Some light is reflected (left) and some continues parallel to the prism surface (top).*

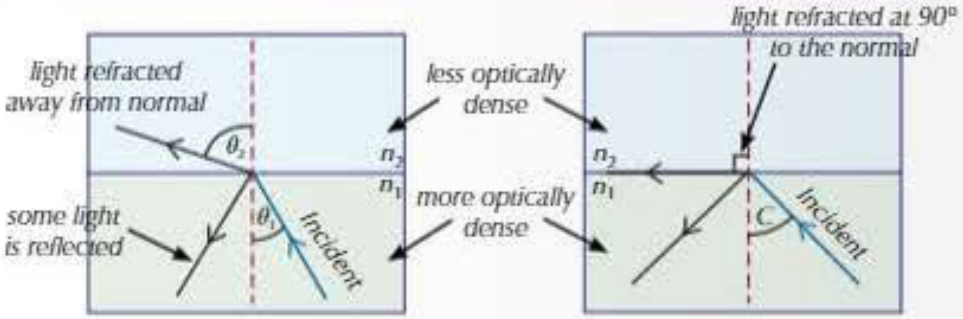


Figure 4: *Refraction of light near to and along a boundary, when incident below and at the critical angle.*

You can work out the critical angle for a certain boundary with the equation:

$$\sin C = \frac{n_2}{n_1} \quad \text{where } n_1 > n_2$$

C = *critical angle*

$\boldsymbol{n_2}$ = *refractive index of less optically dense material*

$\boldsymbol{n_1}$ = *refractive index of more optically dense material*

You can derive this formula by rearranging Snell's law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

First, rearrange it to get the angles on the same side:

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

The angle of incidence is equal to the critical angle when the angle of refraction is 90°, so put these values in:

$$\frac{\sin C}{\sin 90°} = \frac{n_2}{n_1}$$

$\sin 90° = 1$, so the formula becomes:

$$\sin C = \frac{n_2}{n_1}$$

The refractive index of air is 1, so this can be simplified for a material to air boundary by letting $n_2 = 1$:

critical angle for material to air boundary $\sin C = \frac{1}{n}$ *refractive index of material*

Tip: The assumption here is that air is the less optically dense material — this is usually the case, because its refractive index is 1.0002...

Examples — Maths Skills

Find the critical angle of a glass to air boundary if the glass has a refractive index of 1.5.

You've been asked to find the critical angle for a boundary with air, and you know the refractive index of the other material. So you can use the formula for the critical angle of a material to air boundary shown above.
But you need to rearrange it for C first:

$\sin C = \frac{1}{n}$, which gives:

$$C = \sin^{-1}\left(\frac{1}{n}\right) = \sin^{-1}\left(\frac{1}{1.5}\right) = 41.8...° = 42° \text{ (to 2 s.f.)}$$

Tip: Make sure your calculator is set to degrees mode to work out the inverse sin ($\sin^{-1}$) of this angle.

A plastic block is immersed in a liquid. If the refractive index of the liquid is 1.40 and the critical angle for light travelling from the plastic to the liquid is 79.1°, find the refractive index of the plastic.

Write down what you know: $C = 79.1°$, $n_1 = ?$, $n_2 = 1.40$

Then rearrange $\sin C = \frac{n_2}{n_1}$ to find n_1:

$$\sin C = \frac{n_2}{n_1} \Rightarrow n_1 = \frac{n_2}{\sin C} = \frac{1.40}{\sin 79.1°} = 1.425... = 1.43 \text{ (to 3 s.f.)}$$

Total internal reflection

At angles of incidence greater than the critical angle, refraction can't happen. That means all the light is reflected back into the material. This effect is called **total internal reflection** (**TIR**) — see Figures 6 and 7.

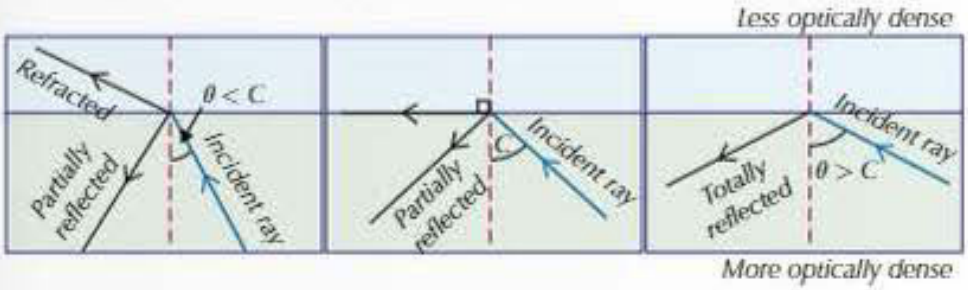


Figure 7: *Light hitting a boundary with a less optically dense material at different angles of incidence.*

Figure 6: *Total internal reflection of a laser beam inside an optical fibre (light travelling from right to left). An optical fibre is used in communications.*

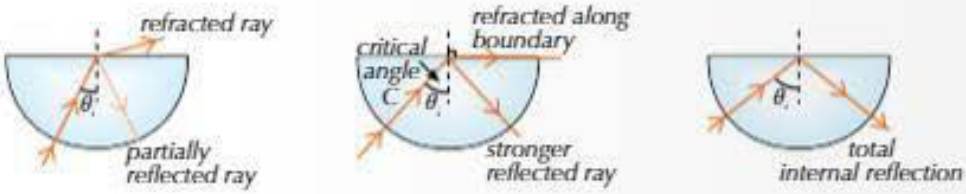
Tip: Carry out a risk assessment before starting this experiment. If you're using a laser, see page 216 for more about how to use them safely.

Investigating critical angles and TIR

PRACTICAL ACTIVITY GROUP 6

You can measure the critical angle of a glass-to-air boundary by finding the angle at which light refracts along the boundary. Shine a light ray into the curved face of a semi-circular glass block so that it always enters at right angles to the edge — this means the ray won't refract as it enters the block, just when it leaves from the straight edge. Vary the angle of incidence, θ_i, until the light beam refracts so much that it exits the block along the straight edge (see Figure 8). This angle of incidence is now the critical angle, C, for the glass-air boundary.

You can then rearrange the formula for the critical angle at a material-air boundary ($\sin C = \frac{1}{n}$) and put in your value for C to find the refractive index of the block. If you increase the angle of incidence so it's greater than C, you'll find the ray is entirely reflected from the straight edge of the block. This is total internal reflection (TIR).



Here the angle of incidence is less than the critical angle.

Here the angle of incidence is equal to the critical angle.

Here the angle of incidence is greater than the critical angle.

Figure 8: *Ray diagrams showing wave behaviour at different angles of incidence.*

Practice Questions — Application

Q1 The speed of light in a material is 1.94×10^8 ms^{-1}.

a) Calculate the refractive index of the material.

b) Light travels from air into the material. Explain whether the light bend towards or away from the normal as it enters the material.

c) Calculate the critical angle for the material to air boundary.

Q2 A student is using a semi-circular glass block to measure the critical angle for the glass-air boundary. Why is it important that light rays enter the block at right angles to the edge?

Q3 A fisherman sees a lobster cage on the sea floor and tries to retrieve it by lowering a winch directly over where the cage appears to be. The winch lands behind the cage.

a) Explain why the cage isn't exactly where he thought it was.

b) The fisherman uses an underwater camera to get a better view. Light travels from the lobster cage and hits the camera lens at an angle of 37.2° to the normal. If the water has a refractive index of 1.38 and the lens has a refractive index of 1.49, at what angle to the normal will the refracted ray in the camera lens be?

Practice Questions — Fact Recall

Q1 What is the refractive index of a material?

Q2 In what way will light bend if it passes at an angle into a medium with a higher refractive index than the material it just left?

Q3 What conditions need to be met for total internal reflection?

7. Superposition and Interference

When waves pass through each other, they combine their displacements — they can make a bigger wave, a smaller wave, or cancel each other out...

Learning Objectives:

- Know the principle of superposition of waves.
- Use graphical methods to illustrate the principle of superposition.
- Understand what is meant by interference, phase difference and coherence.
- Be able to describe constructive interference and destructive interference in terms of phase difference.

Specification Reference 4.4.3

Superposition of waves

Superposition happens when two or more waves pass through each other. At the instant that waves cross, the displacements due to each wave combine. Then each wave continues on its way. You can see this if two pulses are sent simultaneously from each end of a rope, as in Figure 1.

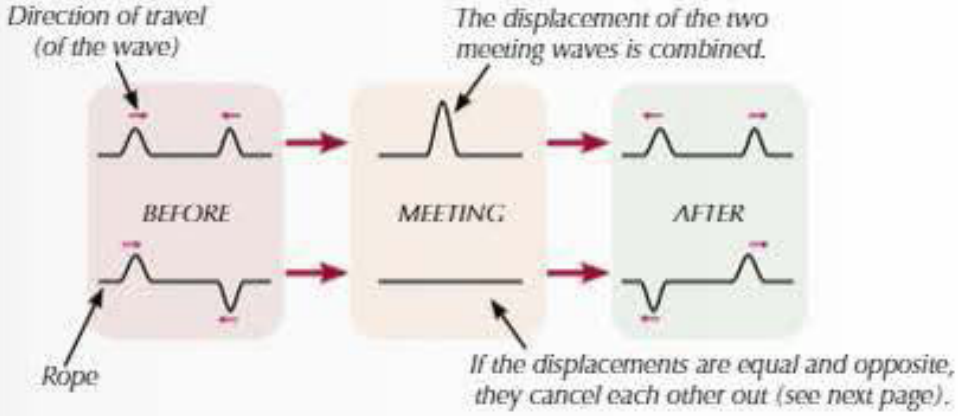

Direction of travel (of the wave)
The displacement of the two meeting waves is combined.
BEFORE
MEETING
AFTER
Rope
If the displacements are equal and opposite, they cancel each other out (see next page).

Figure 1: *Superposition of waves.*

The principle of **superposition** of waves says that when two or more waves cross, the resultant displacement equals the vector sum of the individual displacements.

Tip: The vector sum is just the sum of the two displacements taking into account both magnitude and direction (see page 41).

"Superposition" means "one thing on top of another thing". You can use the same idea in reverse — a complex wave can be separated out mathematically into several simple sine waves of various sizes.

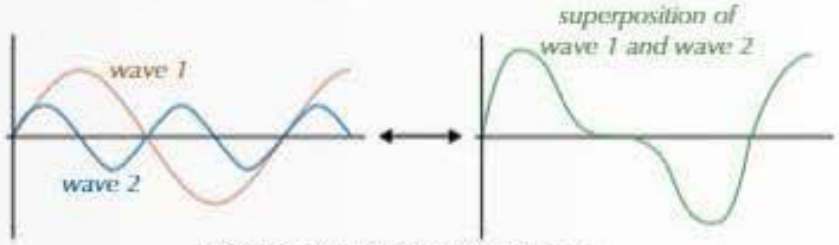


Figure 2: *Two sine waves shown as separate waves and as a superposition.*

Constructive and destructive interference

The superposition of two or more waves can result in **interference**. Interference can either be constructive or destructive.

- When two waves meet, if their displacements are in the same direction, the displacements combine to give a bigger displacement. A crest plus a crest gives a bigger crest. A trough plus a trough gives a bigger trough. This is known as **constructive interference**.
- If a wave with a positive displacement (crest) meets a wave with a negative displacement (trough), they will undergo **destructive interference** and cancel each other out. The displacement of the combined wave is found by adding the displacements of the two waves (see Figure 3).

Tip: Remember, amplitude only has magnitude, but displacement has direction and magnitude. See page 186 for more.

Tip: A trough and a crest won't cancel each other out completely unless they have the same magnitude.

Tip: Graphically, you can superimpose waves by adding the individual displacements at each point along the *x*-axis, and then plot them.

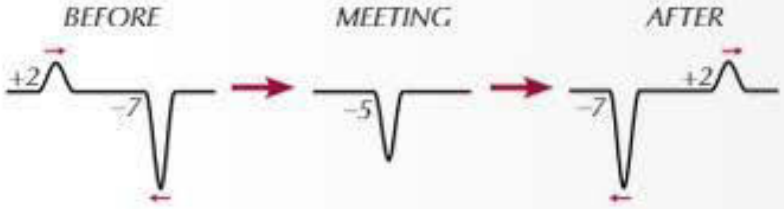


Figure 3: *Destructive interference.*

Figure 4: *Constructive and destructive interference can be shown by water waves in a ripple tank.*

- If two waves with equal and opposite displacements meet (e.g. a crest and a trough with equal magnitudes), they cancel each other out completely. This is total destructive interference.

Phase difference

Two points on a wave are in phase if they are both at the same point in the wave cycle (the same phase — see p.187). Points in phase have the same displacement and velocity.

In Figure 5, points A and B are in phase; points A and C are out of phase; and points A and D are exactly out of phase.

Tip: If two points are exactly out of phase, they're an odd integer of half-cycles apart (1 half-cycle, 3 half-cycles, etc.).

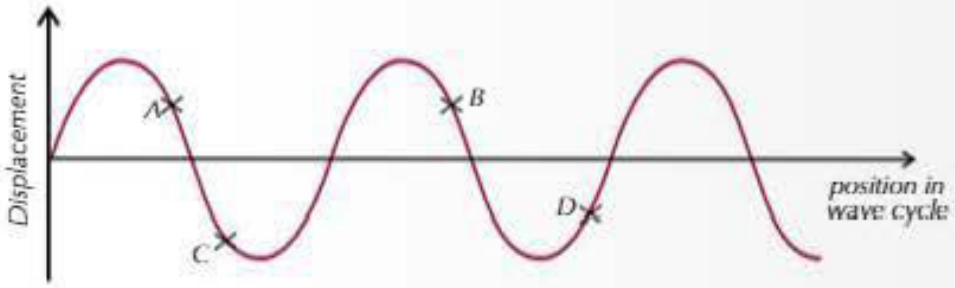


Figure 5: *Points in and out of phase on a wave pattern.*

Tip: The position on the *x*-axis tells you at what point in the wave cycle you are. It's usually measured as an angle or fraction of a cycle (see below).

It's mathematically handy to show one complete cycle of a wave as an angle of 360° (2π radians) — see Figure 6. The phase difference of two points on a wave is the difference in their positions in a wave's cycle, measured in degrees, radians or fractions of a cycle. Two points with a phase difference of zero or a multiple of 360° (i.e. a full cycle) are in phase. Points with a phase difference of odd-number multiples of 180° (π radians, or half a cycle) are exactly out of phase.

Tip: Radians are just a different unit for angle measurement — see page 485.

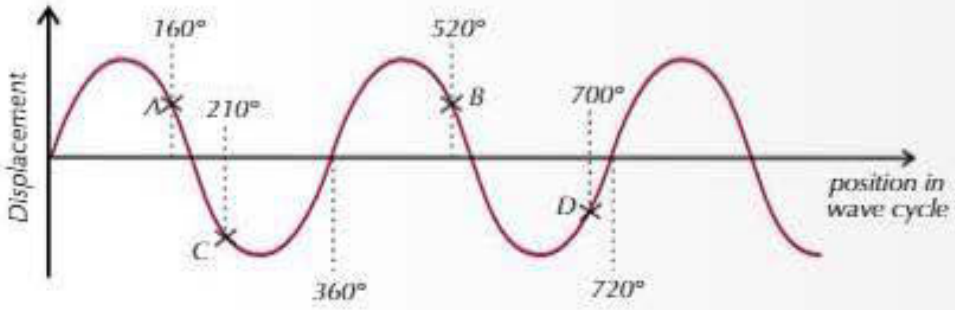


Figure 6: *The points from Figure 5 shown as angles.*

Tip: Points A and B are a full cycle apart (360°), while points A and D are three half-cycles apart (180° × 3 = 540°).

Phase difference between waves

You can also talk about the phase difference between two **coherent** waves (rather than two points on one wave). Two waves are coherent if they have the same wavelength and frequency and a fixed phase difference between them.

The phase difference is the amount by which one wave lags behind the other. So you can also talk about two waves being in or out of phase. Two waves are in phase if their phase difference is 0 or a multiple of 360° — see Figure 7. In practice this is usually because both waves come from the same oscillator. In other situations there will nearly always be a phase difference between two waves.

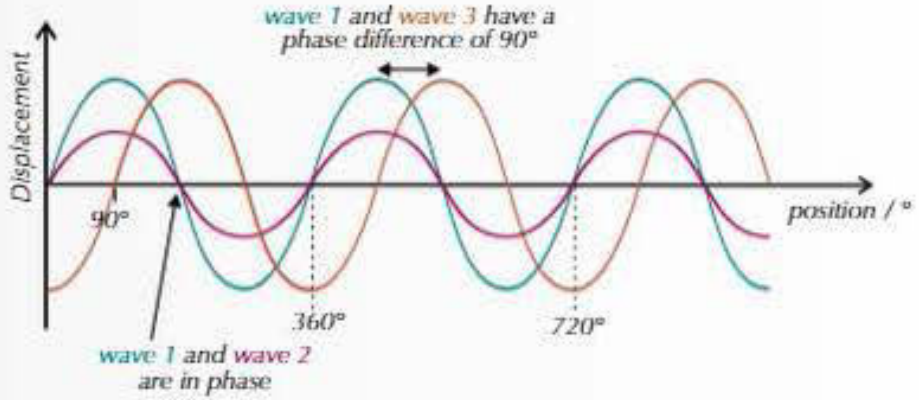


Figure 7: *Three coherent waves. Two waves are in phase and the third wave is out of phase by 90°.*

Tip: Waves don't need to have the same amplitude to be in phase, but they do need to have the same frequency and wavelength.

Tip: Compare the position of two equivalent points on two coherent waves (e.g. the top of two crests) to find the phase difference between them.

When two waves that are in phase with each other are superposed, they will interfere constructively with each other. Destructive interference occurs when two waves that are exactly out of phase with each other are superposed. For any other phase difference, there will be a mixture of constructive and destructive interference.

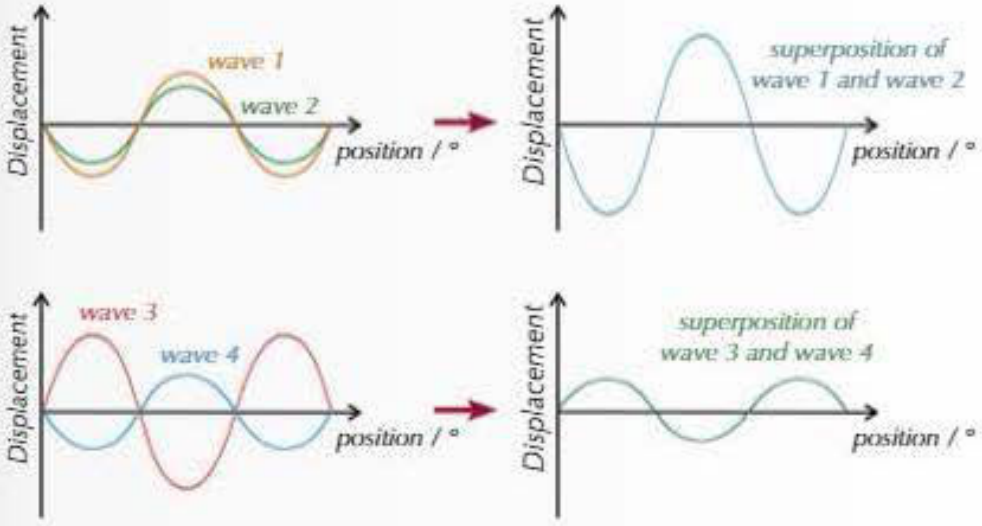
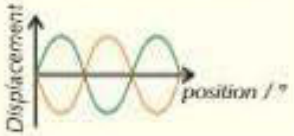


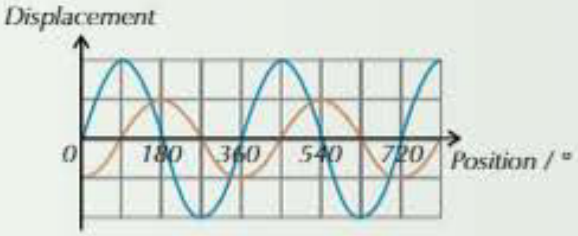
Figure 8: *Waves 1 and 2 are in phase, so interfere constructively. Waves 3 and 4 are exactly out of phase and interfere destructively.*

Tip: Superposing these waves would give total destructive interference:



Practice Questions — Application

Q1 What is the phase difference between the two waves in the diagram below?

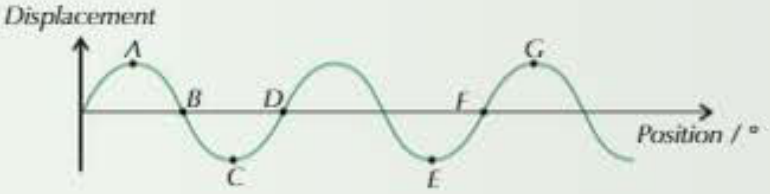


Q2 Sketch two waves that, when superposed, will have total destructive interference.

Q3 a) In the diagram below, which point (if any) is in phase with

i) point A ii) point B iii) point C

b) How many of the points are exactly out of phase with point G?



Practice Questions — Fact Recall

Q1 What does the principle of superposition say?

Q2 Describe constructive interference.

Q3 What is meant by the phase difference of two points on a wave?

Q4 Give three possible units for phase difference.

Q5 When are two points on a wave exactly out of phase?

Q6 What does it mean for two wave sources to be coherent?

Q7 What does it mean for two coherent waves to be in phase?

8. Diffraction

Diffraction is the reason you can hear sound when you're round a corner from the source — it's just waves spreading out through gaps or around obstacles.

Learning Objectives:

- Understand what is meant by diffraction for all waves, including light.
- Know and understand techniques and procedures used to demonstrate diffraction using a ripple tank.

Specification Reference 4.4.1

Diffraction through gaps

The way that waves spread out as they come through a narrow gap or go round obstacles is called **diffraction**. All waves diffract, but it's not always easy to observe. The amount of diffraction depends on the wavelength of the wave compared with the size of the gap.

Diffraction can be shown using a ripple tank. Objects can be placed into the ripple tank to create a barrier with a gap in the middle of it. This gap can be varied to see the effects this has on how the waves spread through the tank:

- When the gap is a lot bigger than the wavelength, diffraction is unnoticeable — see Figure 1, left-hand diagram.
- You get noticeable diffraction through a gap several wavelengths wide (Figure 1, middle diagram).
- The most diffraction is when the gap is the same size as the wavelength (Figure 1, right-hand diagram).
- If the gap is smaller than the wavelength, the waves are mostly just reflected back.

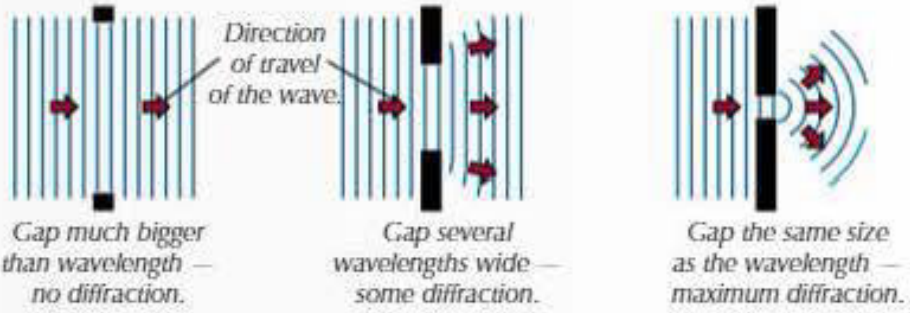

Direction of travel of the wave.
Gap much bigger than wavelength — no diffraction.
Gap several wavelengths wide — some diffraction.
Gap the same size as the wavelength — maximum diffraction.

Figure 1: *The diffraction of water waves in a ripple tank.*

When sound passes through a doorway, the size of the gap and the wavelength are usually roughly equal, so a lot of diffraction occurs. That's why you have no trouble hearing someone through an open door to the next room, even if the other person is out of your line of sight. The reason that you can't see him or her is that when light passes through the doorway, it is passing through a gap around a few million times bigger than its wavelength — the amount of diffraction is tiny. So to get noticeable diffraction with light, you must shine it through a very narrow slit.

Figure 2: *Diffraction in a ripple tank.*

Diffraction around obstacles

When a wave meets an obstacle, you get diffraction around the edges. Behind the obstacle is a 'shadow', where the wave is blocked. The wider the obstacle compared with the wavelength of the wave, the less diffraction you get, and so the longer the shadow.

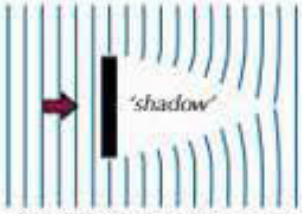


Figure 3: *The diffraction of waves around an obstacle.*

Tip: Radio waves with long wavelengths (or long wave, LW, radio) can reach receivers in valleys and across mountain ranges, because diffraction allows the waves to bend around the hills.

The diffraction of light

Diffraction of light can be demonstrated by shining light through a very narrow slit (or aperture) onto a screen. If the wavelength of a light wave is roughly similar to the size of the aperture, you get a diffraction pattern of light and dark fringes — see Figure 4. You should use monochromatic, coherent light (such as laser light — see page 213).

Tip: This is often called the single-slit experiment — you'll come across the double-slit experiment on page 216.

Monochromatic light is light of a single wavelength, and so a single colour. If you use light that isn't monochromatic, different wavelengths will diffract by different amounts and the pattern produced won't be very clear.

Tip: You need to be careful using lasers. Laser beams are very powerful and could damage your eyesight, so you should take precautions when using them (page 216).

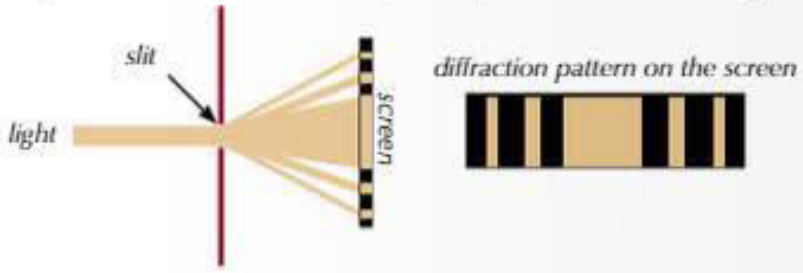


Figure 4: *Diffraction pattern from monochromatic light through a single slit.*

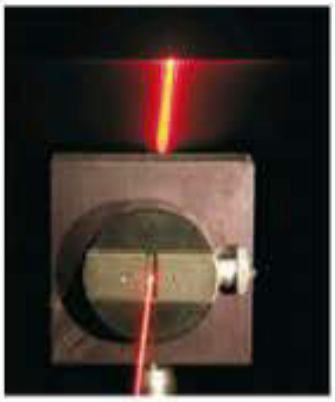


Figure 5: *Diffraction of a laser beam through a single slit.*

The pattern has a bright central fringe with alternating dark and bright fringes on either side of it. You can alter the amount of diffraction by changing the width of the slit. The narrower the slit, the wider the diffraction pattern. The fringe pattern is due to interference:

- The bright fringes are due to constructive interference, where waves from across the width of the slit arrive at the screen in phase.
- The dark fringes are due to total destructive interference, where waves from across the width of the slit arrive at the screen completely out of phase.

You can do a similar experiment using a white light source and a set of colour filters instead of the laser. The size of the slit can be kept constant while the wavelength is varied by putting different colour filters over the slit.

Practice Question — Application

Q1 A water wave with a wavelength of 2 m passes through a gap that is 5 cm wide. Will diffraction effects be noticeable? Explain your answer.

Tip: The wavelength of sound waves is of the order of 1 m.

Practice Questions — Fact Recall

Q1 What sort of waves diffract?

Q2 Explain, in terms of diffraction, why someone stood next to an open doorway can be heard from the other side but not seen.

Q3 What is monochromatic light?

Q4 Describe the diffraction pattern produced when monochromatic light is shone through a single narrow slit.

9. Two-Source Interference

When two wave sources interfere, they can make pretty interference patterns. You need both sources to be pretty much identical...

Learning Objectives:

- Know the meaning of the term path difference.
- Be able to describe constructive interference and destructive interference in terms of path difference.
- Understand two-source interference with sound and microwaves.
- Know techniques and procedures used for superposition experiments involving sound and microwaves (PAG5).

Specification Reference 4.4.3

What is two-source interference?

Two-source interference is when the waves from two sources interfere to produce a pattern. In order to get clear interference patterns, two or more sources must be coherent (page 209). If a light source is coherent and in phase, the troughs and crests line up — this causes constructive interference and a very intense beam.

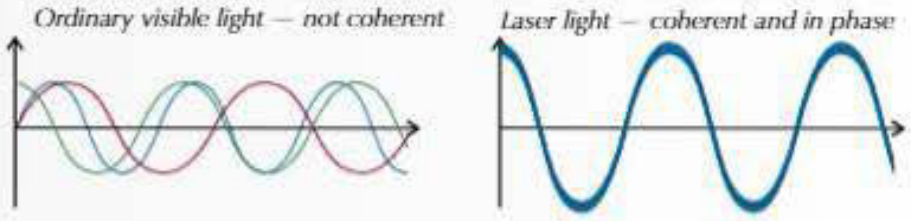


Figure 1: *Coherent and non-coherent light sources.*

Interference still happens when you're observing waves of different wavelength and frequency — but it happens in a jumble. If the sources are coherent, clear patterns of constructive and destructive interference are seen. Whether you get constructive or destructive interference at a point depends on how much further one wave has travelled than the other wave to get to that point. The amount by which the path travelled by one wave is longer than the path travelled by the other wave is called the **path difference**.

Exam Tip
In exam questions, the two sources will almost certainly be in phase. The fixed phase difference will be zero.

At any points an equal distance from two sources in phase you will get constructive interference (see Figure 2). These points are known as **maxima**. You also get constructive interference at any point where the path difference is a whole number of wavelengths. At these points the two waves are in phase and reinforce each other, which is why you get constructive interference.

At points where the path difference is half a wavelength, one and a half wavelengths, two and a half wavelengths, etc., the waves arrive exactly out of phase and you get total destructive interference. These points are known as **minima**.

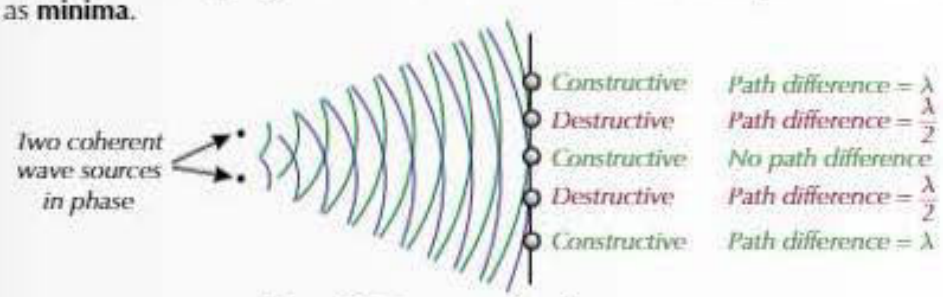
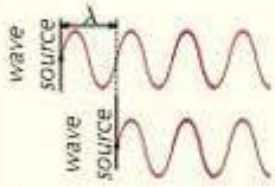


Figure 2: *Two-source interference.*

Tip: Path difference and phase difference are different things. The two waves below are in phase — they have 0 phase difference, but their path difference is λ.



Constructive interference occurs when:

$$\text{path difference} = n\lambda \text{ (where } n = 0, 1, 2, \ldots)$$

Total destructive interference occurs when:

$$\text{path difference} = \frac{(2n+1)\lambda}{2} = \left(n + \frac{1}{2}\right)\lambda$$

Example — **Maths Skills**

Two coherent sources of sound waves each with a wavelength of 1.5 m are set up so that they produce interference fringes. When the path difference is 3.75 m, would you expect constructive or destructive interference?

- Find out how many times the wavelength goes into the path difference:

$$\frac{3.75}{1.5} = 2.5.$$

- So the path difference is $2.5 \times \lambda$ so the interference is destructive.

Figure 3: *Two-source interference of water waves demonstrated by two dippers vibrating at the same frequency in a ripple tank.*

Demonstrating two-source interference

PRACTICAL ACTIVITY GROUP 5

It's easy to demonstrate two-source interference for either sound or water because they've got relatively large wavelengths. This makes it easier to detect interference patterns. The trick for getting them coherent and in phase is to use the same oscillator to drive both sources. For water, one vibrator drives two dippers (see Figure 3). For sound, one amplifier is connected to two loudspeakers (see Figure 4).

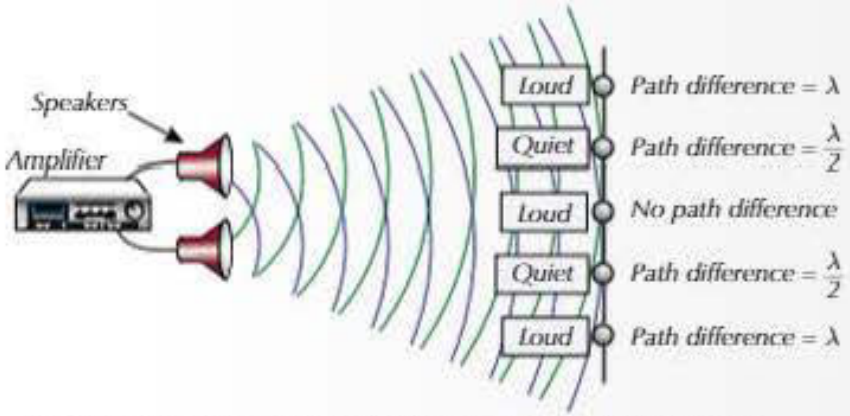


Figure 4: *Demonstrating two-source interference of sound waves.*

Tip: You could use a microphone and an oscilloscope to investigate how interference varies with position along this line.

Tip: You may still hear some sound at the quietest points due to sound being reflected off walls and around the room.

Tip: Make sure you take all necessary safety precautions when doing these two experiments and carry out a risk assessment first.

In Figure 4, an interference pattern is produced at the black line. You can observe this by having someone walk in a straight line, parallel to the line of the speakers. They will observe areas of loud and quiet sound. Constructive interference (at p.d. = $n\lambda$) causes the sound to increase in volume, so you get a loud area. Destructive interference causes the sound to decrease in volume, so you get a quiet area.

To see interference patterns with microwaves, you can use two microwave transmitter cones attached to the same signal generator (see Figure 5 on the next page). You also need a microwave receiver probe (like the one used in the stationary waves experiment on page 225). As you move the probe along the path of the orange arrow in Figure 5, you'll get an alternating pattern of strong and weak signals.

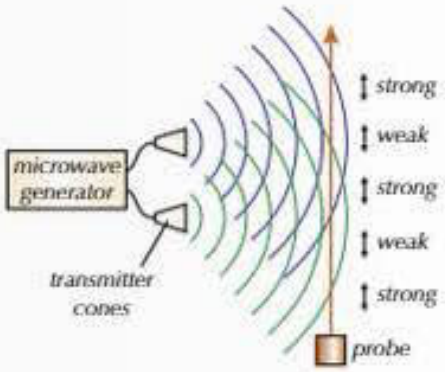


Figure 5: *Two-source interference with microwaves.*

Tip: As usual, strong signals occur where the two sources interfere constructively and weak signals occur where the two sources interfere destructively.

Practice Questions — Application

Q1 A wave, W, is produced and looks as follows:

a) Draw a second wave which is in phase with wave W, where the path difference between them is $\frac{\lambda}{2}$.

b) Draw another wave which also has a path difference of $\frac{\lambda}{2}$ but which is completely out of phase with wave W.

Q2 Two coherent sources produce microwaves which have a wavelength of 12 cm. At a point where the path difference is 42 cm, would there be constructive or destructive interference?

Tip: If you're struggling, go back to page 208 to refresh your memory about phase.

Practice Questions — Fact Recall

Q1 What must be true of two wave sources if they produce a clear, standard two-source interference pattern?

Q2 What is meant by the path difference of two waves?

Q3 For two coherent, in phase waves, at what path differences will you see constructive interference?

Q4 How can you create two coherent sources of sound waves?

Q5 Describe an experiment to produce and observe an interference pattern with sound waves.

Q6 Explain what you would observe if you moved a microwave probe in a straight line parallel to the line of two coherent microwave transmitters.

Learning Objectives:

- Know techniques and procedures used for superposition experiments involving light (PAG5).
- Be able to explain Young's double-slit experiment using visible light.
- Be able to use $\lambda = \frac{ax}{D}$ for all waves where $a << D$.
- Know techniques and procedures used to determine the wavelength of light using a double-slit (PAG5).

Specification Reference 4.4.3

Tip: Create a risk assessment before doing this experiment and follow all of the laser safety precautions given on this page.

Tip: Remember, light that isn't monochromatic is made up of different wavelengths of light, which will diffract by different amounts and makes the fringes much less clear.

Figure 2: *Laser warning signs should be used to alert people of the danger of laser beams.*

10. Young's Double-Slit Experiment

Just like for sound and microwaves, two sources of light can interfere. It's much harder to produce and observe clear diffraction patterns though.

Double-slit interference of light

PRACTICAL ACTIVITY GROUP 5

To see two-source interference with light, you can either use two coherent light sources or you can shine a laser through two slits. Remember, a laser is a source of monochromatic and coherent light. This means you can effectively create two coherent light sources by shining a single laser through a mounted card containing two slits (known as a single-source double-slit set-up).

The slits have to be about the same size as the wavelength of the laser light so that it is diffracted (see page 211). This makes the light from the slits act like two coherent point sources. You get a pattern of light and dark fringes, depending on whether constructive or destructive interference is taking place — see Figure 1.

You might see this experiment referred to as "Young's double-slit experiment" — Thomas Young was the first person to carry it out, although he used a source of white light instead of a laser. He then came up with an equation to work out the wavelength of the light from the experiment (p.217).

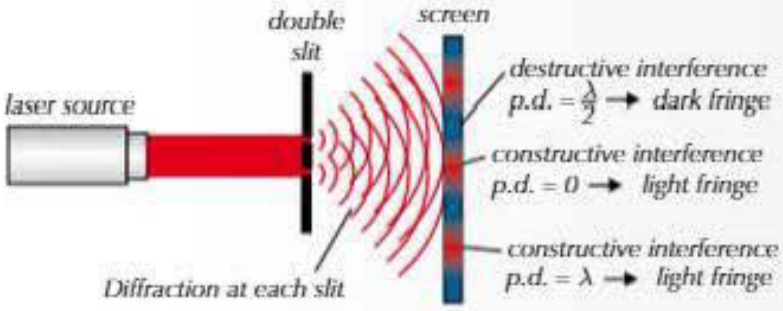


Figure 1: *Two-source interference of laser light.*

If you were to use a white light source instead of a coherent monochromatic laser, the diffraction pattern would be less intense, with wider maxima. The pattern would also contain different colours with a central white fringe, because white light is made up of a mixture of frequencies (see page 222).

Laser safety precautions

Working with lasers is very dangerous because laser light is focused into a very direct, powerful beam of monochromatic light. To make sure you don't cause damage while using lasers, make sure you:

- Never shine the laser towards a person.
- Are wearing laser safety goggles.
- Avoid shining the laser beam at a reflective surface.
- Have a warning sign on display (see Figure 2).
- Turn the laser off when it's not needed.

The double-slit formula

The fringe spacing (x), wavelength (λ), spacing between slits (a) and the distance between the slits and screen (D) (see Figure 3) are all related by Young's double-slit formula, which works for all waves when $a << D$.

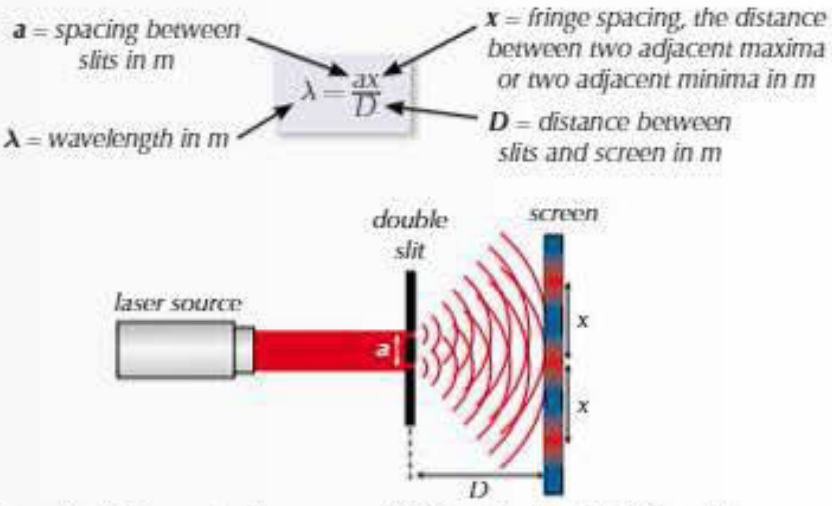

a = spacing between slits in m
λ = wavelength in m
$\lambda = \frac{ax}{D}$
x = fringe spacing, the distance between two adjacent maxima or two adjacent minima in m
D = distance between slits and screen in m
double slit
screen
laser source
a
x
x
D

Figure 3: *A diagram to show a, x and D from the double-slit formula.*

You can derive this equation by considering the path difference between the waves from each slit when the light interferes constructively to form the first bright fringe — see Figure 5. The path difference between the light waves at the first bright fringe is λ.

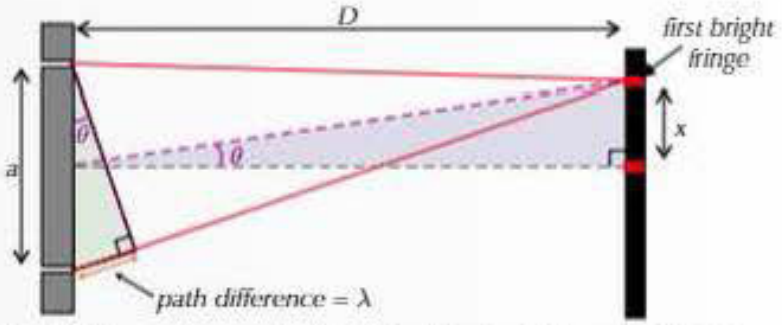


Figure 5: *Light waves from each slit of a double slit, with a path difference of λ, interfering to form the first bright fringe of the diffraction pattern.*

Look at the green and purple triangles in the diagram — the angle θ is the same in each triangle. Using SOH CAH TOA on the green triangle:

$$\sin\theta = \frac{\text{opposite}}{\text{hypoteneuse}} = \frac{\text{path difference}}{\text{slit separation}} = \frac{\lambda}{a}$$

Using SOH CAH TOA on the purple triangle:

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{x}{D}$$

By small angle approximations: $\tan\theta \approx \theta$ and $\sin\theta \approx \theta$. Remember, this is only true if θ is in radians.

So we can say that $\tan\theta = \sin\theta$ for small angles.

$$\frac{x}{D} = \frac{\lambda}{a} \quad \text{so } \lambda = \frac{ax}{D}$$

Exam Tip
You need to be able to use and rearrange this formula, but you don't need to memorise it or know how to derive it.

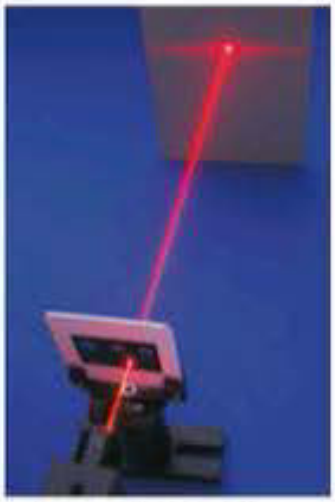


Figure 4: *A laser beam passing through a double slit and forming a diffraction pattern on a screen.*

Tip: Figure 5 is not to scale, x would have to be much bigger than a for the pattern to be easily visible.

Tip: You don't need to know this derivation — it's just to help you understand where the equation comes from.

Tip: As with all experiments, you should carry out a risk assessment before you start.

Finding wavelength using the double-slit formula

You can investigate this formula using the double-slit apparatus on page 216. You'll need to measure D and x using a ruler, and a should be printed on the double slit.

PRACTICAL ACTIVITY GROUP 5

Since the wavelength of light is so small, you can see from the formula that a high ratio of D/a is needed to make the fringe spacing big enough to see. You can use $\lambda = \frac{ax}{D}$ to calculate the wavelength of the light.

The fringes are usually so tiny that it's very hard to get an accurate value of x. It's easier to measure across several fringes then divide by the number of fringe widths between them.

Tip: Measuring across several fringes will reduce the uncertainty. E.g. if you measure the width of one fringe as 1.0 ± 0.1 cm, then the percentage uncertainty is 10%. But if you measure the width of 10 fringes as 10.0 ± 0.1 cm, then the average fringe width is 1.0 ± 0.01 cm (you divide the result and the uncertainty by the number of fringes), so the percentage uncertainty is only 1.0%.

Example — **Maths Skills**

The maxima of an interference pattern produced by shining a laser light through a double slit onto a screen is shown in Figure 6. The slits were 0.20 mm apart and the distance between the slits and the screen was 15 m. Find the wavelength of the laser light.

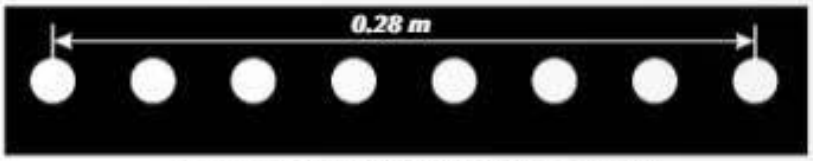


Figure 6: *A double-slit interference pattern.*

You can use Young's double-slit formula to find the wavelength (λ). But first you need to find the fringe spacing of one fringe (x). Seven fringe widths in Figure 6 have a spacing of 0.28 m, so one fringe width has a spacing of $\frac{0.28}{7} = 0.040$ m.

Substitute the information you know into the formula:
$x = 0.040$ m, $a = 0.00020$ m and $D = 15.0$ m.

$$\lambda = \frac{ax}{D} = \frac{0.040 \times 0.00020}{15} = 5.333... \times 10^{-7} = 5.3 \times 10^{-7} \text{ m (to 2 s.f.)}$$

Tip: Don't get confused here. There are 8 bright spots (maxima), but only 7 gaps (fringe widths) between them. So you need to divide the total width by 7 and not 8.

You can investigate a range of relationships using Young's double-slit experiment. Try:

- Varying D to see how it affects x.
- Varying a by using different double-slit systems to see how it affects x.
- Varying the wavelength/colour of the light to see how it affects x.

You should find that all of these change x in line with $\lambda = \frac{ax}{D}$.

Figure 7: *Coloured lenses can be used to limit the range of wavelengths from a light source.*

Tip: It was later discovered that light has properties of both waves and particles. See pages 241-244 for more on wave-particle duality.

Evidence for the wave nature of light

HOW SCIENCE WORKS

Towards the end of the 17th century, two important theories of light were published — one by Isaac Newton and the other by a chap called Huygens. Newton's theory suggested that light was made up of tiny particles, which he called "corpuscles". And Huygens put forward a theory using waves.

The corpuscular theory could explain reflection and refraction, but diffraction and interference are both uniquely wave properties. If it could be shown that light showed interference patterns, that would help settle the argument once and for all.

Young's double-slit experiment (over 100 years later) provided the necessary evidence. It showed that light could both diffract (through the two narrow slits) and interfere (to form the interference pattern on the screen).

Practice Questions — Application

Q1 A blue-violet laser with a wavelength of 450 nm is shone through a double-slit system to produce an interference pattern on a screen, as shown in Figure 8. The screen is 12.0 m from the slits and the slits are 0.30 mm apart.

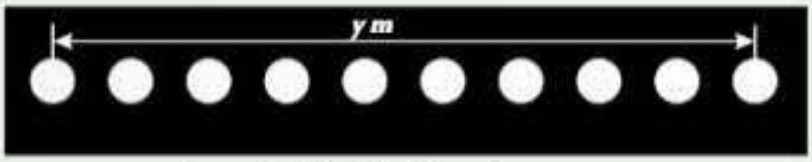


Figure 8: *A double-slit interference pattern.*

a) Give one property of a laser that makes it better than white light in this experiment.

b) Where are light and dark fringes produced?

c) Find the fringe spacing, x.

d) Find the value of y.

Q2 A laser with $\lambda = 615$ nm is shone through a double slit to produce a diffraction pattern on a screen with fringes 1.29 cm apart. If the slits are 0.11 mm apart, find the distance, D, between the slits and the screen.

Tip: A nanometre (nm) is equal to 1×10^{-9} m.

Practice Questions — Fact Recall

Q1 How can you create two coherent sources of light waves?

Q2 Describe an experiment to measure the wavelength of a laser light source by using a double slit.

Q3 Explain how Young's double-slit experiment suggested that light was a wave.

Learning Objective:

- Know techniques and procedures used to determine the wavelength of light using a diffraction grating (PAG5).

Specification Reference 4.4.3

11. Diffraction Gratings

Now you know about double-slit experiments, it's time to look at what happens when there are more than two slits. Diffraction gratings are particularly handy for analysing light from stars.

Interference with a diffraction grating

You can carry out single-source, double-slit type experiments (see p.216) using more than two equally spaced slits. You get basically the same shaped pattern as for two slits — but the bright bands are brighter and narrower and the dark areas between are darker, so the interference pattern produced is sharper.

A **diffraction grating** contains lots of equally spaced slits very close together and so can be used to do this (see Figure 1). When monochromatic light (all of the same wavelength) is passed through a diffraction grating with hundreds of slits per millimetre at normal incidence (right angles to the grating), the interference pattern is really sharp because there are so many beams reinforcing the pattern. Sharper fringes make for more accurate measurements.

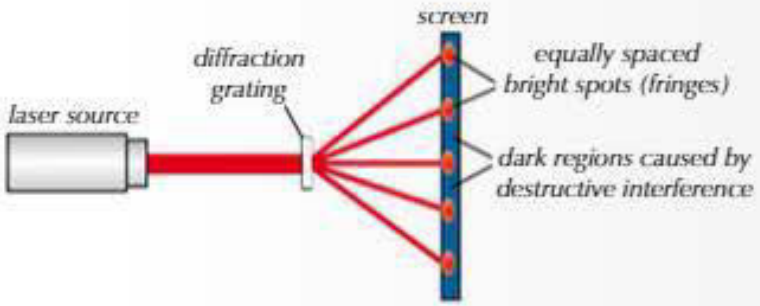


Figure 1: *Laser light passing through a diffraction grating.*

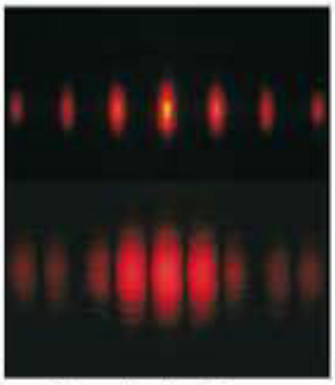


Figure 2: *Interference patterns produced by a laser going through a diffraction grating (top) and a double-slit set-up (bottom).*

For monochromatic light, all the maxima in the diffraction pattern formed are sharp lines. This means the distance between the maxima can be easily measured (fringe width). There's a line of maximum brightness at the centre called the **zero order line** — which is in the same direction as the beam incident on the grating (see Figure 3). The lines just either side of the central one are called first order lines. The next pair out are called second order lines, and so on.

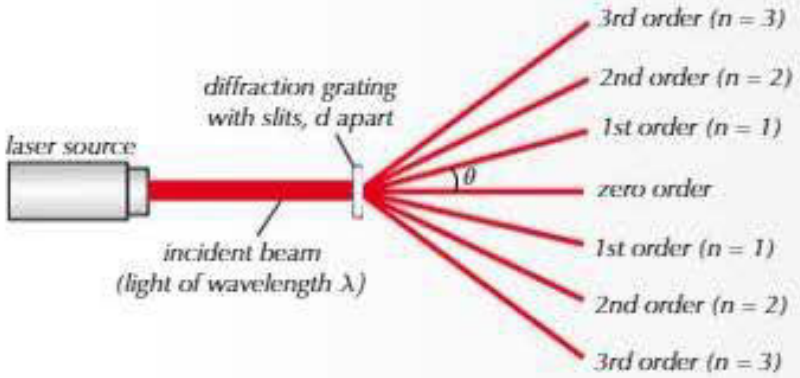

3rd order (n = 3)
2nd order (n = 2)
diffraction grating with slits, d apart
1st order (n = 1)
laser source
θ
zero order
1st order (n = 1)
2nd order (n = 2)
incident beam (light of wavelength λ)
3rd order (n = 3)

Figure 3: *The order of maxima for light passing through a diffraction grating.*

Tip: Be careful here — the space between slits is *d*, but for double-slit experiments it was *a*.

Determining the wavelength of light

PRACTICAL ACTIVITY GROUP 5

You can find the wavelength of a light source from the interference pattern produced when it passes through a diffraction grating. It's just like Young's double-slit experiment, p.216-219, but you use a diffraction grating instead of a mounted double-slit card.

Using the fringe width, x, and the distance to the screen, D, the angle the 1st order fringe makes with the zero order line can be calculated using small angle approximations.

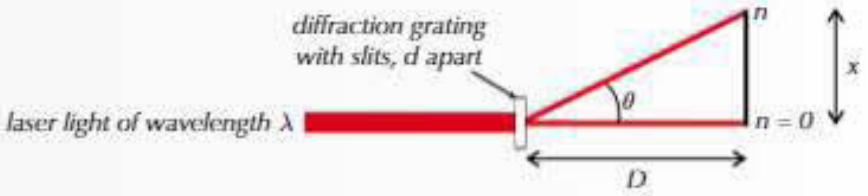
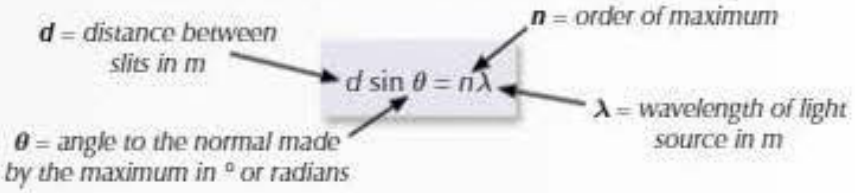


Figure 5: *The 1st order maxima for light passing through a diffraction grating onto a screen.*

For small angles where θ is in radians, $\tan\theta \approx \theta$ and $\tan\theta = \frac{x}{D}$, so:

$$\theta \approx \frac{x}{D}$$

The slit separation, d, for the diffraction grating is usually given. If the grating has N slits per metre, then the slit spacing, d, is just $1/N$ metres. If you know the slit separation, d, what order maximum you're observing, n, and the angle between this maximum and the incident light, θ, you can find the wavelength of the incident light using this equation:


d = distance between slits in m
n = order of maximum
d sin θ = nλ
λ = wavelength of light source in m
θ = angle to the normal made by the maximum in ° or radians

Example — Maths Skills

Green laser light of wavelength 5.00×10^{-7} m is transmitted through a perpendicular diffraction grating with 3.00×10^{5} slits per metre. At what angle to the normal are the second order maxima seen?

There are 3.00×10^{5} slits per metre, so the slit spacing is:

$$d = \frac{1}{3.00 \times 10^{5}} = 3.33... \times 10^{-6}\text{ m}$$

Rearrange the diffraction grating equation, $d \sin\theta = n\lambda$, for θ, and remember $n = 2$, as it's the second order we're after. So:

$$\theta = \sin^{-1}\left(\frac{n\lambda}{d}\right) = \sin^{-1}\left(\frac{2 \times (5.00 \times 10^{-7})}{3.33... \times 10^{-6}}\right)$$

$$= 17.457... = 17.5° \text{ (to 3 s.f.)}$$

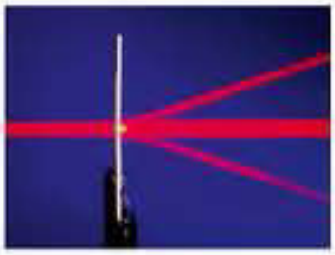


Figure 4: *A laser beam is split into zero order and first order maxima by a diffraction grating.*

Tip: Make sure you follow the laser safety precautions on page 216.

Tip: The slit separation is often written on the slide frame.

Tip: If multiple orders are visible, you could try repeating your measurements for different orders of n.

Tip: Make sure that your calculator is working in the correct mode, depending on whether you want degrees or radians. You'll need the button for $\sin^{-1}$ (inverse sin).

From this equation you can draw a few conclusions:

- If λ is bigger, $\sin\theta$ is bigger, and so θ is bigger. This means that the larger the wavelength, the more the pattern will spread out.
- If d is bigger, $\sin\theta$ is smaller. This means that the coarser the grating, the less the pattern will spread out.
- Values of $\sin\theta$ greater than 1 are impossible. So if for a certain n you get a result of more than 1 for $\sin\theta$, you know that that order doesn't exist.

Tip: Coarser just means fewer slits in a given width.

The diffraction of white light

White light is really a mixture of colours. If you diffract white light through a diffraction grating then the patterns due to different wavelengths within the white light are spread out by different amounts. Each order in the pattern becomes a spectrum, with red on the outside and violet on the inside. The zero order maximum stays white because all the wavelengths just pass straight through.

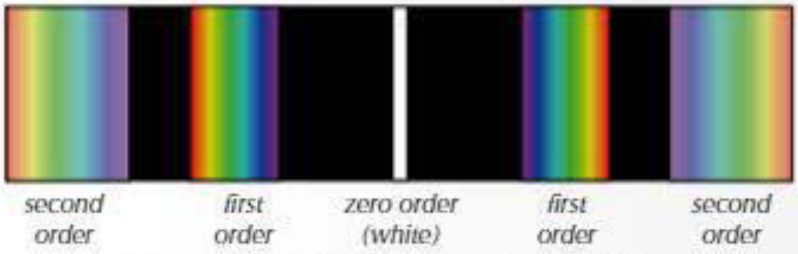


Figure 6: *The diffraction pattern for white light.*

Figure 7: *The surface of a CD contains many tiny grooves that cause it to act in a similar way to a diffraction grating. When white light is shone on it, a spectrum is visible.*

Astronomers and chemists often need to study spectra to help identify elements. They use diffraction gratings rather than prisms because they're more accurate.

Practice Question — Application

Q1 An orange laser beam with a wavelength of 590 nm is transmitted through a perpendicular diffraction grating with 4.5×10^5 slits per metre.

a) At what angle to the normal is the third order maximum seen? Use the diffraction grating formula $d\sin\theta = n\lambda$, where d is the slit spacing, θ is the angle to the normal made by the maximum, n is the order of maximum and λ is the wavelength of the light source.

b) Use the diffraction grating formula to show whether there will be a fourth order maximum.

c) A red laser beam is used instead with a wavelength of 700 nm. Describe what will happen to the interference pattern, and explain why.

Practice Questions — Fact Recall

Q1 Describe an experiment you could carry out to measure the wavelength of a monochromatic light source using a diffraction grating and an observation screen.

Q2 Why are spectra formed when white light passes through a diffraction grating?

12. Stationary Waves

So far you've been looking at progressive waves — waves that move. Now it's time for stationary waves — waves that stay where they are.

What is a stationary wave?

A **stationary (standing) wave** is the superposition of two progressive waves with the same wavelength and frequency, moving in opposite directions. So a stationary wave has all the same properties as a progressive wave, however unlike progressive waves, no energy is transmitted by a stationary wave.

You can demonstrate stationary waves by attaching a vibration transducer (an oscillator) at one end of a stretched string with the other end fixed. The transducer is given a wave frequency by a signal generator and creates that wave by vibrating the string.

The wave generated by the transducer is reflected back and forth. For most frequencies the resultant pattern is a jumble. However, if the transducer happens to produce an exact number of waves in the time it takes for a wave to get to the end and back again, then the original and reflected waves reinforce each other.

The frequencies at which this happens are called **resonant frequencies** and it causes a stationary wave where the overall pattern doesn't move along — it just vibrates up and down, so the string forms oscillating 'loops' — see Figure 1.

These stationary waves are transverse, so each particle vibrates at right angles to the string. **Nodes** are where the amplitude of the vibration is zero — they just stay perfectly still. **Antinodes** are points of maximum amplitude. The separation between any two adjacent nodes (or antinodes) is equal to $\lambda/2$, where λ is the wavelength of the progressive wave.

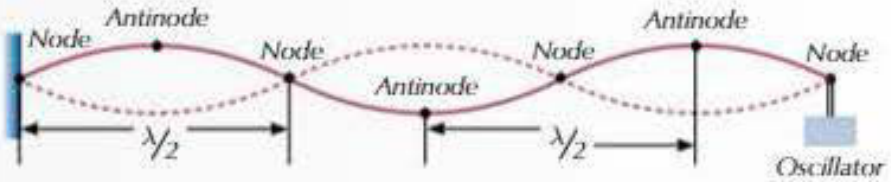


Figure 1: *A stationary wave oscillating on a string.*

Resonant frequencies

A stationary wave is only formed at a resonant frequency (when an exact number of half wavelengths fits on the string). There are some special names for each resonant frequency.

The fundamental mode of vibration

The standing wave in Figure 2 is vibrating at the lowest possible resonant frequency, called the **fundamental mode of vibration** or the first harmonic. It has one "loop" with a node at each end. One half wavelength fits onto the string, and so the wavelength is double the length of the string.

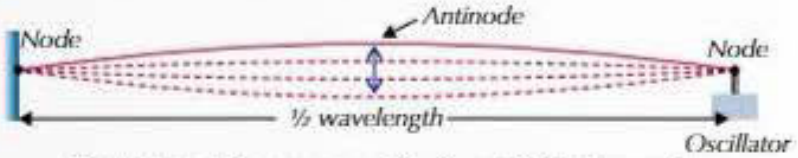


Figure 2: *A stationary wave vibrating at its first harmonic.*

Learning Objectives:

- Know the similarities and differences between stationary and progressive waves.
- Be able draw graphical representations of a stationary wave.
- Know what nodes and antinodes are.
- Understand the idea that the separation between adjacent nodes (or antinodes) is equal to $\lambda/2$, where λ is the wavelength of the progressive wave.
- Know what is meant by the fundamental mode of vibration (1st harmonic) and the harmonics.
- Know the stationary wave patterns for a stretched string.
- Demonstrate understanding of stationary (standing) waves using stretched strings, air columns and microwaves.
- Know the stationary wave patterns for air columns in closed and open tubes.
- Know techniques and procedures used to determine the speed of sound in air by formation of stationary waves in a resonance tube (PAG5).

Specification Reference 4.4.4

Exam Tip

If you're asked to sketch a standing wave, make sure you make it clear where the nodes and antinodes are. You don't need to draw loads of lines like in Figure 2, just show what shape the string is vibrating in.

Figure 4: *A stationary wave vibrating at its first, second, third and fourth harmonics (going from top to bottom).*

Second harmonic

Figure 3 shows the second harmonic. It has twice the frequency of the fundamental mode of vibration. There are two "loops" with a node in the middle and one at each end. Two half wavelengths fit on the string, so the wavelength is the length of the string.

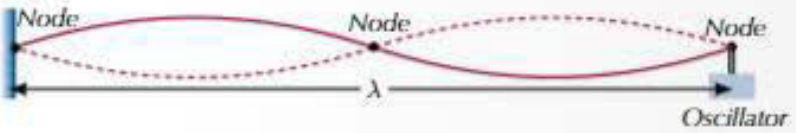


Figure 3: *A stationary wave vibrating at its second harmonic.*

Third harmonic

The third harmonic is three times the frequency of the first harmonic — see Figure 5. One and a half wavelengths fit on the string.

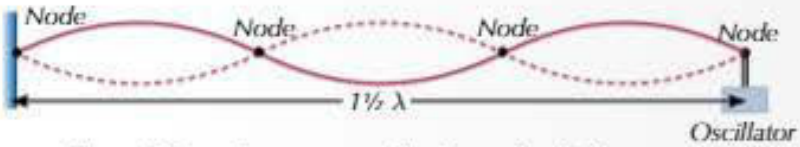


Figure 5: *A stationary wave vibrating at its third harmonic.*

You can have as many harmonics as you like — an extra loop and an extra node are just added with each one, the number of λ that fit goes up by ½, and the frequency increases by the value of the frequency of the first harmonic. For example, Figure 4 shows the fourth harmonic of a stationary wave.

Tip: At the a^{th} harmonic, the number of antinodes is equal to a, and the number of nodes is equal to $a + 1$.

Tip: At the a^{th} harmonic, $\frac{a}{2}$ wavelengths will fit on the string.

Tip: If you're given the first harmonic frequency, you can work out the resonant frequency f at the a^{th} harmonic with $f = a \times$ first harmonic frequency.

Example — **Maths Skills**

A banjo string vibrates with a first harmonic frequency of 290 Hz. Find the frequency of vibration of the string at the third harmonic.

The third harmonic is three times the frequency of the first harmonic, so:
$f = 290 \times 3 = 870$ Hz

A violin string is vibrating at fives times the frequency of its first harmonic. How many wavelengths fit onto the string?

The fifth harmonic is made up of five 'loops'. Each loop is ½λ long.
$5 \times ½ = 2.5$, so there are two and a half wavelengths on the string.

Other examples of stationary waves

Stretched strings

Just like the strings in the demonstration on the previous page, transverse stationary waves form on the strings of stringed instruments like violins and guitars. Your finger or the bow sets the string vibrating at the point of contact. Waves are sent out in both directions and reflected back at both ends.

Air columns

Longitudinal stationary waves form in a wind instrument or other air column. If a source of sound is placed at the open end of a flute, piccolo, oboe or other column of air, there will be some frequencies for which resonance occurs and a stationary wave is set up.

If the instrument has a closed end, a node will form there. You get the lowest resonant frequency when the length, *l*, of the pipe is a quarter wavelength. Antinodes form at the open ends of pipes.

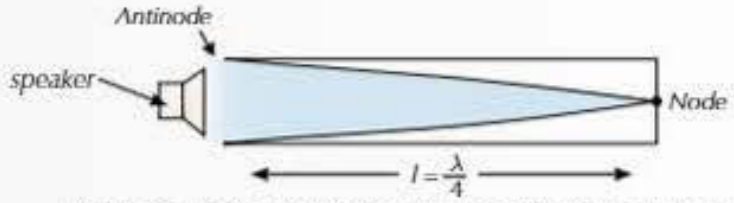


Figure 7: *An air column vibrating at the fundamental mode of vibration in a pipe with one closed end.*

If both ends are open, there are antinodes at both ends. You get the lowest resonant frequency when the length, *l*, of the pipe is a half wavelength.

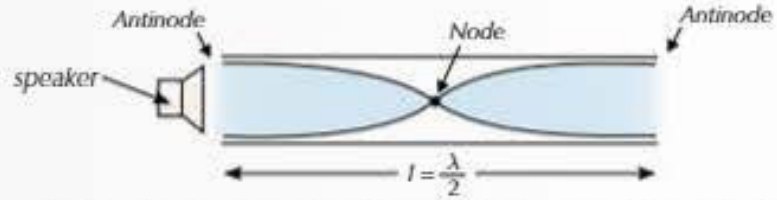


Figure 8: *An air column vibrating at the fundamental mode of vibration in an open-ended pipe.*

All harmonics have nodes at closed ends and antinodes at open ends. For example, the second harmonic for the open-ended pipe is shown in Figure 9.

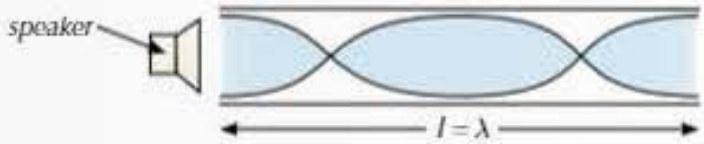


Figure 9: *An air column vibrating at the second harmonic in an open-ended pipe.*

Stationary microwaves

You can set up a stationary wave by reflecting a microwave beam at a metal plate (see Figure 10). The superposition of the wave and its reflection produces a stationary wave. You can find the nodes and antinodes by moving the probe between the transmitter and reflecting plate. The meter or loudspeaker receives no signal at the nodes and maximum signal at the antinodes.

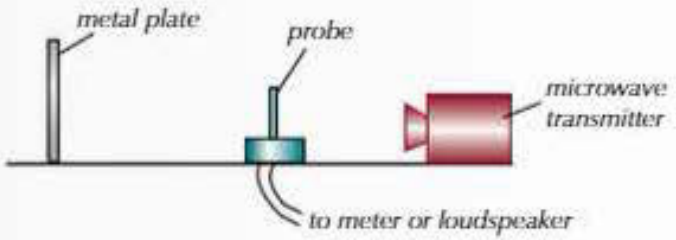


Figure 10: *Apparatus for demonstrating stationary microwaves.*

Figure 6: *The individual pipes in a set of panpipes are cut to different lengths to give them different fundamental frequencies.*

Tip: Remember, the sound waves in wind instruments are longitudinal — they don't actually look like these diagrams.

Tip: You could do a similar experiment using a loud speaker by moving around a microphone connected to a meter.

Tip: The string demonstration on page 223 gives you the best idea of what stationary waves actually look like, but this experiment also shows the presence of stationary waves.

Measuring the speed of sound

PRACTICAL ACTIVITY GROUP 5

You can use the known wavelength of a stationary wave in an air column to calculate the speed of sound in air.

Tip: Make sure you carry out this experiment safely — you should clear up any water spillages that occur to prevent anybody slipping on a wet floor.

Create a closed-end pipe by placing a hollow tube into a measuring cylinder of water (see Figure 11). Choose a tuning fork and note down the frequency of sound it produces (it'll be stamped on the side of it).

Gently tap the tuning fork and hold it just above the hollow tube. The sound waves produced by the fork travel down the tube and get reflected (and form a node) at the air/water surface. Move the tube up and down until you find the shortest distance between the top of the tube and the water level that the sound from the fork resonates at (when the sound is at its loudest). At this point the frequency of the sound wave is equal to the lowest resonant frequency of the closed tube.

Tip: The antinode of the wave actually forms slightly above the top of the tube. You'd add an 'end correction value' on to take this into account, but you don't need to worry about it.

Just like with any closed pipe, the length of the tube at its lowest resonant frequency is a quarter of the wavelength of the stationary sound wave (see page 225). Once you know the frequency and wavelength of the stationary sound wave, you can work out the speed of sound (in air), v, using the equation $v = f\lambda$.

Figure 12: *Tuning forks produce a very pure sound at one frequency.*

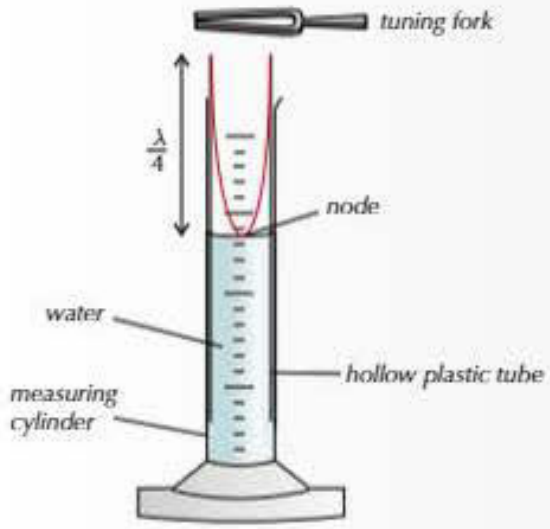


Figure 11: *Apparatus for measuring the speed of sound.*

Repeat the experiment with different tuning forks and calculate an average value of v from your results.

Example — **Maths Skills**

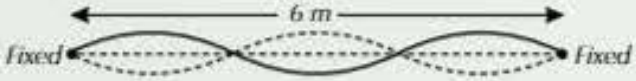
A student conducts an experiment to measure the speed of sound. He holds one end of a hollow tube in water and uses a tuning fork to send sound waves with a frequency of 424 Hz into the tube. He finds that the shortest distance between the water level and the top of the tube when the sound is loudest is 20.1 cm. What is the speed of sound?

Wavelength = 4 × pipe length = 4 × 0.201 = 0.804 m

$v = f\lambda = 424 \times 0.804 = 340.896 = 341$ ms^{-1} (to 3 s.f.)

Practice Questions — Application

Q1 For the stationary wave in the diagram below, find the wavelength.



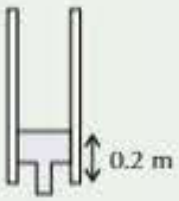
Q2 The diagram below represents a stationary wave on a string which is fixed at A and driven by an oscillator at B.

a) The length of the string AB is 2.5 m and the frequency of the wave is 100 Hz.
Calculate the speed of the waves forming the stationary wave.

b) If the wave is now vibrating at a frequency of 200 Hz, how many wavelengths would fit on the string? What is the name for this resonant frequency?

Q3 A student holds one end of hollow tube in water and uses a tuning fork to send sound waves into the tube. She moves the tube up and down in the water until she finds the lowest resonant frequency of the wave. What is the length of the pipe equal to in terms of the wavelength of the sound waves?

Q4 A speaker is used to set up a stationary sound wave in a pipe which is open at both ends. The wave is at the lowest fundamental frequency and has a wavelength of 3.2 m. The last 0.2 m of the pipe is then blocked up as shown in the diagram on the right. Again, the speaker is used to set up a stationary sound wave in the blocked pipe at the lowest fundamental frequency.
Calculate the wavelength of the new stationary wave.



Not to scale

Tip: Remember $v = f\lambda$.

Practice Questions — Fact Recall

Q1 What is a stationary wave?

Q2 What is the difference between stationary and progressive waves in terms of energy transfer?

Q3 Describe what a resonant frequency of a string is.

Q4 What is the distance between two adjacent nodes on a stationary wave?

Q5 What is the fundamental mode of vibration?

Q6 Sketch a string vibrating at its second harmonic and label the positions of all nodes and antinodes.

Q7 Give an example of a way to observe stationary microwaves.

Section Summary

Make sure you know...

- That a wave which carries energy without transferring any material is called a progressive wave.
- What displacement, amplitude, wavelength, period, frequency and phase difference mean for waves.
- The nature of transverse and longitudinal waves, and be able to draw graphical representations of each.
- How to calculate the period of a wave using $f = \frac{1}{T}$.
- How to determine the frequency of a wave using an oscilloscope.
- How to calculate wave speed using $v = f\lambda$.
- That the intensity of a wave is proportional to the square of its amplitude and how to calculate it using $I = \frac{P}{A}$.
- What the electromagnetic spectrum is and the properties of electromagnetic waves.
- The orders of magnitude of the wavelengths of each of the seven categories of electromagnetic wave.
- That plane polarised waves are waves that only oscillate in one direction, and be able to describe how polarising effects can be observed using light and microwaves.
- That waves can be reflected and refracted, and how to demonstrate their effects using a ripple tank.
- That the refractive index of a material is the ratio between the speed of light in a vacuum and the speed of light in that material, and how to calculate it using $n = \frac{c}{v}$.
- How to use $n \sin\theta$ = constant and $n_1 \sin\theta_1 = n_2 \sin\theta_2$ to solve problems at a boundary between materials.
- How to use ray boxes and transparent rectangular blocks to investigate the refraction of light.
- That the critical angle is the angle of incidence at which the angle of refraction is 90°.
- How to use the equation $\sin C = \frac{1}{n}$ to calculate the critical angle of light at a material-air boundary.
- How to use ray boxes and semi-circular blocks to investigate total internal reflection of light.
- The principle of superposition and how it can be illustrated graphically.
- What the terms interference, phase difference and coherence mean.
- How to describe constructive interference and destructive interference in terms of phase difference.
- That diffraction is the way that waves spread out as they pass through a narrow gap or go round obstacles, and how it can be demonstrated using a ripple tank.
- That all waves can be diffracted and that diffraction effects become significant when the wavelength is comparable to the gap width that a wave is travelling through.
- Know what the term path difference means.
- How to describe constructive interference and destructive interference in terms of path difference.
- How to carry out experiments using sound and microwaves to demonstrate superposition.
- How visible light can be used to carry out Young's double-slit experiment, and how this experiment gave a classical confirmation of the wave-nature of light.
- That Young's double slit formula, $\lambda = \frac{ax}{D}$, can be used for all waves where $a << D$.
- How to determine the wavelength of light using a double-slit or using a diffraction grating.
- The similarities and differences between stationary and progressive waves.
- How a stationary wave can be represented graphically.
- What nodes and antinodes are and that the separation between adjacent nodes (or antinodes) is $\lambda/2$.
- That the resonant frequencies at which stationary waves are formed are called the harmonics, and that the lowest possible resonant frequency is the fundamental mode of vibration (1st harmonic).
- How to demonstrate stationary waves using stretched strings, air columns and microwaves.
- The stationary wave patterns for a stretched string and air columns in closed and open tubes.
- How the speed of sound in air can be found using stationary waves in a resonance tube.

Exam-style Questions

1 Two waves that are in phase and coherent interfere.
Which of the following could be correct?

A The waves have a path difference of λ and interfere destructively.

B The waves have a path difference of $\frac{\lambda}{2}$ and interfere constructively.

C The waves have a path difference of 3λ and interfere destructively.

D The waves have a path difference of 4λ and interfere constructively.

(1 mark)

2 Unpolarised white light is incident on two polarising filters placed one in front of the other. The transmission axes of the two filters are aligned horizontally, then one filter is gradually rotated. At what angle of rotation is all light blocked from passing through the filters?

A 0°

B 45°

C 90°

D 180°

(1 mark)

3 X-rays with a period of 1.7×10^{-16} ms travel through a vacuum.
What is the value of λ?

A 5.1×10^{-11} m

B 1.76×10^{24} m

C 5.1×10^{-8} m

D 1.76×10^{27} m

(1 mark)

4 A wave has an amplitude of 78 cm. The intensity of the wave is reduced from 0.42 Wm^{-2} to 0.11 Wm^{-2}. What is the new amplitude of the wave?

A 5.4 cm

B 20 cm

C 152 cm

D 40 cm

(1 mark)

5 **Fig 5.1** shows a graphical representation of a stationary sound wave in an air column.

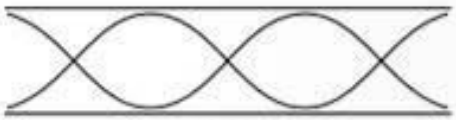


Fig 5.1

(a) What is the name for the resonant frequency displayed?

(1 mark)

(b) The tube is 1.25 m long. Calculate the wavelength of the stationary wave.

(1 mark)

(c)* Describe an experimental method that a student could use to measure the speed of sound using the phenomenon of standing sound waves in air columns. Your answer should include a labelled diagram and one way to improve the accuracy of the experiment.

(6 marks)

6 A scientist is testing a transmitter of electromagnetic (EM) radiation. He connects an oscilloscope to the transmitter to display the EM wave being produced. A sketch of the trace is shown in **Fig 6.1**. The timebase of the oscilloscope is set to 0.500 μs/div.

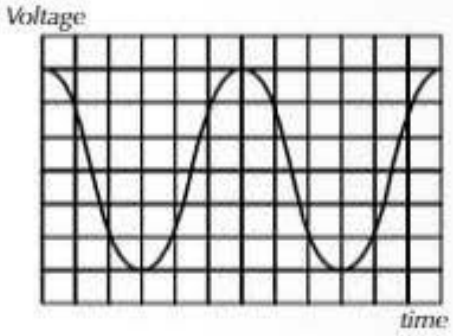


Fig 6.1

(a) Calculate the wavelength of the detected wave. You can assume the wave is travelling at the speed of light in a vacuum.

(3 marks)

(b) Which part of the electromagnetic spectrum does the wave belong to?

(1 mark)

(c) The scientist connects the oscilloscope to a second transmitter, which produces EM waves with the same frequency and wavelength as the first transmitter, but half the amplitude. The waves of both transmitters are displayed on the screen. The scientist notes that the wave from the second transmitter is 90° in front of the wave from the first transmitter. Sketch what the scientist sees on the oscilloscope screen.

(2 marks)

(d) The scientist measures the power of the first wave to be 15.2 W. Calculate the power of the second wave.

(2 marks)

* The quality of your response will be assessed in this question.

7 A student is investigating diffraction patterns. He directs a laser beam through a double-slit system, where the slits are 0.15 mm apart and the screen is 7.5 m from the slits. The interference pattern produced is shown in **Fig 7.1**.

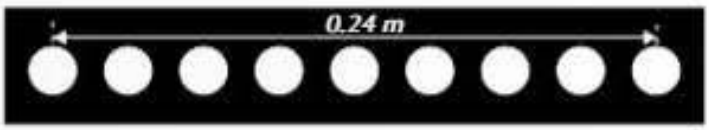


Fig 7.1

(a) Find the wavelength of the laser beam.

(3 marks)

(b) Explain why the student used a laser and not a standard light source.

(2 marks)

(c) Calculate the angle of the first order maxima if the same laser is shone through a diffraction grating with 2.55×10^5 slits per metre.

(4 marks)

(d) Describe the difference you would expect to see in the interference pattern if a diffraction grating was used instead of the double slit in this experiment.

(1 mark)

8 **Fig 8.1** shows two rays of light at a frequency of 5.00×10^{14} Hz entering an optical fibre used for communications.

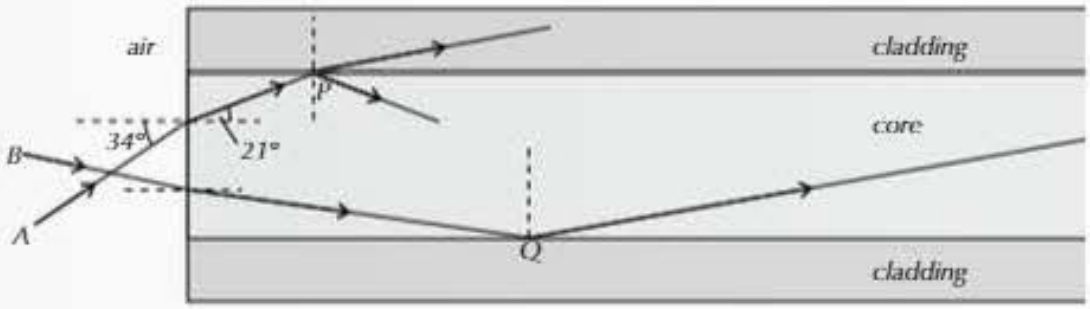


Fig 8.1 ***not drawn to scale***

(a) Light travels at 2.03×10^8 ms^{-1} in the cladding.
Calculate the refractive index of the cladding.

(2 marks)

(b) Find the refractive index of the core of the optical fibre.

(3 marks)

(c) Find the wavelength of the light as it travels through the core.

(3 marks)

(d) Calculate the critical angle of the core-cladding boundary.

(2 marks)

(e) Explain why light ray A enters the cladding of the optical fibre.

(2 marks)

(f) Describe what has happened to ray B, and explain why.

(2 marks)

1. The Photon Model

You might think that light and all other electromagnetic waves are, well, waves. But you can use a particle model called the photon model to describe them...

Learning Objectives:

- Understand the particulate nature (photon model) of electromagnetic radiation.
- Know that a photon is a quantum of energy of electromagnetic radiation.
- Know how to calculate the energy of a photon, using $E = hf$ and $E = \frac{hc}{\lambda}$.
- Know and be able to use the electronvolt (eV) as a unit of energy.

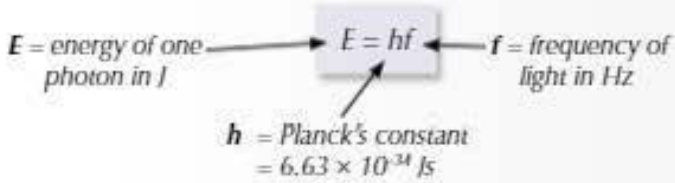
Specification Reference 4.5.1

Photons

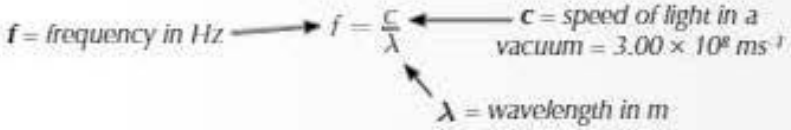
When Max Planck was investigating black body radiation (don't worry, you don't need to know about this), he suggested that electromagnetic (EM) waves can only be released in discrete packets. This meant that the energy carried by a wave could only be released in fixed 'lumps' of energy, called quanta.

Einstein went further by suggesting that EM waves (and the energy they carry) can only exist in discrete packets. He called these wave-packets **photons**. One photon is one quantum of energy of electromagnetic radiation.

He believed that a photon acted as a neutral particle. He said that it would either transfer all or none of its energy when interacting with one other particle (this is called a **one-to-one interaction**). The energy, E, carried by one of these photons had to be:



So, the higher the frequency of the electromagnetic radiation, the more energy its photons carry. The frequency, wavelength and speed of light are related by the equation:



You can substitute this equation into $E = hf$ to give another equation for the energy of one photon:

$$E = hf = \frac{hc}{\lambda}$$

Tip: The photon model is needed to explain the photoelectric effect, which is on pages 237-240.

Tip: This is just the wave speed equation from page 191.

Exam Tip
In the exam you'll be given a data and formulae booklet that has these equations, as well as the values of c and h. So you don't have to worry about learning them — just make sure you can use them.

Example **Maths Skills**

Calculate the energy of a photon with a wavelength of 3.9×10^{-9} m.

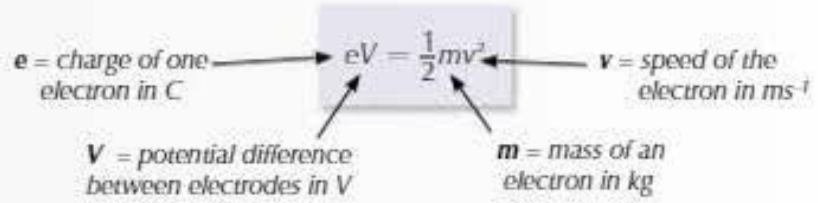
Substitute $\lambda = 3.9 \times 10^{-9}$ m, $h = 6.63 \times 10^{-34}$ Js and $c = 3.00 \times 10^{8}$ ms^{-1} into the equation $E = \frac{hc}{\lambda}$ to find the energy, E, of the photon.

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}) \times (3.00 \times 10^{8})}{3.9 \times 10^{-9}} = 5.1 \times 10^{-17} \text{J}$$

The electronvolt

The energies involved when you're talking about photons are usually so tiny that it makes sense to use a more appropriate unit than the joule. So the electronvolt (eV) is used.

When you accelerate an electron in a potential difference (p.154), it transfers some electrical potential energy (= eV) into kinetic energy (= $\frac{1}{2}mv^2$).


e = charge of one electron in C
eV = ½mv²
v = speed of the electron in ms⁻¹
V = potential difference between electrodes in V
m = mass of an electron in kg

The electronvolt is defined as the kinetic energy gained by an electron when it is accelerated through a potential difference of one volt. So the value of one electronvolt can be found using:

$$1 \text{ electronvolt} = e \times V = 1.60 \times 10^{-19} \text{ C} \times 1 \text{ JC}^{-1}$$

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

Figure 1: *Electron tubes are used to produce and accelerate electrons.*

Tip: To convert energy from electronvolts to joules, just multiply it by 1.60×10^{-19}. To convert from joules to electronvolts, divide by 1.60×10^{-19}.

Tip: Prefixes like 'M' are covered on page 32.

Example — Maths Skills

A photon has a frequency of 100 MHz. Calculate the energy of the photon in eV.

100 MHz = $100 \times 10^6 = 1 \times 10^8$ Hz,

Substitute $f = 1 \times 10^8$ Hz and $h = 6.63 \times 10^{-34}$ Js into $E = hf$:

$E = 6.63 \times 10^{-34} \times 1 \times 10^8 = 6.63 \times 10^{-26}$ J

Divide this by 1.60×10^{-19} to get the energy in eV:

$(6.63 \times 10^{-26}) \div (1.60 \times 10^{-19}) = 4.14... \times 10^{-7}$ eV = 4.1×10^{-7} eV (to 2 s.f.)

Practice Questions — Application

Q1 A photon has an energy of 2.20 eV. Calculate its energy in J.

Q2 A photon has a frequency of 6.0×10^{13} Hz. Calculate its energy in J.

Q3 A photon has an energy of 8.2×10^{-20} J. Calculate its wavelength.

Practice Questions — Fact Recall

Q1 What is a photon?

Q2 What equation would you use to calculate the energy of a photon from its wavelength?

Q3 What is an electronvolt a measure of?

Learning Objectives:

- Be able to use LEDs and the equation $eV = \frac{hc}{\lambda}$ to estimate the value of the Planck constant, h.
- Determine the Planck constant using different coloured LEDs (PAG6).

Specification Reference 4.5.1

2. The Planck Constant

The Planck constant is used in loads of equations in quantum physics. It's one of the smallest physical constants, and you need to know how to find it...

Threshold voltage of LEDs

You can find the value of Planck's constant by doing a simple experiment with light-emitting diodes (LEDs) — see below.

LEDs, like all electrical components, have a potential difference across them when they're part of a complete circuit. This potential difference accelerates the electrons flowing around the circuit (the current) and causes them to gain kinetic energy, like you saw on the last page. This kinetic energy is transferred into a photon that is emitted by the LED.

However, current will only pass through an LED after a minimum voltage (potential difference) is placed across it — the **threshold voltage** V_0. This is the voltage needed to give the electrons the same energy as a photon emitted by the LED.

Figure 1: *LEDs come in a range of sizes and colours. They're often used because they produce light at a lower potential difference than traditional filament bulbs.*

Tip: You may see this written as $eV = \frac{hc}{\lambda}$, but don't panic, V is still the threshold voltage.

Tip: Make sure you do a risk assessment before you start this experiment, to ensure your experiment is carried out as safely as possible.

Tip: A milliammeter is most suitable (p.17) for this experiment, as it will allow you to more accurately tell when a current has just begun to flow.

Finding the Planck constant

PRACTICAL ACTIVITY GROUP 6

You can use the threshold voltage of LEDs to find the Planck constant. To do this, you'll need to use monochromatic LEDs — i.e. LEDs that emit a single colour. The colour of light depends on its wavelength, so monochromatic LEDs only emit a single wavelength of light. You should know the wavelength of the LED from the manufacturer.

At the threshold voltage, the kinetic energy of each electron is equal to eV_0 (from the equation on page 233) where V_0 is the threshold voltage. All of this energy is transferred into one photon that is emitted by the LED. The energy of any photon is given by $E = \frac{hc}{\lambda}$ (page 232). So to find the Planck constant, equate the energy of one electron to the energy of a photon emitted:

e = charge of one electron in C

V_0 = threshold voltage in V

$$eV_0 = \frac{hc}{\lambda}$$

energy of one photon

Then rearrange this for the Planck constant to get:

$$h = \frac{eV_0\lambda}{c}$$

To determine the Planck constant from an LED, you will need to set up the circuit shown in Figure 2.

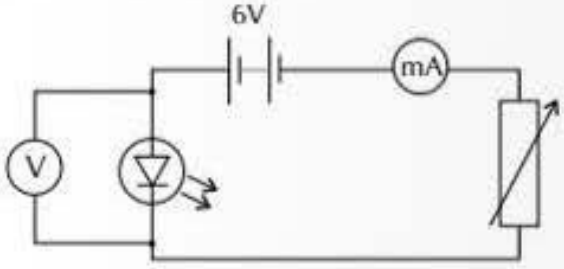


Figure 2: *A circuit used to find the Planck constant from an LED.*

- Start off with the variable resistor set to its maximum resistance, so no current can flow through the circuit.
- Adjust the variable resistor until a current just begins to flow through the circuit (and the LED lights up).
- Record the voltage across the LED. This is the threshold voltage, V_0.
- Record the wavelength of the light emitted by the LED.
- Disconnect the circuit to allow all of the components to cool.
- Repeat the experiment several times and take an average of your result for V_0.
- Then repeat the experiment for a range of LEDs.

Tip: Doing this experiment in a dark room, or placing a tube of paper around the LED, makes it easier to see when the LED just begins to light up.

Tip: If your LED doesn't light up, swap over the wires connected to it. LEDs have a positive and a negative end, and they won't work unless they're connected the right way round.

If you look at $eV_0 = \frac{hc}{\lambda}$ from the previous page, you can see that $V_0 \propto 1/\lambda$ (as e, h and c are all constants). So a graph of the threshold voltages against $1/\lambda$ will give you a straight line of best fit. You can use the gradient of this line to find the Planck constant (see below).

You should get a straight line graph similar to the one shown in Figure 3. Your line of best fit must go through the origin.

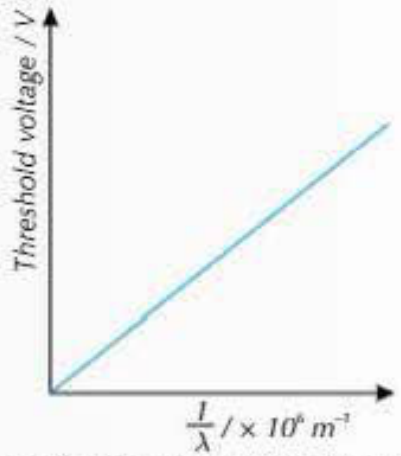


Figure 3: *A graph of threshold voltage against $1/\lambda$ for LEDs.*

Tip: Make sure you use the same equipment for each repeat, and control variables like temperature to make the experiment a fair test that will give you valid results (p.18).

The gradient of this graph is equal to:

$$\text{gradient} = \frac{\Delta y}{\Delta x} = V_0\lambda$$

Tip: There's more about graphs and how to calculate a gradient on page 14.

Substitute this into the equation from the previous page:

$$h = \frac{eV_0\lambda}{c} = \frac{e \times \text{gradient}}{c}$$

Example — **Maths Skills**

A student investigates the threshold voltages of some LEDs. They plot a graph of threshold voltage against $1/\lambda$. The gradient of the graph is equal to 1.24×10^{-6} Vm. Use this data to calculate Planck's constant.

Substitute the gradient into $h = \frac{e \times \text{gradient}}{c}$:

$$h = \frac{(1.60 \times 10^{-19}) \times (1.24 \times 10^{-6})}{3.00 \times 10^{8}}$$

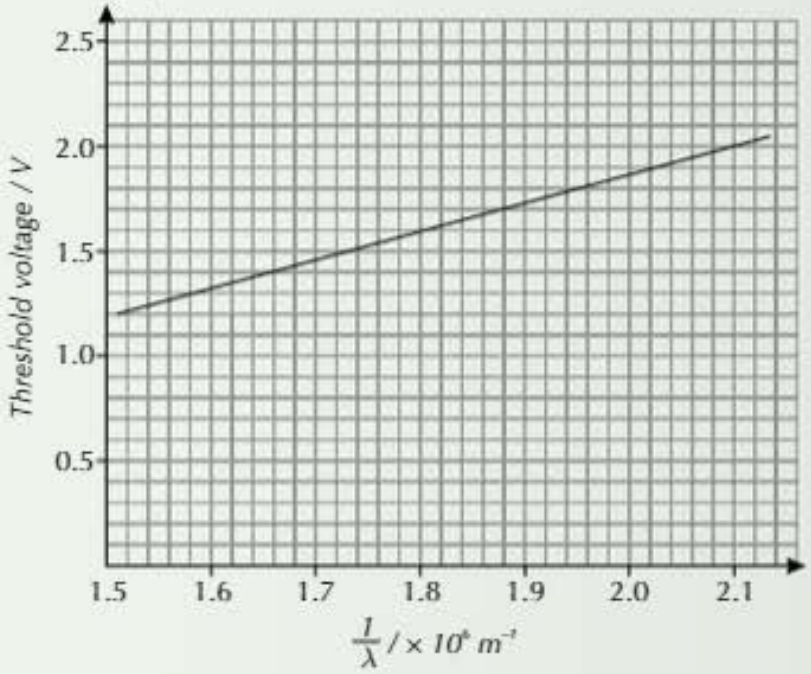
$h = 6.613.... \times 10^{-34}$ Js $= 6.61 \times 10^{-34}$ Js (to 3 s.f.)

Tip: All of the data here is to three significant figures. As the question doesn't specify the number of significant figures for the answer, you should match the data given in the question (page 9).

Practice Questions — Application

Q1 An LED emits light at a wavelength of 600 nm.
Calculate the threshold voltage for the LED, to 2 significant figures.

Q2 A student investigates the Planck constant using LEDs. He measures the threshold voltage for a range of monochromatic LEDs of different wavelengths. He then produces a graph of his results, shown below. Use the graph to determine the Planck constant.



Tip: Remember to watch out for prefixes and scaling factors on axis labels.

Practice Questions — Fact Recall

Q1 What is the threshold voltage of an LED?

Q2 State the equation that links threshold voltage to the Planck constant.

Q3 a) Draw a circuit diagram for the circuit you could use to determine the Planck constant using LEDs.

b) Explain why a milliammeter is more suitable than an ammeter in this circuit.

3. The Photoelectric Effect

One of the most famous cases of light acting like a particle is the photoelectric effect. Read on to find out more...

Learning Objectives:

- Understand the term work function.
- Understand the photoelectric effect, including a simple experiment to demonstrate this effect, e.g. using a gold-leaf electroscope and a zinc plate.
- Understand the term threshold frequency.
- Know that there is a one-to-one interaction between a photon and a surface electron.
- Know how to use Einstein's photoelectric equation $hf = \phi + KE_{max}$.
- Understand the idea that the maximum kinetic energy of photoelectrons is independent of the intensity of the incident radiation.
- Understand the idea that rate of emission of photoelectrons above the threshold frequency is directly proportional to the intensity of the incident radiation.

Specification Reference 4.5.2

What is the photoelectric effect?

If you shine radiation of a high enough frequency onto the surface of a metal, it will instantly eject electrons (see Figure 1). For most metals, the necessary frequency falls in the ultraviolet range.

Because of the way atoms are bonded together in metals, metals contain 'free electrons' that are able to move about the metal. The free electrons on or near the surface of the metal absorb energy from the radiation.

Before an electron can leave the surface of the metal, it needs enough energy to break the bonds holding it there. This energy is called the **work function** energy (ϕ) and its value depends on the metal. If an electron absorbs this amount of energy (or more), it is released. This is called the **photoelectric effect** and the electrons emitted are called **photoelectrons**.

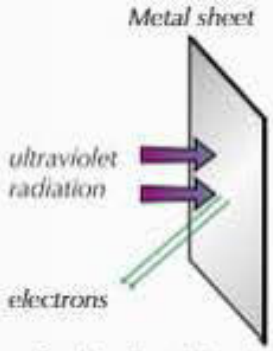


Figure 1: *The photoelectric effect.*

The photoelectric effect can be demonstrated with a simple experiment that uses a gold-leaf electroscope (a box containing a piece of metal with a strip of gold leaf attached).

A zinc plate is attached to the top of the electroscope (see Figure 3). The zinc plate is given a negative charge (which in turn means the metal and the gold leaf in the box are negatively charged). The negatively charged metal repels the negatively charged gold leaf, causing it to rise up. UV light is then shone onto the zinc plate. The energy of the light causes electrons to be lost from the zinc plate via the photoelectric effect. As the zinc plate, the metal and the gold leaf lose their negative charge, the gold leaf is no longer repelled and so falls back down.

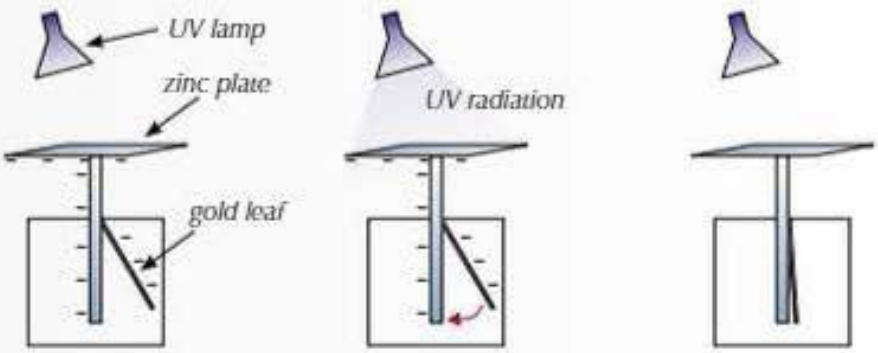


Figure 3: *Demonstration of the photoelectric effect using a UV lamp and a zinc plate attached to the top of an electroscope.*

Figure 2: *Solar cells use the photoelectric effect to convert light energy into electricity.*

Conclusions from the photoelectric effect

From photoelectric experiments, scientists came up with these conclusions:

Conclusion 1 For a given metal, no photoelectrons are emitted if the radiation has a frequency below a certain value — called the **threshold frequency**.

Conclusion 2 The photoelectrons are emitted with a variety of kinetic energies ranging from zero to some maximum value. This value of maximum kinetic energy increases with the frequency of the radiation, but is independent of the intensity of the incident radiation.

Conclusion 3 The number of photoelectrons emitted per second is directly proportional to the intensity of the radiation.

Tip: The key thing about the photoelectric effect is that it shows that light can't just act as a wave. Certain observations of the photoelectric effect can't be explained by classical wave theory.

But they couldn't explain the first two conclusions if EM radiation only acts as a wave...

Threshold frequency

Wave theory says that for a particular frequency of EM wave, the energy carried should be proportional to the intensity of the beam. The energy carried by the EM wave would also be spread evenly over the wavefront.

This means that if an EM wave were shone on to a metal, each free electron on the surface of the metal would gain a bit of energy from each incoming wavefront. Gradually, each electron would gain energy until its energy equalled the work function and it could leave the metal. If the EM wave had a lower frequency (i.e. was carrying less energy) it would take longer for the electrons to gain this energy, but it would happen eventually. However, electrons are never emitted unless the wave is above a threshold frequency — so wave theory can't explain the threshold frequency.

Kinetic energy of photoelectrons

The higher the intensity of the wave, the more energy it should transfer to each electron — the kinetic energy of the electrons should increase with intensity.

Wave theory can't explain the fact that the kinetic energy depends only on the frequency in the photoelectric effect.

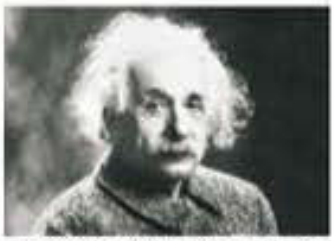


Figure 4: *Albert Einstein, the physicist who explained the photoelectric effect using photons.*

Explaining the photoelectric effect

According to the photon model, when EM radiation hits a metal, the metal's surface is bombarded by photons. If one of these photons collides with a free electron, there is a one-to-one interaction between the photon and the surface electron. The electron gains energy equal to hf (as all of the photon's energy is transferred to the electron, p.232). This idea could be used to explain the conclusions from the photoelectric effect, meaning that the photoelectric effect supported the particulate (particle-like) nature of EM radiation.

Tip: Remember — Einstein saw photons of light as having particle-like interactions. So they either transfer all of their energy, or none of it (page 232).

Threshold frequency

If the energy gained by an electron (on the surface of the metal) from a photon is greater than the work function, the electron is emitted. If the energy is lower, the electron will just shake about a bit, then release the energy as another photon. The metal will heat up, but no electrons will be emitted.

So for electrons to be released $hf \geq \phi$. The value of f is the threshold frequency when $hf = \phi$, as this is the minimum requirement for electrons to be emitted.

So you can find the threshold frequency, f_0, by rearranging $hf_0 = \phi$ to get:

$$f_0 = \frac{\phi}{h}$$

Example — **Maths Skills**

A metal has a work function of 7.2×10^{-19} J. Calculate the minimum frequency of EM radiation needed for a photoelectron to be released.

$$f_0 = \frac{\phi}{h} = \frac{7.2 \times 10^{-19}}{6.63 \times 10^{-34}} = 1.1 \times 10^{15}\,\text{Hz} \text{ (to 2 s.f.)}$$

Exam Tip
Asking you to explain why there's a threshold frequency and how this shows light acts as a particle is an exam favourite — make sure you know it.

Exam Tip
You won't get this equation in the exam, but you are given $hf = \phi + KE_{max}$. At the threshold frequency, $KE_{max} = 0$. This leaves you with $hf = \phi$, which you can rearrange to get the threshold frequency.

Maximum kinetic energy

The energy transferred from EM radiation to an electron is the energy it absorbs from one photon, hf. The kinetic energy it will be carrying when it leaves the metal is hf minus any energy losses. These energy losses are the reason the electrons emitted from a metal have a range of kinetic energies.

The minimum amount of energy an electron can lose is the work function energy, so the maximum kinetic energy, KE_{max}, is given by the equation $KE_{max} = hf - \phi$. Rearranging this equation gives you Einstein's photoelectric equation:

$$hf = \phi + KE_{max}$$

Tip: There are loads of ways electrons leaving the metal can lose energy, e.g. they might have to do work to get to the surface of the metal.

Kinetic energy = ½ × mass × speed², so the maximum kinetic energy a photoelectron can have is:

$$KE_{max} = \frac{1}{2}mv_{max}^2$$

KE_{max} = maximum kinetic energy of a photoelectron

v_{max} = maximum speed of an emitted electron

m = mass of an electron = 9.11×10^{-31} kg

You can use this to write the photoelectric equation as:

$$hf = \phi + \frac{1}{2}mv_{max}^2$$

The intensity of radiation is the power transferred by the radiation over a given area. In the photon model, this is the number of photons that hit a given area every second. Increasing the intensity just means more photons per second on an area — each photon has the same energy as before. The kinetic energy of the electrons is independent of the intensity, as they can only absorb one photon at a time and the energy of each photon has not changed.

Example — **Maths Skills**

The threshold frequency of radiation needed to cause the photoelectric effect in aluminium is 1.03×10^{15} Hz. Radiation with a frequency of 3.45×10^{15} Hz is shone on an aluminium sheet. Calculate the maximum kinetic energy of a photoelectron emitted from the surface of this sheet.

To work out the maximum kinetic energy you need to rearrange and use the photoelectric equation.

First, use the threshold frequency to calculate the work function, ϕ.

$\phi = hf_0 = (6.63 \times 10^{-34}) \times (1.03 \times 10^{15}) = 6.82... \times 10^{-19}$ J

Then substitute this, Planck's constant and the frequency of the radiation being shone on the metal into the photoelectric equation $hf = \phi + KE_{max}$, so

$KE_{max} = hf - \phi$

$= (6.63 \times 10^{-34} \times 3.45 \times 10^{15}) - (6.83 \times 10^{-19})$

$= 1.60 \times 10^{-18}$ J (to 3 s.f.)

Rate of emission of photoelectrons

Providing the incident radiation is above the threshold frequency of a metal, the number of photoelectrons emitted every second (the rate of emission) is directly proportional to the intensity of the radiation. This is because each collision liberates an electron, so if more collisions occur in a given time, more photoelectrons must be released in that time.

Tip: These calculations involve a lot of numbers written in standard form — be careful when you're punching them into your calculator that silly mistakes don't creep into your calculations.

Tip: If you get asked to find the maximum speed of an emitted electron — just rearrange good old $KE_{max} = \frac{1}{2}mv_{max}^2$.

Practice Questions — Application

Q1 Photons each with an energy 6.0×10^{-18} J strike the surface of a sheet of zinc. The work function of zinc is 5.82×10^{-19} J. Will any photoelectrons be emitted from the surface of the zinc sheet? Explain your answer.

Q2 Electrons are emitted from a metal's surface when it is irradiated with radiation with a frequency of 1.20×10^{16} Hz.

a) What effect would increasing the frequency of the incident radiation have on the electrons emitted by the metal? Explain your answer.

b) Calculate the energy of a single photon of incident radiation.

c) The maximum kinetic energy of an electron emitted from the metal's surface is 7.26×10^{-18} J. Calculate the work function of the metal.

Practice Questions — Fact Recall

Q1 Describe what is meant by the 'photoelectric effect'.

Q2 Explain what happens when UV radiation is incident on a negatively charged zinc plate which is attached to a gold-leaf electroscope.

Q3 Describe what will happen to the electrons emitted by a metal if the intensity of the light shining on it is increased.

Q4 Explain why there is a threshold frequency below which no electrons will be emitted by a metal.

Q5 Explain why electrons emitted due to the photoelectric effect have a maximum possible kinetic energy.

4. Wave-Particle Duality

The photoelectric effect shows that light can act as a particle, but there's plenty of evidence that it acts as a wave too. It turns out that not only can waves act like particles, but particles can also act like waves.

Learning Objectives:

- Be able to use the de Broglie equation, $\lambda = \frac{h}{p}$.
- Know about electron diffraction, including experimental evidence of this effect.
- Know that diffraction of electrons occurs for electrons travelling through a thin slice of polycrystalline graphite, caused by the atoms of the graphite and the spacing between the atoms.

Specification Reference 4.5.3

Is light a particle or a wave?

Diffraction

When a beam of light passes through a narrow gap, it spreads out. This is called diffraction (see page 211). Diffraction can only be explained using waves. If the light was acting as a particle, the light particles in the beam would either not get through the gap (if they were too big), or just pass straight through and the beam would be unchanged.

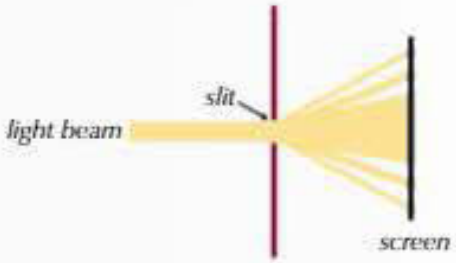


Figure 1: *Diffraction of light waves as they pass through a narrow slit.*

The photoelectric effect

The results of photoelectric effect experiments (see p.238) can only be explained by thinking of light as a series of particle-like photons. If a photon of light is a discrete bundle of energy, then it can interact with an electron in a one-to-one way. All the energy in the photon is given to one electron.

The photoelectric effect and diffraction show that light behaves as both a particle and a wave — this is an example of a phenomenon known as **wave-particle duality**.

Exam Tip
Make sure you know the two examples that show light acts as both a wave and a particle — they might just come up in the exam.

Wave-particle duality theory

Louis de Broglie made a bold suggestion in his PhD thesis. He said if 'wave-like' light showed particle properties (photons), 'particles' like electrons should be expected to show wave-like properties.

The de Broglie equation relates a wave property (wavelength, λ) to a moving particle property (momentum, p).

λ = *de Broglie wavelength in m*

$$\lambda = \frac{h}{p}$$

h = *the Planck constant* $= 6.63 \times 10^{-34}$ *Js.*

p = *momentum in kgms*$^{-1}$

Tip: Remember that momentum is the mass multiplied by the velocity of an object (page 138).

Tip: The derivation of the de Broglie equation is beyond the scope of this course, so you don't need to worry about it.

Figure 2: *Louis de Broglie, the physicist who first suggested the idea of wave-particle duality.*

The de Broglie wave of a particle can be interpreted as a 'probability wave'. Many physicists at the time weren't very impressed — his theory wasn't accepted straight away. Other scientists had to evaluate de Broglie's theory (by a process known as peer review, p.1) before he published it, and then it was tested with experiments, such as electron diffraction (see below). Once enough evidence was found to back it up, the theory was accepted as validated by the scientific community.

Scientists' understanding of the nature of matter has changed over time through this process of hypothesis and validation. De Broglie's theory is accepted to be true — at least until any new conflicting evidence comes along.

Electron diffraction

In 1927, two American physicists, Clinton Davisson and Lester Germer succeeded in diffracting electrons using an electron diffraction tube. In the tube, electrons are accelerated to high velocities in a vacuum and then passed through a thin sheet of polycrystalline graphite. As they pass through the spaces between the carbon atoms of the graphite, they diffract just like waves passing through a narrow slit and produce a pattern of rings. This provides evidence that electrons have wave properties, supporting de Broglie's theory.

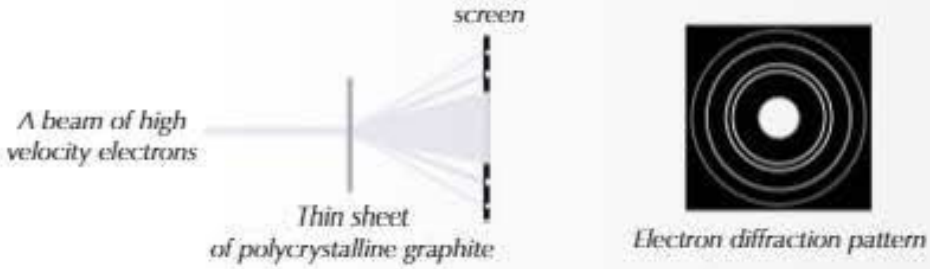


Figure 3: *An electron diffraction tube experiment shows electrons have wave properties.*

You only get diffraction if a particle interacts with an object of about the same size as its de Broglie wavelength. Electrons with a wavelength of around 1×10^{-10} m are likely to be diffracted by the atoms in polycrystalline structures, like the ones used by Davisson and Germer.

Example

A 0.058 kg tennis ball travelling at 100 ms^{-1} has a de Broglie wavelength of 10^{-34} m. That's 10^{19} times smaller than an atom's nucleus. There's nothing that small for it to interact with, and so it only acts as a particle.

According to wave theory, the spread of the lines in the diffraction pattern increases if the wavelength of the wave is greater. In electron diffraction experiments, a smaller accelerating voltage, i.e. slower electrons, gives widely spaced rings. Increase the electron speed and the diffraction pattern circles squash together towards the middle. This fits in with the de Broglie equation — if the velocity is higher, the wavelength is shorter and the spread of lines is smaller.

If particles with a greater mass (e.g. neutrons) were travelling at the same speed as the electrons, they would show a more tightly-packed diffraction pattern. That's because a neutron's mass (and therefore its momentum) is much greater than an electron's, and so a neutron has a shorter de Broglie wavelength.

Tip: There's more about diffraction on page 211.

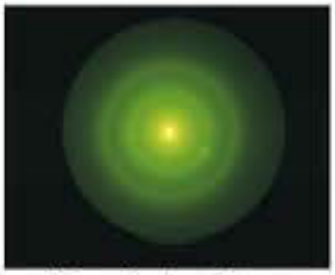


Figure 4: *An electron diffraction pattern from an electron diffraction tube.*

Tip: Electrons can be used to investigate the spacing between atoms in a crystal — an electron beam will diffract when the de Broglie wavelength of the electrons is roughly the same size as the spaces between the atoms.

Example — **Maths Skills**

An electron of mass 9.11×10^{-31} kg is fired from an electron gun at 7.0×10^{6} ms^{-1}. What size object will the electron need to interact with in order to diffract?

An electron will diffract when the size of the object is roughly the same size as its de Broglie wavelength, so you need to find λ.

Momentum of electron, $\boldsymbol{p} = m\boldsymbol{v}$

$= 9.11 \times 10^{-31} \times 7.0 \times 10^{6}$

$= 6.377 \times 10^{-24}$ kg ms^{-1}

Substitute this into de Broglie's equation:

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{6.377 \times 10^{-24}}$$

$= 1.03... \times 10^{-10}$ m $= 1.0 \times 10^{-10}$ m (to 2 s.f.)

So, only crystals with atom layer spacing around this size are likely to cause the diffraction of this electron.

Tip: Momentum is a vector quantity, but we're only interested in its magnitude here — which is why p isn't bolded in the de Broglie equation.

Example — **Maths Skills**

Electrons with a wavelength of 0.170 nm are diffracted as they pass between atoms in a crystal lattice. Calculate the velocity of the electrons.

To convert from nm to m, multiply by 10^{-9}:

$\lambda = 0.170 \times 10^{-9} = 1.70 \times 10^{-10}$ m

Substitute $\lambda = 1.70 \times 10^{-10}$ m, $h = 6.63 \times 10^{-34}$ Js, and $m = 9.11 \times 10^{-31}$ kg into the de Broglie equation.

$$\lambda = \frac{h}{p} = \frac{h}{mv}, \text{ so } v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 1.70 \times 10^{-10}}$$

$= 4\,281\,009.8...$ ms^{-1}

$= 4.28 \times 10^{6}$ ms^{-1} (to 3 s.f.)

Tip: Remember — nm is nanometres and should be converted to m before using the de Broglie equation.

Tip: You could write the answer to this calculation in standard form as 4.28×10^{6} ms^{-1}.

Example — **Maths Skills**

An electron and a neutron are travelling with velocities such that they have equal wavelengths. The mass of an electron is 9.11×10^{-31} kg and the mass of a neutron is 1.675×10^{-27} kg. If the electron has velocity 5.00×10^{6} ms^{-1}, what is the velocity of the neutron?

Substitute $v = 5.00 \times 10^{6}$ ms^{-1} and $m = 9.11 \times 10^{-31}$ kg into the de Broglie equation to find the wavelength of the electron:

$$\lambda = \frac{h}{p} = \frac{h}{mv}, \text{ so } \lambda = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 5.00 \times 10^{6}} = 1.455... \times 10^{-10} \text{ m}$$

Use this wavelength in the de Broglie equation to find the velocity of the neutron:

$$\lambda = \frac{h}{p} = \frac{h}{mv}, \text{ so } v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34}}{1.675 \times 10^{-27} \times 1.455... \times 10^{-10}}$$

$= 2719.4...$ ms^{-1} $= 2.72 \times 10^{3}$ ms^{-1} (to 3 s.f.)

Exam Tip
The mass of an electron and the mass of a neutron will be given in the data and formulae booklet in the exam.

Figure 5: *A laboratory technician using an electron microscope.*

Tip: Remember kinetic energy $E_k = \frac{1}{2}mv^2$.

Electron microscopes

A shorter wavelength gives smaller diffraction effects. This fact is used in the electron microscope. Diffraction effects blur detail on an image. If you want to resolve tiny detail in an image, you need a shorter wavelength.

Light blurs out detail more than 'electron-waves' do, so an electron microscope can resolve finer detail than a light microscope. They can let you look at things as tiny as a single strand of DNA.

Practice Questions — Application

Q1 Explain how a particle's velocity affects its de Broglie wavelength.

Q2 An electron has a de Broglie wavelength of 0.162 nm. The mass of an electron is 9.11×10^{-31} kg.

a) Calculate the momentum of the electron.

b) Calculate the kinetic energy of the electron.

Q3 An alpha particle has a mass of 6.646×10^{-27} kg.

a) Calculate the de Broglie wavelength of an alpha particle travelling at a velocity of 60.0 ms^{-1}.

b) Calculate the speed of an electron that has the same de Broglie wavelength as the alpha particle in part a). (The mass of an electron is 9.11×10^{-31} kg.)

Practice Questions — Fact Recall

Q1 Name one effect demonstrating that electromagnetic waves have wave properties and one demonstrating that they have particle properties.

Q2 Describe what is meant by wave-particle duality.

Q3 What phenomenon shows that electrons have wave-like properties?

Section Summary

Make sure you know...

- That electromagnetic radiation exists in discrete packets, called quanta.
- That Einstein called these packets photons.
- That photons behave like particles — they have one-to-one interactions with other particles.
- That the energy of a photon can be found using $E = hf$ or $E = \frac{hc}{\lambda}$.
- That the electronvolt is a unit of energy.
- What the term threshold voltage means and the equation relating it to the Planck constant, $h = \frac{eV_0\lambda}{c}$.
- How to use LEDs to determine the value of the Planck constant.
- That the work function is the minimum energy a surface electron needs to absorb to be released from a metal.
- What the photoelectric effect is and how it can be demonstrated, e.g. using a gold-leaf electroscope and a zinc plate.
- That for a given metal, if the incident radiation is below the threshold frequency, photoelectrons won't be emitted.
- That the kinetic energy of photoelectrons ranges from zero to a maximum value, and that this maximum value increases with the frequency of the radiation.
- That the number of photoelectrons emitted each second is directly proportional to the intensity of the incident radiation.
- That wave theory cannot explain the idea of a threshold frequency, or why the kinetic energy of photoelectrons doesn't depend on the intensity of the incident radiation.
- That ideas about photons can be used to explain the photoelectric effect.
- That photons have a one-to-one interaction with surface electrons, so all of a photon's energy is transferred to a single electron in the photoelectric effect.
- That one-to-one interactions explain why there is a threshold frequency in the photoelectric effect and why the kinetic energy of the electrons is independent of intensity.
- How to calculate the threshold frequency of a metal, using $f_0 = \frac{\phi}{h}$.
- How to use Einstein's photoelectric equation, $hf = \phi + KE_{max}$.
- That diffraction shows that light behaves like a wave, but the photoelectric effect shows that light behaves like a particle.
- The idea of wave-particle duality — that waves display particle-like properties and particles display wave-like properties.
- How to use the de Broglie equation, $\lambda = \frac{h}{p}$, which relates wavelength (a wave property) to momentum (a particle property).
- That electrons are diffracted when they travel through a thin sheet of polycrystalline graphite.
- That electron diffraction supports the theory of wave-particle duality.
- That you only get diffraction if a particle interacts with an object of about the same size as its de Broglie wavelength.

Exam-style Questions

1 Radiation is made of photons that each have an energy of 2.0 GeV. What is the frequency of the radiation?

A 3.0×10^{42} Hz

B 4.8×10^{14} Hz

C 4.8×10^{23} Hz

D 2.1×10^{-24} Hz

(1 mark)

2 The work function of a metal is 4.33 eV. What is the maximum wavelength of radiation that could liberate electrons from the metal?

A 7.35×10^{-45} m

B 2.87×10^{-7} m

C 3.47×10^{6} m

D 4.59×10^{-26} m

(1 mark)

3 A monochromatic LED emits light with a frequency of 5.8×10^{14} Hz. What is the threshold voltage for this LED?

A 0.4 V

B 1.2 V

C 2.2 V

D 2.4 V

(1 mark)

4 A beam of neutrons is fired at a grating. The neutrons travel at 3200 ms^{-1}. The spacing of the grating is 2 μm. Which of the following statements is/are correct?

1 The neutrons diffract when they go through the grating.

2 If the speed of the neutrons was doubled, they would diffract as they went through the grating.

3 If the speed of the neutrons was halved, they would diffract as they went through the grating.

A 1, 2 and 3

B Only 1 and 2

C Only 3

D None of the statements are correct.

(1 mark)

5 An LED emits light at a wavelength of 5.00×10^{-7} m.

(a) Calculate the energy carried by each photon emitted from the LED in electronvolts.

(2 marks)

(b) The LED is connected in a circuit. The potential difference across the LED is 2.0 V. Determine whether or not the LED will light up.

(2 marks)

6 Louis de Broglie was the first scientist to propose wave-particle duality. His ideas were later supported by electron diffraction experiments.

(a) Describe what is meant by the term 'wave-particle duality'.

(1 mark)

(b) An electron gun contains two electrodes. The potential difference across the electrodes causes electrons to be accelerated between them, so that each electron is fired out of the electron gun with a kinetic energy of 1.02×10^{-26} J.

(i) Calculate the velocity of one electron.

(2 marks)

(ii) Calculate the de Broglie wavelength of each electron.

(2 marks)

(iii) By what factor would the de Broglie wavelength of the electrons increase or decrease if the potential difference across the electrodes was halved?

(1 mark)

7 A metal surface emits electrons when a certain frequency of light is shone onto it. This effect is called the photoelectric effect.

(a) State what is meant by the work function of a metal.

(1 mark)

(b) Explain why there is a threshold frequency for the light being shone onto the metal, below which no electrons are emitted.

(2 marks)

(c) Describe an experiment using a gold-leaf electroscope that illustrates the photoelectric effect.

(5 marks)

(d) A beam of light is shone onto a sheet of lead. Each photon has an energy of 3.0×10^{-18} J. The threshold frequency of lead is 1.03×10^{15} Hz. Calculate the maximum kinetic energy of an electron emitted from this surface.

(3 marks)

(e) The intensity of the radiation is increased.

(i) Explain how this affects the maximum speed of the photoelectrons emitted.

(2 mark)

(ii) Explain how this affects the rate at which photoelectrons are emitted.

(1 mark)

Learning Objectives:

- Know the simple kinetic model that describes solids, liquids and gases in terms of the spacing, ordering and motion of atoms or molecules.
- Be able to explain Brownian motion in terms of the kinetic model and describe a simple demonstration using smoke particles suspended in air.
- Know the meaning of 'internal energy'.
- Know that the internal energy of a body increases as its temperature rises.
- Know that during a change of phase, the internal energy of a substance changes but the temperature is constant.
- Know that the absolute scale of temperature does not depend on the property of any particular substance.
- Know that absolute zero (0 K) is the lowest limit for temperature and is the temperature at which a substance has minimum internal energy.
- Know that temperature measurements can be recorded in both degrees Celsius (°C) and kelvin (K), where $T(\text{K}) \approx \theta\,(^\circ\text{C}) + 273$.
- Know the meaning of 'thermal equilibrium'.

Specification References 5.1.1 and 5.1.2

1. Phases of Matter and Temperature

You should remember the three phases of matter from GCSE. Solids, liquids and gases are described by the kinetic model of matter.

Three phases of matter

Solids, liquids and gases are three different phases that matter can exist in. Particles behave differently in each phase (see Figure 1):

- Particles in solids vibrate about fixed positions in a regular lattice. They're close together, and are held in position by strong forces of attraction.
- Particles in liquids are constantly moving around and are free to move past one another. The attraction between particles in liquids is weaker than for particles in solids. They're fairly close together but have an irregular arrangement.
- Particles in gases are far apart and free to move around with constant random motion, and so are not in any particular order. There are no forces of attraction between particles in an **ideal gas** (p.264).

***Figure 1**: The solid, liquid and gas phases of matter.*

The idea that solids, liquids and gases are made up of tiny moving or vibrating particles is called the **kinetic model of matter** (or **kinetic theory**). It seems obvious now, but this wasn't always accepted by the scientific community. It took several scientists and hundreds of years to develop a controversial idea into an accepted theory.

Brownian motion

In 1827, botanist Robert Brown noticed that tiny particles of pollen suspended in water moved with a zigzag, random motion. This type of movement of any particles suspended in a fluid is known as **Brownian motion**.

Brown couldn't explain this, but nearly 80 years later Einstein showed that this provided evidence for the existence of atoms or molecules in the water (the kinetic model of matter). The randomly moving water particles were hitting the pollen particles unevenly, causing this motion.

You can observe Brownian motion in the lab. Start by putting some smoke in a glass jar. Use a glass rod to focus light from a lamp into the glass jar to illuminate it, and observe the particles using a microscope — see Figure 3. The smoke particles appear as bright specks moving haphazardly from side to side, and up and down (see Figure 4).

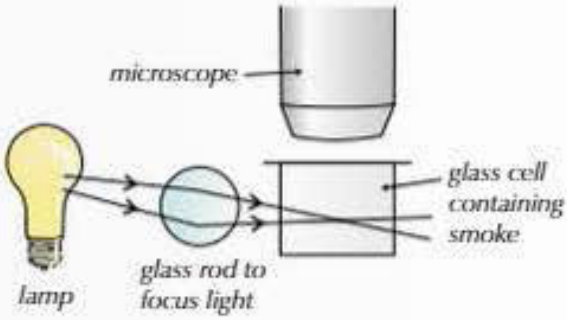


Figure 3: *Apparatus used to observe the Brownian motion of smoke particles in air.*

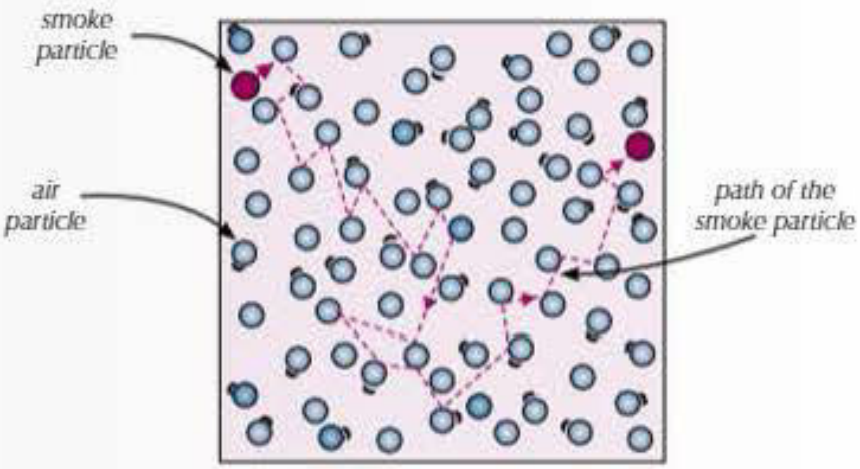


Figure 4: *Brownian motion of a smoke particle in air.*

Figure 2: *British botanist Robert Brown, after whom Brownian motion is named.*

Tip: Brownian motion can happen in any fluid — i.e. any liquid or gas.

Internal energy

Particles in a gas have a range of kinetic and potential energies. The kinetic energy of a particle depends on its mass and speed. Potential energy is caused by interactions between particles and is based on their positions relative to each other. These energies are randomly distributed amongst the particles.

The amount of energy contained in a system is called its **internal energy** — it's found by summing the kinetic and potential energies of all the particles within it.

Internal energy is the sum of the random distribution of kinetic and potential energies associated with the molecules of a system.

Heating a substance means supplying that substance with thermal energy. This energy can be transferred to the kinetic and potential energies associated with the molecules, which increases the temperature of the substance. So as a body's temperature increases, its internal energy increases.

The particles in an ideal gas don't have potential energy, so the internal energy of an ideal gas depends only on the average kinetic energy of its particles — see p.268-269.

Tip: The hotter a gas is, the faster its particles move. Remember — the kinetic energy of an object is equal to $\frac{1}{2}mv^2$, where m is the object's mass and v is the object's speed. So the average kinetic energy of the particles in a gas increases with temperature.

Change of phase

If you heat or cool a substance, a change of phase may take place. A change of phase, sometimes also referred to as a change of state, occurs when a substance changes between a solid, liquid or gas. When a substance changes phase its internal energy changes, but the total kinetic energy of the particles and its temperature stay the same (see Figure 6). This is because the change of phase is altering the bonds between, and therefore potential energies of, the particles, while the kinetic energy of the particles stays constant.

For example, in a pan of boiling water, the sum of the potential energies of the water molecules increases as they break free of the liquid. But the water in both phases is at 100 °C.

Figure 5: *When boiling water, the energy is used to convert the water into steam rather than to increase the temperature of the water.*

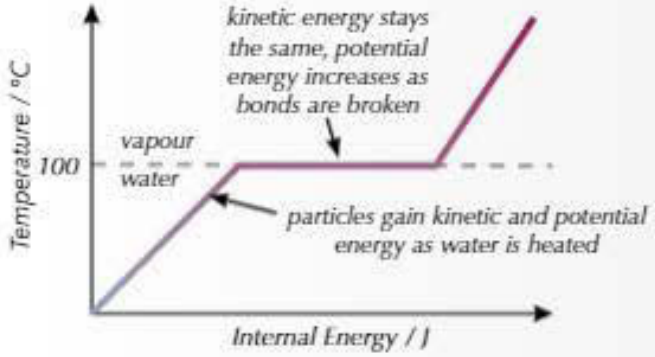

kinetic energy stays the same, potential energy increases as bonds are broken
Temperature / °C
100
vapour
water
particles gain kinetic and potential energy as water is heated
Internal Energy / J

Figure 6: *Graph of temperature against internal energy for a sample of water being heated at a constant rate to its boiling point and beyond.*

The absolute scale of temperature

The Celsius scale (with the freezing and boiling points of water at 0 °C and 100 °C) can easily be used for day-to-day activities. However, scientists use the Kelvin scale (the **absolute scale of temperature**) for all equations in thermal physics. It's also known as the thermodynamic scale, and it does not depend on the properties of any particular substance (like the freezing and boiling points of water). The unit of this scale is the kelvin (K).

Zero kelvins is the lowest possible temperature and is called **absolute zero**. At 0 K (or –273 °C, to 3 s.f.) all particles have the minimum possible internal energy. At higher temperatures, particles have more energy. In fact, with the Kelvin scale, a particle's energy is proportional to its temperature (see page 268).

Tip: Absolute zero is rounded to 273 K (3 s.f.) here, so we use an approximately equal sign (≈) rather than an equal sign (=).

The Kelvin scale is named after Lord Kelvin who first suggested it. A change of 1 K equals a change of 1 °C. To change from degrees Celsius into kelvins you add 273 (or subtract 273 to go the other way).

Tip: You don't use the degrees sign (°) when writing temperatures in kelvins.

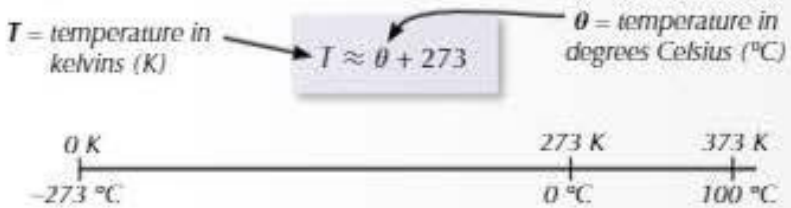


Figure 7: *Equivalent temperatures in the Kelvin and degrees Celsius scales.*

Example — **Maths Skills**

An object is heated to 75 °C. What is this temperature in kelvin?

$T \approx \theta + 273 = 75 + 273 = 348$ K

Thermal equilibrium

Suppose A, B and C in Figure 8 are three identical metal blocks. A has been in a warm oven, B has come from a refrigerator and C is at room temperature.

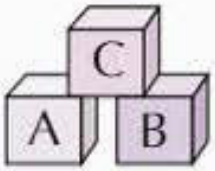


Figure 8: *Three identical metal blocks.*

There is a net flow of thermal energy from A to C and C to B until they all reach **thermal equilibrium** and the net flow of energy stops. This happens when the three blocks are at the same temperature.

If body A and body B are both in thermal equilibrium with body C, then body A and body B must be in thermal equilibrium with each other.

Thermal energy is always transferred from regions of higher temperature to regions of lower temperature.

Tip: Be careful — at thermal equilibrium, energy is still flowing between the bodies, but it's the net flow that stops.

Practice Questions — Application

Q1 A student needs to heat a flask of liquid to 345 K. Calculate the temperature the student needs to heat the flask to in degrees Celsius.

Q2 a) A person making a pie encases some hot filling in cool pastry and leaves it to rest. State the direction of the net flow of thermal energy between the pastry and the filling.

b) The whole pie eventually comes to thermal equilibrium with the room. State how you know that the filling and the pastry are now the same temperature.

Practice Questions — Fact Recall

Q1 Describe the order, spacing and movement of the particles in:
a) a solid b) a liquid c) an ideal gas

Q2 When pollen particles are suspended in water, they appear to move with a zigzag, random motion. State the name of this type of motion and explain why this occurs.

Q3 Explain why the average kinetic energy of particles in a substance remains constant when the substance changes phase whilst being heated.

Q4 Sketch a graph of temperature against internal energy to show how the internal energy of water changes as it is heated at a constant rate to its boiling point and beyond. Label the energy changes of the particles on the graph.

Q5 a) What is the lowest possible temperature an object can theoretically reach called?

b) What is the value of this temperature in kelvins and degrees Celsius?

Q6 What is meant by the internal energy of a body?

Q7 When are two objects said to be in thermal equilibrium?

Tip: It's easier to draw the graph in Q4 using degrees Celsius, but you could also use kelvins.

Learning Objectives:

- Know what is meant by the specific heat capacity of a substance and be able to use $E = mc\Delta\theta$.
- Be able to describe an electrical experiment to determine the specific heat capacity of a metal block or liquid.
- Know what specific latent heat of fusion and specific latent heat of vaporisation are, and be able to use the formula $E = mL$.
- Be able to describe an electrical experiment to determine the specific latent heat of fusion and vaporisation.

Specification Reference 5.1.3

2. Thermal Properties of Materials

Specific heat capacity and specific latent heat are quantities used to calculate the energy required to heat up or change the state of different substances.

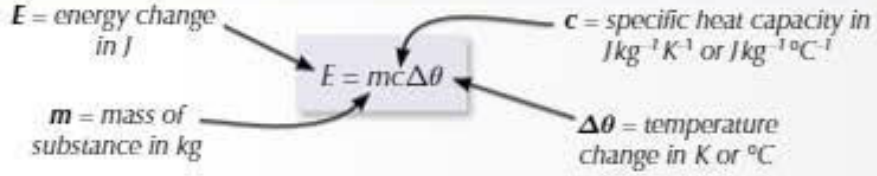
Specific heat capacity

When you heat something, the amount of energy needed to raise its temperature depends on its specific heat capacity.

The **specific heat capacity** (c) of a substance is the amount of energy needed to raise the temperature of 1 kg of the substance by 1 K (or 1°C).

Or put another way:

energy change = mass × specific heat capacity × change in temperature


E = energy change in J
c = specific heat capacity in Jkg⁻¹K⁻¹ or Jkg⁻¹°C⁻¹
E = mcΔθ
m = mass of substance in kg
Δθ = temperature change in K or °C

Example **Maths Skills**

The specific heat capacity of water is 4180 Jkg^{-1}K^{-1}. If 172 kJ of energy is supplied to 5.00 kg of water at 300 K (correct to 3 significant figures), what will its final temperature be?

First find the value of $\Delta\theta$:

$$E = mc\Delta\theta \Rightarrow \Delta\theta = \frac{E}{mc} = \frac{172 \times 10^3}{5.00 \times 4180}$$
$$= 8.229... \text{ K}$$

Then add this to the initial temperature:

$$\text{final temperature} = 300 + 8.229... = 308.229...$$
$$= 308 \text{ K (to 2 s.f.)}$$

Tip: Q is sometimes used instead of E for the energy change. You might also see $\Delta\theta$ written as ΔT.

Tip: Energy is supplied to the water, so the temperature goes up.

Measuring specific heat capacity

You can use this experiment to measure specific heat capacity yourself. The method is the same for solids and liquids, but the set-up is a little bit different:

Tip: As always, do a full risk assessment before beginning this experiment, and take extra care if you are placing an electrical heater in water.

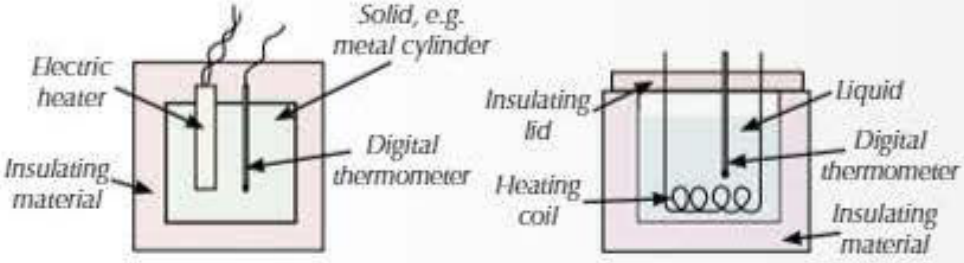


Figure 1: *Set-ups used to determine the specific heat capacity of a solid or liquid.*

Measure the mass of the substance and its initial temperature. Heat the substance with the heater — you need a temperature rise of about 10 K. Measure the new temperature of the substance and subtract the initial temperature from it to get the change in temperature. Use an ammeter and voltmeter attached to your electric heater to measure the current through, I, and voltage across, V, the heater, and time how long the heater is on for, t, using a stopwatch. You can then calculate the energy (E) supplied by the heater using $E = W = VIt$. Plug your data into $E = mc\Delta\theta$ to calculate c.

Tip: The value you end up with for c will probably be too high by quite a long way. That's because some of the energy from the heater gets transferred to the air and the container. You can reduce the error by starting below and finishing above room temperature to cancel out gains and losses.

Example — **Maths Skills**

0.250 kg of water is heated from 12.1 °C to 22.9 °C with an electric immersion heater. The heater has a voltage of 11.2 V and a current of 5.30 A, and is on for 205 s. Calculate the specific heat capacity of water.

$E = VIt = 11.2 \times 5.30 \times 205 = 12\,168.8$ J

$\Delta\theta = 22.9 - 12.1 = 10.8$ °C

$E = mc\Delta\theta$ so $c = \frac{E}{m\Delta\theta} = \frac{12168.8}{0.250 \times 10.8} = 4506.9...$

$= 4510$ J kg^{-1} °C^{-1} (to 3 s.f.)

(or 4510 J kg^{-1} K^{-1})

Tip: You met the equation $W = VIt$ in Module 4.

Tip: You don't need to convert between °C and K here, since the change in temperature will be the same for both.

Estimating specific heat capacity

If it's difficult to use the method above for a particular object, you could try using the method of mixtures to estimate the specific heat capacity. It works on the principle that when a hot substance is mixed, or combined, with a cold substance, they will eventually reach thermal equilibrium and end up at the same temperature. It relies on the assumption that no heat is lost from the system to the surroundings. Of course, some heat will be lost, so the specific heat capacity obtained is only an estimate.

Tip: Go to page 251 if you need a refresher on thermal equilibrium.

In order to use this method to find the specific heat capacity of a substance, you must know the specific heat capacity of the other substance it's being mixed with. For example, to find the specific heat capacity of a metal block you can use water, which has a specific heat capacity of 4180 J kg^{-1} K^{-1}:

- Heat the metal block of known mass, m_b, up to a temperature T_b.
- Quickly transfer this block into an insulated container containing a mass of water, m_w, at a temperature T_w.
- The hot block will heat the water. Measure the temperature of the water once it has reached a steady value, T_s.

The heat (energy) gained by the water is equal to the heat lost by the block, so:

$$m_w c_w \Delta\theta_w = m_b c_b \Delta\theta_b$$

which becomes:

$$m_w c_w (T_s - T_w) = m_b c_b (T_b - T_s)$$

Rearrange for c_b, the specific heat capacity of the block:

$$c_b = \frac{m_w c_w (T_s - T_w)}{m_b (T_b - T_s)}$$

So you can plug the specific heat capacity of water into the formula with your measurements to find c_b.

Figure 2: *Water has a very high specific heat capacity, so its temperature goes down very slowly as it gives out energy. That's why a mug of tea stays warm long enough for you to drink it.*

Tip: You could also use this experiment to estimate the specific heat capacity of a liquid if you know the specific heat capacity of a metal block.

Example — **Maths Skills**

A 0.982 kg steel bar is heated to 75.0 °C then placed in water, which is initially at a temperature of 20.5 °C. The water has a mass of 3.12 kg and a specific heat capacity of 4180 J kg^{-1} K^{-1}. The steel and water come to thermal equilibrium at a temperature of 22.4 °C. Estimate the specific heat capacity of steel.

$$c_b = \frac{m_w c_w (T_f - T_w)}{m_b (T_b - T_f)} = \frac{3.12 \times 4180 \times (22.4 - 20.5)}{0.982 \times (75.0 - 22.4)} = 479.719...$$
$$= 480 \text{ J kg}^{-1} \text{ K}^{-1} \text{ (to 3 s.f.)}$$

Specific latent heat

To melt a solid, you need to break the bonds that hold the particles in place. The energy needed for this is called the latent heat of fusion. Similarly, when you boil or evaporate a liquid, energy is needed to pull the particles apart completely. This is the latent heat of vaporisation.

The larger the mass of the substance, the more energy it takes to change its state. That's why the specific latent heat is defined per kg:

The **specific latent heat** (L) of fusion or vaporisation is the quantity of thermal energy required to change the state of 1 kg of a substance.

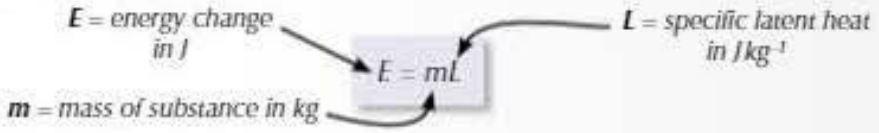
Tip: The specific latent heat of fusion is used when a substance is melting or freezing. The specific latent heat of vaporisation is used when a substance is boiling or condensing.

Which gives:

energy change = mass × specific latent heat

Tip: Energy has to be added for a material to melt or boil. When a material freezes or condenses, energy is lost from the material.

Or in symbols:



Tip: You'll usually see the latent heat of vaporisation written L_v and the latent heat of fusion written L_f.

Example — **Maths Skills**

Find the energy needed to turn 1.00 kg of water at 90.0 °C to 1.00 kg of steam at 110.0 °C. c_{water} = 4180 J kg^{-1} K^{-1}, c_{steam} = 1890 J kg^{-1} K^{-1}, and the latent heat of vaporisation of water is 2.26 × 10^6 J kg^{-1}.

First find the energy needed to heat the water by 10.0 °C:

$$E = mc\Delta\theta = 1.00 \times 4180 \times 10.0$$
$$= 41\,800 \text{ J (or 41.8 kJ)}$$

Then find the energy needed to turn the water to steam:

$$E = mL = 1.00 \times (2.26 \times 10^6)$$
$$= 2.26 \times 10^6 \text{ J (or 2260 kJ)}$$

Tip: Water turns to steam at 100 °C.

Then find the energy needed to heat the steam by 10.0 °C:

$$E = mc\Delta\theta = 1.00 \times 1890 \times 10.0$$
$$= 18\,900 \text{ J (or 18.9 kJ)}$$

Then just add all these numbers together:

Energy needed = 41.8 + 2260 + 18.9 = 2320.7 = 2320 kJ (to 3 s.f.)

Measuring specific latent heat of fusion

Figure 4 shows a set-up you can use to measure the specific latent heat of fusion of a substance such as ice. First connect a heating coil up to an ammeter and a voltmeter, and put equal masses of ice in two identical funnels above beakers. Put the heating coil in one of the funnels and turn it on for three minutes, using a stopwatch to time it. Measure the current through, I, and voltage across, V, the coil during this time. Calculate the energy transferred by the heater in the three minutes using the equation $E = W = VIt$.

At the end of the three minutes, measure the mass of water collected in the beakers. The unheated funnel of ice is there so you can measure how much ice melts due to the ambient temperature of the room. Subtract the mass of water collected from the unheated funnel from the mass of water collected from the heated funnel. This gives the mass of ice, m, that melted solely due to the presence of the heater. $E = mL$, so to find the specific latent heat of fusion for water just divide the energy supplied by the heater by the mass of ice that the heater melted: $L = E \div m$.

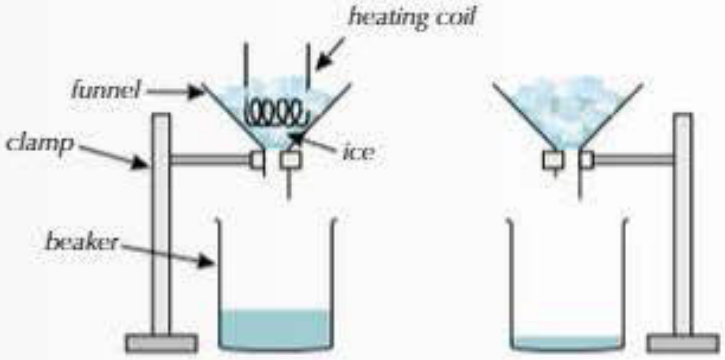


Figure 4: *Experimental set-up for determining the specific latent heat of fusion of water.*

Tip: Make sure that the ammeter is in series with the coil and that the voltmeter is in parallel with the coil.

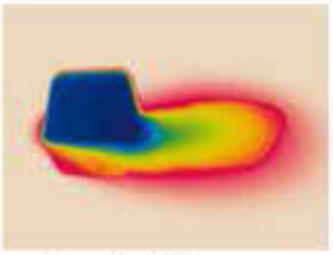


Figure 3: *A thermogram of a melting ice cube shows cool areas in blue and warm areas in red and yellow. Once the specific latent heat of fusion has been transferred to the ice, it melts into water and begins to rise to the same temperature as its surroundings.*

Measuring specific latent heat of vaporisation

Another experiment can be carried out to determine the specific latent heat of vaporisation of a liquid such as water. Figure 5 shows the set-up for the experiment.

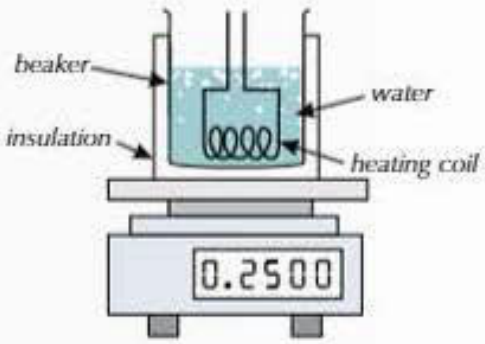


Figure 5: *Experimental set-up for determining the specific latent heat of vaporisation of water.*

Tip: This experiment doesn't require a control apparatus (like the extra funnel of ice in the experiment above) since water won't boil at room temperature.

Place insulation around the outside of a small beaker, but leave the top of the beaker open to the air. Fill the beaker part-way up with water. Connect a voltmeter and an ammeter to a heating coil and insert the coil into the water. Then place the beaker and its contents on a balance.

Tip: Ensure that the balance remains on for the duration of the experiment.

Tip: Your value for L_v will probably be too high, since some of the heat energy supplied will be lost to the surroundings.

Tip: Head back to page 250 for a reminder about the Kelvin temperature scale.

Without removing the beaker from the balance, switch the heating coil on. Once the water is boiling, record the mass on the mass balance and start the timer. Measure the voltage across and current through the heating coil. When the mass has decreased by approximately 15 g, stop the timer and turn the heating coil off.

Record the new mass of the beaker and its contents. Subtract this from the original mass to get the decrease in mass of the water. Calculate the energy transferred by the heater to the water by multiplying the recorded voltage and current together with the time taken (since $E = W = VIt$). Rearrange $E = mL$ to $L = E \div m$, then plug in the energy transferred and the change in the mass of water to get the latent heat of vaporisation of water.

Practice Questions — Application

Q1 How much energy is required to raise the temperature of 0.45 kg of a liquid with specific heat capacity 244 J kg^{-1} K^{-1} by 3.0 kelvins?

Q2 A 270 g metal block with a temperature of 82 °C and a specific heat capacity of 890 J kg^{-1} K^{-1} is placed into a beaker of liquid of mass 910 g and temperature 34 °C. If the specific heat capacity of the liquid is 3600 J kg^{-1} K^{-1}, calculate the temperature at which the solid and liquid reach thermal equilibrium.

Q3 A bowl containing 100.0 g of water at 25.0 °C is placed in a freezer. A few hours later, all of the water has become ice at –5.00 °C. Find how much energy the water lost to its surroundings. $c_{water} = 4180$ J kg^{-1} K^{-1}, $c_{ice} = 2110$ J kg^{-1} K^{-1}, $L_f = 334\,000$ J kg^{-1}.

Practice Questions — Fact Recall

Q1 What is the specific heat capacity of a substance?

Q2 Describe an experiment that can be used to measure the specific heat capacity of a solid material.

Q3 What is the specific latent heat of fusion?

Q4 Describe an experiment that can be used to find the specific latent heat of fusion for water.

3. The Gas Laws

An 'ideal gas' follows the three gas laws, which describe how a fixed mass of gas behaves when you change its temperature, pressure or volume. Be careful though — they work on the Kelvin temperature scale, not traditional °C.

Learning Objectives:

- Know techniques and procedures used to investigate PV = constant (Boyle's Law) and $\frac{P}{T}$ = constant (PAG8).
- Know how to make an estimation of absolute zero using the variation of gas temperature with pressure (PAG8).

Specification Reference 5.1.4

Boyle's law

A (theoretical) ideal gas obeys all three gas laws. Each of the three gas laws was worked out independently by careful experiment with fixed masses of gases. The first of them is **Boyle's law**, which says that:

> At a constant temperature the pressure p and volume V of an ideal gas are inversely proportional.

Inversely proportional means that as one variable increases, the other decreases by the same proportion, i.e. $p \propto \frac{1}{V}$. For example, if you reduce the volume of a gas, its particles will be closer together and will collide with each other and the container more often, so the pressure increases.

Boyle's law means that at any given temperature the product of p and V will always be the same:

$$pV = \text{constant}$$

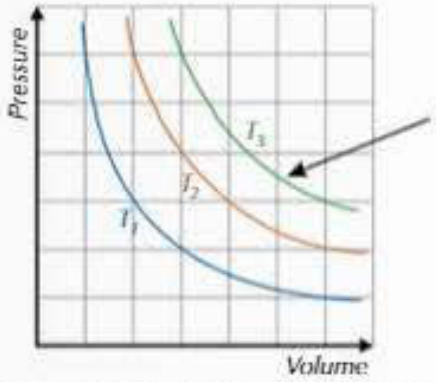


Figure 1: *Graphs of pressure against volume for an ideal gas at different temperatures where $T_1 < T_2 < T_3$.*

Tip: Most gases can usually be assumed to act as ideal gases — see the assumptions on page 264.

Tip: If the volume halves, the pressure doubles, and so on.

Figure 2: *Irish chemist and physicist Robert Boyle.*

Example — **Maths Skills**

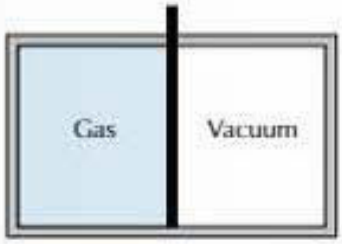
The diagram shows a sealed container with a divider at its centre. An ideal gas fills the left half of the container, and the right half is a vacuum. The gas is contained at a pressure of 350 kPa. The divider is then removed so that the gas fills the whole container. Calculate the new pressure of the gas, assuming that the temperature of the gas remains constant.

pV = constant so $p_1V_1 = p_2V_2$.

Rearranging: $p_2 = p_1\left(\frac{V_1}{V_2}\right)$

The volume containing the gas is now twice as big, so $V_2 = 2V_1 \Rightarrow \frac{V_1}{V_2} = \frac{1}{2}$.

So $p_2 = 350 \times 10^3 \times \left(\frac{1}{2}\right) = 175 \times 10^3 = 175\,\text{kPa}$



Experiment to investigate Boyle's law

PRACTICAL ACTIVITY GROUP 8

You can investigate the effect of pressure on volume by setting up the experiment shown in Figure 3.

- The oil traps a pocket of air in a sealed tube with fixed dimensions.
- Use a tyre pump to increase the pressure on the oil.
- Use the Bourdon gauge to record the pressure. As the pressure increases, more oil will be pushed into the tube, the oil level will rise, and the air will compress. The volume occupied by air in the tube will reduce.
- Measure the volume of air when the system is at atmospheric pressure by multiplying the length of the part of the tube containing air by $\pi \times$ radius of tube squared.
- Gradually increase the pressure by a set interval. Make sure you wait a few moments each time you change the pressure — this will give the temperature time to stabilise before you take any readings.
- Note down both the pressure and the volume of air as it changes. Multiplying these together at any point should give the same value.
- Repeat the experiment twice more and take a mean for each reading.

If you plot a graph of p against $\frac{1}{V}$ you should get a straight line.

Tip: Do a full risk assessment before starting any experiments involving gases (including the experiment on the next page). This experiment involves pressurising a glass container — so wear goggles and don't use too high a pressure.

Tip: If your gauge has units of bar, you can multiply by 10^5 to get your readings in Pa.

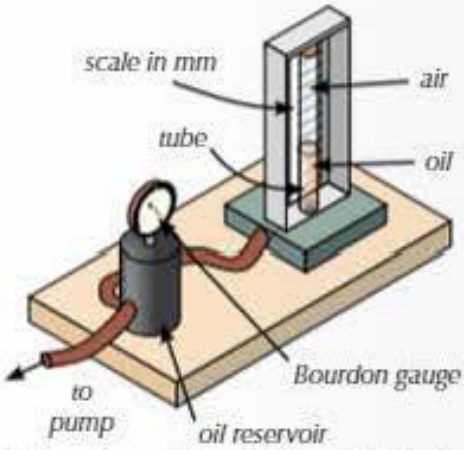


Figure 3: *Experimental set-up for investigating Boyle's Law.*

Figure 4: *The low pressure in a plane cabin causes a crisp packet sealed at atmospheric pressure to expand to a larger volume.*

The pressure law

The **pressure law** states that:

> At constant volume, the pressure p of an ideal gas is directly proportional to its absolute temperature T.

For example, if you heat a gas, the particles gain kinetic energy. This means they move faster. If the volume doesn't change, the particles will collide with each other and their container more often and at higher speeds, increasing the pressure inside the container.

At absolute zero the pressure is also zero. If the pressure law is obeyed, the pressure divided by the temperature for a fixed volume is a constant:

$$\frac{p}{T} = \text{constant}$$

Tip: If you use this relation, make sure the temperature is in K.

So a graph of pressure against temperature will be a straight line (its gradient will be constant). This is shown in Figure 5.

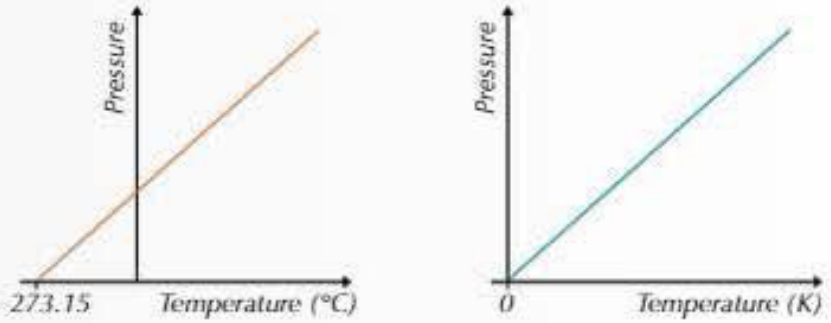


Figure 5: *Graphs of pressure against temperature for an ideal gas at constant volume on the degrees Celsius scale and on the Kelvin scale.*

Tip: The lines always reach the temperature axis at absolute zero.

Example — **Maths Skills**

A gas is contained in a sealed box with a fixed volume at a temperature of 21°C and pressure of 110 000 Pa. Calculate the temperature the gas would need to be heated to in order to increase the pressure by 15 000 Pa.

$\frac{p}{T} = \text{constant so } \frac{p_1}{T_1} = \frac{p_2}{T_2} \Rightarrow T_2 = \frac{p_2}{p_1} T_1$

$T_1 \approx \theta_1 + 273 = 21 + 273 = 294 \text{ K}$

$p_2 = 110\,000 + 15\,000 = 125\,000 \text{ Pa}$

So $T_2 = \frac{125\,000 \times 294}{110\,000} = 334.09\ldots = \mathbf{330\ K\ (= 57^\circ C)\ (to\ 2\ s.f.)}$

Tip: To roughly convert from degrees Celsius to kelvins, add 273 (see page 250).

Experiment to investigate the pressure law and estimate absolute zero

PRACTICAL ACTIVITY GROUP 8

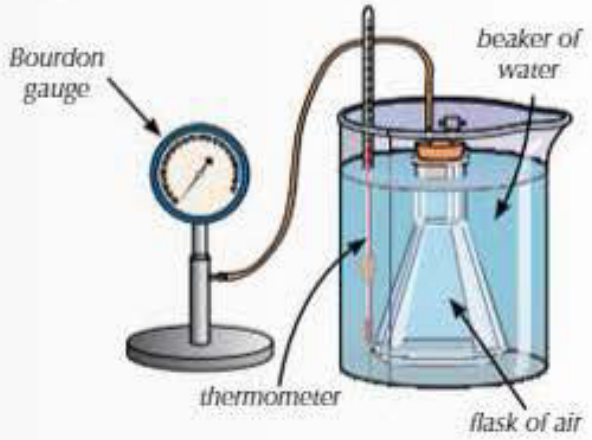


Figure 6: *Experimental set-up for investigating the pressure law.*

- Immerse a stoppered flask of air in a beaker of water so that as much of the flask as possible is submerged.
- Connect the stopper to a Bourdon gauge using a short length of tube — the volume of the tubing must be much smaller than the volume of the flask.
- Record the temperature of the water and the pressure on the gauge.

Tip: Absolute zero is the lowest possible temperature, where all particles have the minimum possible internal energy (see page 250).

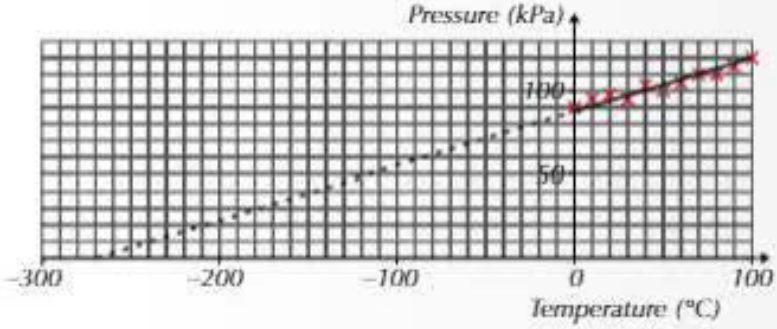
Tip: In practice a real (non-ideal) gas would condense before reaching 0 K.

Tip: If the markings on your thermometer are quite far apart, you could interpolate between them (e.g. if the temperature is halfway between the markings for 24 °C and 25 °C you could record it as 24.5 °C). But it's better to use one with a finer scale if you can.

- Insert an electric heater and switch on for a few minutes to heat the water, then remove it. Stir the water to ensure it is at a uniform temperature and allow some time for the heat to be transferred from the water to the air.
- Record the pressure on the gauge and the temperature, then repeat several times until the water starts to boil.
- Repeat your experiment twice more with fresh cool water.
- Multiplying pressure and temperature together at any point should give you a constant.
- Plot your results on a graph of pressure against temperature. Draw a line of best fit.
- Estimate the value of absolute zero by continuing (extrapolating) your line of best fit until it crosses the *x*-axis.

Example

A student heats a beaker of water that contains a sealed flask of air. He measures the water temperature and air pressure at several points as he heats the water from 0 °C to 100 °C. He plots his results on a graph as shown below. The student then extrapolates back his line of best fit until it intercepts the *x*-axis (shown by the dashed line in the graph below). At this point of interception the pressure is zero and the temperature is therefore at its theoretical minimum value, i.e. absolute zero.



So, using his results, the student finds the value of absolute zero to be –270 °C.

Charles' law

An ideal gas also obeys Charles' law. Charles' law states that:

> At constant pressure, the volume V of an ideal gas is directly proportional to its absolute temperature T.

Tip: If the temperature doubles, the volume doubles, and so on.

At the lowest theoretically possible temperature (0 K, see page 250) the volume is zero. If Charles' law is obeyed, the volume divided by the temperature is a constant:

$$\frac{V}{T} = \text{constant}$$

Practice Questions — Application

Q1 A student conducts an experiment to estimate absolute zero. She immerses a flask of air in water and gradually heats the water. She takes measurements of the temperature of the water and the pressure of the air. The table below shows her results.

Temperature (°C)	Pressure (kPa)
20	102
40	107
60	114
80	122

Plot a graph of her results and use it to estimate absolute zero.

Q2 A gas syringe is filled with 30.0 cm^3 of an ideal gas at 27 °C and the pressure inside the syringe is measured as 1.4×10^5 Pa.

a) The plunger of the syringe is pushed in so that the volume inside the syringe becomes 15 cm^3. If the temperature remains constant, what will the pressure inside the syringe be?

b) The syringe is then cooled to –173 °C. Assuming the volume remains constant, what will the pressure inside the syringe be now?

Q3 A pair of students trap a pocket of air in a sealed tube of radius 3.0 mm by filling the bottom with oil. They use a pump to increase the pressure on the oil which pushes more oil into the tube and decreases the volume of the trapped air. They use a Bourdon gauge to measure the pressure of the oil at certain intervals and measure the length of tube occupied by the air at these pressures.

a) At a pressure of 350 kPa, the length of the trapped air is 92 mm. Calculate the volume occupied by the air at this pressure.

b) Calculate the volume of the air at a pressure of 420 kPa.

c) Explain why the students should wait a few moments each time they increase the pressure before taking any readings.

d) Describe the type of graph the students would expect to see if they plotted their values of p against $\frac{1}{V}$.

e) Which gas law is this experiment investigating?

Practice Questions — Fact Recall

Q1 What must be true of a gas's mass for it to obey the three gas laws?

Q2 What is the pressure law?

Learning Objectives:

- Know that the amount of a substance can be measured in moles.
- Know that the Avogadro constant N_A is equal to 6.02×10^{23} mol^{-1} and is the number of particles in one mole.
- Be able to use the equation of state of an ideal gas $pV = nRT$, where n is the number of moles.
- Know that the Boltzmann constant, k, is calculated using $k = \frac{R}{N_A}$.
- Be able to use the equation $pV = NkT$.

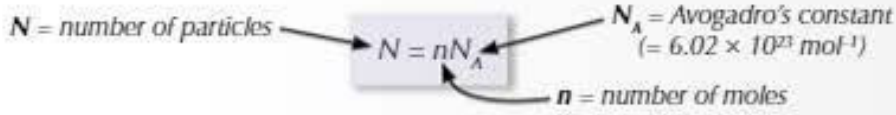
Specification Reference 5.1.4

4. The Ideal Gas Equation

You can use the gas laws from the last topic to form the equation of state of an ideal gas. It's pretty important in thermal physics, so you'll see it quite a lot.

Avogadro's constant

One **mole** of any material contains the same number of particles, no matter what the material is. This number is called **Avogadro's constant** and has the symbol N_A. The value of N_A is 6.02×10^{23} particles per mole. The number of particles, N, in an amount of gas is given by the number of moles, n, multiplied by Avogadro's constant, N_A:



n is measured in 'mol', so N_A has the unit mol^{-1}.

The equation of state

The three gas laws from the previous topic can be combined to give the equation:

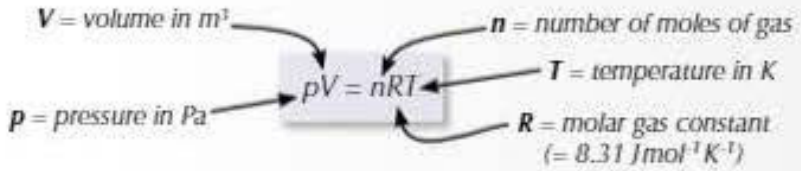
$$\frac{pV}{T} = \text{constant}$$

The constant in the equation depends on the amount of gas used. The amount of gas can be measured in moles. Putting in values for 1 mole of an ideal gas at room temperature and atmospheric pressure gives the constant a value of 8.31 JK^{-1}mol^{-1}. This is the molar gas constant, R.

The value of $\frac{pV}{T}$ increases or decreases if there's more or less gas present — the more gas you have, the more space it takes up. The amount of gas is measured in moles, n, so the constant in the equation above becomes nR, where n is the number of moles of gas present. Plugging this into the equation gives:

$$p\frac{V}{T} = nR$$

Which can be rearranged to give the **equation of state of an ideal gas**:



This equation works well (i.e. a real gas approximates to an ideal gas) for gases at low pressures and fairly high temperatures.

Tip: Don't forget to convert the units — remember the Kelvin scale is used for temperature in ideal gas calculations.

Example **Maths Skills**

What's the volume occupied by exactly one mole of an ideal gas at room temperature (20.0 °C) and at a pressure of 1.00×10^5 Pa?

Rearranging the equation of state: $pV = nRT \Rightarrow V = \frac{nRT}{p}$

So $V = \frac{1 \times 8.31 \times (20.0 + 273)}{1.00 \times 10^5} = 0.0243483 = 2.43 \times 10^{-2}\,\text{m}^3$ (to 3 s.f.)

The Boltzmann constant

The **Boltzmann constant**, k, is given by:

$$k = \frac{R}{N_A}$$

k = the Boltzmann constant ($= 1.38 \times 10^{-23}$ JK^{-1})

R = molar gas constant ($= 8.31$ JK^{-1}mol^{-1})

N_A = Avogadro's constant ($= 6.02 \times 10^{23}$ mol^{-1})

If you combine this with $N = nN_A$ from the previous page, you'll see that $Nk = nR$. First rearrange $N = nN_A$ for N_A to get $N_A = \frac{N}{n}$. Then substitute this into the equation for Boltzmann's constant:

$$k = \frac{R}{N_A} \quad \Rightarrow \quad k = \frac{R}{\left(\frac{N}{n}\right)} = \frac{nR}{N} \quad \Rightarrow \quad nR = Nk$$

Substituting this into the equation of state (from the previous page) gives the equation of state for N molecules:

$$pV = NkT$$

V = volume in m^3

p = pressure in Pa

N = number of molecules of gas

T = temperature in K

k = the Boltzmann constant ($= 1.38 \times 10^{-23}$ JK^{-1})

Example — **Maths Skills**

An ideal gas at 303 K and 1.00×10^5 Pa occupies 23.2 litres. Find how many molecules of the gas are present.

Rearrange equation: $N = \frac{pV}{kT} = \frac{(1.00 \times 10^5) \times (23.2 \times 10^{-3})}{(1.38 \times 10^{-23}) \times 303}$

$= 5.5483... \times 10^{23} = 5.55 \times 10^{23}$ (to 3 s.f.)

Practice Questions — Application

Q1 Find the volume occupied by 23 moles of an ideal gas at 25 °C and a pressure of 2.4×10^5 Pa.

Q2 A sealed, airtight container contains 8.21×10^{24} molecules of an ideal gas at a fixed volume of 4.05 m^3. The gas is heated to 500 K (correct to 3 s.f.). What's the pressure inside the container?

Q3 A sealed, airtight container is filled with 1.44×10^{25} molecules of an ideal gas and kept at a constant pressure of 1.29×10^5 Pa. The container is heated so that its volume expands to 0.539 m^3. What temperature is the gas inside the container at this point?

Q4 A sealed, airtight container with a fixed volume of 0.39 m^3 is filled with 20 moles of an ideal gas. The maximum pressure the container can withstand is 2.3×10^5 Pa. What's the highest temperature the container can be heated to before the pressure on it would be too high?

Practice Questions — Fact Recall

Q1 What is Avogadro's constant?

Q2 Write down the equation of state for n moles of an ideal gas, and state what each term in the equation represents.

Q3 Give the equation for calculating the Boltzmann constant, stating what each symbol represents.

Q4 Write down the equation of state for N molecules of an ideal gas, and state what each term in the equation represents.

Tip: You can think of the Boltzmann constant as the gas constant for one particle of gas, while R is the gas constant for one mole of gas.

Figure 1: *Austrian physicist Ludwig Boltzmann.*

Tip: There are 1000 litres in 1 m^3.

Tip: $R = 8.31$ Jmol^{-1}K^{-1} and $k = 1.38 \times 10^{-23}$ JK^{-1}. You'll be given these in the data and formulae booklet in the exam.

Learning Objectives:

- Know the model of kinetic theory of gases.
- Be able to explain pressure in terms of this model.
- Be able to use the equation $pV = \frac{1}{3}Nm\overline{c^2}$, where N is the number of particles (atoms or molecules) and $\overline{c^2}$ is the mean square speed.
- Know what root mean square (r.m.s.) speed and mean square speed are.

Specification Reference 5.1.4

Tip: Perfectly elastic collisions are collisions in which no kinetic energy is lost.

Tip: Remember — particles can mean atoms or molecules in a gas.

Tip: This is an example of using a model to develop a scientific explanation.

5. The Pressure of an Ideal Gas

Kinetic theory (p.248) can be used to derive another formula for the pressure of an ideal gas, but first we need some assumptions of ideal gases.

Assumptions made in kinetic theory

In kinetic theory, physicists picture gas particles moving at high speed in random directions. To obtain equations that describe the properties of a gas, some simplifying assumptions are needed:

- The gas contains a large number of particles.
- The particles move rapidly and randomly.
- The volume of the particles is negligible when compared to the volume of the gas.
- Collisions between particles themselves or between particles and the walls of the container are perfectly elastic.
- The duration of each collision is negligible when compared to the time between collisions.
- There are no forces between particles except for the moment when they are in a collision.

An ideal gas obeys all of these assumptions. Real gases behave like ideal gases as long as the pressure isn't too big and the temperature is reasonably high (compared with their boiling points).

Figure 1: *A visual representation of an ideal gas. All particles are identical, have negligible volume and move randomly.*

Newton's laws and ideal gas pressure

You can use Newton's laws and the assumptions of kinetic theory to explain the pressure of an ideal gas. Imagine a cubic box containing N particles of an ideal gas, each with a mass m.

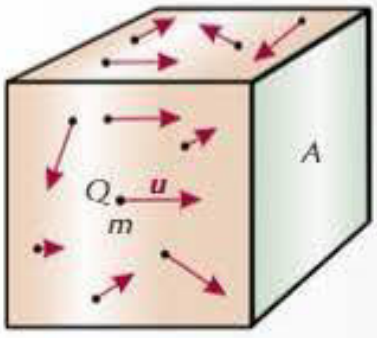


Figure 2: *A cubic box with sides of area A, containing N particles each of mass m.*

The particles of the gas are free to move around with constant random motion. There are no forces of attraction between the particles, so according to Newton's 1st law, they continue to move with constant velocities until they collide with another particle or the box itself.

When a particle collides with a wall of the box, it exerts a force on the wall, and the wall exerts an equal and opposite force on the particle. This is Newton's 3rd law in action. The size of the force exerted by the particle on the wall can be calculated using Newton's 2nd law, which says the force is equal to the rate of change of momentum.

For example, if particle Q is travelling directly towards wall A with velocity $\boldsymbol{u}$, its momentum is $m\boldsymbol{u}$. When it hits the wall, the force of the impact causes it to rebound in the opposite direction, at the same speed. Its momentum is now $-m\boldsymbol{u}$, which means the change in momentum is $-2m\boldsymbol{u}$.

So, the force a particle exerts is proportional to its mass and its velocity. The mass of a single gas particle is tiny (for example, an atom of helium gas is only 6.6×10^{-27} kg), so each particle can only exert a minuscule force.

But, there isn't just one particle in the box — there's probably billions of them. The combined force from so many tiny particles is much bigger than the contribution from any individual particle. Because there are so many particles in the box, a significant number will be colliding with each wall of the box at any given moment. And because the particles' motion is random, the collisions will be spread all over the surface of each wall. The result is a steady, even force on all the walls of the box — this is pressure.

So, the pressure in a gas is a result of all the collisions between particles and the walls of the container. The pressure of an ideal gas can be calculated using this equation:

$$pV = \frac{1}{3}Nm\overline{c^2}$$

$\boldsymbol{p}$ = pressure in Pa

$\boldsymbol{V}$ = volume in m^3

$\boldsymbol{N}$ = number of particles of gas

$\boldsymbol{m}$ = mass of a gas particle in kg

$\overline{c^2}$ = mean square speed of gas particles in m^2s^{-2}

The equation shows that the pressure exerted by a gas depends on four things:

- The volume, V, of the container — increasing the volume of the container decreases the frequency of collisions because the particles have further to travel in between collisions. This decreases the pressure.
- The number of particles, N — increasing the number of particles increases the frequency of collisions between the particles and the container, so increases the total force exerted by all the collisions.
- The mass, m, of the particles — according to Newton's 2nd law, force is proportional to mass, so heavier particles will exert a greater force.
- The speed, c, of the particles — the faster the particles are going when they hit the walls, the greater the change in momentum and force exerted.

Tip: You covered Newton's laws in Module 3.

Tip: Imagine putting a single grain of sugar into a lunchbox and shaking it around — the box might as well be empty. Now put a couple of spoonfuls of sugar in, and shake that around — this time you'll hear the sugar thumping against the walls of the box as it collides. You don't notice the effect of the individual grains, only the combined action of loads of them.

Tip: The mean square speed represents the mean of the squared speeds of all the particles, i.e. if 3 particles had velocities v_1, v_2 and v_3, their mean square speed would be $(v_1^2 + v_2^2 + v_3^2) \div 3$.

Figure 3: *Pumping more air into a ball increases the number of particles in the ball, which raises the pressure inside the ball.*

Tip: N_A is 6.02×10^{23} mol^{-1} (see page 262).

Example — **Maths Skills**

61.0 moles of a gas are enclosed in a 0.750 m³ container. If the pressure in the container is 101 kPa and each particle has a mass of 2.65×10^{-26} kg, calculate the mean square speed of the gas particles.

Rearrange $pV = \frac{1}{3}Nm\overline{c^2}$ to $\overline{c^2} = \frac{3pV}{Nm}$

Number of particles in the gas $N = nN_A$ (see page 262), so

$$\overline{c^2} = \frac{3pV}{nN_Am} = \frac{3 \times 101 \times 10^3 \times 0.750}{61.0 \times 6.02 \times 10^{23} \times 2.65 \times 10^{-26}} = 2.335... \times 10^5$$

$$= 2.34 \times 10^5 \text{ m}^2\text{s}^{-2} \text{ (to 3 s.f.)}$$

Root mean square speed

It often helps to think about the motion of a typical particle in kinetic theory. $\overline{c^2}$ is the **mean square speed** and has units m²s⁻². $\overline{c^2}$ is the average of the squared speeds of all the particles, so the square root of it gives you the typical speed.

This is called the **root mean square speed** or, usually, the r.m.s. speed. It's often written as c_{rms}. The unit is the same as any speed — ms⁻¹.

$$c_{rms} = \sqrt{\text{mean square speed}} = \sqrt{\overline{c^2}}$$

Example — **Maths Skills**

The particles in a gas have a mean square speed of 2.4×10^5 m²s⁻². Calculate the r.m.s. speed of the particles.

$c_{rms} = \sqrt{\overline{c^2}} = \sqrt{2.4 \times 10^5} = 489.89... = 490$ ms⁻¹ (to 2 s.f.)

Practice Questions — Application

Q1 A sealed, rigid container with a volume of 1.44 m³ is filled with exactly 5 moles of an ideal gas. If the molecules of the gas have a mean square speed of 8.11×10^6 m²s⁻² and a mass of 5.31×10^{-26} kg, find the pressure inside the container.

Q2 A container with a volume of 905 cm³ is filled with 0.310 moles of gas. The gas exerts a pressure of 7.40×10^5 Pa on the container walls. Calculate the root mean square speed of the gas molecules if they each have a mass of 5.10×10^{-26} kg.

Practice Questions — Fact Recall

Q1 State four assumptions made about an ideal gas in kinetic theory.

Q2 Explain why a contained gas exerts pressure on the walls of its container.

Q3 What does c_{rms} represent?

6. Internal Energy of an Ideal Gas

Particles in gases all have different amounts of kinetic energy, and it all depends on the absolute temperature.

Learning Objectives:

- Know what is meant by the internal energy of an ideal gas.
- Be able to derive and use $\frac{1}{2}m\overline{c^2} = \frac{3}{2}kT$.

Specification Reference 5.1.4

Maxwell-Boltzmann distribution

The particles in a gas don't all have the same kinetic energy. Some particles have a lot of kinetic energy and move at high speeds, but others don't have much kinetic energy and move much more slowly. Most will travel around the average speed.

If you plot a graph of the numbers of molecules in an ideal gas with different kinetic energies you get a Maxwell-Boltzmann distribution. It looks like this:

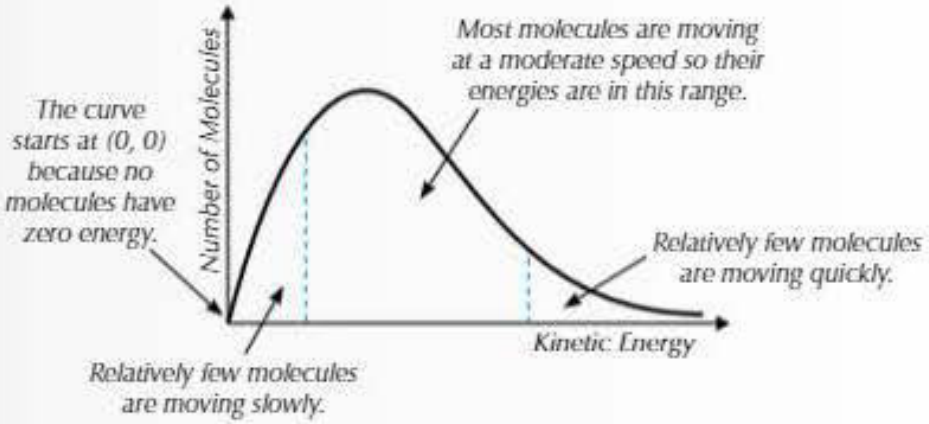
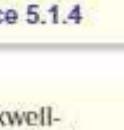

Number of Molecules
Most molecules are moving at a moderate speed so their energies are in this range.
The curve starts at (0, 0) because no molecules have zero energy.
Relatively few molecules are moving quickly.
Kinetic Energy
Relatively few molecules are moving slowly.

Figure 1: *The Maxwell-Boltzmann distribution of kinetic energies in a gas.*

Tip: The Maxwell-Boltzmann distribution is a theoretical model that has been developed to explain scientific observations.



Tip: You get pretty much the same distribution in solids and liquids too.

Speed distribution

Kinetic energy is proportional to (speed)^2, so you can also plot the speed distribution of gas particles, which is also a Maxwell-Boltzmann distribution. The shape of the Maxwell-Boltzmann distribution depends on the temperature of the gas. Figure 3 shows two different speed distribution curves. Both curves represent the same number of particles, but the cooler curve has a higher, steeper peak at a lower speed. As the temperature of the gas increases:

- The average particle speed increases.
- The maximum particle speed increases.
- The distribution curve becomes more spread out.

Figure 2: *The speed distributions of particles in a gas can be compared to shoppers on a busy street—some will be walking slowly, others will be in a hurry, but most will be walking with a moderate speed.*

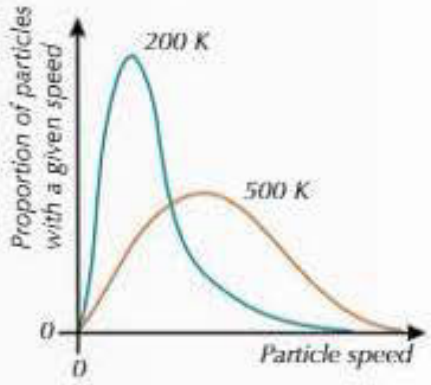


Figure 3: *The Maxwell-Boltzmann distribution of particle speeds for a gas at 200 K and at 500 K.*

Tip: A kinetic energy curve would differ with temperature in the same way.

Energy changes between particles

The particles of a gas collide with each other all the time. Some of these collisions will be 'head-on' (particles moving in opposite directions) while others will be 'shunts from behind' (particles moving in the same direction). As a result of the collisions, energy will be transferred between particles. Some particles will gain speed in a collision and others will slow down. Between collisions, the particles will travel at constant speed.

Although the energy of an individual particle changes at each collision, the collisions don't alter the total energy of the system. So, the average speed of the particles will stay the same provided the temperature of the gas stays the same.

Average kinetic energy and internal energy

You already know that $pV = NkT$ (from page 263) and that $pV = \frac{1}{3}Nm\overline{c^2}$ (page 265). You can combine these to derive the kinetic energy of gas particles in an ideal gas:

$$\frac{1}{3}Nm\overline{c^2} = NkT$$

and you can cancel N to give:

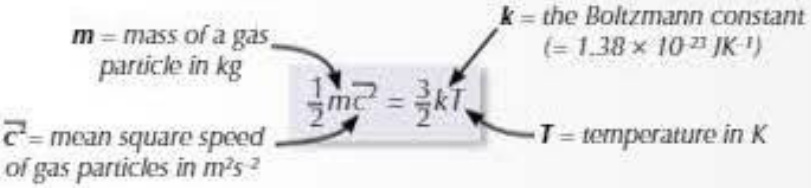
$$\frac{1}{3}m\overline{c^2} = kT$$

which you can rearrange to give:

$$m\overline{c^2} = 3kT$$

$\frac{1}{2}m\overline{c^2}$ is the average kinetic energy of an individual particle. (Remember $\overline{c^2}$ is a measure of speed squared, so this is just like the equation for kinetic energy, $E_k = \frac{1}{2}mv^2$, that you'll know already.)

So multiplying both sides by $\frac{1}{2}$ gives you:


m = mass of a gas particle in kg
k = the Boltzmann constant (= 1.38 × 10⁻²³ JK⁻¹)
½mc² = ³⁄₂kT
c² = mean square speed of gas particles in m²s⁻²
T = temperature in K

So the average kinetic energy, E, of one gas particle is given by:

$$E = \frac{3}{2}kT$$

Exam Tip
You could be tested on this derivation, so make sure you know it.

Tip: The kinetic energy of gas particles is always an average value because the particles are all travelling at different speeds.

Example — **Maths Skills**

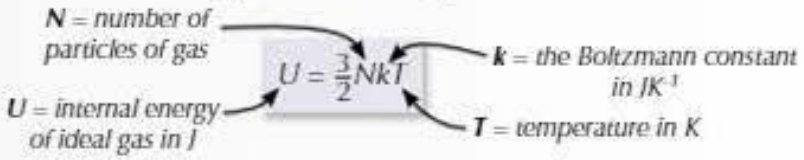
What's the average kinetic energy of the molecules in an ideal gas at 100 °C (correct to 3 significant figures)?

Just put the numbers into the equation above, making sure to convert the temperature to kelvin.

$$\frac{1}{2}m\overline{c^2} = \frac{3}{2}kT$$
$$= \frac{3}{2} \times (1.38 \times 10^{-23}) \times (100 + 273)$$
$$= 7.7211 \times 10^{-21} = 7.72 \times 10^{-21}\text{ J (to 3 s.f.)}$$

In an ideal gas the potential energy is 0 J because there are no forces between the particles. This means that the internal energy is equal to the total random kinetic energy only (see p.249).

So you can multiply E by the number of gas particles, N, to get an equation for the internal energy, U, of an ideal gas:


N = number of particles of gas
U = internal energy of ideal gas in J
$U = \frac{3}{2}NkT$
k = the Boltzmann constant in JK⁻¹
T = temperature in K

Example — **Maths Skills**

A container is filled with 2.5 moles of an ideal gas. The temperature of the gas is 310 K. Calculate the internal energy of the gas.

Number of particles = $N = nN_A = 2.5 \times 6.02 \times 10^{23} = 1.505 \times 10^{24}$

$U = \frac{3}{2}NkT = \frac{3}{2} \times 1.505 \times 10^{24} \times 1.38 \times 10^{-23} \times 310 = 9657.585$ J

$= 9.7$ kJ (to 2 s.f.)

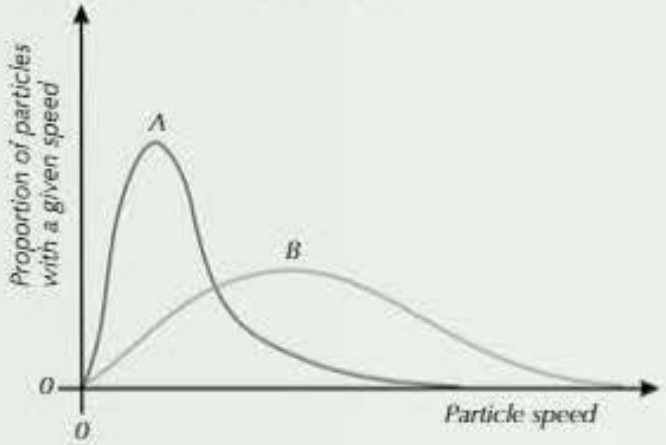
Tip: $N = nN_A$ was covered on page 262.

The equation above and the equation on the previous page show that average kinetic energy and internal energy are both directly proportional to the absolute temperature — a rise in absolute temperature will cause an increase in the kinetic energy of the particles, meaning a rise in internal energy.

Tip: Remember — absolute temperature is just the temperature in kelvin — see page 250.

Practice Questions — Application

Q1 The graph below shows the Maxwell-Boltzmann distributions for a gas at two different temperatures. State whether the higher temperature is shown by curve A or curve B.



Q2 What's the average kinetic energy of the molecules of an ideal gas at 112 K?

Practice Questions — Fact Recall

Q1 State the two equations from which $\frac{1}{2}m\overline{c^2} = \frac{3}{2}kT$ can be derived.

Q2 For an ideal gas, what can you assume about its potential energy?

Section Summary

Make sure you know...

- That solids, liquids and gases can be described by a simple kinetic model in terms of the spacing, ordering and motion of particles.
- That Brownian motion describes the zigzag, random motion of small particles in air, and be able to describe how a microscope can be used to observe Brownian motion in smoke particles.
- That internal energy is the sum of the random distribution of kinetic and potential energies associated with the molecules of a system, and that it increases with temperature.
- That internal energy changes during a change of phase while temperature remains constant.
- That the absolute scale of temperature does not depend on the property of any particular substance.
- That the lowest limit for temperature is absolute zero (equal to 0 K or –273 °C), and that at this temperature a substance has minimum internal energy.
- That temperature measurements can be in Celsius (°C) or kelvin (K), and how to convert between them using $T(K) \approx \theta\,(°C) + 273$.
- That two objects in thermal equilibrium are at the same temperature with no net flow of energy between them.
- That if body A and body B are both in thermal equilibrium with body C, then body A and body B must be in thermal equilibrium with each other.
- That the specific heat capacity of a substance is the amount of energy needed to raise the temperature of 1 kg of the substance by 1 K, and how to calculate the energy change using $E = mc\Delta\theta$.
- How the specific heat capacity of a metal or liquid can be determined using a stopwatch and an electrical heater connected to a voltmeter and an ammeter.
- How the method of mixtures can be used to estimate the specific heat capacity of a metal block and a liquid.
- That the specific latent heat of fusion or vaporisation is the quantity of thermal energy required to change the state of 1 kg of a substance, and how to use $E = mL$ to calculate the energy needed to change the state of a substance.
- How the specific latent heat of fusion or the specific latent heat of vaporisation of a substance can be determined using a stopwatch and a heating coil connected to a voltmeter and an ammeter.
- That Boyle's law is PV = constant and experimental techniques that can be used to investigate it.
- That the pressure law is $\frac{P}{T}$ = constant and experimental techniques that can be used to investigate it.
- How the variation of gas temperature with pressure can be used to estimate the value for absolute zero.
- That Avogadro's constant, N_A, is the number of particles contained in one mole of a substance and is equal to 6.02×10^{23} mol^{-1}.
- How to use the equation of state of an ideal gas $pV = nRT$, where n is the number of moles and R is the molar gas constant, equal to 8.31 J mol^{-1} K^{-1}.
- That the Boltzmann constant, k, can be calculated using $k = \frac{R}{N_A}$.
- How to use the equation of state for N particles of gas, $pV = NkT$.
- The assumptions for the model of kinetic theory of gases, and be able to explain pressure in terms of this model.
- How to use the equation $pV = \frac{1}{3}Nm\overline{c^2}$.
- That the mean square speed, $\overline{c^2}$, is the average of the squared speeds of all the particles in a gas, and that the square root of it gives you the root mean square (r.m.s.) speed, c_{rms}.
- That the distribution of the kinetic energies of particles in an ideal gas can be described by a Maxwell-Boltzmann curve.
- How to use $\frac{1}{2}m\overline{c^2} = \frac{3}{2}kT$ and be able to derive it from $pV = NkT$ and $pV = \frac{1}{3}Nm\overline{c^2}$.

Exam-style Questions

1 A syringe contains a fixed amount of an ideal gas at a pressure of 120 kPa. The plunger is pulled out, extending the tube length containing the gas from 30 mm to 50 mm, whilst the temperature is kept constant. What is the new gas pressure?

A 43 kPa

B 72 kPa

C 200 kPa

D 330 kPa

(1 mark)

2 A cold spoon is put in some hot water. Which of the following is true?

A There is no net energy flow from the water to the spoon.

B The spoon and water can never reach thermal equilibrium.

C Energy won't flow from the water to the spoon when they reach the same temperature.

D The spoon and water are in thermal equilibrium when they reach the same temperature.

(1 mark)

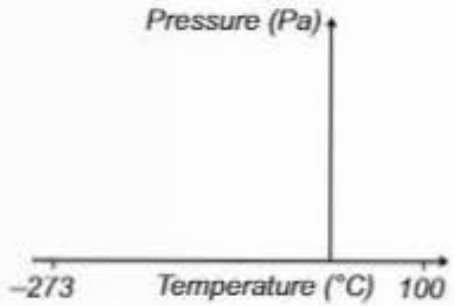
3 A sealed container with a fixed volume of 0.51 m^3 is filled with an ideal gas at low temperature. The gas is heated so that the pressure inside the container increases. At 0.0 °C the pressure inside the container is 8.1×10^5 Pa.

(a) By considering the molecules of the gas, explain why increasing the temperature leads to an increase in pressure inside the container.

(3 marks)

(b) Using the axes below, show how the pressure of the gas varies with temperature between –273 °C and 100 °C.

(2 marks)



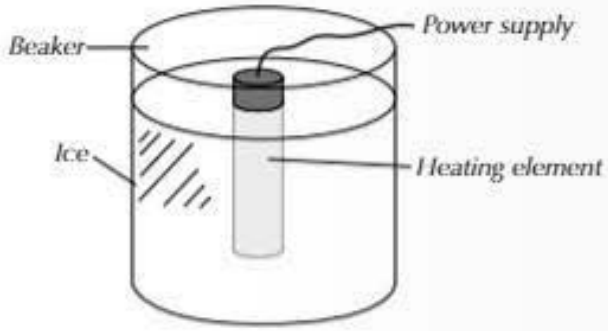
(c) Calculate the number of molecules of gas present in the container.

(2 marks)

(d) The specific heat capacity of the gas is 2.2×10^3 $J kg^{-1} K^{-1}$ and the mass of a molecule of the gas is 2.7×10^{-26} kg. If the gas was heated from –150 °C to 0.0 °C, calculate the energy transferred to the gas.

(2 marks)

4 A glass beaker contains a cylindrical block of ice at –25 °C. A heating element is placed inside the block of ice and turned on, supplying energy at a rate of 50.0 $J s^{-1}$. The ice has a mass of 92 g, a specific heat capacity of 2110 $J kg^{-1} K^{-1}$ and a specific latent heat of fusion of 3.3×10^5 $J kg^{-1}$.



(a) Calculate the energy needed to heat the ice from –25 °C to its melting point, 0.0 °C.

(1 mark)

(b) Calculate the energy needed to melt all of the ice once it reaches 0.0 °C.

(1 mark)

(c) Assuming the ice is heated uniformly and there is no heat transfer between the ice and the surroundings, calculate how long it will take for all of the ice to melt.

(2 marks)

(d) In practice, heat can be transferred between the ice and the surroundings. State whether insulating the beaker would speed up or slow down the melting of the ice. Explain your answer.

(2 marks)

5 The kinetic model of matter is used to describe substances in different states.

(a)* Describe solids, liquids and gases in terms of the kinetic model of matter. Include an explanation of Brownian motion and describe a simple demonstration that can be used to observe the Brownian motion of smoke particles in air.

(6 marks)

(b) A sealed container with a fixed volume of 4.18 m^3 is filled with 54.0 moles of an ideal gas. Each particle has a mass of 3.40×10^{-26} kg.

(i) Calculate the total number of molecules of gas in the container.

(1 mark)

(ii) Calculate the temperature of the gas if the pressure is 1.00×10^5 Pa.

(2 marks)

(iii) The container is uniformly heated to 855 °C. Calculate the r.m.s. speed of the gas particles.

(3 marks)

(iv) Sketch a graph showing the distribution of the particle speeds in the gas at the temperature found in **(ii)** and at 855 °C, and identify each curve.

(2 marks)

* The quality of your response will be assessed in this question.

1. Circular Motion

In circular motion, you'll see that all angle measurements use radians, so make sure you're comfortable with them before tackling this section.

Radians

Objects in circular motion travel through angles — these angles are usually measured in radians.

The angle in **radians**, θ, is equal to the arc-length divided by the radius of the circle (see Figure 1).

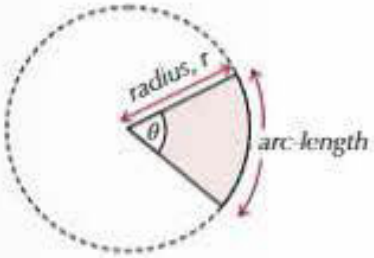


Figure 1: *The radius of a circle, and the angle and arc-length of a sector.*

In other words:

$$\text{arc length} = r\theta$$

For a complete circle (360°), the arc-length is just the circumference of the circle ($2\pi r$). Dividing this by the radius (r) gives 2π. So there are 2π radians in a complete circle (see Figure 2). 1 radian is equal to about 57°.

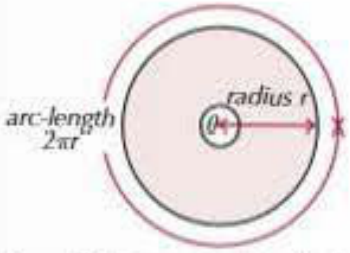


Figure 2: *For a complete circle, the angle $\theta = 2\pi$ rad.*

To convert from degrees to radians, you multiply the angle by $\frac{\pi}{180°}$:

$$\text{angle in radians} = \frac{\pi}{180^\circ} \times \text{angle in degrees}$$

Figure 3 shows some common angles, given in both degrees and radians.

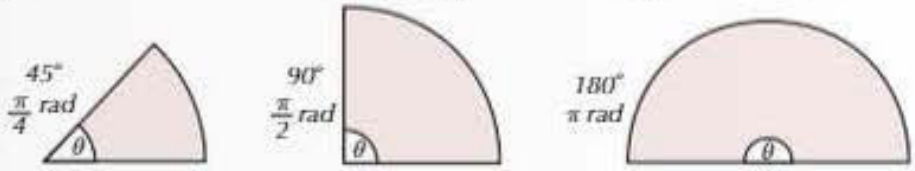


Figure 3: *Angles of sectors in degrees and radians.*

Angular velocity

Angular velocity is the angle an object rotates through per second. Just as linear velocity, $\boldsymbol{v}$, is defined as displacement ÷ time, the angular velocity, $\boldsymbol{\omega}$, is defined as angle ÷ time. The unit is rad s^{-1} (radians per second).

ω = angular velocity in rad s^{-1} → $\omega = \frac{\theta}{t}$ ← θ = angle that the object turns through in rad; t = time in s

Learning Objectives:

- Know how to use radians as a measure of angle.
- Know what is meant by angular velocity, ω.
- Be able to use the formula for constant speed in a circle, $v = \omega r$.
- Understand what is meant by the period and frequency of an object in circular motion.
- Know how to calculate angular velocity using $\omega = \frac{2\pi}{T}$ and $\omega = 2\pi f$.

Specification References 5.2.1 and 5.2.2

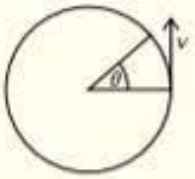
Exam Tip
You're given formulas for the circumference (and other properties) of a circle in the data and formulae booklet.

Exam Tip
You should remember the equation to convert between radians and degrees, but it's a good idea to memorise these common angles too.

Tip: Angular velocity is a vector quantity (i.e. it has a size and a direction). However, you're not usually interested in its direction — just the size.

Tip: Angular velocity is the same at any point on a solid rotating object.

Tip: The linear speed v is how fast an object would be travelling in a straight line if it broke off from circular motion.



Tip: You don't need to know the derivation of $v = \omega r$ for your exam. And you'll only need to use it in situations where the speed v is constant.

Figure 5: *One of the first cyclotrons, built by Ernest Lawrence at the University of California.*

The magnitude of the angular velocity (angular speed) can be written in terms of the magnitude of the linear velocity (linear speed), v. Consider an object moving in a circular path of radius r that moves an angle of θ radians in t seconds. The linear speed is equal to distance ÷ time, where the distance travelled is the arc length that the object moves through in its circular motion. So the equation for speed can be written as:

$$v = \frac{\text{arc length}}{t}$$

The arc length of a sector of angle θ was given on the previous page as:

$$\text{arc length} = r\theta$$

Substituting this into the equation for v gives:

$$v = \frac{r\theta}{t}$$

From the equation on the previous page, you can see that angular speed (ω) can be written as $\omega = \frac{\theta}{t}$, so the equation for linear speed can be written as:

v = *linear speed in ms*$^{-1}$ → $v = \omega r$ ← r = *radius of circle of rotation in m*; ω = *angular speed in rad s*$^{-1}$

Example

A cyclotron is a type of particle accelerator. Particles start off in the centre of the accelerator, and electric and magnetic fields cause them to move in circles of increasing size.

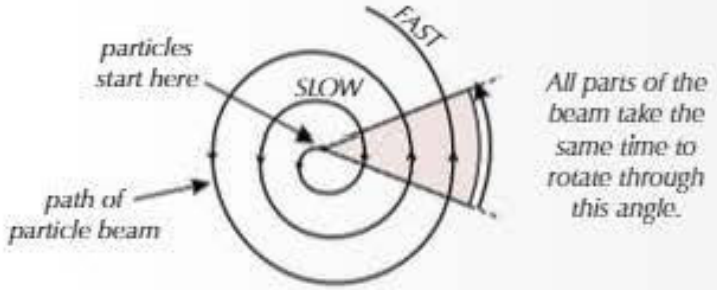

FAST
particles start here
SLOW
All parts of the beam take the same time to rotate through this angle.
path of particle beam

Figure 4: *The path of a particle in a cyclotron.*

Different parts of the particle beam are rotating at different linear speeds, v. But all the parts rotate through the same angle in the same time — so they have the same angular velocity, ω.

Example — Maths Skills

A cyclist is travelling at a speed of 28.8 km h^{-1} along a road. The diameter of his front wheel is 68 cm. What is the angular speed of the wheel? How long does it take for the wheel to turn one complete revolution?

Linear speed in ms^{-1} = 28.8 × 1000 ÷ 3600 = 8.00 ms^{-1}

Angular speed = $\omega = \frac{v}{r}$,

where $r = \frac{1}{2} \times$ diameter $= \frac{1}{2} \times 0.68 = 0.34$ m

So angular speed: $\omega = \frac{8.00}{0.34} = 23.5294... = 24$ rad s^{-1} (to 2 s.f.)

Time to complete one revolution can be found by rearranging $\omega = \frac{\theta}{t}$:

$$t = \frac{\theta}{\omega} = \frac{2\pi}{23.5294...} = 0.2670... = 0.27 \text{ s (to 2 s.f.)}$$

Tip: The cyclist's speed is the same as the linear speed of the tyre, as long as there's enough friction between the tyre and ground so that the wheel doesn't slip.

Frequency and period

Circular motion has a frequency and period. The **frequency**, f, is the number of complete revolutions per second (measured in hertz, Hz, or s^{-1}).
The **period**, T, is the time taken for a complete revolution (in seconds).
Frequency and period are linked by the equation:

f = frequency in Hz → $f = \frac{1}{T}$ ← T = period in s

For a complete circle, an object turns through 2π radians in a time T. So the equation for angular velocity becomes:

ω = angular velocity in rad s^{-1} → $\omega = \frac{2\pi}{T}$ ← T = period in s

By replacing $\frac{1}{T}$ with frequency, f, you get an equation that relates ω and f:

ω = angular velocity in rad s^{-1} → $\omega = 2\pi f$ ← f = frequency in Hz

Exam Tip
The equations $\omega = \frac{2\pi}{T}$ and $\omega = 2\pi f$ will be given in your data and formulae booklet, but make sure you know how to use them.

Example — **Maths Skills**

A wheel is turning at a frequency of 20 revolutions per second. Calculate the period and angular velocity of its rotation. Leave your answer in terms of π.

$f = \frac{1}{T}$, so $T = \frac{1}{f} = \frac{1}{20} = 0.05$ s

Angular velocity $= 2\pi f = 2 \times \pi \times 20 = 40\pi$ rad s^{-1}

Exam Tip
Exam questions may ask you to leave an answer in terms of π rather than converting it to a horrible fraction or decimal. Always read the question carefully.

Practice Questions — Application

Q1 The Moon orbits the Earth at a distance of roughly 384 000 km. If it takes the Moon 28 days to complete a full orbit, find its angular velocity and its linear speed.

Q2 An observation wheel of diameter 125 m turns exactly four times every hour. What angle does it rotate through each minute? What is the linear speed of one of the passenger cars?

Q3 A CD is spinning at a frequency of 460 rpm. What are the angular and linear speeds at points 2.0 and 4.0 cm from the centre of the CD?

Q4 A ball with mass m is spun at a constant speed in a circle on the end of a string of length l with time period T. Find the kinetic energy of the ball in terms of m, l and T.

Tip: The radii of the Earth and Moon are much smaller than the distance between them, so you can just treat them as two points.

Tip: 'rpm' is 'revolutions per minute'.

Tip: The equation for kinetic energy is $\frac{1}{2}mv^2$.

Practice Questions — Fact Recall

Q1 How would you convert an angle from degrees to radians?

Q2 What is the definition of angular velocity?

Q3 Write down the formula that links angular velocity and linear speed, defining any symbols you use.

Q4 What is meant by the period and frequency of rotation?

Q5 How would you calculate the angular velocity of an object in circular motion using its frequency? Define any symbols you use.

Learning Objectives:

- Understand that a constant net force perpendicular to the velocity of an object causes it to travel in a circular path.
- Know how to use the equations for centripetal acceleration, $a = \frac{v^2}{r}$ and $a = \omega^2 r$, and centripetal force, $F = \frac{mv^2}{r}$ and $F = m\omega^2 r$.
- Know and understand techniques and procedures used to investigate circular motion using a whirling bung.

Specification Reference 5.2.2

2. Centripetal Force and Acceleration

Objects moving in a circle are accelerating even if their speed isn't changing. This might sound strange, but it's because velocity is a vector quantity and the direction of the object's velocity is constantly changing.

Centripetal acceleration

Objects travelling in circles are accelerating since their velocity is changing. Even if the car shown in Figure 1 below is going at a constant speed, its velocity is changing since its direction is changing. Since acceleration is defined as the rate of change of velocity, the car is accelerating even though it isn't going any faster. This acceleration is called the **centripetal acceleration** and is always directed towards the centre of the circle.

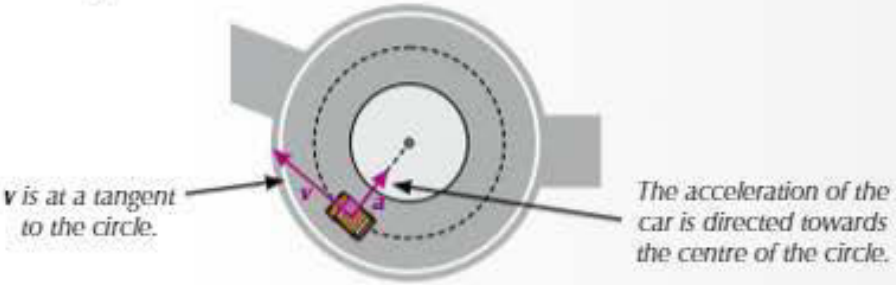


Figure 1: *A car moving in a circle.*

Derivation of formula for centripetal acceleration

A ball is moving at a constant speed in a circle (see Figure 2). Because the ball is always changing direction, the linear velocity is always changing. However, the magnitude of the linear velocity, v, is always the same (sometimes this is called linear speed). During time Δt, the ball moves from a point on the circle, A, to another point, B. The ball turns through the angle θ (the angle between the lines OA and OB).

Tip: You don't need to know this derivation for the exam, it's just here so you can see where the equations for centripetal acceleration come from. These equations are on the next page.

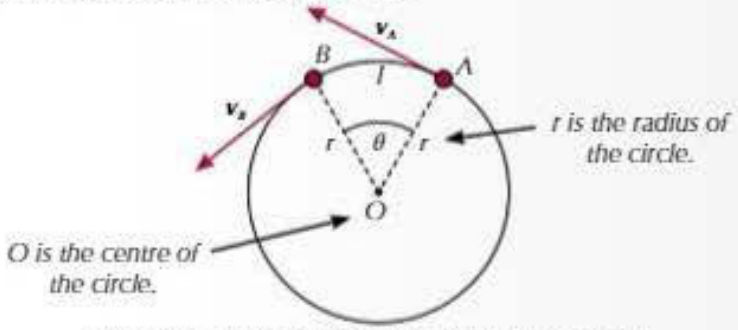


Figure 2: *A ball travelling in a circular path travels through angle θ when it moves from point A to point B.*

Tip: Linear velocity has the same magnitude as linear speed, but is a vector that is a tangent to the circle. v_A and v_B have the same magnitude but different directions.

The distance, l, that the ball travels along the circle from A to B equals the ball's linear speed multiplied by the time it takes to move that distance:

$$l = v\Delta t$$

At point A the ball has linear velocity $\boldsymbol{v}_A$ and at point B it has linear velocity $\boldsymbol{v}_B$. The change in linear velocity, $\Delta\boldsymbol{v}$, is:

$$\Delta\boldsymbol{v} = \boldsymbol{v}_B - \boldsymbol{v}_A$$

Tip: Isosceles triangles have two sides the same length.

You can draw a triangle made up of the velocity vectors $\boldsymbol{v}_A$, $\boldsymbol{v}_B$ and $\Delta\boldsymbol{v}$ (see Figure 3). The linear velocity is always at a tangent to the radius, so the angle between $\boldsymbol{v}_A$ and $\boldsymbol{v}_B$ is also θ. This triangle is the same shape as the triangle ABO, since both are isosceles triangles with the same angle θ between the two sides of identical length.

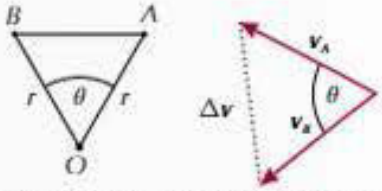


Figure 3: *The velocity vectors $\mathbf{v}_A$, $\mathbf{v}_B$ and $\Delta\mathbf{v}$ form an isosceles triangle.*

If θ is small, then the length of the straight line AB is approximately equal to the length of the arc l. Because the two triangles in Figure 3 have the same shape, the ratio of l and r is the same as the ratio of $\Delta\mathbf{v}$ and $\mathbf{v}_A$.

$$\frac{l}{r} = \frac{\Delta\mathbf{v}}{\mathbf{v}_A} = \frac{\Delta\mathbf{v}}{v}$$

The previous equation, $l = v\Delta t$, can be substituted in:

$$\frac{v\Delta t}{r} = \frac{\Delta\mathbf{v}}{v} \Rightarrow \frac{\Delta\mathbf{v}}{\Delta t} = \frac{v \times v}{r} = \frac{v^2}{r}$$

Since acceleration, $\mathbf{a}$, is equal to the change in velocity over time, this gives:

$$\mathbf{a} = \frac{\Delta\mathbf{v}}{\Delta t} = \frac{v^2}{r}$$

The formula for centripetal acceleration can then be written in terms of either the linear speed or angular speed:

$\mathbf{a}$ = centripetal acceleration in ms^{-2} → $\mathbf{a} = \frac{v^2}{r}$ ← v = linear speed in ms^{-1}; ← r = radius in m

$\mathbf{a}$ = centripetal acceleration in ms^{-2} → $\mathbf{a} = \omega^2 r$ ← r = radius in m; ← ω = angular speed in rads^{-1}

Centripetal force

Newton's first law of motion says that an object's velocity will stay the same unless there's a force acting on it. Since an object travelling in a circle has a centripetal acceleration, there must be a force causing this acceleration. Circular motion is caused by a constant net force perpendicular to the velocity (i.e. acting towards the centre of the circle), called a **centripetal force**.

Although the force changes the direction of the motion, the object never moves towards or away from the centre of the circle, so there is no motion in the direction of the force. Hence no work is done on the object, and the object's kinetic energy (and therefore speed) remains constant.

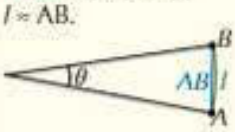
Newton's second law says $\mathbf{F} = m\mathbf{a}$, so substituting this into the equations above for the centripetal acceleration gives you equations for the centripetal force:

m = mass in kg; $\mathbf{F}$ = centripetal force in N → $F = \frac{mv^2}{r}$ ← v = linear speed in ms^{-1}; ← r = radius in m

$\mathbf{F}$ = centripetal force in N → $F = m\omega^2 r$ ← r = radius in m; m = mass in kg → ← ω = angular speed in rads^{-1}

Tip: These two triangles are similar — they're the same shape, just different sizes and orientations.

Tip: If θ is small, $l \approx$ AB.



Tip: The length of each of the vectors $\mathbf{v}_A$ and $\mathbf{v}_B$ is just v.

Tip: Remember — linear speed and the magnitude of the linear velocity are the same thing.

Exam Tip
You'll be given both of these formulas in the exam data and formulae booklet.

Tip: $v = r\omega$, so $v^2 = \omega^2 r^2$. This gives $\frac{v^2}{r} = \frac{\omega^2 r^2}{r} = \omega^2 r$.

Tip: You met Newton's laws of motion in Module 3.

Tip: The centripetal force is what keeps the object moving in a circle — remove the force and the object would fly off at a tangent with velocity $\mathbf{v}$.

Tip: Don't confuse the centripetal force with the centrifugal force. The centrifugal force is the outwards reaction force you experience when you're spinning.

Example — **Maths Skills**

A car of mass 890 kg is moving at 15 ms^{-1}. It travels around a circular bend of radius 18 m. Calculate the centripetal acceleration and centripetal force experienced by the car.

Acceleration = $\boldsymbol{a} = \frac{v^2}{r} = \frac{15^2}{18} = 12.5 = 13$ ms^{-2} (to 2 s.f.)

Force = $\boldsymbol{F} = \frac{mv^2}{r} = m\boldsymbol{a} = 890 \times 12.5 = 11\,125 = 11\,000$ N (to 2 s.f.)

Example — **Maths Skills**

A satellite orbits the Earth twice every 24 hours. The acceleration due to the Earth's gravity is 0.57 ms^{-2} at the satellite's orbiting altitude. How far above the Earth's surface is it orbiting? The radius of the Earth is 6400 km.

1 orbit takes 12 hours, so period in seconds = $T = 12 \times 3600 = 43\,200$ s

Angular speed = $\omega = \frac{2\pi}{T}$

$= \frac{2\pi}{43\,200} = 1.4544... \times 10^{-4}$ rad s^{-1}

Acceleration $\boldsymbol{a} = \omega^2 r$ so radius $r = \frac{\boldsymbol{a}}{\omega^2}$

$= \frac{0.57}{(1.4544... \times 10^{-4})^2} = 26\,945.2...$ km

Height above Earth = radius of orbit − radius of Earth

$= 26\,945.2... - 6400 = 20\,545.2... = 21\,000$ km (to 2 s.f.)

Tip: The acceleration due to the Earth's gravity decreases as you move further away from the centre of the planet — only at the surface is it 9.81 ms^{-2}. See page 300 for more.

Investigating circular motion

You can investigate circular motion using a rubber bung, some washers, some string and a clear plastic tube, as shown in Figure 4.

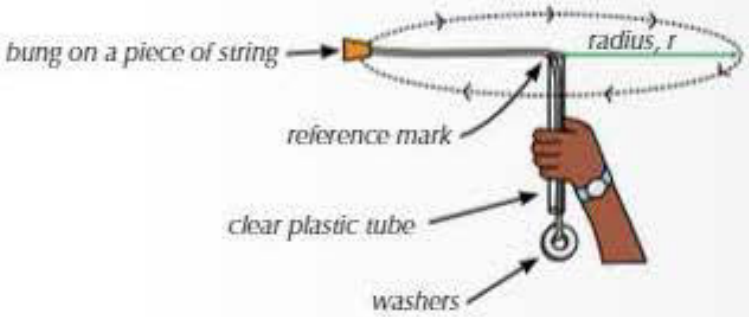


Figure 4: *The experimental set-up for investigating circular motion.*

Tip: Make sure that the bung and the washers are securely fastened to the string, that you're not standing too close to anyone and that there's nothing breakable nearby. And remember to wear safety goggles.

Measure the mass of the bung (m_b) and the mass of the washers (m_w), then attach the bung to the string. Thread the string through the plastic tube, and weigh down the free end using the washers.

Make a reference mark on the string, then measure the distance from the mark to the centre of the bung. Pull the string taut to make sure this measurement is as accurate as possible.

Line the reference mark up with the top of the tube, then begin to spin the bung in a horizontal circle, as shown in Figure 4. You'll need to spin it at the right speed to keep the reference mark level with the top of the tube (if you spin too quickly it'll move outwards, but if you spin too slowly it'll move down). Try to keep your hand as still as possible whist you spin.

Measure the time taken for the bung to make one complete circle. This is the time period, T. In practice, this may be too small to time accurately, so you might need to measure the time taken to complete ten circles and divide to get an average.

Once you've got an accurate value for T, use the formula $\omega = \frac{2\pi}{T}$ to find the angular speed of the bung. You can then find the centripetal force exerted on the bung using $\boldsymbol{F} = m_b\omega^2 r$. In this equation, r is the radius of the bung's circular path, which will equal the distance from the reference mark on the string to the centre of the bung.

The centripetal force should be equal to the weight of the washers ($\boldsymbol{W} = m_w\boldsymbol{g}$). This weight is what causes the tension in the string, which acts as the centripetal force.

Repeat this experiment for different values of r (i.e. different distances between the bung and the reference mark) — you should find that as r gets bigger, the time period gets longer but the centripetal force stays the same.

Tip: Keeping the reference mark level with the top of the tube is a bit tricky, so you may need to repeat the experiment quite a few times to get an accurate measurement of T.

Figure 5: *When a hammer thrower is spinning, the centripetal force acting along the chain keeps the hammer moving in a circle. When the thrower lets go, this force vanishes and the hammer flies off with an initial speed equal to its linear speed.*

Tip: For Q2, start by finding expressions for ω in terms of $\boldsymbol{a}$ and r and in terms of T. You can then rearrange and combine them to find the answer.

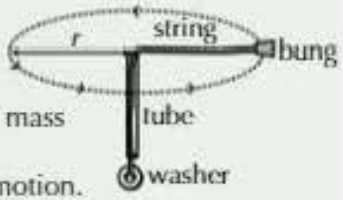
Practice Questions — Application

Q1 On a ride at a theme park, riders are strapped into seats attached to the edge of a horizontal wheel of diameter 8.5 m. The wheel rotates 15 times a minute. Calculate the force felt by a rider of mass 60.0 kg.

Q2 A planet follows a circular orbit of radius r around a star. The planet experiences a constant centripetal acceleration $\boldsymbol{a}$. How long does it take for the planet to orbit the star 3 times? Choose the correct option.

A $3\pi\sqrt{\frac{\boldsymbol{a}}{r}}$ B $6\pi\sqrt{\frac{\boldsymbol{a}}{r}}$ C $6\pi\sqrt{\frac{r}{\boldsymbol{a}}}$ D $\frac{2}{3}\pi\sqrt{\boldsymbol{a}r}$

Q3 A student is investigating circular motion using the experimental set-up shown. She attaches a washer of mass $m_w = 310$ g to the end of the string, and spins a bung of mass $m_b = 22$ g in a circle of radius $r = 0.60$ m. Calculate the time period (T) of the bung's motion.



Q4 a) A car is being driven in a circle of radius 56.8 m with a linear speed of 31.1 ms^{-1}. Calculate the centripetal acceleration of the car.

b) The linear speed of the car decreases to half its original value. If the original centripetal force experienced by the car was F, what is the new centripetal force experienced by the car in terms of F?

Q5 A biker rides in a vertical circle around the inside of a cylinder of radius 5.0 m (so he's upside down at the top of the cylinder). For the biker to not fall off at the top of the loop, he must be going fast enough that his centripetal acceleration does not drop below the acceleration due to gravity, 9.81 ms^{-1}. What's the minimum speed he must be travelling at the top of the loop?

Practice Questions — Fact Recall

Q1 What is meant by centripetal acceleration and centripetal force?

Q2 Give the equation for centripetal acceleration in terms of angular velocity. Define all the symbols you use.

Q3 Give the equation for centripetal force in terms of linear speed. Define all the symbols you use.

Learning Objectives:

- Know what is meant by simple harmonic motion.
- Know that the defining equation of SHM is $\boldsymbol{a} = -\omega^2 \boldsymbol{x}$.
- Understand the terms displacement, amplitude, period, frequency, angular frequency and phase difference in the context of simple harmonic motion.
- Know what angular frequency (ω) is, and be able to calculate it using $\omega = 2\pi f$ and $\omega = \frac{2\pi}{T}$.
- Be able to use graphical methods to relate the changes in displacement, velocity and acceleration during simple harmonic motion.
- Know that the period of a simple harmonic oscillator is independent of its amplitude (isochronous oscillator).
- Be able to describe the interchange between kinetic and potential energy during simple harmonic motion.
- Understand energy-displacement graphs for a simple harmonic oscillator.

Specification References 5.3.1 and 5.3.2

Tip: As acceleration is proportional to displacement, the force is also proportional to displacement.

3. Simple Harmonic Motion

A swinging pendulum moves with simple harmonic motion — this topic is all about what simple harmonic motion is and where you might see it occurring.

What is simple harmonic motion?

An object moving with **simple harmonic motion** (SHM) oscillates to and fro, either side of an equilibrium position (see Figure 1). This equilibrium position is the midpoint of the object's motion. The distance of the object from the equilibrium is called its displacement.

Figure 1: *A metronome moves with simple harmonic motion about an equilibrium position.*

There is always a restoring force pulling or pushing the object back towards the equilibrium position. The size of the restoring force depends on the displacement (see Figure 2). The restoring force makes the object accelerate towards the equilibrium.

Figure 2: *The size of the restoring force for an object moving with simple harmonic motion depends on its displacement from its equilibrium position.*

SHM can be defined as:

> An oscillation in which the acceleration of an object is directly proportional to its displacement from the midpoint, and is directed towards the midpoint.

This definition can be written as the relation:

$$\boldsymbol{a} = -\omega^2 \boldsymbol{x}$$

Here, $\boldsymbol{a}$ is the acceleration, $\boldsymbol{x}$ is the displacement, and ω^2 is a constant. The minus sign shows that the acceleration is always opposing the displacement (that's the "directed towards the midpoint" bit in the definition).

Graphs of simple harmonic motion

You can draw graphs to show how the displacement, velocity and acceleration of an object oscillating with SHM change with time (see Figure 3). There's more information about where these graphs come from on pages 284-285.

Displacement

Displacement, $\boldsymbol{x}$, varies as a cosine or sine wave with a maximum value, A (the amplitude).

Velocity

Velocity, $\boldsymbol{v}$, is the gradient of the displacement-time graph. It has a maximum value of ωA, where ω is the angular frequency of the oscillation. The angular frequency can be calculated using $\omega = 2\pi f$ or $\omega = \frac{2\pi}{T}$ (see page 275).

Acceleration

Acceleration, $\boldsymbol{a}$, is the gradient of the velocity-time graph. It has a maximum value of $\omega^2 A$.

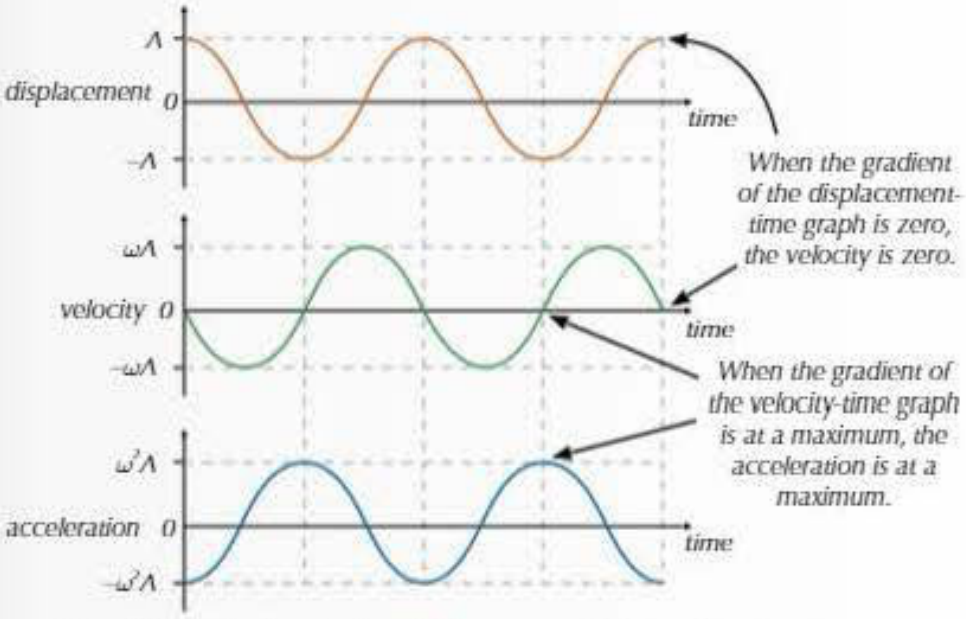


Figure 3: *Graphs showing how displacement, velocity and acceleration of an object experiencing SHM change with time.*

It might seem odd that the acceleration should be a maximum at zero velocity, but it might be helpful to remember that acceleration is the rate of change of velocity — acceleration is found from the gradient of the velocity-time graph. The steepest part of the velocity-time graph (and therefore the largest acceleration) is when the curve is passing through zero.

Also, remember that $\boldsymbol{a} = -\omega^2\boldsymbol{x}$. So the maximum acceleration occurs at the maximum negative displacement (the turning points of the displacement-time graph), which coincides with where velocity is zero.

Phase difference

Phase difference is a measure of how much one wave lags behind another wave, and can be measured in radians, degrees, or fractions of a cycle. Two waves that are in phase with each other have a phase difference of zero (or 2π radians) — i.e. the maxima and minima in each wave occur at the same time. If two waves are exactly out of phase ('in antiphase'), they have a phase difference of π radians or 180° — one wave's maximum occurs at the same time as the other's minimum.

Tip: Sine and cosine waves are graphs plotted of the functions sin and cos of some changing value (like time).

Tip: ω is called angular frequency in SHM, and angular velocity in circular motion (see page 273).

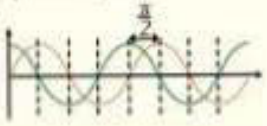
Tip: The equations that determine the displacement-time graph are on p.284. The velocity-time graph is derived from the gradient of the displacement-time graph because $\boldsymbol{v} = \frac{\Delta \boldsymbol{x}}{\Delta t}$. Similarly, $\boldsymbol{a} = \frac{\Delta \boldsymbol{v}}{\Delta t}$.

Tip: See page 285 to see where these maximum values of velocity and acceleration come from.

Tip: Kinetic energy (E_k) will vary in a similar way to $\boldsymbol{v}$, since $E_k = \frac{1}{2}mv^2$ — except the graph for E_k will always be positive (as v^2 is always positive).

Tip: These graphs can be plotted from experiments by using position sensors and data loggers to measure the motion of an object undergoing SHM — see page 287.

Tip: Remember there are 2π radians in a full cycle, so $\frac{\pi}{2}$ radians is a quarter cycle.



The velocity is a quarter of a cycle in front of the displacement in the velocity-time and displacement-time graphs for an object in SHM (see Figure 3) — it's $\frac{\pi}{2}$ radians out of phase. The acceleration is another $\frac{\pi}{2}$ radians ahead of the velocity, and so is in antiphase with the displacement.

Frequency and period

From maximum positive displacement (e.g. maximum displacement to the right) to maximum negative displacement (e.g. maximum displacement to the left) and back again is called a cycle of oscillation. The frequency, f, of the SHM is the number of cycles per second (measured in hertz, Hz). The period, T, is the time taken for a complete cycle (in seconds).

Tip: Remember, $f = \frac{1}{T}$ (see p.275).

The amplitude of an oscillation is the maximum magnitude of the displacement. In SHM, the frequency and period are independent of the amplitude (i.e. they're constant for a given oscillation). So a pendulum clock will keep ticking in regular time intervals even if its swing becomes very small. This kind of oscillator is called an isochronous oscillator.

Potential and kinetic energy

An object in SHM exchanges **potential energy** and **kinetic energy** as it oscillates. The type of potential energy (E_p) depends on what it is that's providing the restoring force. This will be gravitational E_p for pendulums and elastic E_p (elastic potential energy) and possibly gravitational E_p for masses on springs.

Tip: Gravitational potential energy is gained by moving away from a mass (page 306), and elastic potential energy is stored by elastic objects when they're taut.

- As the object moves towards the equilibrium position, the restoring force does work on the object and so transfers some E_p to E_k.
- When the object is moving away from the equilibrium, all that E_k is transferred back to E_p again.
- At the equilibrium, the object's E_p is said to be zero and its E_k is maximum — therefore its velocity is maximum.
- At the maximum displacement (the amplitude) on both sides of the equilibrium, the object's E_p is maximum, and its E_k is zero — so its velocity is zero.

Figure 4: *An oscillating pendulum moves fastest when it passes through its equilibrium position — when all its energy is kinetic.*

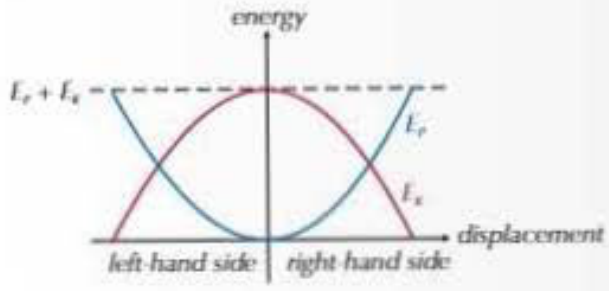


Figure 5: *Graph to show how the kinetic and potential energy of an object in SHM change with displacement.*

The sum of the potential and kinetic energy is called the mechanical energy and stays constant (as long as the motion isn't damped — see page 291). The energy transfer for one complete cycle of oscillation (see Figure 6) is: E_p to E_k to E_p to E_k to E_p ... and then the process repeats...

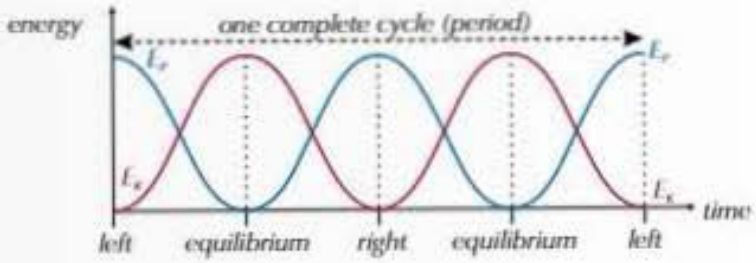
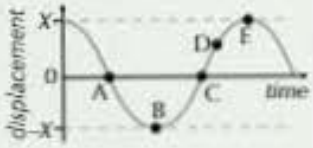


Figure 6: *Graph to show how the kinetic and potential energy of an object in SHM change with time.*

Exam Tip
Make sure you know the graphs in Figures 5 and 6 well — you could be asked to recognise, sketch and apply them to other things.

Practice Questions — Application

Q1 This graph shows displacement against time for a mass on the end of a spring. Which of the point(s) A-E shows:



a) when the magnitude of the velocity of the mass is at a maximum?

b) when the magnitude of the acceleration of the mass is at a maximum?

Q2 A girl is sitting on a swing undergoing simple harmonic motion. To start the motion, she raised the swing so it was higher than its lowest position, then lifted her feet off the ground. Sketch a graph of the girl's kinetic energy against time for two complete oscillations of the swing, where $t = 0$ at the point where she lifted her off the ground. You can assume no energy is lost to the surroundings.

Q3 A pendulum is undergoing simple harmonic motion. The maximum displacement of the pendulum is 0.60 m and the maximum speed of the pendulum is 0.90 ms^{-1}.

a) Calculate the frequency of the oscillation.

b) Calculate the time taken for the pendulum to complete one oscillation.

c) Calculate the maximum acceleration of the pendulum.

Tip: Remember that speed has a maximum value of ωA or $2\pi fA$.

Practice Questions — Fact Recall

Q1 Write down the defining equation of simple harmonic motion.

Q2 What is meant by the frequency and period of an oscillation?

Q3 What is the phase difference between the displacement and the velocity of an object moving with SHM?

Q4 For an object undergoing simple harmonic motion, state whether the following occur at equilibrium or maximum displacement:

a) maximum velocity

b) minimum velocity

c) maximum acceleration

d) minimum acceleration

Q5 Sketch the graphs of displacement, velocity and acceleration against time for an object moving with SHM. Mark on your graphs the maximum and minimum values that each can take.

Q6 Describe how the kinetic and potential energy of an object moving with SHM change with time.

Learning Objectives:

- Know solutions to the equation $\boldsymbol{a} = -\omega^2\boldsymbol{x}$, e.g. $\boldsymbol{x} = A\cos(\omega t)$ or $\boldsymbol{x} = A\sin(\omega t)$.
- Be able to use the equation $\boldsymbol{v} = \pm\omega\sqrt{A^2 - \boldsymbol{x}^2}$.
- Know that $v_{max} = \omega A$.

Specification Reference 5.3.1

4. Calculations with SHM

You saw the defining equation of simple harmonic motion ($\boldsymbol{a} = -\omega^2\boldsymbol{x}$) on p.280. The next few pages are about solutions to that equation, which will be equations for $\boldsymbol{x}$ in terms of time (t). There's some maths here, but it's hopefully not too bad.

Equations for simple harmonic motion

To understand the maths behind SHM, it's useful to think about what circular motion in a horizontal plane would look like when viewed from the side. Imagine a ball spinning in a horizontal circle (see Figure 1). From above, you'd see the ball's circular path, but from the side (in the plane of rotation) it will look like the ball's oscillating from side to side, and moving with SHM. Its speed, v, will appear fastest when $\boldsymbol{x} = 0$ and appear slowest when $\boldsymbol{x} = \pm A$.

Tip: See pages 273-279 for more about circular motion.

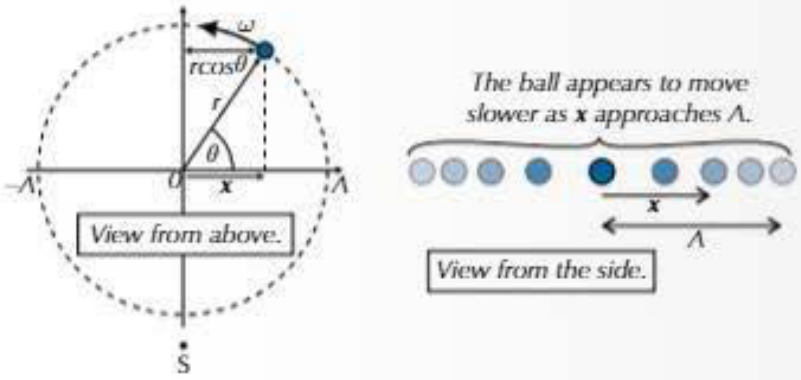


Figure 1: *A ball following a circular path, pictured from above (left) and from the side at point S (right).*

Displacement

- Figure 1 shows that $\boldsymbol{x}$, the displacement of the ball when viewed from point S, is just equal to the 'left-to-right' component of the ball's position, i.e. $\boldsymbol{x} = r\cos\theta$.
- When viewed from the side, the amplitude of the ball's simple harmonic motion, A, is the same as the radius of the circle, r.
- If the ball's displacement (as seen from the side) is at its maximum when $t = 0$ (i.e. when $t = 0$, $\theta = 0$ and $\boldsymbol{x} = A$) and its angular speed is ω, then $\theta = \omega t$. Note that the equivalent 'component' to ω in SHM is called **angular frequency**. It still has the same value, the same units and the same symbol, it's just got a different name.
- Combining these equations gives you an expression for the displacement, $\boldsymbol{x}$, of an object undergoing SHM:

$$\boldsymbol{x} = r\cos\theta = A\cos(\omega t)$$

- If the displacement (as seen from the side) is <u>not</u> at its maximum when $t = 0$, then the displacement graph (see p.281) would just be 'shifted' horizontally. In particular, if the ball is passing through its equilibrium position ($\boldsymbol{x} = 0$) when $t = 0$, the equation for displacement becomes:

$$\boldsymbol{x} = A\sin(\omega t)$$

- So for an object undergoing SHM, the displacement can be described by:

$\boldsymbol{x}$ = *displacement in m* → $\boldsymbol{x} = A\cos(\omega t)$ or $\boldsymbol{x} = A\sin(\omega t)$ ← t = *time in s*

A = *amplitude in m* ↑ ↑ ω = *angular frequency in rads*$^{-1}$

Tip: A is the maximum displacement — it's not acceleration.

Tip: Angular frequency (ω) and frequency (f) are similar but not the same. ω is measured in rads^{-1} and f is measured in Hz (or s^{-1}). The relation between the two is $\omega = 2\pi f$.

Tip: Remember to set your calculator to radians when using the equations here and on the next page.

Tip: Remember, the only difference between the graphs for sin and cos is a 'horizontal shift'.

Acceleration

The acceleration of an object moving with SHM can also be related to the acceleration of an object in circular motion. Look again at Figure 1.

- The ball is moving in a circle, so its acceleration is $\omega^2 r$.
- Seen from point S (and assuming $\theta = 0$ at $t = 0$), the acceleration of the ball, $\boldsymbol{a}$, would appear to be just the 'left-to-right' component of $\omega^2 r$:

$$\boldsymbol{a} = -\omega^2 r\cos\theta = -\omega^2 A\cos(\omega t)$$

  There's a minus sign because the acceleration is always acting towards the centre of the circle (e.g. if $\boldsymbol{x}$ is positive, then $\boldsymbol{a}$ is in the negative direction).
- But $A\cos(\omega t)$ is equal to the ball's displacement, $\boldsymbol{x}$.
  So the acceleration of the object is:

$\boldsymbol{a}$ = acceleration in ms^{-2} → $\boldsymbol{a} = -\omega^2 \boldsymbol{x}$ ← $\boldsymbol{x}$ = displacement in m; ω = angular frequency in rads^{-1}

- This is the defining equation of SHM (see page 280), which means the formula $\boldsymbol{x} = A\cos(\omega t)$ is a solution to this equation.
- Similarly, if at $t = 0$ the ball had been passing through its equilibrium position, its acceleration would still be given by the 'left-to-right' component of $\omega^2 r$. But this would now be $\boldsymbol{a} = -\omega^2 A\sin(\omega t) = -\omega^2 \boldsymbol{x}$.
  So the formula $\boldsymbol{x} = A\sin(\omega t)$ is also a solution to the SHM equation.
- Since $\boldsymbol{a} = -\omega^2 A\cos(\omega t)$ or $\boldsymbol{a} = -\omega^2 A\sin(\omega t)$, the object's acceleration has a maximum magnitude of $a_{max} = \omega^2 A$. This occurs when the object's displacement is at its maximum magnitude — i.e. when $\boldsymbol{x} = \pm A$:

$\boldsymbol{a}_{max}$ = magnitude of the max acceleration in ms^{-2} → $a_{max} = \omega^2 A$ ← $\boldsymbol{A}$ = amplitude in m; ω = angular frequency in rads^{-1}

Tip: You don't need to know the derivations of all these equations — they're just here to help you understand SHM a little more.

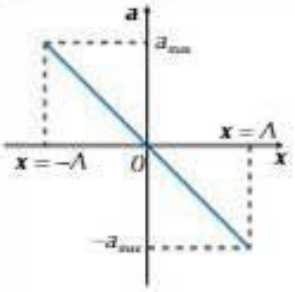


Figure 2: *Graph showing how acceleration varies with displacement for an object moving with SHM.*

Tip: You'll be given the formulas for $\boldsymbol{x}$, $\boldsymbol{v}$ and $\boldsymbol{a}$ in the data and formulae booklet, but you need to understand what they mean and how to use them.

You won't be given the formulas for v_{max} and a_{max} though, so you either need to learn them, or be able to work them out.

Velocity

You also need to know the equation for the velocity of an object moving with SHM. Don't worry about the derivation of $\boldsymbol{v}$, it's pretty complicated. You just need to know that it is given by:

$\boldsymbol{v}$ = velocity in ms^{-1} → $\boldsymbol{v} = \pm\omega\sqrt{A^2 - x^2}$ ← $\boldsymbol{x}$ = displacement in m; ω = angular frequency in rads^{-1}; $\boldsymbol{A}$ = amplitude in m

The velocity is positive if the object is moving in the positive direction (e.g. to the right), and negative if it's moving in the negative direction (e.g. to the left) — that's why there is a ± sign in there. The maximum speed (magnitude of velocity) is when the object is passing through the equilibrium, where $\boldsymbol{x} = 0$:

$\boldsymbol{v}_{max}$ = max speed in ms^{-1}; Max speed = $v_{max} = \omega A$ ← $\boldsymbol{A}$ = amplitude in m; ω = angular frequency in rads^{-1}

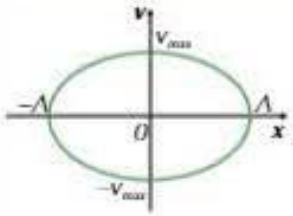


Figure 3: *Graph showing how velocity varies with displacement for an object moving with SHM.*

Tip: Remember — ω is called angular velocity in circular motion and angular frequency in SHM. It's just the name that's different, so the equation for ω is always: $\omega = 2\pi f$ or $\omega = \frac{2\pi}{T}$.

Tip: It's always helpful to draw a quick sketch for questions like this — it can help you visualise what's happening.

Tip: Part c) of this example asks you to find the speed. You can use $v = \pm\omega\sqrt{A^2 - x^2}$ and just ignore the ± sign, as this shows the direction of the motion — which is only important for velocity, not speed.

Tip: Remember the time period is just the inverse of the frequency.

Tip: The speed is at its maximum value when the mass passes through its equilibrium position.

Tip: See pages 288-289 for more on pendulums.

Example — **Maths Skills**

A mass is attached to a horizontal spring. It is pulled 7.5 cm from its equilibrium position and released. It begins oscillating with SHM, and takes 1.2 s to complete a full cycle.

a) What is the frequency of oscillation of the mass?

b) What is the magnitude of its maximum acceleration?

c) What is its speed when it is 2.0 cm from its equilibrium position?

a) Frequency $= \frac{1}{T} = \frac{1}{1.2} = 0.8333... = 0.83$ Hz (to 2 s.f.)

b) Amplitude in m $= 7.5 \div 100 = 0.075$ m

$\omega = 2\pi f = 2 \times \pi \times 0.8333... = 5.2359...$ rads^{-1}

$a_{max} = \omega^2 A = (5.2359...)^2 \times 0.075 = 2.056... = 2.1$ ms^{-2} (to 2 s.f.)

c) Speed at $x = 0.02$ m

$v = \omega\sqrt{A^2 - x^2} = 5.2359... \times \sqrt{0.075^2 - 0.02^2} = 0.3784...$

$= 0.38$ ms^{-1} (to 2 s.f.)

Practice Questions — Application

Q1 A pendulum oscillating with SHM has an angular frequency of 1.5 rads^{-1} and an amplitude of 1.6 m.

a) Calculate its acceleration when it has a displacement of 1.6 m.

b) How long does it take to complete 15 oscillations?

Q2 A mass attached to a spring oscillates with SHM. It has a period of 0.75 s, and moves with speed 0.85 ms^{-1} when passing through its equilibrium position.

a) What is the amplitude of its oscillation?

b) What will its velocity be when it is 8.0 cm to the right of its equilibrium position?

Q3 A pendulum is pulled a distance 0.45 m from its equilibrium position and is released at time $t = 0$. If it takes 15.5 s to complete exactly 5 oscillations, how far will it be from its equilibrium position after 10.0 s?

Q4 A mechanical metronome produces a ticking sound every time a pendulum arm moving with SHM passes through its equilibrium position. Its maximum displacement from its equilibrium position is 6.2 cm. If it is set to produce 120 ticks per minute, what is the magnitude of the arm's maximum acceleration?

Practice Questions — Fact Recall

Q1 Write down the solution to the SHM equation $a = -\omega^2 x$, assuming that at $t = 0$, the object is at its maximum displacement. Give your answer as a function of time, and define any symbols you use.

Q2 Give the equations for the acceleration and velocity of an object moving with SHM as a function of displacement. Define any symbols you use.

Q3 What is the maximum speed of an object moving with SHM? Define any symbols you use.

Q4 What is the magnitude of the maximum acceleration of an object moving with SHM? Define any symbols you use.

5. Investigating SHM

PRACTICAL ACTIVITY GROUP 10

Simple harmonic motion (SHM) is very common. This means there are various different experimental set-ups you can use to investigate it — for example, with a mass on a spring, or with a pendulum.

Using sensors and a data logger

Data loggers and sensors are a useful way of investigating simple harmonic motion, as they allow you to make very precise measurements. To investigate the motion of a mass oscillating on a spring, set up the equipment as shown in Figure 1. If you don't have a long spring you can connect a few shorter ones together.

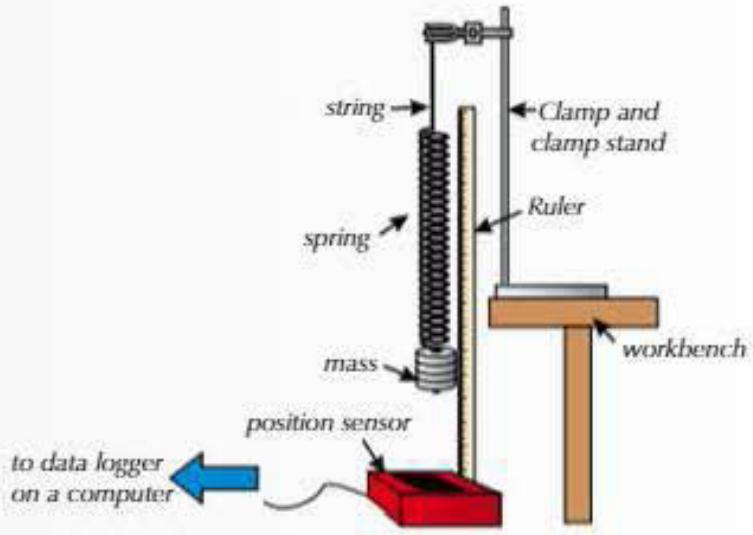


Figure 1: *Experimental set-up for investigating the motion of a mass oscillating on a spring.*

- Lift the mass slightly and release it — this will cause the mass-spring system to start oscillating with simple harmonic motion.
- To make sure your experiment is repeatable, place a ruler behind the spring to measure how far you raise the mass. Make sure your eye is level with the mass when you take the measurement.
- As the mass oscillates, the position sensor will measure the displacement of the mass over time. The computer can be set to record this data automatically.
- Let the experiment run until you've got a good amount of data (at least ten complete oscillations).

Once you've collected your data, you can use the computer to generate a displacement-time graph, like the one in Figure 2. From the graph, you can measure T, the time period of the oscillation, and A, the amplitude of the oscillation. The frequency of the oscillations (f) is given by $f = \frac{1}{T}$.

You should find that the amplitude of the oscillations gets smaller over time, but the time period and the frequency remain constant. This is because energy is lost to overcoming air resistance as the mass moves up and down.

Learning Objective:

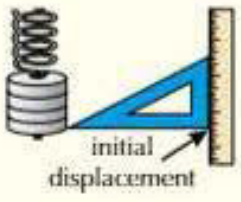
- Understand techniques and procedures used to determine the period and frequency of simple harmonic oscillations (PAG10).

Specification Reference 5.3.1

Tip: Before carrying out any experiment, you should always carry out a risk assessment.

Tip: You should try to lift the mass straight up, to stop the mass from swinging from side to side. The string also helps to stop the mass from swinging from side to side.

Tip: You could also use a set square to help get an accurate reading for the initial displacement of the mass. Using a set square helps you read the mark on the ruler that's exactly level with the mass.



Tip: Depending on how your position sensor is set up, the rest position may not be shown as zero.

Tip: If you leave the system oscillating for long enough, the amplitude will decrease until the mass eventually comes to rest.

Tip: If the oscillations happen so quickly that it's difficult to measure the time period accurately, then you could use a stroboscope. A stroboscope is a light that flashes at a frequency which the user can set. When the frequency of the stroboscope matches the frequency of the oscillation, then the mass will appear to be stationary. You can find the time period using $T = \frac{1}{f}$, where f is the frequency of the stroboscope (and therefore the frequency of the oscillation).

Tip: A reference mark like the one shown in Figure 3 is called a fiducial marker.

Figure 4: *To get a perfectly vertical reference line, you could use a plumb bob — a mass on the end of a string.*

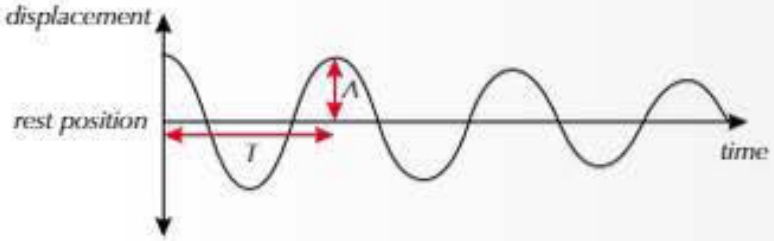


Figure 2: *A displacement-time graph for a mass oscillating on a spring. You can use the graph to measure the time period of the oscillations (T) and their amplitude (A).*

You can use this system to investigate how different variables affect the time period of the oscillation. You just have to change one variable at a time (the independent variable), while keeping all the others fixed. You should repeat the experiment for each different value of the independent variable you're investigating. For example, you could:

- Change the weight of the mass.
- Change the stiffness of the spring you're using.
- Change the size of the initial displacement.

You should find that a heavier mass leads to a longer time period and a stiffer spring leads to a shorter time period, while the initial displacement has no effect on the time period.

Without using sensors and a data logger

If you don't have a data logger, you won't be able to generate a displacement-time graph, but you can still investigate the time period of an oscillation. For example, you can investigate the simple harmonic motion of a pendulum using the experimental set-up in Figure 3 and a stopwatch.

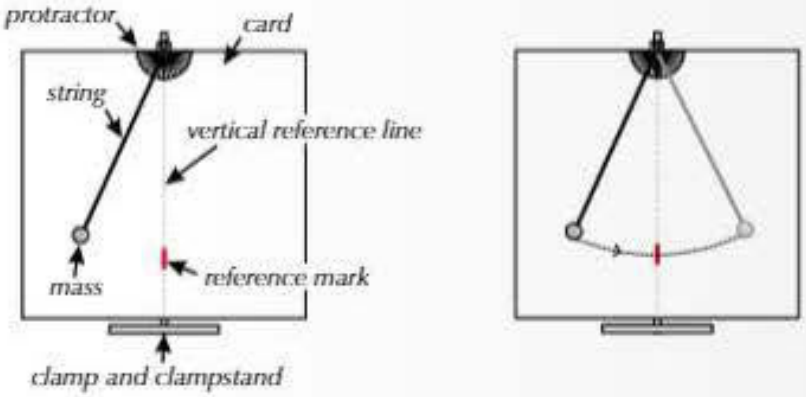


Figure 3: *Experimental set-up for investigating the motion of a pendulum.*

- Measure the weight of the mass, and use a ruler to find the string's length.
- Move the mass to the side, keeping the string taut. Measure the angle between the string and the vertical using the protractor. Make sure it's less than 10°, or the mass won't swing with simple harmonic motion when you release it.
- Release the mass. Position your eye level with the reference mark on the card, and start the stopwatch when the mass passes in front of it.

- Record the time when the mass passes the mark again, moving from the same direction — this is the time period of the oscillator. Depending on your pendulum, T might be too short to measure accurately from one swing. If so, measure the total time for a number of complete oscillations combined (say 5 or 10) and take an average (or you could use a stroboscope — see previous page).
- Keep recording T at regular intervals as the motion dies away. You can use these values of T to calculate the frequency of the oscillations using the formula $f = \frac{1}{T}$. You should find that T and f remain constant as the amplitude of the swing decreases.

Tip: To get the time period of a complete oscillation, only record times when the mass passes in front of the reference mark from the same direction (e.g. from left to right).

You can investigate how different factors affect the motion of the pendulum. Measure all the variables as accurately as possible, and only change one variable at a time.

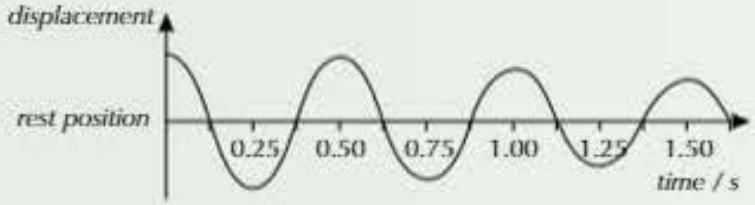
- Change the length of the piece of string you use for your pendulum.
- Change the weight of the mass on the end of the pendulum.
- Change the amount by which you initially move the mass to the side before releasing it (making sure you keep the angle below 10°).

Tip: Measuring the total time for a number of complete oscillations and taking an average means any human error introduced by starting or stopping the stopwatch too late (or too early) will be 'shared' over several oscillations. This means the time period you calculate for a single oscillation will be more accurate.

You should find that as the length of the string increases, the time period also increases, but that the weight of the mass and the angle of the initial displacement have no effect on the time period of the pendulum.

Practice Question — Application

Q1 The graph below was plotted by a student investigating simple harmonic motion. The graph shows the displacement of a mass on a spring against time.



a) Find the time period (T) of the oscillations.

b) Find the frequency (f) of the oscillations.

c) Calculate the angular frequency (ω) of the oscillations.

Tip: Your data will contain more random errors if you don't use a data logger, so you'll need to do more repeats.

Practice Questions — Fact Recall

Q1 Describe an experimental set-up you could use to investigate simple harmonic motion.

Q2 If you're investigating simple harmonic motion without using data loggers and sensors, why is it a good idea to:

a) find the time period of a single oscillation by recording the time for multiple oscillations, then dividing by the total number of oscillations?

b) use fiducial markers when recording the time to complete a number of oscillations?

Learning Objectives:

- Understand what is meant by free and forced oscillations.
- Know what resonance is in terms of the natural frequency.
- Understand and be able to sketch amplitude-driving frequency graphs for forced oscillators.
- Be able to give practical examples of forced oscillations and resonance.
- Understand the effects of damping on an oscillatory system.
- Understand observations of forced and damped oscillations for a range of systems.

Specification Reference 5.3.3

6. Free and Forced Oscillations

An object can be forced to oscillate by providing a driving frequency. If this is near the object's natural frequency, the object will start to resonate — which can be good or bad news, depending on what the object is needed for.

Free vibrations

Free vibrations involve no transfer of energy to or from the surroundings. If you stretch and release a mass on a spring, it oscillates at its **resonant frequency** (or **natural frequency**). The same happens if you strike a metal object — the sound you hear is caused by vibrations at the object's natural frequency. If no energy's transferred to or from the surroundings, it will keep oscillating with the same amplitude forever. In practice this never happens, but a spring vibrating in air is called a free vibration (or oscillation) anyway.

Forced vibrations

Forced vibrations happen when there's an external driving force. A system can be forced to vibrate by a periodic external force. The frequency of this force is called the **driving frequency**.

Resonance

When the driving frequency approaches the natural frequency, the system gains more and more energy from the driving force and so vibrates with a rapidly increasing amplitude. When this happens the system is **resonating**. At resonance the phase difference between the driver and oscillator is 90°. Figures 1 and 2 show how the relationship between amplitude and driving frequency can be investigated by experiment.

Tip: You should carry out a risk assessment before starting this experiment.

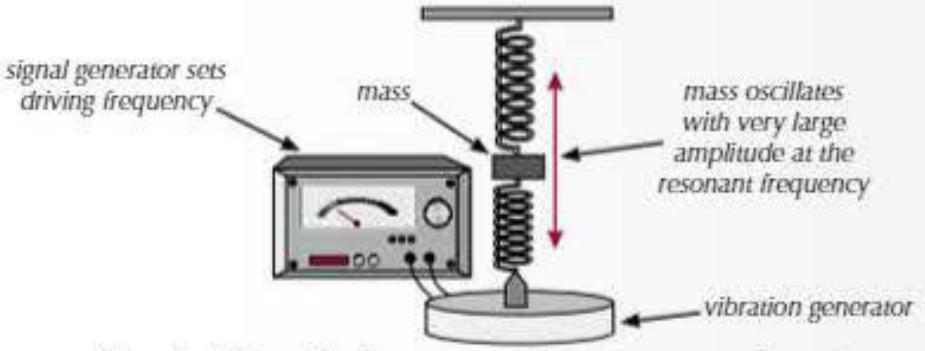
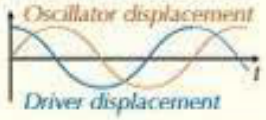

signal generator sets driving frequency
mass
mass oscillates with very large amplitude at the resonant frequency
vibration generator

Figure 1: *Using a vibration generator to oscillate a mass-spring system. The system resonates when the driving frequency equals the natural frequency.*

Tip: At resonance, the driver displacement is 90° out of phase to the displacement of the oscillator — i.e. when the displacement of the driver is at its maximum, the oscillator is passing through its equilibrium point.



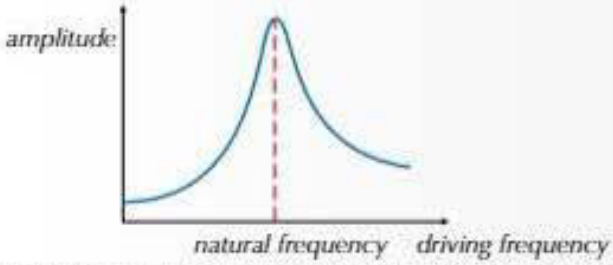


Figure 2: *Graph showing how the amplitude of oscillation of a system changes with driving frequency.*

Examples

Here are some examples of resonance:

A radio is tuned so the electric circuit resonates at the same frequency as the radio station you want to listen to.

A glass resonates when driven by a sound wave at the right frequency.

The column of air resonates in an organ pipe, driven by the motion of air at the base. This creates a stationary wave in the pipe.

A swing resonates if it's driven by someone pushing it at its natural frequency.

Tip: Make sure you know some examples of resonance.

Tip: Remember, a stationary wave is created by the superposition of two waves (with the same frequency and amplitude) moving in opposite directions.

Damping

In practice, any oscillating system loses energy to its surroundings. This is usually down to frictional forces like air resistance. These are called **damping** forces. Systems are often deliberately damped to stop them oscillating or to minimise the effect of resonance.

Tip: Another name for damping forces is 'dissipative forces', since they dissipate the energy of the oscillator to its surroundings.

Example

Shock absorbers in a car's suspension provide a damping force by squashing oil through a hole when compressed.

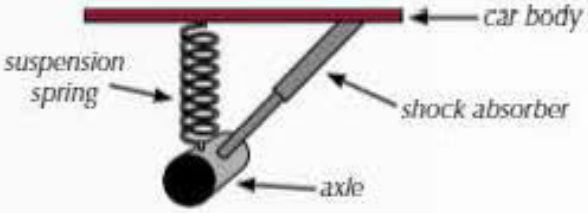


Figure 3: *Damping in a car suspension system.*

Figure 4: *In this car suspension system, the shock absorber is inside the spring. It quickly reduces the amplitude of the spring's oscillations.*

The degree of damping can vary from light damping (where the damping force is small) to overdamping. Damping reduces the amplitude of the oscillation over time. Generally, the heavier the damping, the quicker the amplitude is reduced to zero (although overdamping is an exception). Figure 5 shows how different degrees of damping reduce the amplitude to zero at different speeds.

Light and heavy damping

Lightly damped systems take a long time to stop oscillating, and their amplitude only reduces a small amount each period. Heavily damped systems take less time to stop oscillating, and their amplitude gets much smaller each period.

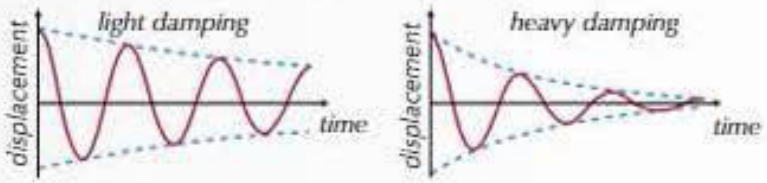


Figure 5: *Graphs showing the effect of light and heavy damping.*

Exam Tip
Make sure you're able to describe the four different types of damping (the next two are on the next page), and sketch these graphs showing how the amplitude of oscillation changes with time.

Tip: The period of a pendulum doesn't depend on its mass, so the mass of the book wouldn't affect the SHM in any way.

Example

A pendulum formed of a small bob on a rod is an example of a lightly damped system — air resistance will cause the pendulum to slow down only very slightly each period. If the pendulum bob was removed and replaced with this book, the larger surface area of the book would increase air resistance, and the damping forces would be larger, slowing the oscillation more quickly.

Critical damping

Critical damping reduces the amplitude (i.e. stops the system oscillating) in the shortest possible time (see Figure 6).

Example

Car suspension systems are critically damped so that they don't oscillate but return to equilibrium as quickly as possible.

Overdamping

Systems with even heavier damping are **overdamped**. They take longer to return to equilibrium than a critically damped system (see Figure 6).

Example

Some heavy doors are overdamped, so that they don't slam shut too quickly, but instead close slowly, giving people time to walk through them.

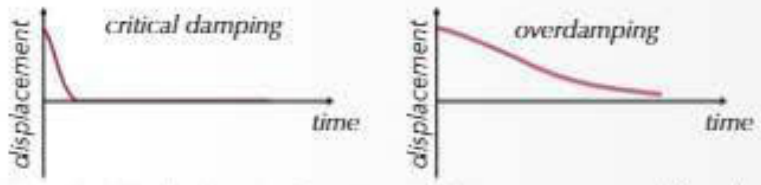


Figure 6: *Graphs showing the effects of different amounts of damping.*

Tip: You met plastic deformation in Module 3 — it's when a material is permanently stretched, beyond its elastic limit.

Oscillations can also be damped using objects made from ductile materials. As the shape of the object changes and the object plastically deforms, it absorbs energy and reduces the amplitude of the oscillations (i.e. it damps the oscillations).

Damping of forced systems

Tip: Structures are damped to avoid being damaged by resonance. Loudspeakers are also made to have as flat a response as possible so that they don't 'colour' the sound.

Lightly damped systems have a very sharp resonance peak. Their amplitude only increases dramatically when the driving frequency is very close to the natural frequency. Heavily damped systems have a flatter response. Their amplitude doesn't increase very much near the natural frequency and they aren't as sensitive to the driving frequency. Figure 7 shows the effect of increasing levels of damping on oscillations near the natural frequency.

Tip: In general, the more damped a system is, the flatter the graph of amplitude of oscillation against driving frequency.

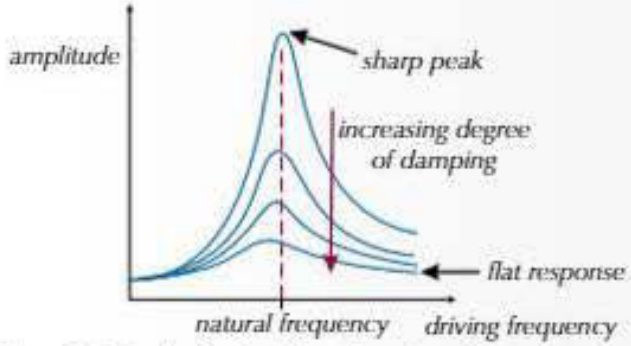


Figure 7: *Graph showing how damping affects resonance.*

Tip: For heavier damping, resonance occurs at a slightly lower driving frequency than the natural frequency.

Notice from Figure 7 that when the driving frequency approaches zero, the amplitude of the oscillations are the same, however much the system is damped. If the driving frequency is close to zero, the amplitude of the oscillation must be the same as the amplitude of whatever's driving the oscillation.

Example

Some structures are damped to avoid being damaged by resonance. Some buildings in regions prone to earthquakes have a large mass called a tuned mass damper. When an earthquake causes the building to shake, the mass moves in the opposite direction to the building, damping its oscillation (see Figure 8). This is an example of critical damping.

Figure 8: *The Taipei 101 building in Taiwan, standing at 508 m tall. A 660-tonne pendulum suspended down the centre of the building acts as a tuned mass damper, reducing the amplitude of the building's oscillations in earthquakes.*

Figure 9 shows apparatus that can be used to demonstrate the effect of damping on the resonance of a spring-mass system. A flat disc is attached to the set-up you saw in Figure 1. As the mass oscillates, air resistance on the disc acts as a damping force, reducing the amplitude of the oscillation. The larger the disc, the larger the damping force and the smaller the amplitude of oscillation of the system at resonance.

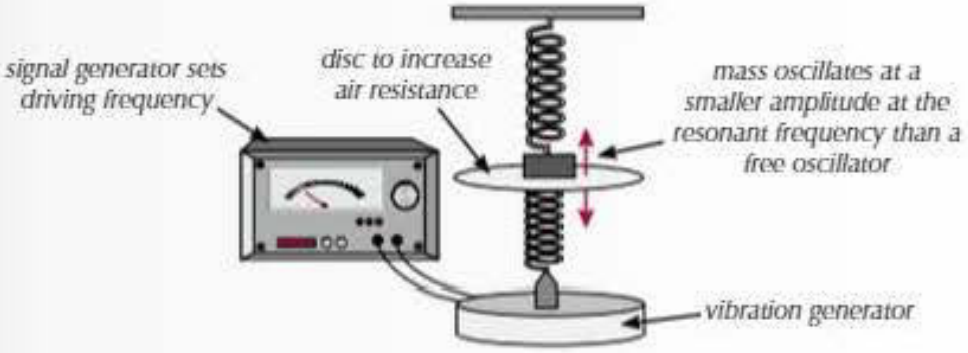

signal generator sets driving frequency
disc to increase air resistance
mass oscillates at a smaller amplitude at the resonant frequency than a free oscillator
vibration generator

Figure 9: *Experiment to show how damping affects resonance.*

Tip: Before carrying out this experiment, you should be aware of any safety risks that are associated with the equipment.

Practice Questions — Fact Recall

Q1 What is the difference between a free vibration and a forced vibration?

Q2 What is resonance, and when does it occur?

Q3 Give three examples of situations where resonance can occur.

Q4 What is meant by a damping force?

Q5 Name and briefly describe the four types of damping.

Q6 Sketch a graph of amplitude against driving frequency for a system to show how the level of damping affects the sharpness of the resonance peak at the natural frequency.

Section Summary

Make sure you know...

- How to use radians as a measure of angles, and how to convert between radians and degrees.
- That angular velocity is the angle an object moving with circular motion rotates through per second.
- How to use $v = \omega r$ to calculate the angular speed of an object moving with circular motion.
- That the frequency of rotation is the number of complete revolutions per second.
- That the period of rotation is the time taken for one complete revolution in seconds.
- The relationship between frequency and period.
- How to use the formulas $\omega = \frac{2\pi}{T}$ and $\omega = 2\pi f$ to calculate angular velocity.
- Objects undergoing circular motion experience a centripetal acceleration towards the circle's centre.
- That a constant net force perpendicular to an object's velocity causes it to travel in a circular path.
- How to use the formulas $\boldsymbol{a} = \frac{v^2}{r}$, $\boldsymbol{a} = \omega^2 r$, $\boldsymbol{F} = \frac{mv^2}{r}$ and $\boldsymbol{F} = m\omega^2 r$ to calculate centripetal acceleration and force.
- Techniques and procedures used to investigate circular motion using a whirling bung.
- That simple harmonic motion is the oscillation of an object with an acceleration that is proportional to its displacement from the midpoint and that is always directed towards the midpoint ($\boldsymbol{a} = -\omega^2 \boldsymbol{x}$).
- What is meant by the displacement and amplitude of an object undergoing SHM.
- How to use $\omega = \frac{2\pi}{T}$ and $\omega = 2\pi f$ to calculate the angular frequency of an object undergoing SHM.
- How to sketch the graphs of displacement, velocity and acceleration against time for SHM.
- That the velocity of an object undergoing SHM is given by the gradient of a displacement-time graph and acceleration is given by the gradient of a velocity-time graph.
- The phase differences between displacement, velocity and acceleration for SHM.
- That a cycle of oscillation in SHM is from maximum positive displacement to maximum negative displacement and back again, and how this relates to the frequency and period of SHM.
- That frequency and period are independent of amplitude for an object moving with SHM (i.e. it is an isochronous oscillator).
- How the kinetic and potential energy of an object moving with SHM vary with time and displacement.
- That mechanical energy is the sum of the kinetic and potential energy of an object and that it stays constant for undamped oscillations.
- That solutions to the equation $\boldsymbol{a} = -\omega^2 \boldsymbol{x}$ are of the form $\boldsymbol{x} = A\cos(\omega t)$ or $\boldsymbol{x} = A\sin(\omega t)$.
- How to use the formulas $\boldsymbol{x} = A\cos(\omega t)$ and $\boldsymbol{x} = A\sin(\omega t)$ for the displacement of an object in SHM.
- How to use $\boldsymbol{a} = -\omega^2 \boldsymbol{x}$ and $\boldsymbol{v} = \pm\omega\sqrt{A^2 - \boldsymbol{x}^2}$ for the acceleration and velocity of an object in SHM.
- How to use the formulas for the magnitude of maximum acceleration and maximum speed.
- How to determine the frequency and period of an object undergoing simple harmonic motion using, e.g. position sensors and data loggers, or by using a stopwatch.
- That free vibrations involve no transfer of energy between an object and its surroundings, and that an object oscillating freely does so at its natural frequency.
- That a forced vibration is driven by a periodic, external driving force at a driving frequency.
- That resonance is a rapid increase in the amplitude of oscillation of an object, and that it usually occurs at the object's natural frequency.
- Some practical examples of forced oscillations and resonance.
- That a damping force causes an oscillator to lose energy to its surroundings and reduce the amplitude of its oscillations.
- How the shape of an amplitude-frequency graph changes as you change the degree of damping.

Exam-style Questions

1 A roundabout with radius 3.0 m is spinning at a rate of 6 revolutions per minute. What is the linear speed of someone standing on the edge of the roundabout?

A 0.3 ms^{-1}

B 18 ms^{-1}

C 0.63 ms^{-1}

D 1.9 ms^{-1}

(1 mark)

2 A mass is attached to a spring and set oscillating with simple harmonic motion. The system is damped. Which of the following is false?

A If there was no damping at all, the mass would oscillate forever.

B The maximum displacement of the mass decreases with each oscillation.

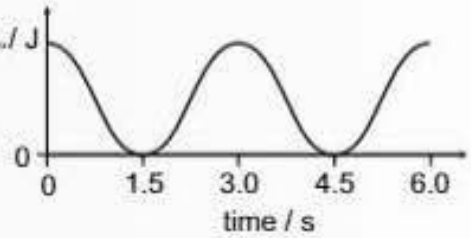
C The mass would take longer to come to rest if it was critically damped compared to if it was overdamped.

D When the amplitude decreases, the frequency remains the same.

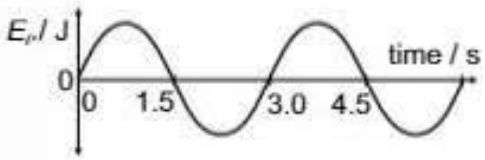
(1 mark)

3 A pendulum is oscillating with simple harmonic motion. At time $t = 0$, the pendulum passes through its equilibrium position, and passes through it again 1.5 s later. Which of the following graphs shows how the pendulum's potential energy, E_P, varies with time?

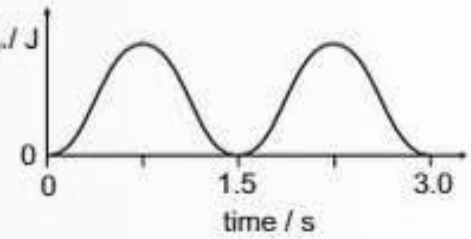
A



B



C



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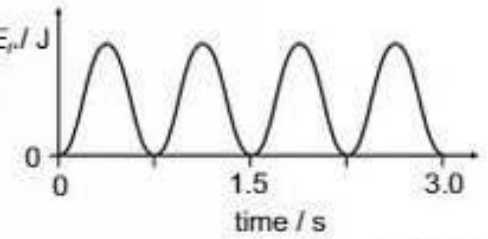


(1 mark)

4 A block of wood on a smooth surface is attached to one end of a horizontal spring. The other end of the spring is attached to a wall. The mass of the block is 0.60 kg and the restoring force (F, in newtons) produced by the spring is given by $F = -18x$, where x is the displacement of the block (in m) from its equilibrium position. When the block is displaced a distance of 20 cm from its equilibrium position and released, it begins to oscillate with a time period of 1.147 s.

(a) Find the velocity of the block when it is 15 cm away from the equilibrium position.

(2 marks)

(b) Calculate the maximum kinetic energy of the block.

(3 marks)

(c) As the block oscillates, a stopwatch is started as the block passes through the equilibrium position. Find the distance of the block from its equilibrium position 3.0 s after the stopwatch is started.

(2 marks)

5 A motorcyclist is riding around a vertical cylindrical track with radius 5.0 m, as shown in **Fig. 5.1**. To avoid sliding down the track and falling off, the centripetal force acting on her must be at least 1500 N.

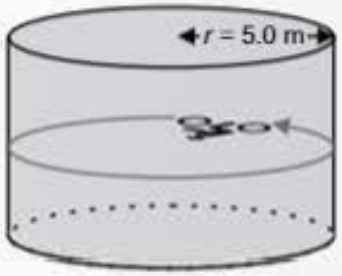


Fig. 5.1

(a) The combined mass of the motorcyclist and her motorcycle is 210 kg. Calculate the minimum speed (v_{min}) that the motorcyclist needs to ride at to avoid sliding down the track.

(2 marks)

(b) Find the motorcyclist's angular speed if she were to ride at speed v_{min}.

(2 marks)

(c) Find the time period of the motorcyclist's motion at speed v_{min}.

(2 marks)

6 A simple pendulum is made to oscillate in a vacuum by a periodic driving force, at a frequency below its natural frequency.

(a) Describe what happens as the driving frequency is increased to the pendulum's natural frequency. Give the name of this phenomenon.

(2 marks)

(b) The same pendulum is placed in a tank of water, and made to oscillate again. Describe and explain what happens this time as the driving frequency is increased up to the pendulum's natural frequency.

(3 marks)

1. Gravitational Fields

So far you've probably only considered forces acting at a specific point, with a specific cause (e.g. the pushing of a swing). Gravitational fields, on the other hand, are regions in which a mass will experience a force.

Learning Objectives:

- Understand the concept of gravitational fields as being one of a number of forms of field giving rise to a force.
- Know that gravitational fields are due to objects having mass.
- Be able to use gravitational field lines to map gravitational fields.
- Be able to model the mass of a spherical object as a point mass at its centre.
- Be able to apply Newton's law of gravitation for the force between two point masses, $F = -\frac{GMm}{r^2}$.

Specification References 5.4.1 and 5.4.2

What is a gravitational field?

A **gravitational field** is a **force field** — a region where an object will experience a non-contact force. Force fields cause interactions between objects or particles — e.g. static or moving charges interact through electric fields (p.368) and objects with mass interact through gravitational fields.

A gravitational field is a force field generated by any object with mass which causes any other object with mass to experience an attractive force.

Any object with mass has a gravitational field (even you have a gravitational field). Any object with mass will experience an attractive force if you put it in the gravitational field of another object. Only objects with a large mass, such as stars and planets, have gravitational fields that produce a significant effect. For example, the gravitational fields of the Moon and the Sun are noticeable here on Earth — they're the main cause of our tides. Smaller objects do still have gravitational fields that attract other masses, but the effect is too weak to detect without specialised equipment.

Force fields can be represented as vectors, showing the direction of the force they would exert on an object placed in that field. **Gravitational field lines**, or "lines of force", are arrows showing the direction of the force that masses would feel in a gravitational field (see Figure 2 below).

Figure 1: *The gravitational fields of the Moon and the Sun are the main causes of our tides.*

Radial and uniform fields

Point masses have radial gravitational fields, where all the field lines meet at the point where the mass is concentrated. Spherical objects of uniform density can be treated as point masses with all of their mass concentrated at their centre.

The Earth's gravitational field is radial — if you extended the lines of force, they would meet at the centre of the Earth (see Figure 2). If you put a small mass, m, anywhere in the Earth's gravitational field, it will always be attracted towards the Earth.

The lines can be used to show the strength of the field at each point, where the lines being closer together shows a stronger gravitational field. The stronger the gravitational field, the larger the force on a mass, m, due to the field (see page 300). If you move mass m further away from the Earth — where the lines of force are further apart — the force it experiences decreases.

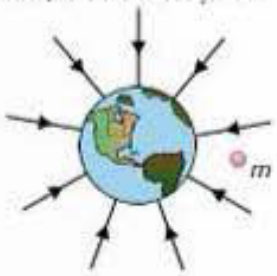


Figure 2: *A small mass, m, in Earth's gravitational field.*

Tip: The smaller mass, m, has a gravitational field of its own. This doesn't have a noticeable effect on Earth though, because the Earth is so much more massive.

Close to Earth's surface, the field is (almost) uniform — the field lines are (almost) parallel and equally spaced.

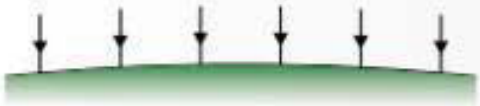
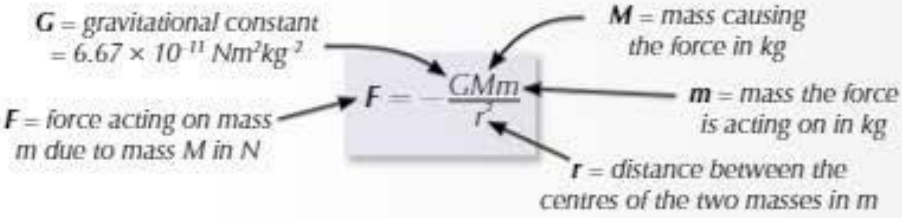


Figure 3: *The gravitational field at Earth's surface is roughly uniform.*

Newton's law of gravitation

Newton's law of gravitation says that the force acting between two point masses (or spherical masses) is proportional to the product of their masses and inversely proportional to the square of the distance between their centres of mass:


G = gravitational constant
= 6.67 × 10⁻¹¹ Nm²kg⁻²
M = mass causing the force in kg
F = force acting on mass m due to mass M in N
F = −GMm/r²
m = mass the force is acting on in kg
r = distance between the centres of the two masses in m

Remember, force is a vector quantity, so direction is important. For this equation, the positive direction is always defined as from M (the mass causing the force) to m (the mass feeling the force). So, the force on m is negative (and has a minus sign) because its direction is always towards the centre of the M. The force on m due to M is equal and opposite to the force on M due to m.

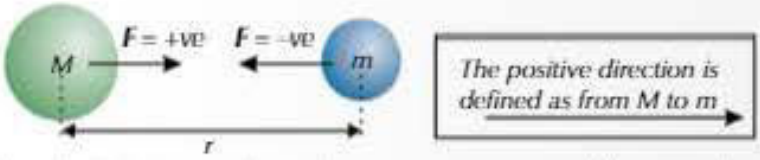


Figure 4: *The forces acting on the two masses are equal but opposite.*

We sometimes only consider the force acting on the smaller object because that's the one that experiences a greater acceleration — $\boldsymbol{a} = \frac{F}{m}$, so as m becomes bigger, $\boldsymbol{a}$ becomes smaller.

Example **Maths Skills**

Two planets have masses of 7.55×10^{24} kg and 9.04×10^{24} kg respectively. If the force due to gravity between the two planets is 6.69×10^{17} N, how far apart are the planets?

You want to find r, so you'll need Newton's law of gravitation.

We're not interested in the direction of a particular force in this example, so you can ignore the minus sign in the formula:

$$F = \frac{GMm}{r^2} \Rightarrow r^2 = \frac{GMm}{F} \Rightarrow r = \sqrt{\frac{GMm}{F}}$$

Then just put the numbers in:

$$r = \sqrt{\frac{GMm}{F}} = \sqrt{\frac{(6.67 \times 10^{-11}) \times (7.55 \times 10^{24}) \times (9.04 \times 10^{24})}{6.69 \times 10^{17}}}$$

$$= 8.2491... \times 10^{10} \text{ m}$$

$$= 82.5 \text{ million km (to 3 s.f.)}$$

Tip: Remember, spherical objects can be treated as point masses (see previous page).

Tip: G is the gravitational constant — don't get this confused with $\boldsymbol{g}$, the gravitational field strength (see p.300).

Tip: For these calculations, you have to arbitrarily pick which mass is M and which is m. The bigger mass is usually M. The equation gives the force on m due to M, where the positive direction is from M to m.

Tip: Note that r is the distance between the centres of mass of the objects, not the edges.

Exam Tip
An exam question might ask you to work out the mass of one of the objects first. E.g. you might need to use density $= \frac{\text{mass}}{\text{volume}}$.

Tip: If you'd have used the minus sign in the equation, you'd have had to use a minus value for the force too, so the minus signs would cancel out and you'd get the same answer.

Inverse square laws

The law of gravitation is an **inverse square law**: $F \propto \frac{1}{r^2}$

This means if the distance r between the masses increases, then the force F will decrease. Because it's r^2 and not just r, if the distance doubles then the force will be one quarter the strength of the original force.

Tip: The $\propto$ symbol means "is proportional to".

Example — Maths Skills

The gravitational force between two objects 10 m apart (to 2 s.f.) is 0.291 N. What will the gravitational force between them be if they move to 25 m apart?

25 m is $\frac{25}{10} = 2.5$ times larger than 10 m. So to find the new gravitational force, divide 0.291 N by 2.5^2 (because of the inverse square law):

$$\frac{0.291}{2.5^2} = 4.656 \times 10^{-2}\ \text{N} = 4.7 \times 10^{-2}\ \text{N (to 2 s.f.)}$$

Practice Questions — Application

Q1 Two stars are orbiting each other with a constant separation of 100 million km. If their masses are 2.15×10^{30} kg and 2.91×10^{30} kg, show that the force they are exerting on each other is approximately 4.2×10^{28} N.

Q2 Two asteroids 2.5 km apart exert a gravitational force on each other of 25 N. Calculate the magnitude of the force they will exert on each other when they're 0.5 km apart.

Q3 An aircraft with a mass of 2500 kg is hovering 10 000 m above ground.

a) If the radius of Earth is 6370 km, how far is the aircraft from the centre of the Earth?

b) How much upwards force must be acting on the aircraft to keep it hovering at a constant altitude?
The mass of Earth is 5.97×10^{24} kg.

Tip: Don't forget to convert any distance measurements into metres before using Newton's law of gravitation.

Tip: Remember, you have to assume that the Earth is a point mass, so r is the distance of the aircraft from the Earth's centre.

Practice Questions — Fact Recall

Q1 What's a gravitational field?

Q2 Draw a diagram showing the gravitational field lines of Earth:

a) Looking at Earth from a distance.

b) At Earth's surface.

Q3 Give the formula for Newton's law of gravitation.
Define any symbols you use.

Learning Objectives:

- Know what gravitational field strength is.
- Be able to use $g = \frac{F}{m}$ to calculate gravitational field strength.
- Know that gravitational field strength is uniform close to the surface of the Earth and numerically equal to the acceleration of free fall.
- Be able to use $g = -\frac{GM}{r^2}$ to calculate gravitational field strength for a point mass.

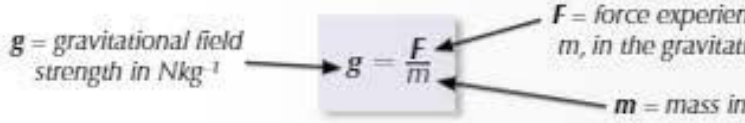
Specification References 5.4.1 and 5.4.2

2. Gravitational Field Strength

It's no use being able to draw gravitational field lines if you can't use them to work anything out. The gravitational field strength, g, tells you how strong the force due to gravity is at any point in a gravitational field.

The gravitational field strength, *g*

Gravitational field strength, ***g***, is the force per unit mass. Its value depends on where you are in the field. There's a really simple equation for working it out:



F is the force experienced by a mass *m* when it's placed in the gravitational field. Divide ***F*** by *m* and you get the force per unit mass.

g is a vector quantity — it has a magnitude and a direction. It's always pointing towards the centre of mass of the object whose field you're describing. Depending on the direction defined to be positive, it could be negative. Since the gravitational field is almost uniform at the Earth's surface, you can assume ***g*** is uniform near the Earth's surface.

The value of ***g*** at the Earth's surface is approximately 9.81 Nkg^{-1}

g can also be seen as the acceleration of a mass in a gravitational field. It's often called the acceleration due to gravity. On Earth, this is approximately 9.81 ms^{-2}.

Tip: From $F = ma$ you can get $a = F/m$, which has the units Nkg^{-1} — just another way of measuring acceleration.

Example **Maths Skills**

An 80.0 kg astronaut feels a force of 130.0 N due to gravity on the Moon. What's the value of *g* on the moon?

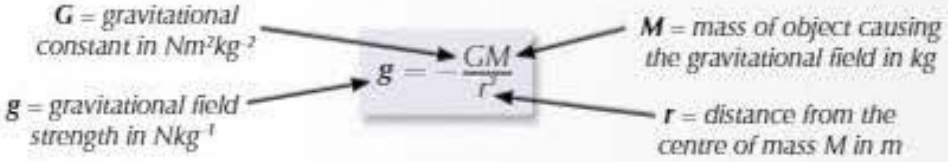
Just put the numbers into the formula:

$$g = \frac{F}{m} = \frac{130}{80} = 1.625 = 1.63\,\text{Nkg}^{-1} \text{ (to 3 s.f.)}$$

Radial fields

Point masses (or spherical masses) have radial gravitational fields (see page 297). The value of ***g*** depends on the distance *r* from the centre of the mass *M*.

Tip: Remember, a spherical mass of uniform density can be treated as a point mass with all of its mass concentrated at its centre (p.297).


G = gravitational constant in Nm²kg⁻²
g = gravitational field strength in Nkg⁻¹
g = −GM/r²
M = mass of object causing the gravitational field in kg
r = distance from the centre of mass M in m

Tip: Remember, for a spherical mass, *M*, *r* is the distance from its centre of mass, not from its surface.

You can derive this formula by looking at Newton's law of gravitation (p.298): Start with $F = -\frac{GMm}{r^2}$ and substitute this into $g = \frac{F}{m}$, cancelling down where possible:

$$g = \frac{F}{m} = \frac{\left(-\frac{GMm}{r^2}\right)}{m} = -\frac{GM}{r^2}$$

g is negative because the positive direction is defined, again, to be away from *M* (see p.298). ***g*** is a vector towards the centre of *M*, so ***g*** is negative.

Example — **Maths Skills**

The mass of the Earth is 5.97×10^{24} kg and its radius is 6.37×10^{6} m. Find the value of g at the Earth's surface.

Just put the numbers into the equation:

$$g = -\frac{GM}{r^2} = -\frac{(6.67 \times 10^{-11}) \times (5.97 \times 10^{24})}{(6.37 \times 10^{6})^2}$$

$$= -9.8134...$$

$$= -9.81 \text{ Nkg}^{-1} \text{ (to 3 s.f.)}$$

This is another case of the inverse square law (page 299) — as r doubles, g decreases to a quarter of its original value.

If you plot a graph of g against r for the Earth, you get a curve like this:

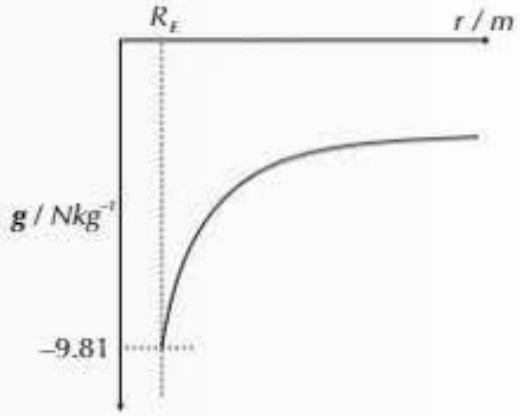


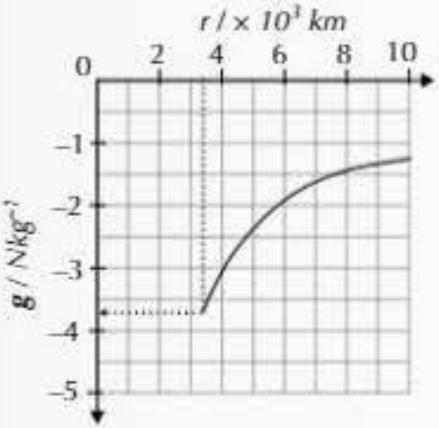
Figure 1: *Graph showing the relationship between g and r for Earth.*

Figure 2: *The value of g on top of a mountain is slightly lower than at sea level.*

It shows that g is greatest at the surface of the Earth (R_E), but decreases rapidly as r increases and you move further from the centre of the Earth.

Example — **Maths Skills**

The graph shows how the gravitational field strength, g, varies with distance, r, from the centre of the planet Mars. The radius of Mars is approximately 3.4×10^{3} km. Estimate the mass of Mars.



You can see from the graph that the value of g at the surface of Mars is about –3.7 Nkg^{-1}.

Rearrange the formula: $g = -\frac{GM}{r^2}$ to find M, then stick the values in — don't forget to convert to standard units first.

So, $M = -\frac{gr^2}{G}$

$$= -\frac{-3.7 \times (3.4 \times 10^{6})^2}{6.67 \times 10^{-11}}$$

$$= 6.4 \times 10^{23} \text{ kg (to 2 s.f.)}$$

Tip: The mass of an object is always the same wherever it is.

Practice Questions — Application

Q1 A 105 kg object experiences an attractive force due to gravity of 581 N. What's the gravitational field strength?

Q2 Why would an astronaut find it easier to pick up a rock with a mass of 20 kg on the Moon than a rock with a mass of 20 kg on Earth?

Q3 Venus has a radius of 6050 km and a mass of 4.87×10^{24} kg. Calculate the gravitational field strength at the surface of Venus.

Q4 a) A person standing on the surface of the Moon experiences a gravitational force of magnitude 105 N. If the mass of the Moon is 7.34×10^{22} kg and the mass of the person is 65 kg, what is the radius of the Moon?

b) What is the gravitational field strength 640 km above the surface of the Moon?

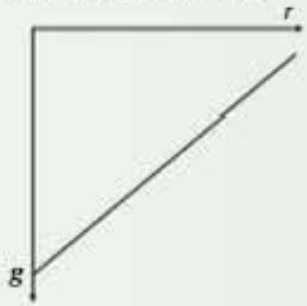
Practice Questions — Fact Recall

Q1 Other than the acceleration due to gravity, how is $\boldsymbol{g}$ defined and what are its units?

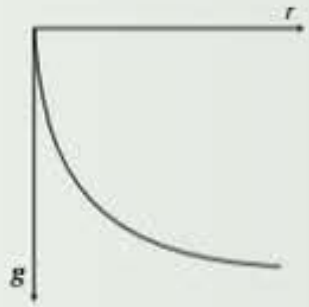
Q2 What does M represent in the formula for gravitational field strength in a radial field?

Q3 Which of the following shows how gravitational field strength changes with distance r?

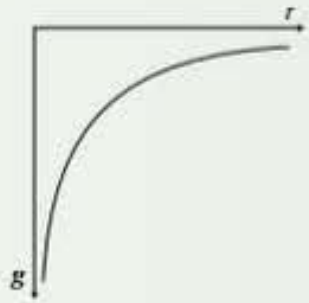
a)



b)



c)



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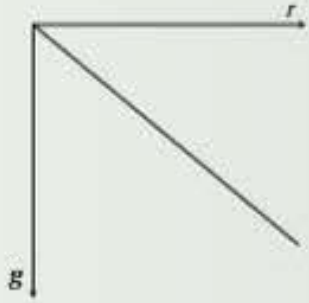
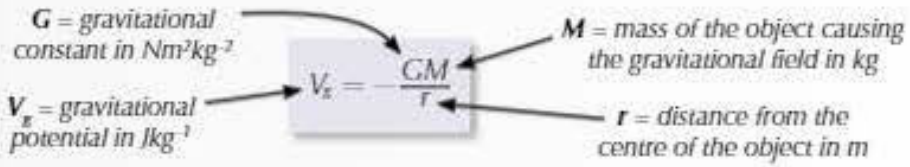


3. Gravitational Potential and Energy

All objects in a gravitational field have a gravitational potential that depends on how far they are from the centre of the field. It can be hard to get your head round at first, but try not to get it confused with gravitational potential energy.

What is gravitational potential?

The **gravitational potential**, V_g, at a point is the work done in moving a unit mass from infinity to that point. In a radial field (like the Earth's), the equation for gravitational potential is:


G = gravitational constant in Nm²kg⁻²
V_g = gravitational potential in Jkg⁻¹
V_g = −GM/r
M = mass of the object causing the gravitational field in kg
r = distance from the centre of the object in m

Gravitational potential is negative — you have to do work against the gravitational field to move an object out of it. The further you are from the centre of a radial field, the smaller the magnitude of V_g. At an infinite distance from the mass, the gravitational potential will be zero.

Example — **Maths Skills**

Find the gravitational potential at the surface of the Earth. The Earth's mass is 5.97×10^{24} kg and its radius is 6.37×10^{6} m.

Just put the numbers into the equation:

$$V_g = -\frac{GM}{r} = -\frac{(6.67 \times 10^{-11}) \times (5.97 \times 10^{24})}{6.37 \times 10^{6}}$$
$$= -6.2511... \times 10^{7}$$
$$= -6.25 \times 10^{7}\ \text{Jkg}^{-1}\ \text{(to 3 s.f.)}\ (= -62.5\ \text{MJkg}^{-1})$$

Figure 1 shows how gravitational potential varies with distance from the Earth. The graph is the same for any sphere, where R_E is the radius of the sphere.

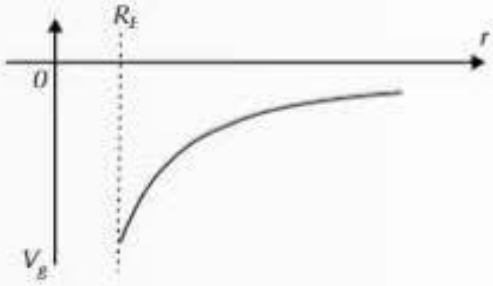


Figure 1: *A graph of gravitational potential against distance for the Earth.*

If you find the gradient of a V_g-r graph at a particular point using a tangent, you get the value of $-g$ at that point. In other words $g = -\frac{\Delta V_g}{\Delta r}$.

Learning Objectives:

- Know that gravitational potential at a point is the work done in bringing a unit mass from infinity to the point.
- Know that gravitational potential is zero at infinity.
- Be able to calculate gravitational potential using $V_g = -\frac{GM}{r}$ at a distance r from a point mass M.
- Be able to calculate changes in gravitational potential.
- Be able to sketch a force-distance graph for a point or spherical mass.
- Know that the area under the force-distance graph for a point or spherical mass is the work done.
- Be able to calculate gravitational potential energy using $E = mV_g = -\frac{GMm}{r}$ at a distance r from a point mass M.
- Know what escape velocity is.

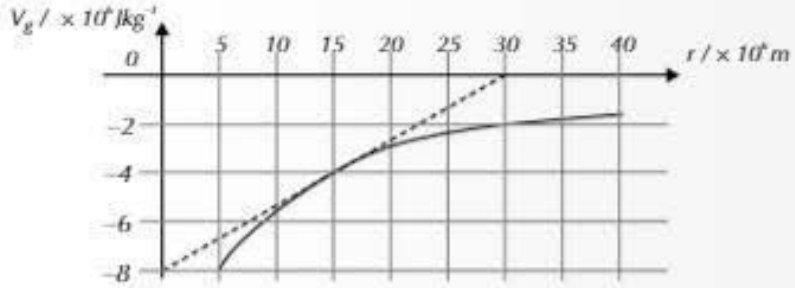
Specification Reference 5.4.4

Tip: The answer to the example means a 1 kg mass needs 62.6 MJ of energy to be able to fully escape the Earth's gravitational pull.

Tip: Note that V is proportional to $1/r$, not $1/r^2$ like the gravitational field strength.

Example — **Maths Skills**

The graph below shows the gravitational potential V_g against the distance r from the centre of a planet. Find the gravitational field strength g at $r = 15 \times 10^6$ m.



The gravitational field strength is given by the negative of the gradient:

$$g = -\frac{\Delta V_g}{\Delta r} = -\frac{0-(-8\times10^6)}{(30\times10^6)-0} = -0.2666 = -0.3\,\text{Nkg}^{-1}\ \text{(to 1 s.f.)}$$

Changes in gravitational potential

Two points at different distances from a mass will have different gravitational potentials (because the magnitude of the gravitational potential decreases with distance) — this means that there is a gravitational potential difference between these two points.

When you move an object against gravity to change the gravitational potential, you do work — the amount of energy you need depends on the mass of the object and the gravitational potential difference you move it through:

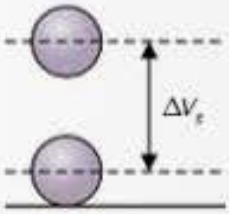
ΔW = *work done in J* ← $\Delta W = m\Delta V_g$ → ΔV_g = *gravitational potential difference in Jkg⁻¹*

m = *mass of the object in kg*

Tip: In physics, doing work means using a force to transfer energy from one type to another. For example, if you drop a ball from a height, gravitational potential energy is converted into kinetic energy.

Example — **Maths Skills**

Show that the work done to move a mass m through a gravitational potential difference of ΔV_g can be derived from the equation for gravitational field strength. The mass m is being moved away from the mass creating the gravitational field.



The gravitational field strength is given by:

g in a uniform field → $g = -\frac{\Delta V_g}{\Delta r} = \frac{F}{m}$ ← *g defined as force per unit mass (page 300).*

This rearranges to give:

$$m\Delta V_g = -F\Delta r$$

By definition, the work done is equal to force × distance moved, where the force is equal and opposite to F, since the work's being done against gravity. So the work done is given by $-F\Delta r$ and so you can write:

$$\text{work done} = -F\Delta r = m\Delta V_g$$

So the energy needed to move a mass m against a gravitational potential difference is the same as the work done, which is given by $m\Delta V_g$.

Figure 2: *A woman does work to kick a ball high into the air, where the ball has a higher gravitational potential.*

Example — **Maths Skills**

A forklift truck lifts an 80 kg pig 2.5 m from the surface of the Earth. The mass of the Earth is 5.97×10^{24} kg and its radius is 6370 km. Calculate the difference in potential, and therefore how much work is done.

$\Delta W = m\Delta V_g$, and $m = 80$ kg, so you need to find ΔV_g.

$$\Delta V_g = -\frac{GM}{r_2} - -\frac{GM}{r_1} = -\frac{GM}{r_2} + \frac{GM}{r_1}$$

$$= -\frac{(6.67\times10^{-11})\times(5.97\times10^{24})}{(6370\times10^{3}) + 2.5} + \frac{(6.67\times10^{-11})\times(5.97\times10^{24})}{6370\times10^{3}}$$

$$= 24.533... = 25 \text{ Jkg}^{-1} \text{ (to 2 s.f.)}$$

So $\Delta W = 80 \times 24.533... = 1962.68... = 2000$ J (to 2 s.f.)

Tip: Check you've got the sign right. The pig is being lifted further from the Earth, so it's gravitational potential should increase. So ΔV_g should be positive. If it isn't you've probably done the subtraction the wrong way round.

The graphs in Figure 3 show how the magnitude of the force on an object, due to the gravitational field of a point mass and a spherical mass, varies with the object's distance, r, from the mass. The area under the curve between two values of r gives the work done to move the object between those two points.

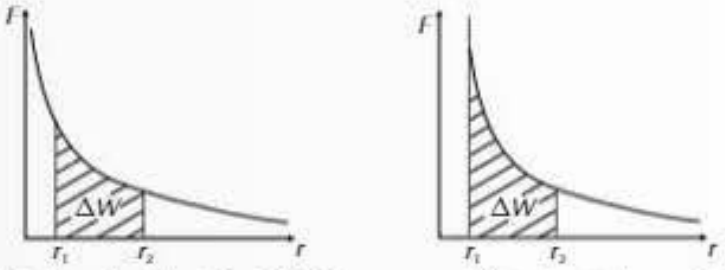


Figure 3: A graph of the size of the force on an object due to the gravitational field of a point mass (left) or a spherical mass (right) against distance.

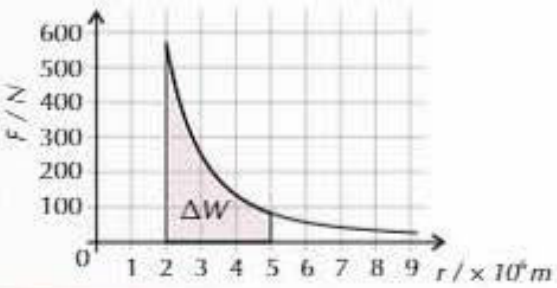
Exam Tip
Since **F** is usually defined to be negative (see page 298), you might see a graph of **F** against r. This would have the same shape as the graphs in Figure 3, but reflected in the r axis instead. Figure 3 just shows the magnitude of **F**.

Example — **Maths Skills**

The graph below shows the force on an object due to a planet's gravitational field as a function of the distance r away from the centre of the planet. The radius of the planet is approximately 2×10^6 m. Use the graph to estimate the work done to move the object from the surface of the planet to a point 3×10^5 m above the planet's surface.

The work done is equal to the area under the graph. The question is asking for an estimate, so the best way to do this is to count the number of squares. 3×10^6 m above the surface is equal to 5×10^6 m from the planet's centre.

The total number of squares under the graph between 2×10^6 m and 5×10^6 m is approximately 6.5. Each square has an area of $100 \times (1 \times 10^6) = 1 \times 10^8$ Nm, so the work done $= 6.5 \times (1 \times 10^8) = 6.5 \times 10^8$ J.



Tip: Remember, joules (J) are equal to Nm. Make sure the area you've calculated is in Nm before writing it in J.

Gravitational potential energy

Gravitational potential is work done per unit mass, so the **gravitational potential energy** (E) of an object at a point in a gravitational field is its gravitational potential multiplied by its mass. You know the formula for V_g so you can substitute it in to get the following formula:

$$E = mV_g = -\frac{GMm}{r}$$

m = *mass of the object in kg*

G = *gravitational constant* $= 6.67 \times 10^{-11}\ Nm^2kg^{-2}$

E = *gravitational potential energy of object of mass m in J*

M = *mass of the object causing the gravitational field in kg*

r = *the distance between centres of the two masses in m*

Tip: E is negative because V_g is negative. It makes sense if you think about it. An object at infinity has zero gravitational potential, and so zero gravitational potential energy. As it is attracted towards the mass, it loses gravitational potential energy. Think of a ball falling to Earth, it's losing g.p.e.

Example — **Maths Skills**

Calculate the gravitational potential energy of a person of mass 70.0 kg standing on the surface of the Earth. The mass of the Earth is 5.97×10^{24} kg and its radius is 6370 km.

$$E = -\frac{GMm}{r} = -\frac{6.67\times10^{-11}\times5.97\times10^{24}\times70.0}{6370\times10^{3}}$$

$$= -4.375...\times10^{9} = -4.38\times10^{9}\ \text{J (to 3 s.f.)}$$

Figure 4: *Rockets burn a lot of fuel to provide the kinetic energy needed to escape Earth.*

Escape velocity

The **escape velocity** is defined as the velocity needed so an object has just enough kinetic energy to escape a gravitational field. In other words, it's the minimum speed an unpowered object needs in order to leave the gravitational field of a planet and not fall back towards the planet due to gravitational attraction.

To derive the equation for escape velocity, you need to think about the energies involved. At an infinite distance (i.e. the point at which an object has escaped the gravitational field of the planet), the gravitational potential energy of the object is zero. At the surface of the planet, the gravitational potential energy is negative.

The increase in the gravitational potential energy of the object (from the surface to infinity) comes from the initial kinetic energy of the object. If the object is given just enough kinetic energy to escape the gravitational field of the planet, you know from conservation of energy that all of its initial kinetic energy will be converted into gravitational potential energy.

So kinetic energy lost = gravitational potential energy gained, and so:

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

Cancelling out m and rearranging for v gives the escape velocity:

$$v = \sqrt{\frac{2GM}{r}}$$

v = *escape velocity in* ms^{-1}

G = *gravitational constant* $= 6.67 \times 10^{-11}\ Nm^2kg^{-2}$

r = *distance of the object from the centre of mass of the planet in m*

M = *mass of the planet in kg*

Tip: The escape velocity equation is not dependent on the mass of the object. So, whether you were throwing a tennis ball or projecting a double decker bus into the air, they'd need the same velocity to escape the Earth's gravitational field (assuming there was no air resistance).

Be careful with the value of r in this equation — it's the radial distance from the centre of the planet to the object. So if the object is initially at the planet's surface, then r is just the planet's radius. But if the object is initially in orbit, then r will be the planet's radius plus the orbital distance.

Example — **Maths Skills**

Calculate the escape velocity for an object on the surface of the Earth.

Earth's mass = 5.97×10^{24} kg and Earth's radius = 6.37×10^{6} m.

Substitute these values into the equation for escape velocity:

$$v = \sqrt{\frac{2GM}{r}}$$

$$= \sqrt{\frac{2 \times (6.67 \times 10^{-11}) \times (5.97 \times 10^{24})}{(6.37 \times 10^{6})}}$$

$$= 1.1181... \times 10^{4}$$

$$= 1.11 \times 10^{4} \text{ ms}^{-1} \text{ (to 3 s.f.) (or 11.1 kms}^{-1})$$

Tip: The direction of the escape velocity is always directly away from the mass creating the gravitational field.

Tip: This means you'd have to throw a ball upwards at 11.1 km s^{-1} for it to fully escape Earth's pull.

Practice Questions — Application

Q1 A satellite is orbiting Earth. What's the effect (if any) on the following values of halving the satellite's orbital radius? Explain your answers.

a) G b) V_g at the satellite's orbital radius

c) m d) $\boldsymbol{g}$ at the satellite's orbital radius

Q2 A 1.72 kg brick is dropped off the side of a cliff. If its gravitational potential changes by 531 Jkg^{-1}, how much work is done by gravity on the brick?

Q3 The graph below shows the force on an object due to the gravitational field of a planet against the radial distance from the centre of the planet. The object is moved upwards from 2×10^{6} m to 5×10^{6} m. Use the graph to estimate the work done moving the object.

Q4 a) If the mass of the Moon is 7.34×10^{22} kg and the radius of the Moon is 1.74×10^{6} m, what is the gravitational potential energy of a 95.0 kg astronaut standing on its surface?

b) What is the minimum velocity with which the astronaut must project a piece of Moon rock so that it is able to escape the Moon's gravitational field?

Practice Questions — Fact Recall

Q1 What is gravitational potential, V_g? What are its units?

Q2 What is the significance of the negative sign in the equation $V_g = -\frac{GM}{r}$?

Q3 Sketch a graph of the gravitational potential (V_g) against distance (r) for an object in Earth's gravitational field.

Q4 Define escape velocity.

Tip: Make sure your graph for Q3 starts at the radius of Earth and not at its centre.

Learning Objectives:

- Know that the centripetal force on a planet is provided by the gravitational force between it and the Sun.
- Know and be able to derive the equation $T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$.
- Know what is meant by a geostationary orbit, and uses of geostationary satellites.
- Know Kepler's three laws of planetary motion.
- Know the relationship for Kepler's third law $T^2 \propto r^3$ and that it can be applied to systems other than our solar system.

Specification Reference 5.4.3

4. Motion of Masses in Gravitational Fields

You saw circular motion earlier on pages 273-279, and here it's covered again for objects in gravitational fields. Gravity provides the centripetal force that keeps objects in orbit around a much larger body.

Satellites

Satellites are kept in orbit by gravitational forces. A satellite is just any smaller mass which orbits a much larger mass — the Moon is a satellite of the Earth, planets are satellites of the Sun, etc.

Satellites are kept in orbit by the gravitational 'pull' of the mass they're orbiting. In our Solar System, the planets have nearly circular orbits, so you can use the equations of circular motion (pages 273-279) to investigate their **orbital speed** and **orbital period**.

Orbital speed

Any object undergoing circular motion is kept in its path by a centripetal force. What causes this force depends on the object — in the case of satellites it's the gravitational attraction of the mass they're orbiting. This means that in this case the centripetal force is the gravitational force.

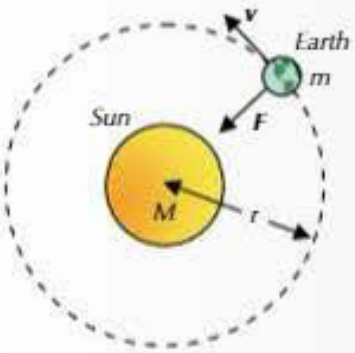


Figure 1: *Diagram showing the Earth's orbit around the Sun.*

Tip: The minus sign is missing in Newton's law of gravitation because it's just the magnitude of the force.

The Earth feels a force due to the gravitational 'pull' of the Sun — see Figure 1. The magnitude of this force is given by Newton's law of gravitation (p.298):

$$F = \frac{GMm}{r^2}$$

The Earth has velocity $\boldsymbol{v}$. Its linear speed is constant but its direction is not — so it's accelerating. The magnitude of the centripetal force (p.277) causing this acceleration is:

$$F = \frac{mv^2}{r}$$

The centripetal force on the Earth must be a result of the gravitational force due to the Sun, and so these forces must be equal:

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \Rightarrow v^2 = \frac{GMm\cancel{v}}{r^{\cancel{2}}\cancel{m}}$$

So the orbital speed is:

$$v = \sqrt{\frac{GM}{r}}$$

$\boldsymbol{G}$ = gravitational constant $= 6.67 \times 10^{-11}\ Nm^2kg^{-2}$

$\boldsymbol{v}$ = the orbital speed in ms^{-1}

$\boldsymbol{M}$ = mass of the object being orbited in kg

$\boldsymbol{r}$ = distance from the centre of the object being orbited to the centre of the orbiting satellite in m

Figure 2: *Near-infrared image of Uranus and some of its moons. The moons are natural satellites of Uranus, kept in orbit by the planet's gravity.*

Orbital period

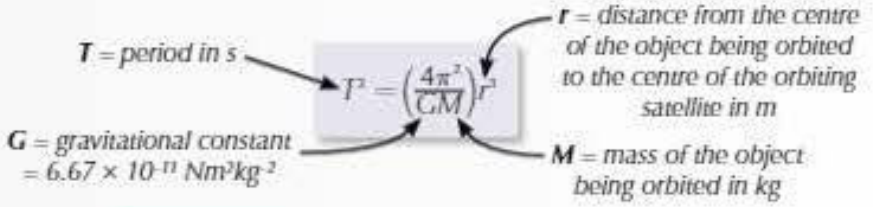
The time taken for a satellite to make one orbit is called the orbital period, T. Remember, speed = $\frac{\text{distance}}{\text{time}}$, and the distance for a circular orbit is $2\pi r$, so $v = \frac{2\pi r}{T}$:

$$v = \frac{2\pi r}{T} \quad \Rightarrow \quad T = \frac{2\pi r}{v}$$

Then substitute the expression for v on the previous page and rearrange:

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\left(\sqrt{\frac{GM}{r}}\right)} = \frac{2\pi r\sqrt{r}}{\sqrt{GM}}$$

Taking the square of the expression gives:


T = period in s
r = distance from the centre of the object being orbited to the centre of the orbiting satellite in m
$T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$
G = gravitational constant = 6.67 × 10⁻¹¹ Nm²kg⁻²
M = mass of the object being orbited in kg

Tip: This is where Kepler's third law comes from (see next page).

Example — **Maths Skills**

The Moon takes 27.3 days to orbit the Earth. Calculate its distance from the Earth. Take the mass of the Earth to be 5.97×10^{24} kg.

You're trying to find the radius of the orbit, r. Use the formula for period, T, and rearrange for r^3:

$$T^2 = \frac{4\pi^2 r^3}{GM} \quad \Rightarrow \quad r^3 = \frac{T^2 GM}{4\pi^2}$$

Then just put the numbers in:

$$r^3 = \frac{(2.35... \times 10^6)^2 \times (6.67 \times 10^{-11}) \times (5.97 \times 10^{24})}{4\pi^2}$$

$$= 5.6116... \times 10^{25}\ \text{m}^3$$

$$r = \sqrt[3]{5.6116... \times 10^{25}}$$

$$= 3.8285... \times 10^8\ \text{m} = 3.83 \times 10^5\ \text{km}\ \text{(to 3 s.f.)}$$

Tip: This is the distance from the centre of the Earth to the centre of the Moon.

Tip: T is given in days, so you need to convert it to seconds first. 1 day = 8.64×10^4 s (this will be in your data and formulae booklet in the exam), so 27.3 days = $2.35... \times 10^6$ s.

Tip: Just take the cube root of r^3 here — you'll probably have a button for it on your calculator ($\sqrt[3]{\square}$).

Geostationary satellites

Geostationary satellites orbit directly over the equator and are always above the same point on Earth. A geostationary satellite travels at the same angular speed as the Earth turns below it and in the same direction (west to east). A geostationary orbit takes exactly one day.

These satellites are really useful for sending TV and telephone signals and have improved communication around the world. Communications satellites are stationary relative to a certain point on the Earth, so you don't have to alter the angle of your receiver (or transmitter) to keep up. Geostationary satellites are also used to monitor and track the weather from above.

There are downsides though — they are expensive and pose a small risk of something going wrong and the satellite falling back to Earth.

Figure 3: *Geostationary satellites remain in a fixed position relative to a point on the Earth's surface.*

Example — **Maths Skills**

Calculate the height above the Earth's surface that a geostationary satellite orbits. The mass of the Earth is 5.97×10^{24} kg and the radius is 6370 km.

You know that $T = 24 \times 60 \times 60 = 86\,400$ s and $M = 5.97 \times 10^{24}$ kg,

so rearrange $T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$ to find r:

$$r = \sqrt[3]{\frac{T^2GM}{4\pi^2}} = \sqrt[3]{\frac{(86\,400)^2 \times 6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{4\pi^2}} = 4.22... \times 10^7 \text{ m}$$

r is the orbital radius from the Earth's centre, so subtract the Earth's radius:

Height of orbit $= 4.22... \times 10^7 - 6370 \times 10^3$

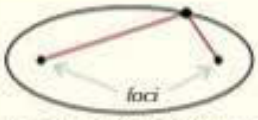
$= 3.585... \times 10^7 = 3.59 \times 10^7$ m (to 3 s.f.)

Tip: All geostationary satellites orbiting Earth are at the same height — regardless of their mass.

Kepler's three laws of planetary motion

Kepler came up with these three laws around 1600, about 80 years before Newton developed his law of gravitation. They're usually used to describe the planets in our solar system, but can be used for any object and its satellite.

Tip: An ellipse is a curve determined by two focal points (foci) inside it.



At any point on the edge of the ellipse, the total length of the red line shown is constant. In a circle, the foci are both at its centre.

- **Kepler's first law**: Each planet moves in an ellipse around the Sun, with the Sun at one focus (a circle is just a special kind of ellipse).
- **Kepler's second law**: A line joining the Sun to a planet will sweep out equal areas in equal times. So in Figure 4, if moving from A to B takes the same amount of time as moving from C to D, the two shaded sections will have equal areas.

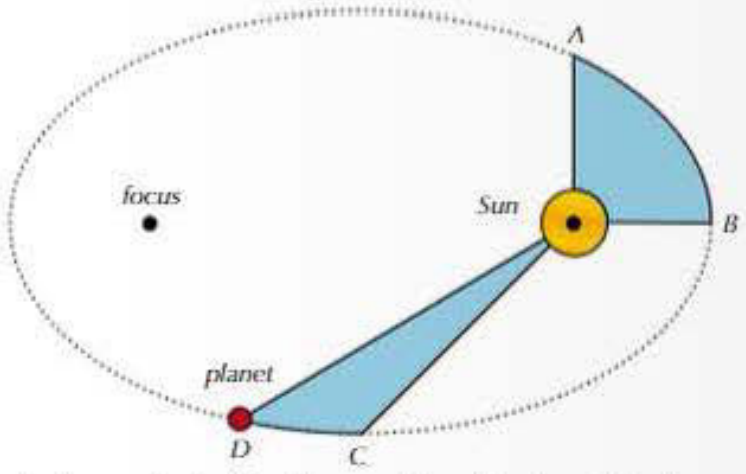


Figure 4: *A diagram showing Kepler's second law. The planet takes the same amount of time to move from A to B, as from C to D, and the shaded areas are equal.*

- **Kepler's third law**: The period of the orbit and the mean distance between the Sun and the planet are related by Kepler's third law:

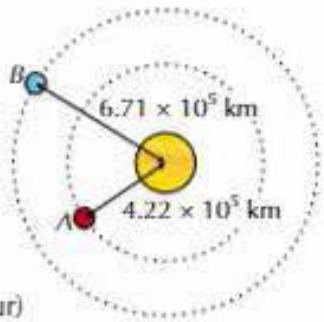
$$T^2 \propto r^3$$

Tip: You'll usually assume that orbits are circular when using this relation.

So the greater the radius of a satellite's orbit, the slower it will travel and the longer it will take to complete one orbit. You can see this relationship in the expression for the orbital period derived on the previous page.

Example — **Maths Skills**

The diagram shows the orbits of two exoplanets around a star in a nearby solar system. Exoplanet A completes one orbit of the star in 42.5 hours. Find the orbital period of exoplanet B to the nearest hour, assuming both orbits are circular.



Using Kepler's third law:

$$\frac{T_A^2}{r_A^3} = \frac{T_B^2}{r_B^3}$$

Rearranging:

$$T_B = \sqrt{\frac{T_A^2 r_B^3}{r_A^3}} = \sqrt{\frac{(42.5)^2 \times (6.71 \times 10^5)^3}{(4.22 \times 10^5)^3}}$$

$$= 85.212... = 85 \text{ hours (to the nearest hour)}$$

Tip: An exoplanet is just a name for a planet that orbits a different star to our own Sun.

Tip: You can leave the orbital period of A in hours in your calculation, as you're asked to give your answer in hours too.

Atmospheric thickness

As well as predicting the motion of satellites, Newton's law of gravitation can also help to explain how thick a planet's atmosphere is. The planet's gravitational field exerts a force on everything around it, including the particles which make up its atmosphere. Otherwise, the particles would float off into space.

The thickness of a planet's atmosphere is dependent on the gravitational force on the particles of the atmosphere around the planet's surface. You've already seen that this force is dependent on the mass of the planet and the distance from the planet (see p.298). So, for example, if two planets have the same volume but different masses, the planet with the larger mass will have a thicker atmosphere. This is because, at any distance above its surface, the gravitational force will be greater than at the same distance above the planet with the lower mass. So the larger mass planet can stop more atmosphere particles escaping into space, leading to a thicker atmosphere.

Tip: An atmosphere is the layer of gases that surround a planet or other large body.

Figure 5: *The thickness of the Earth's atmosphere is relatively tiny compared to the Earth's radius.*

Practice Questions — Application

Q1 Using the expressions $F = \frac{GMm}{r^2}$ and $F = \frac{mv^2}{r}$, derive an expression for the speed of a planet orbiting a star.

Q2 A satellite orbiting a planet has orbital period $T = 42$ hours and orbital radius $r = 3.95 \times 10^5$ km. Calculate the mass of the planet.

Q3 A satellite with orbital radius r has orbital period T. The satellite is moved so that its orbital radius is halved. What is its new orbital period in terms of T?

Practice Questions — Fact Recall

Q1 Define a satellite.

Q2 How is the orbital speed of a satellite related to the radius of its orbit?

Q3 How is the orbital period of a satellite related to the radius of its orbit?

Q4 What is a geostationary satellite and why are they useful for transmitting TV and telephone signals?

Q5 State Kepler's three laws of planetary motion.

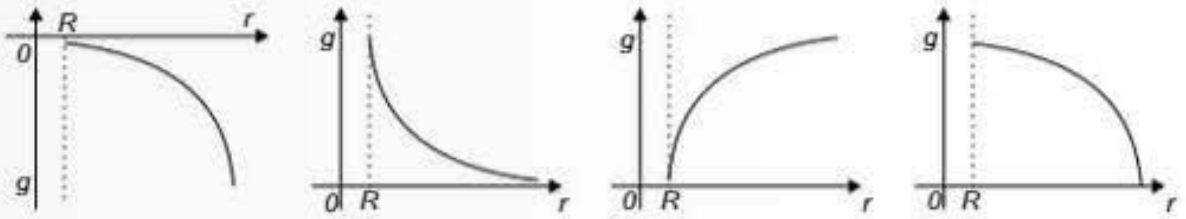
Section Summary

Make sure you know...

- That a force field is a region where an object will experience a non-contact force, and that force fields cause interactions between objects or particles.
- That gravitational fields are a type of force field that are caused by objects having mass.
- How to use gravitational field lines to map gravitational fields, including for a point mass and for a uniform field.
- That spherical objects can be treated as point masses, where all of their mass is concentrated at their centre.
- How to use Newton's law of gravitation, $\boldsymbol{F} = -\frac{GMm}{r^2}$, to calculate the force between two point masses.
- That gravitational field strength, $\boldsymbol{g}$, tells you the force per unit mass due to gravity at a point in a gravitational field.
- How to use $\boldsymbol{g} = \frac{\boldsymbol{F}}{m}$ to calculate gravitational field strength.
- That gravitational field strength is uniform close to the surface of the Earth and equal to 9.81 Nkg^{-1}.
- That $\boldsymbol{g}$ is also numerically equal to acceleration due to gravity in ms^{-2}.
- How to use $\boldsymbol{g} = -\frac{GM}{r^2}$ to calculate gravitational field strength for a point mass.
- That gravitational potential at a point is the work done in bringing a unit mass from infinity to the point, and that it is always negative, and is zero at infinity.
- How to calculate gravitational potential using $V_g = -\frac{GM}{r}$.
- That two points at different distances from a mass will have different gravitational potentials.
- That work has to be done to move an object to change its gravitational potential.
- How to sketch a force-distance graph for a point mass or spherical mass.
- That the area under the force-distance graph for a point mass or spherical mass between two distances gives the work done moving the object between those two distances.
- How to use $E = mV_g = -\frac{GMm}{r}$ to calculate the gravitational potential energy of a mass m in the gravitational field of a mass M.
- That escape velocity is the velocity needed so an object has just enough kinetic energy to escape a gravitational field, and that it can be calculated using $\boldsymbol{v} = \sqrt{\frac{2GM}{r}}$.
- That the centripetal force on a planet is provided by the gravitational force between it and the Sun.
- How to use and derive the equation $T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$, where T is the period of an orbit and r is the distance from the centre of the object being orbited to the centre of the orbiting satellite.
- That each planet moves in an ellipse around the Sun, with the Sun at one focus (Kepler's first law).
- That a line joining the Sun to a planet will sweep out equal areas in equal times (Kepler's second law).
- That the period of the orbit and the mean distance between the Sun and the planet are related by $T^2 \propto r^3$ (Kepler's third law).
- That the relationship for Kepler's third law, $T^2 \propto r^3$, can be applied to systems other than our solar system.
- That a geostationary orbit of the Earth takes exactly 24 hours, follows the same direction as the rotation of the Earth and is located directly over the equator.
- Some uses of geostationary satellites, such as for sending TV and telephone signals.

Exam-style Questions

1 Which of the following graphs shows how the magnitude of the gravitational field strength varies with distance from a spherical mass with a radius R?



(1 mark)

2 Two identical objects of mass m, are a distance of r away from each other. Which of the following is an expression for the magnitude of the force F acting on each of them?

A $\frac{Gm^2}{r^2}$ **B** $\frac{Gm}{r^2}$ **C** $\frac{Gm}{r}$ **D** $\frac{Gm^2}{r}$

(1 mark)

3 The four largest moons of Jupiter are known as the Galilean moons. They can be considered to be uniformly spherical in shape and to have regular circular orbits.

(a) The orbital speed, v, of a satellite undergoing circular motion can be calculated using:

$$v = \sqrt{\frac{GM}{r}}$$

where G is the gravitational constant, M is the mass of the object being orbited, and r is the distance from the centre of the object being orbited to the centre of the orbiting satellite.

Use the orbital speed equation to derive this expression for the orbital period:

$$T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$

(2 marks)

(b) The closest of the Galilean moons, Io, takes 1.8 days to orbit Jupiter at an average orbital radius of 420 000 km. Europa is another Galilean moon. Europa takes 3.6 days to orbit Jupiter.
Calculate Europa's average orbital radius.

(2 marks)

(c) The gravitational field strength on the surface of Io is 1.8 Nkg^{-1}.
Calculate the gravitational force that would act on a 65 kg person stood on the surface of Io, assuming the gravitational force from Jupiter is negligible.

(1 mark)

(d) Carpo is a smaller moon of Jupiter with an elliptical orbit. State what Kepler's second law of planetary motion says about the orbit of the moon Carpo.

(1 mark)

4 (a) Define a force field.

(1 mark)

(b) Sketch the gravitational field lines for the gravitational field around Earth.

(1 mark)

A student is conducting an experiment to determine the local value of ***g***.
He finds the magnitude of ***g*** to be 9.83 Nkg^{-1}, to 3 significant figures.
Another student conducts the same experiment and finds the value to be 6.31 Nkg^{-1}.
He claims his value is about 1/3 lower because his work bench is about 1/3 taller than the other student's.

(c) Explain whether the student's claim is true or not.

(1 mark)

(d) If the Earth has radius 6370 km and mass 5.97×10^{24} kg, calculate the altitude at which the local value of ***g*** is $-6.31\ Nkg^{-1}$. (Altitude is measured from sea level.)

(2 marks)

A satellite is in orbit around Earth at a gravitational potential of $-20.6\ MJkg^{-1}$.

(e) Calculate the value of ***g*** at this point.

(3 marks)

(f) Calculate the speed that the satellite would need in order to escape the gravitational field of Earth from its current position.

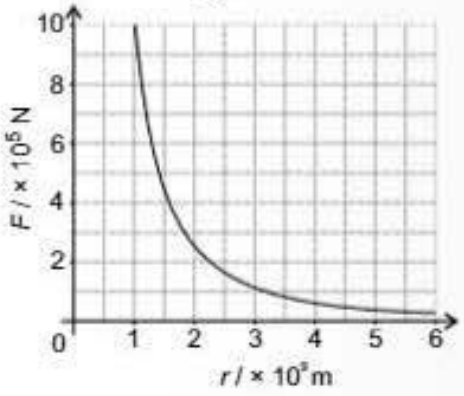
(2 marks)

(g) Another satellite orbits the Earth once every 24 hours in the same plane as the equator, and in the same direction as the Earth's rotation. Name this type of orbit.

(1 mark)

5 An asteroid passes close to a planet of mass 2.14×10^{24} kg then continues travelling away. **Fig 5.1** shows the gravitational force acting on the asteroid against its distance from the centre of the planet.

Fig 5.1



(a) Use the graph to calculate the mass of the asteroid.

(2 marks)

(b) Explain why the gravitational potential energy of the asteroid, *E*, always has a negative value at a distance, *r*, from the planet.

(2 marks)

(c) Sketch a graph of the gravitational potential energy of the asteroid as it gets further away from the planet.

(1 mark)

1. The Solar System

A solar system is made up of a star (e.g. the Sun) at the centre, all the objects that orbit around it — and everything else which orbits around them.

Learning Objectives:

- Understand the terms 'universe', 'galaxy', 'solar system', 'planet', 'planetary satellite' and 'comet'.
- Understand distances measured in astronomical units, AU.

Specification References 5.5.1 and 5.5.3

Our place in the universe

The **universe** is everything that exists — this includes plenty you can see, like **stars** and **galaxies**, and plenty that you can't see, like microwave radiation (page 336), **dark energy** and **dark matter** (pages 339-340). Galaxies, like our Milky Way galaxy, are clusters of stars and planets that are held together by gravity.

Inside the Milky Way is our **solar system**. A solar system consists of a star and all the objects that orbit it. In our solar system, the star is the Sun. The Sun is orbited by:

- **Planets** — these are large objects which orbit a star. There are eight of them orbiting the Sun. They have to be large enough to have "cleared their neighbourhood". This means that their gravity is strong enough to have pulled in any nearby objects apart from their satellites.
- **Dwarf planets** — e.g. Pluto. These are planet-like objects that orbit stars, but are too small to meet all of the rules for being a planet.
- **Planetary satellites** — these are objects that orbit a planet. For example:
  1. Moons — these orbit planets. They're natural satellites.
  2. Artificial satellites are satellites that humans have built. There are lots orbiting the Earth and some orbiting other planets.

Tip: There are a lot of other conditions that an object has to meet to be considered a planet. Size is just one of them.

Figure 1: *A photograph of the planet Jupiter. The two black spots are caused by two of Jupiter's moons passing in front of the planet.*

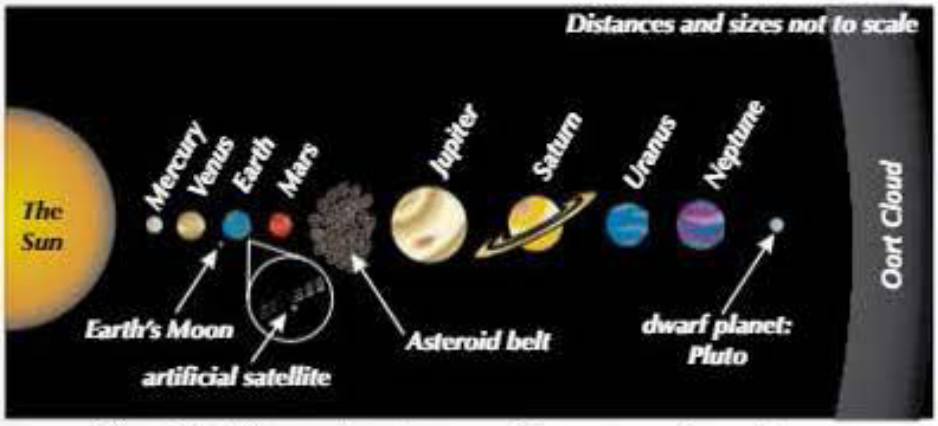


Figure 2: *A diagram showing some of the contents of our solar system, including the eight planets in order.*

- **Asteroids** — these are chunks of rock and mineral that orbit the Sun. They range in size, but can be anywhere from just over 1 m in diameter to the size of a dwarf planet. The asteroid belt is a disc of many asteroids that all sit in orbits between Mars and Jupiter.

Figure 3: *A comet seen travelling across the night sky, as it passes the Earth.*

- **Comets** — these are "dirty snowballs", made of ice, dust and rock. We think most comets usually orbit the Sun about 1000 times further away than Pluto does (in the "Oort cloud"). Occasionally one gets dislodged and heads towards the Sun. It then follows a new elliptical orbit, which can take millions of years to complete. The orbits of the comets we see are highly elliptical. Some comets (from closer in than the Oort cloud) follow a smaller orbit and swing round the Sun more often. The most famous is Halley's comet, which orbits in 76 years.

The scale of the solar system

From Copernicus onwards, astronomers have tried to work out the distances of the planets from the Sun. They could not work out the actual distances in standard units, but they could work out relative distances by comparing the orbits of other planets to the orbit of the Earth. This led to the creation of the **astronomical unit** (AU).

> One astronomical unit (AU) is defined as the mean distance between the Earth and the Sun.

The size of the AU wasn't known accurately until 1769 — when it was carefully measured during a transit of Venus (when Venus passed between the Earth and the Sun). We now know that 1 AU is equal to about 150 million km (1.5×10^{11} m). Neptune, the outermost planet in our solar system, is about 30 AU from the Sun.

Practice Questions — Fact Recall

Q1 What is a galaxy?

Q2 What is a solar system?

Q3 Where do most comets usually orbit the Sun?

Q4 State what is meant by an astronomical unit.

2. Astronomical Distances

The distances to stars can be difficult to calculate. Astronomers have several methods to work them out — but not all of the methods work for all distances. Astronomers also use a few different units to measure distance.

Learning Objectives:

- Understand stellar parallax, and how it can be used to find distances in parsecs.
- Understand distances measured in parsecs (pc).
- Be able to use the equation $p = \frac{1}{d}$ to make parallax calculations.
- Understand distances measured in light-years (ly).

Specification Reference 5.5.3

Parallax

Imagine you're in a moving car. You see that (stationary) objects in the foreground seem to be moving faster than objects in the distance. This is an example of **parallax**.

The distance to nearby stars can be calculated by observing how they appear to move relative to stars that are so distant that they appear not to move at all — background stars. This is done by comparing the position of the nearby star in relation to the background stars at different parts of the Earth's orbit.

Parallax is measured in terms of the angle of parallax. If you observe the position of the star from opposite points of the Earth's orbit (6 months apart), the angle of parallax is half the angle that the star appears to move in relation to the background stars. The greater the angle, the nearer the object is to you.

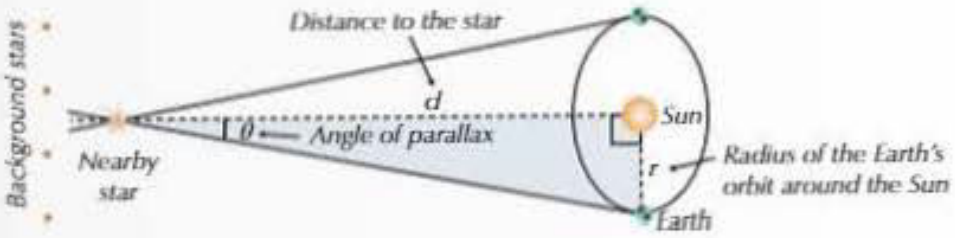


Figure 1: *The distance of a nearby star can be calculated by measuring the angle of parallax and using the diameter of the Earth's orbit.*

Using the shaded triangle in Figure 1, you can calculate the distance to the nearby star, d, using trigonometry, if you know the angle of parallax and the radius of the Earth's orbit:

$$\tan\theta = \frac{r}{d}$$

$$\Rightarrow d = \frac{r}{\tan\theta}$$

For small angles $\tan\theta \approx \theta$, where θ is in radians. Because the angles used in astronomy are so tiny, you can use this assumption for calculations of parallax (as long as you're working in radians). So:

$$d \approx \frac{r}{\theta}$$

d = distance to the star

r = radius of the Earth's orbit

θ = angle of parallax in radians

Remember, the angle in radians = angle in degrees $\times \frac{\pi}{180}$ (see page 485).

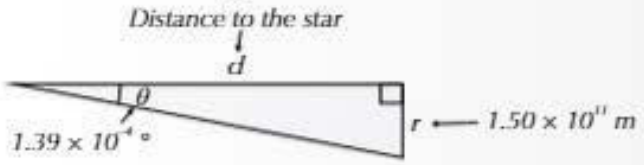
Tip: You can see parallax at work by holding your hand out in front of your face and closing just your right eye, followed by just your left eye. Your hand will move in relation to the background. The closer your hand is to your face, the more it moves in relation to the background.

Tip: Remember, trigonometry says that $\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$.

Tip: You can use any units for d and r, but you must use the same unit for both.

Example — **Maths Skills**

A nearby star has an angle of parallax of 1.39×10^{-4} °. Calculate the distance to the star. The average radius of the Earth's orbit is 1.50×10^{11} m.



Convert the angle into radians:

$$1.39 \times 10^{-4}\,^\circ \times \frac{\pi}{180} = 2.42... \times 10^{-6} \text{ rad}$$

$$d = \frac{r}{\theta} = \frac{1.50 \times 10^{11}}{2.42... \times 10^{-6}} = 6.182... \times 10^{16} \text{ m}$$
$$= 6.18 \times 10^{16} \text{ m (to 3 s.f.)}$$

Parsecs

The angle of parallax is used to define a unit of distance called a **parsec** (pc).

A star is exactly one parsec (pc) away if the angle of parallax,
$\theta = 1$ arcsecond $= \left(\frac{1}{3600}\right)^\circ$

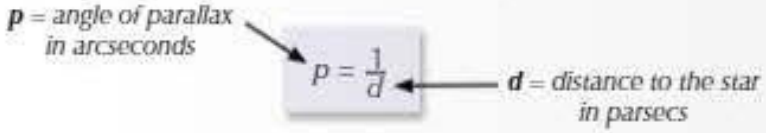
Tip: An arcsecond is just a measure of angle like degrees and radians. It's equivalent to $\left(\frac{1}{3600}\right)^\circ$, so it's useful when measuring the really small angles that astronomers use.

The distances measured in astronomy are usually huge — even the nearest large galaxy to the Milky Way is 780 000 parsecs away. Astronomers often use parsecs (pc) or megaparsecs (Mpc) to measure these large distances. You need to be able to use this conversion:

$$1 \text{ pc} = 3.1 \times 10^{16} \text{ m}$$

Exam Tip
This conversion between parsecs and metres will be in the exam data and formulae booklet, so don't worry about learning it.

When you're calculating distances in parsecs, you can use a special form of the parallax equation from the previous page:



Example — **Maths Skills**

Proxima Centauri has an angle of parallax of 0.77 arcseconds. Calculate the distance to Proxima Centauri in parsecs.

$$p = \frac{1}{d}$$

Rearrange the parsec parallax equation for distance, d:

$$d = \frac{1}{p} = \frac{1}{0.77} = 1.298... \text{ pc}$$
$$= 1.3 \text{ pc (to 2 s.f.)}$$

Light years

All electromagnetic waves travel at the speed of light, c, in a vacuum (where $c = 3.00 \times 10^8$ ms^{-1}). The distance that electromagnetic waves travel through a vacuum in one year is called a **light year** (ly).

$$1 \text{ light year} = 9.5 \times 10^{15} \text{ m}$$

If we see the light from a star that is, say, 10 light years away then we are actually seeing it as it was 10 years ago. The further away the object is, the further back in time we are actually seeing it.

Examples

- Light from the Sun takes around 8 minutes to reach Earth, so the light that we see from the Sun actually left the Sun 8 minutes earlier.
- The star Proxima Centauri is 1.3 pc away from Earth, which is approximately 4.3 light years. So the light from Proxima Centauri will take around 4 years and 4 months to reach us.

The light from very distant galaxies has taken billions of years to reach us. Astronomers search for distant galaxies so that they can 'look into the past' at what the Universe was like billions of years ago.

Practice Questions — Application

Q1 The star Alpha Tauri in the constellation Taurus is 20 pc away. Calculate this distance in light years.

Q2 Scientists at an observatory are studying the star Sirius from Earth. One image of Sirius is captured, and another is captured 6 months later. The angle of parallax is recorded to be 0.37 arcseconds.

a) The average radius of the Earth's orbit is 1.50×10^{11} m. Calculate the distance to Sirius in metres.

b) How long will it take for light from Sirius to reach Earth in years and months?

Practice Questions — Fact Recall

Q1 What is meant by the angle of parallax?

Q2 What is the definition of a parsec?

Q3 What is the definition of a light year?

Learning Objectives:

- Know that a star is formed from interstellar dust and gas.
- Understand how gravitational collapse, nuclear fusion and a balance between gravitational forces and radiation and gas pressure lead to the creation of a stable main sequence star.
- Understand how low-mass stars, like our Sun, evolve into red giants.
- Know that a low-mass red giant will collapse into a white dwarf and a planetary nebula.
- Understand that a star core with mass less than the Chandrasekhar limit becomes a white dwarf because electron degeneracy pressure prevents further collapse.
- Know that stars much more massive than the Sun evolve into red super giants, which will eventually explode in a supernova leaving behind either a neutron star or black hole.
- Know the characteristics of neutron stars and black holes.

Specification Reference 5.5.1

Tip: As a star contracts, the temperature increases due to conservation of energy — gravitational potential energy is converted to thermal energy.

3. Stellar Evolution

'Stellar' means 'to do with stars', so this topic's all about how stars change. Stars go through several different stages in their lives. The Sun is a main sequence star right now, but it's still got a lot of life left in it.

Formation

All stars are born in a cloud of dust and gas, most of which was left when previous stars blew themselves apart in supernovae (see page 322). The denser clumps of the cloud contract (very slowly) under the force of gravity.

When these clumps get dense enough, the cloud fragments into regions called **protostars** that continue to contract, and heat up as they do. Eventually the temperature at the centre of the protostar reaches a few million degrees, and hydrogen nuclei start to fuse together to form helium (see page 445).

As the star's temperature increases and its volume decreases (remember, it's contracting), the gas pressure increases (p.262). There is also radiation pressure in the star — a pressure exerted by electromagnetic radiation on any surface it hits. Radiation pressure is usually too tiny to notice, but becomes significant in stars because of the enormous amount of electromagnetic radiation released by fusion. The combination of gas pressure and radiation pressure counteract the force of gravity, preventing the star from contracting further. The star has now reached the **main sequence** and will stay there, relatively unchanged, while it fuses hydrogen into helium.

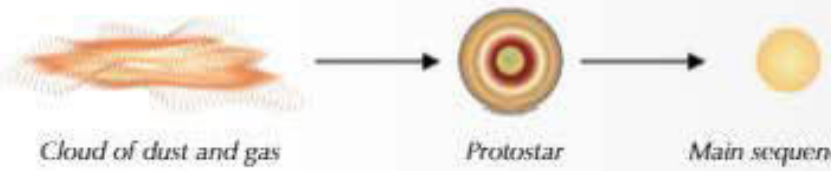


Figure 1: *A star in the early stages of its stellar evolution.*

Core and shell burning sequence

Stars spend most of their lives as main sequence stars. The pressure produced from hydrogen fusion in their core balances the gravitational force trying to compress them. This stage is called core hydrogen burning.

When all the hydrogen in the core has fused into helium, fusion stops, meaning the outward pressure also stops. The helium core contracts and heats up under the weight of the star. As a result, the outer layers expand and cool, and the star becomes a **red giant**.

The material surrounding the core still has plenty of hydrogen. The heat from the contracting helium core raises the temperature of this material enough for the hydrogen to fuse. This is called shell hydrogen burning. (Very low-mass stars stop at this point. They use up their fuel and slowly fade away...)

The helium core continues to contract until, eventually, it gets hot enough and dense enough for helium to fuse into carbon and oxygen. This is called core helium burning. This releases a huge amount of energy, which pushes the outer layers of the star further outwards.

When the core helium runs out, the carbon-oxygen core contracts again and heats a shell around it so that helium can fuse in this region — shell helium burning.

Figure 3 shows a summary of the stages a star goes through during the main sequence and red giant phases.

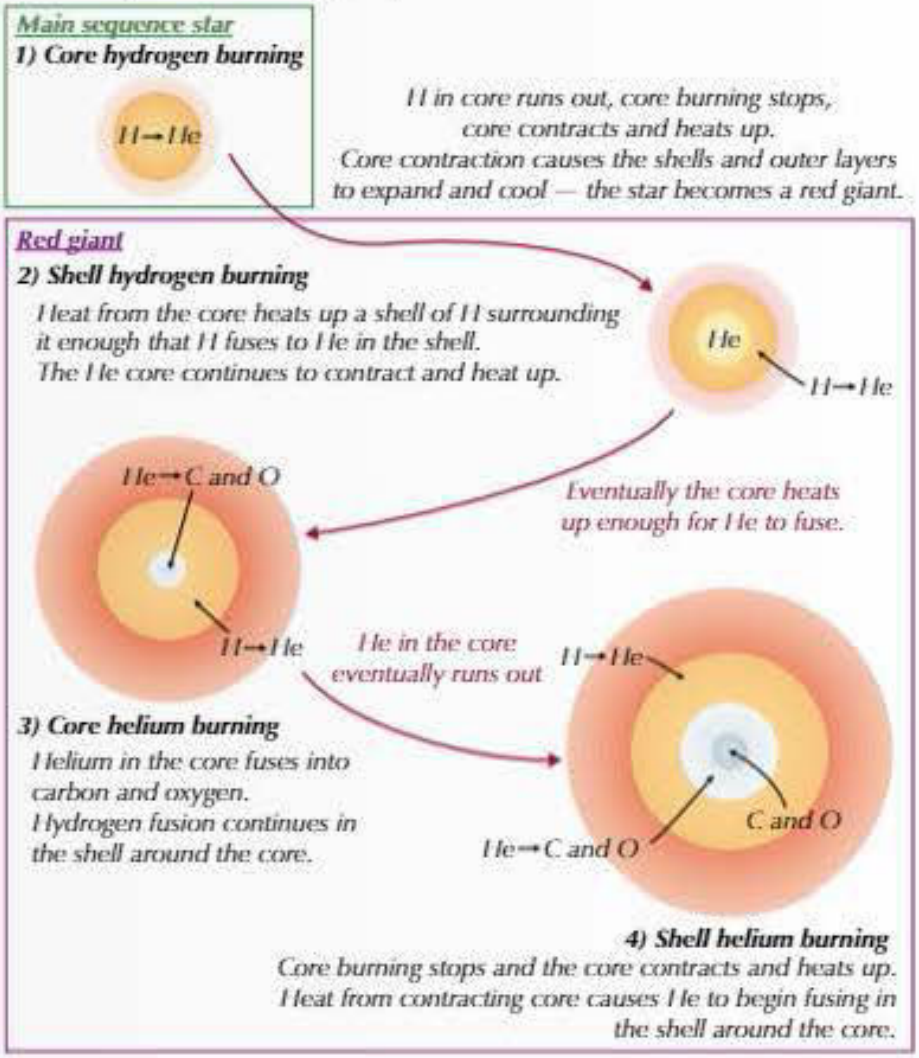

Main sequence star
1) Core hydrogen burning
H→He
H in core runs out, core burning stops, core contracts and heats up.
Core contraction causes the shells and outer layers to expand and cool — the star becomes a red giant.
Red giant
2) Shell hydrogen burning
Heat from the core heats up a shell of H surrounding it enough that H fuses to He in the shell.
The He core continues to contract and heat up.
He
H→He
He→C and O
Eventually the core heats up enough for He to fuse.
H→He
He in the core eventually runs out
H→He
3) Core helium burning
Helium in the core fuses into carbon and oxygen.
Hydrogen fusion continues in the shell around the core.
C and O
He→C and O
4) Shell helium burning
Core burning stops and the core contracts and heats up.
Heat from contracting core causes He to begin fusing in the shell around the core.

Figure 3: *The helium and hydrogen burning sequence of a star as it transitions from a main sequence star to a red giant.*

Tip: This process is just a load of contracting, heating and fusing of successively bigger nuclei.

Figure 2: *A red giant with a radius of around 1 billion km.*

Tip: The cooling of the outer layers of the star makes the star's colour change to become redder — this is why we call them red giants.

White dwarfs

In low-mass stars, like our Sun, the carbon-oxygen core won't get hot enough for any further fusion and so it continues to contract under its own weight. Once the core has shrunk to about the size of the Earth, electrons exert enough pressure (**electron degeneracy pressure**) to stop it collapsing any more (don't worry — you don't have to know how).

The above is only true for stars with a core mass under about 1.4 times the mass of the Sun though — in bigger stars the electron degeneracy pressure isn't high enough to counteract the gravitational force and the star collapses (see next page). The maximum mass for which the electron degeneracy pressure can counteract the gravitational force is called the **Chandrasekhar limit**.

For stars below the Chandrasekhar limit, the helium shell becomes more and more unstable as the core contracts. The star pulsates and ejects its outer layers into space as a **planetary nebula**, leaving behind the dense core. The star is now a very hot, dense solid called a **white dwarf**, which will simply cool down and fade away.

Figure 4: *When a low-mass red giant cools down, the shells are ejected and create a beautiful planetary nebula, like this one, leaving a white dwarf at the centre.*

Tip: The term 'planetary nebula' doesn't actually have anything to do with planets. They were called that because they tend to be roughly spherical, and so look like enormous planets.

Supernovae

High-mass stars have a shorter life and a more exciting death than lower-mass stars like the Sun. Even though stars with a large mass have a lot of fuel, they use it up more quickly and don't spend so long as main-sequence stars.

When they are red giants the 'core burning to shell burning' process can continue beyond the fusion of helium, building up layers of different fusing elements in an onion-like structure, to become **red super giants** (or super red giants). For really massive stars this can go all the way up to iron. Nuclear fusion beyond iron isn't energetically favourable (p.447), so once an iron core is formed then very quickly it's goodbye star...

When the core of a star runs out of fuel, it starts to contract. If this core's mass is larger than the Chandrasekhar limit, electron degeneracy can't stop the core contracting. This happens when the mass of the core is more than 1.4 times the mass of the Sun.

The core of the star continues to contract, and as it does, the outer layers of the star fall in and rebound off the core, setting up huge shockwaves. These shockwaves cause the star to explode cataclysmically in a **supernova**, leaving behind the core, which will either be a **neutron star** or (if the star was massive enough) a **black hole**.

Tip: Supernovae release a huge amount of energy — a supernova even as far as 1000 ly away could seriously damage the Earth's atmosphere.

Figure 6: *An optical image of the Tarantula Nebula (top left) and a nearby supernova (bottom right). The Tarantula Nebula is an incredibly bright region, where many stars are being formed, but the supernova is able to reach a comparable level of brightness.*

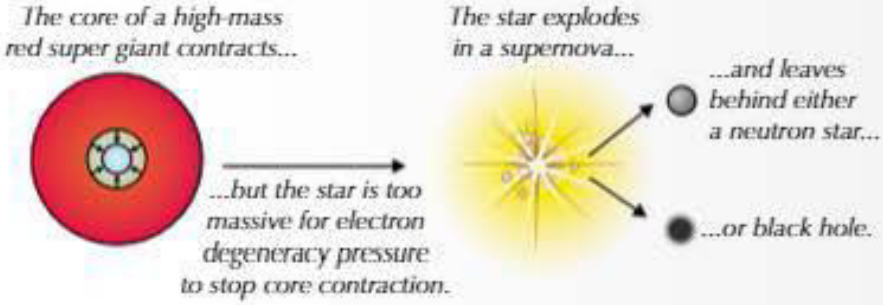

The core of a high-mass red super giant contracts...
...but the star is too massive for electron degeneracy pressure to stop core contraction.
The star explodes in a supernova...
...and leaves behind either a neutron star...
...or black hole.

Figure 5: *The possible evolution paths of a high-mass star from its red super giant phase.*

When the star explodes in a supernova, it will experience a brief and rapid increase in brightness. The light from a supernova can briefly outshine an entire galaxy, before fading over the next few weeks or months.

Neutron stars

As the core of a massive star contracts, the electrons in the core material get squashed onto the atomic nuclei and combine with protons to form neutrons and neutrinos (hence the name neutron star).

If the star's core is between 1.4 and 3 solar masses, this is as far as the star can contract — the core suddenly collapses to become a neutron star made mostly of neutrons. The outer layers of the star fall onto the neutron star, which causes shockwaves in these layers and leads to a supernova. After the supernova, the neutron star is left behind.

Neutron stars are incredibly dense (about 4×10^{17} kg m^{-3}). They're also very small, typically about 20 km across, and they can rotate very fast (up to 600 times a second).

Neutron starts emit two beams of radio waves as they rotate. These beams sometimes sweep past the Earth and can be observed as radio pulses rather like the flashes of a lighthouse. These pulsing neutron stars are called pulsars.

Figure 7: *The Crab Nebula — a supernova remnant with a pulsar (rotating neutron star) in the centre.*

Black holes

If the core of the star is more than 3 times the Sun's mass, the core will contract until neutrons are formed, but now the gravitational force on the core is greater. The neutrons can't withstand this gravitational force, so the star continues to collapse. For something of this size, there are no known mechanisms left to stop the core collapsing to an infinitely dense point, called a singularity. At that point, the laws of physics break down completely.

The **escape velocity** (p.306) is the velocity that an object would need to travel at to have enough kinetic energy to escape a gravitational field. When a massive star collapses into an infinitely dense point, a region around it has such a strong gravitational field that it becomes a black hole — an object whose escape velocity is greater than the speed of light, c. If you enter this region, there's absolutely no escape — not even light can escape it.

The boundary of this region is called the **event horizon**. At the event horizon, the escape velocity is equal to c, so light has just enough kinetic energy to escape the black hole's gravitational pull. The radius of a black hole is considered to be the radius of the event horizon. Past that point, everything, including light, can do nothing but travel further into the black hole.

Tip: Remember, the kinetic energy of an object is given by $E_k = \frac{1}{2}mv^2$.

Tip: Nothing can travel faster than the speed of light, c.

Inside the event horizon, nothing escapes as the escape velocity > c
Infinitely dense point
At the event horizon, escape velocity = c

Figure 8: *The structure of a black hole.*

Figure 9: *Astronomers think the intense radiation seen at the centre of galaxies is caused by matter falling into a supermassive black hole.*

Astronomers now believe that there is a supermassive black hole (more than 10^6 times more massive than our Sun) at the centre of every galaxy. As stars get closer to the event horizon of the black hole, and begin to be consumed, they get very hot and produce intense radiation, making the centre of galaxies very bright.

Practice Questions — Fact Recall

Q1 What causes a cloud of dust and gas to contract and form a star?

Q2 In what stage of evolution does a star spend most of its life?

Q3 Explain how the process of core hydrogen burning stops a main sequence star from compressing under the gravitational force.

Q4 What happens to the core of a star when it runs out of hydrogen? What effect does this have on the hydrogen layer surrounding the core of the star?

Q5 What causes a main-sequence star to become a red giant?

Q6 How is a white dwarf formed?

Q7 What is a planetary nebula?

Q8 Why can stars with a core mass greater than 1.4 solar masses not form white dwarfs?

Q9 Describe how the contracting core of a star creates a supernova.

Q10 What is left after a supernova if the core of the star is not massive enough to form a black hole? What are they mostly made of?

Q11 What is a black hole? How is one formed?

Learning Objectives:

- Know how Wien's displacement law can be used to estimate the peak surface temperature of a star.
- Know what is meant by the luminosity of a star.
- Be able to use Stefan's law to calculate luminosity.
- Be able to use Wien's displacement law and Stefan's law to estimate the radius of a star.
- Know that the Hertzsprung-Russell (HR) diagram is a plot of luminosity against temperature for stars.
- Be able to identify the regions of the HR diagram that correspond to main sequence stars, red giants, super red giants and white dwarfs.

Specification References 5.5.1 and 5.5.2

4. Stellar Radiation and Luminosity

Astronomers can estimate a star's temperature based on the radiation it emits. They can then use this information to help identify what type of star it is.

Wien's displacement law

Objects emit electromagnetic radiation due to their temperature. At everyday temperatures this is mostly in the infrared part of the spectrum (which we can't see). But heat something up enough and it will start to glow.

Stars can be assumed to emit radiation in a continuous spectrum. The relationship between intensity ('power per unit area') and wavelength for this radiation varies with temperature. As shown in Figure 1, a hotter star will emit more radiation than a cooler one, and so have a higher total power output (assuming it has the same surface area). The most common wavelength of EM wave emitted becomes shorter as the surface temperature of the star increases. This is called the peak wavelength, λ_{max}, shown by the dotted lines on Figure 1.

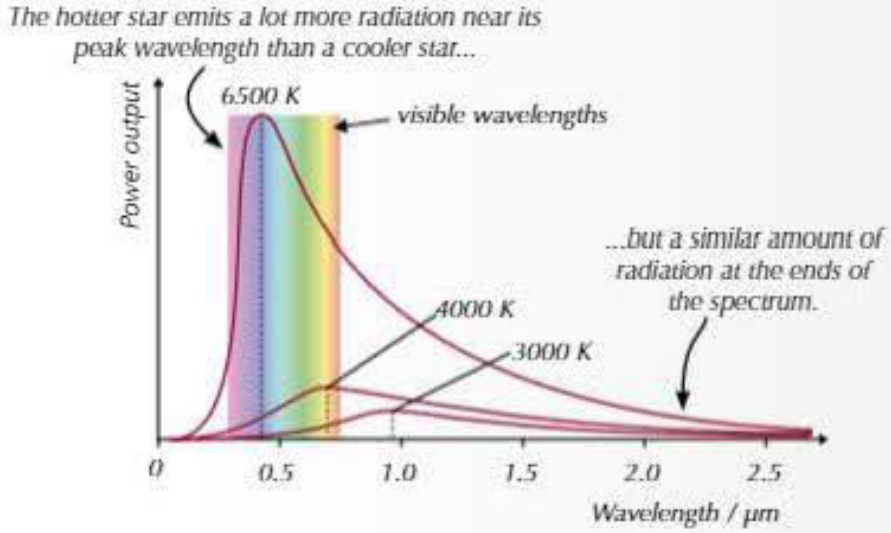


Figure 1: *Three curves showing the different range and intensity of wavelengths of radiation emitted by stars of different temperatures.*

You can use λ_{max} to estimate a star's peak surface temperature using **Wien's displacement law**:

λ_{max} = peak wavelength in m

$$\lambda_{max} \propto \frac{1}{T}$$

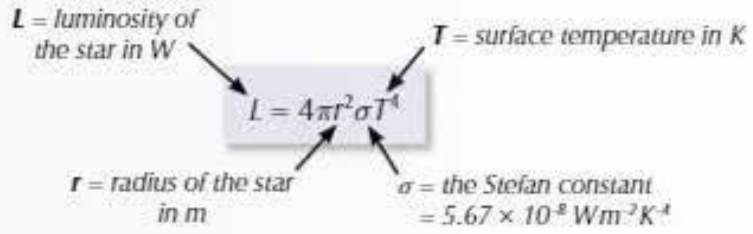
T = temperature in K

We can use this to estimate the temperature of stars from their colour. A star that appears red will emit light in the red part of the visible spectrum with the highest intensity. From the shape of the intensity-temperature curve, this means its peak wavelength can be no smaller than the red part of the visible spectrum. A star that appears white will emit all frequencies of visible light with a similar power output. This means they must have a shorter peak wavelength, around the middle of the EM spectrum. So a white star must have a higher temperature than a red star.

Figure 2: *Wilhelm Wien was a German physicist who was awarded the 1911 Nobel Prize in physics for his work on radiation emission.*

Stefan's law

The **luminosity** of a star is a measure of its brightness. It is the total energy it emits per second, i.e. its power output. The luminosity of a star is directly proportional to the fourth power of the star's surface temperature and is also directly proportional to the surface area. This is **Stefan's law**:


L = luminosity of the star in W
T = surface temperature in K
$L = 4\pi r^2 \sigma T^4$
r = radius of the star in m
σ = the Stefan constant = 5.67 × 10⁻⁸ Wm⁻²K⁻⁴

Tip: The surface area of a sphere is given by $4\pi r^2$. Since stars are spherical, this is the surface area of a star.

Exam Tip
Stefan's law and the Stefan constant are given in the data and formulae booklet.

Example — **Maths Skills**

The Sun has a surface temperature of 5800 K, and a radius of 6.96×10^8 m. What surface temperature would a star have if it had a radius of 9.42×10^7 m but the same luminosity as the Sun?

$L = 4\pi r^2 \sigma T^4$. The luminosities of the star and the Sun are the same, so:

$$4\pi(r_{Sun})^2\sigma(T_{Sun})^4 = 4\pi(r_{star})^2\sigma(T_{star})^4$$

$$\Rightarrow T_{star} = \sqrt[4]{\frac{(r_{Sun})^2(T_{Sun})^4}{(r_{star})^2}} \quad \textit{(cancelling } 4\pi\sigma \textit{ from both sides and rearranging)}$$

$$\Rightarrow T_{star} = \sqrt[4]{\frac{(6.96 \times 10^8)^2 \times (5800)^4}{(9.42 \times 10^7)^2}}$$

$$= 15\,765.47... = 16\,000 \text{ K (to 2 s.f.)}$$

Tip: A smaller star needs to be a lot hotter to have the same power output.

You can combine Stefan's law and Wien's displacement law (see previous page) to estimate a star's radius:

Example — **Maths Skills**

Sirius A is a main sequence star, and Sirius B is a white dwarf. Sirius A has a surface temperature of 9800 K and produces electromagnetic radiation with a peak wavelength of 300 nm (to 2 s.f.). Sirius B has a luminosity of 9.4×10^{24} W and produces electromagnetic radiation with a peak wavelength of 115 nm. Estimate the radius of Sirius B.

First, find the temperature of Sirius B. $\lambda_{max} \propto \frac{1}{T}$, so $T\lambda_{max}$ = constant.

This means $T_A \lambda_{max\,A} = T_B \lambda_{max\,B}$ so $9800 \times 300 \times 10^{-9} = (115 \times 10^{-9})T_B$

So $T_B = \frac{9800 \times (300 \times 10^{-9})}{115 \times 10^{-9}} = 25\,565.2...$ K

Then use Stefan's law to find the star's radius:

$$L = 4\pi r^2 \sigma T^4 \text{ so } r = \sqrt{\frac{L}{4\pi\sigma T^4}}$$

$$= \sqrt{\frac{9.4 \times 10^{24}}{4\pi \times (5.67 \times 10^{-8}) \times (25\,565.2...)^4}}$$

$$= 5.557... \times 10^6 \text{ m} = 5.6 \times 10^6 \text{ m (to 2 s.f.)}$$

The Hertzsprung-Russell diagram

You can plot a graph of luminosity against temperature for stars. In the 1910s, this was done with data from a survey of a large number of stars. Independently, two scientists, Hertzsprung and Russell, noticed that the graph didn't just throw up a random scattering of points, but showed points clustered in distinct areas. These distinct areas turned out to correspond to different stages of a star's life cycle.

This diagram ended up being really important for studying how stars evolve (p.320-323) and became known as the **Hertzsprung-Russell (HR) diagram** (Figure 3).

Tip: A 'solar luminosity unit' is just the luminosity of the Sun. So, a star with a luminosity of 10^2 luminosity units will be 100 times more luminous than the Sun.

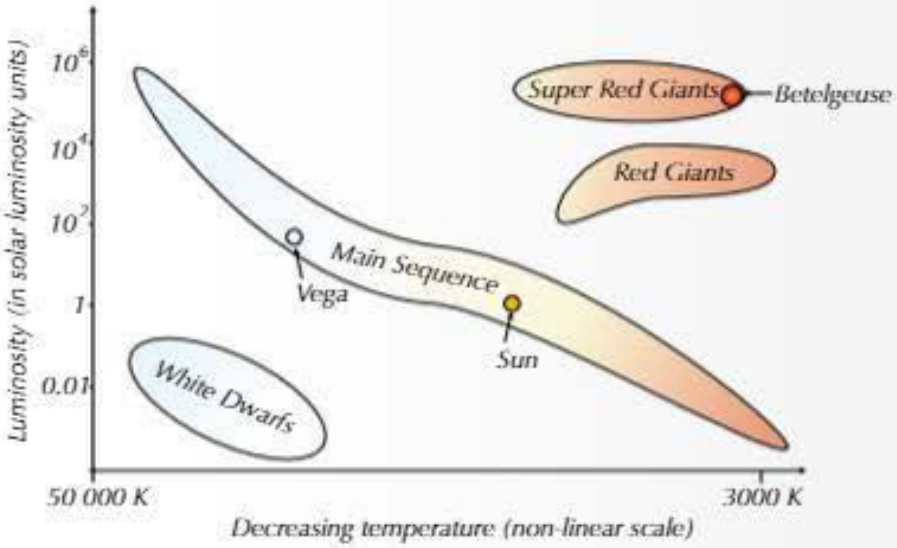


Figure 3: *The Hertzsprung-Russell diagram with the positions of the Sun and the stars Vega and Betelgeuse labelled.*

Tip: Be careful with the axes on an HR diagram. The horizontal axis goes from high temperatures to low temperatures from left to right.

Distinct areas of the HR diagram

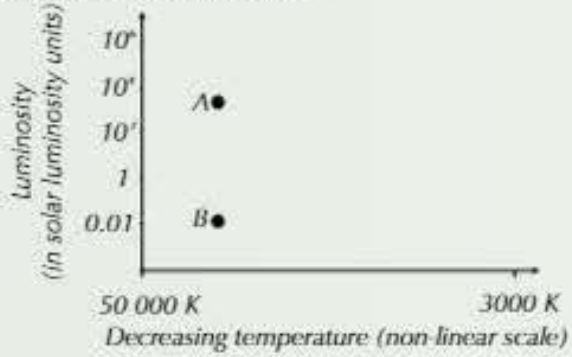
The distinct areas in which stars fall on the HR diagram correspond to four main types of stars:

- There is a long, diagonal band along the middle of the graph, where luminosity increases with temperature. This corresponds to main sequence stars.
- There is a section with high luminosity (10^2-10^4), but relatively low surface temperature. From Wien's displacement law, we can tell these stars should appear reddish. And from Stefan's law we know they must be very large to produce the same luminosity as much hotter main sequence stars. Therefore, this section must correspond to red giants.
- There is another section, at similar temperatures to the red giants, but even higher luminosity (~10^6). These stars must be even bigger than red giants, and so this section corresponds to super red giants.
- There is a fourth section with very high temperatures, but low luminosity. From Stefan's law, we know they must be small to be so hot while only being as luminous as low temperature main sequence stars. And from Wien's displacement law we can tell these stars should appear white. So this section corresponds to white dwarfs.

Tip: In several billion years, the Sun will become a red giant (see pages 320-321). It will expand to be around 20% larger than the Earth's current orbit and shine 3000 times brighter than it does now.

Practice Questions — Application

Q1 The diagram below shows two stars, A and B, marked on a luminosity-temperature graph.



a) Which star you would expect to be bigger, and why?

b) What type of star is Star A?

c) What type of star is Star B?

d) Which star is further along in its evolution sequence?

Q2 The star Rigel, in the constellation Orion, is measured to have a surface temperature of 12 000 K. The Sun has a surface temperature of 5800 K.

a) Suggest how Rigel's peak wavelength will differ to that of the Sun.

b) The luminosity of Rigel is measured as 4.5×10^{31} W. Calculate the radius of Rigel.

Practice Questions — Fact Recall

Q1 Which would you expect to be hotter, a white star or a red star?

Q2 Write down Stefan's law. Define all the symbols you use.

Q3 Sketch the HR diagram. Mark on your diagram the regions corresponding to the main sequence, red giants, super red giants and white dwarfs. Make sure you label both your axes.

Learning Objectives:

- Understand how a transmission diffraction grating can be used to determine the wavelength of light (PAG5).
- Know and understand that $d\sin\theta = n\lambda$ is the condition for maxima in an interference pattern produced by a diffraction grating with spacing d.
- Know that electrons in isolated gas atoms orbit the nucleus in distinct energy levels.
- Know that electron energy levels are given negative values by convention.
- Be able to use the equations $\Delta E = hf$ and $\Delta E = \frac{hc}{\lambda}$ to calculate the energy of emitted photons.
- Understand how the emission of photons by electrons moving between energy levels leads to the creation of emission spectra.
- Understand what is meant by a continuous spectrum and an absorption line spectrum.
- Understand how the spectral lines of different atoms can be used to identify elements within stars.

Specification Reference 5.5.2

5. Stellar Spectra

One way astronomers analyse stars is by splitting the light received from them using a diffraction grating. By comparing the spectrum produced to the emission spectra of elements, they can work out what a star is made of.

Diffraction gratings

PRACTICAL ACTIVITY GROUP 5

You can pass light through a **transmission diffraction grating** to form an interference pattern. Since different wavelengths of light diffract differently, you can use this interference pattern to analyse the light, and determine some of its properties. As always, before you start any investigation, make sure you carry out a risk assessment.

Determining wavelength

If you shine monochromatic light (light with a single wavelength or frequency) through a diffraction grating, you'll get a pattern of bright lines (maxima) on a dark background. This is a result of the light interfering with itself constructively and destructively (you met interference in Section 2 of Module 4).

The line of maximum brightness at the centre is called the zero order line, the next lines on each side are called first order lines, and so on.

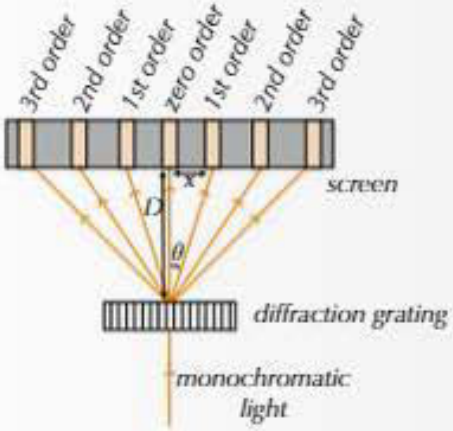


Figure 1: *A diagram of light passing through a diffraction grating and forming an interference pattern.*

Using the fringe width (the distance between the maxima), x, and the distance to the screen, D, the angle the first order line makes with the zero order line can be calculated using the small angle approximation:

θ = angle between the zero and first order fringes in radians → $\theta = \tan\theta = \frac{x}{D}$ ← x = fringe width in m; D = distance to the screen in m

If you know the slit separation, d, the order of the maximum you're observing, n, and the angle between this maximum and the incident light, θ, you can find the wavelength of the incident light:

d = slit spacing in m → $d\sin\theta = n\lambda$ ← λ = wavelength of incident light in m

θ = angle between the zero and nth order fringes in radians → $d\sin\theta = n\lambda$ ← n = number of fringe order

Example — **Maths Skills**

A laser beam is passed through a diffraction grating with grating spacing of 2.0×10^{-5} m. The interference pattern it produces is displayed on a screen 1.6 m away. The first order fringe is located 0.05 m away from the zero order fringe. Calculate the wavelength of the laser light.

First, calculate θ: $\tan\theta = \frac{0.05}{1.6} \Rightarrow \theta = \frac{0.05}{1.6} = 0.03125$

Then find λ, when $n = 1$: $\lambda = d\sin\theta = 2.0 \times 10^{-5} \times \sin(0.03125)$

$= 6.24898... \times 10^{-7}$ m $= 6.2 \times 10^{-7}$ m (to 2 s.f.)

White light and diffraction gratings

White light is a continuous spectrum of all the wavelengths of visible light. If you pass white light through a diffraction grating, it will be split up, as the different wavelengths within the white light are diffracted by different amounts.

Each order in the pattern becomes a spectrum, with red on the outside and violet on the inside, as shown in Figure 2. The zero order maximum stays white because all the wavelengths produce a maximum when $\theta = 0$.

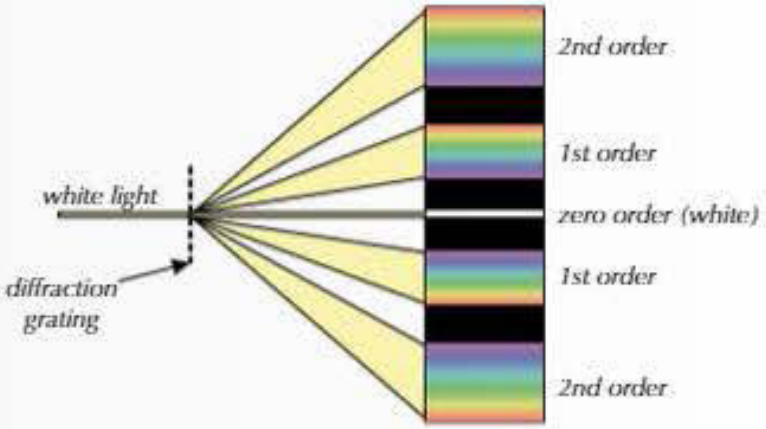


Figure 2: *White light being split into spectra as it passes through a diffraction grating.*

Electron energy levels and photons

Electrons in an isolated gas atom can only exist in certain well-defined energy levels. Each level is given a number, with $n = 1$ representing the ground state (i.e. the lowest energy level).

When energy is transferred to a substance (e.g. by heating), electrons in its atoms can be 'excited' — they move to higher energy levels. Electrons can move back down energy levels by emitting a photon. Since these transitions are between discrete energy levels, the energy of each photon emitted can only take a certain allowed value.

Figure 3 shows the energy levels for atomic hydrogen. The energies are all negative because of how 'zero energy' is defined. The energy carried by each photon emitted is equal to the difference in energies between the two levels. For example, a photon produced by an electron moving from the $n = 3$ to the $n = 2$ level would have energy equal to $\Delta E = -1.5 - (-3.4) = 1.9$ eV.

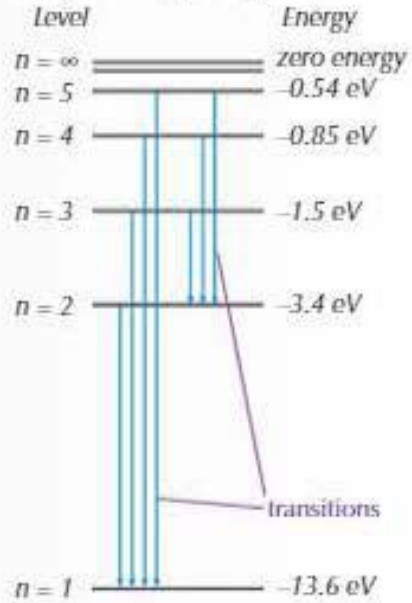


Figure 3: *The electron energy levels in atomic hydrogen.*

Electrons can also move up energy levels (be excited) by absorbing photons. For an electron to absorb a photon and become excited, the photon must be carrying an amount of energy that exactly matches the difference between the electron's current energy level and a higher energy level.

Tip: When we say 'isolated gas atom', we mean an atom of a gas that is not interacting in any way with the other atoms of the gas.

Tip: Remember, an electron volt, eV, is equal to 1.60×10^{-19} J.

You can relate the energy change of an electron in an atom to the frequency of the photon it absorbs or emits using the following equation:

$$\Delta E = hf$$

ΔE = energy carried by photon (or change in electron energy) in J

f = frequency of the photon in Hz

h = Planck's constant in Js = 6.63×10^{-34} Js

Tip: This uses the fact that $c = f\lambda$.

You can also write this in terms of wavelength:

$$\Delta E = \frac{hc}{\lambda}$$

c = speed of light in ms^{-1} = 3×10^{8} ms^{-1}

λ = wavelength of the photon in m

Examples — Maths Skills

An electron in the ground state of a hydrogen atom ($E = -13.6$ eV) absorbs a photon and is excited to a higher energy level with an energy of –0.850 eV. Calculate the frequency of the absorbed photon.

Tip: You'll usually be given electron energy level values in eV. Make sure that you convert them to J before you try and use any energy values in these equations.

First, calculate ΔE, and convert it to J.

$\Delta E = (-0.850 - (-13.6)) = 12.75$ eV $= (12.75 \times 1.6 \times 10^{-19}) = 2.04 \times 10^{-18}$ J

Rearranging the equation for frequency:

$$f = \frac{\Delta E}{h} = \frac{2.04 \times 10^{-18}}{6.63 \times 10^{-34}} = 3.07692... \times 10^{15} \text{ Hz} = 3.08 \times 10^{15} \text{ Hz (to 3 s.f.)}$$

The electron emits a photon, and drops from the –0.850 eV energy level to an energy level with energy –3.40 eV. Calculate the wavelength of the photon emitted in this transition.

$\Delta E = (-0.850 - (-3.40)) = 2.55$ eV $= (2.55 \times 1.6 \times 10^{-19}) = 4.08 \times 10^{-19}$ J

Rearranging the energy equation for wavelength:

$$\lambda = \frac{hc}{\Delta E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{4.08 \times 10^{-19}} = 4.875 \times 10^{-7} \text{ m} = 4.88 \times 10^{-7} \text{ m (to 3 s.f.)}$$

Emission line spectra

Tip: You can assume that all the atoms in these gases are isolated.

If you heat a gas to a high temperature, many of its electrons move to higher energy levels — the gas is said to be 'excited'. As they fall back to the ground state, these electrons emit energy as photons. If you split the light from a hot gas with a diffraction grating, you get an **emission line spectrum** — see Figure 4.

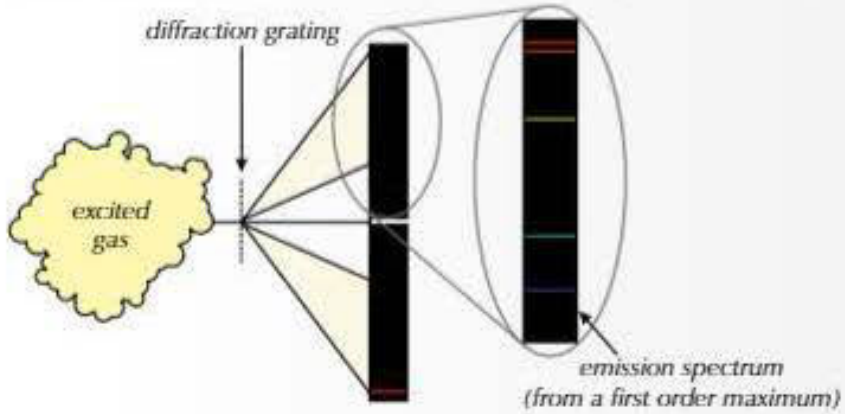


Figure 4: *A line emission spectrum, produced by passing light emitted by a hot gas through a diffraction grating.*

Each line on the spectrum corresponds to a particular wavelength of light emitted by electrons in the gas as they drop down energy levels. Since only certain photon energies are allowed, you only see the corresponding wavelengths. You can calculate the wavelength, λ, of each line in a line emission spectrum using $d\sin\theta = n\lambda$ (see page 328), if you know which order maximum the spectrum is.

Different atoms have different electron energy levels and so different sets of emission spectra. This means you can identify a gas from its emission spectrum.

Absorption line spectra

You get an **absorption line spectrum** when light with a continuous spectrum of energy (white light) passes through a cool gas:

- At low temperatures, most of the electrons in the gas atoms will be in their ground states.
- Photons of the correct wavelength are absorbed by the electrons to excite them to higher energy levels.
- These wavelengths are missing from the continuous spectrum when it comes out on the other side.
- You see a continuous spectrum with black lines in it corresponding to the absorbed wavelengths — see Figure 5.

Tip: Remember, the change in energy of an electron that absorbs a photon is the same change as if it had emitted a photon of the same frequency, but in reverse.

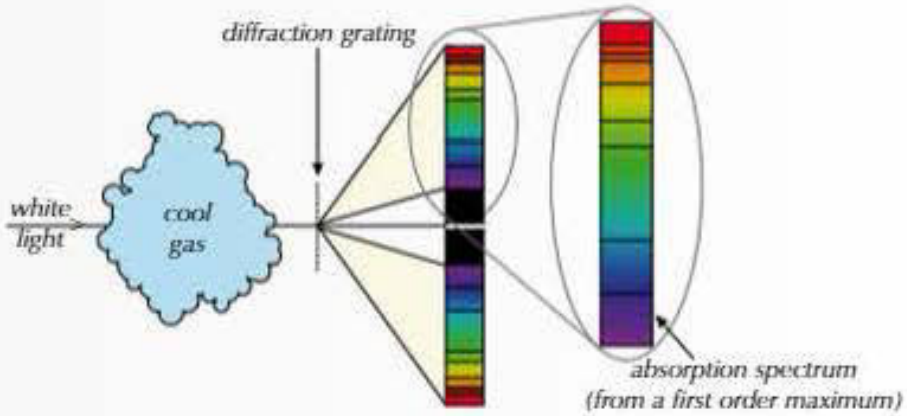


Figure 5: *An absorption spectrum, produced by passing white light through a cold gas and then through a diffraction grating.*

Tip: You can use $d\sin\theta = n\lambda$ to find the wavelength of absorption lines too.

If you compare the absorption and emission spectra of a particular gas, the black lines in the absorption spectrum match up to the bright lines in the emission spectrum — as seen in Figure 6.

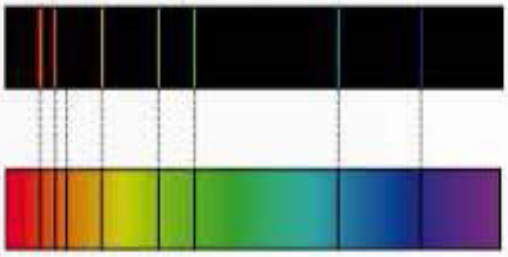


Figure 6: *An emission spectrum and absorption spectrum of the same substance. The lines in the emission spectrum correspond to the missing parts of the full spectrum in the absorption spectrum.*

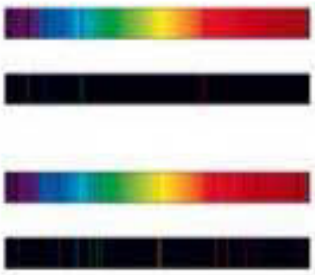


Figure 7: *The absorption and emission spectra of hydrogen (top) and helium (bottom).*

You get absorption lines in the spectra of light from stars. Stars can be assumed to emit radiation in a continuous spectrum. This radiation has to pass through a large amount of gas at the surface of the star (the star's 'atmosphere') before travelling to Earth. This gas absorbs particular wavelengths of light depending on the elements it consists of.

Comparing the absorption spectra of stars to sets of emission spectra from the lab therefore allows you identify elements within a star. The most common element in most stars is hydrogen, so the spectral lines for hydrogen are usually the clearest. This makes these lines the easiest to identify and measure.

Practice Questions — Application

Q1 The diagram below shows three line emission spectra produced by elements in a lab, and an absorption spectrum of a star.

Which element, A, B or C, is not present in the star?

Q2 An electron in a hydrogen atom emits a photon as it drops from the –3.4 eV energy level to the –13.6 eV level. Calculate the frequency of the emitted photon.

Q3 An electron in an atom absorbs a photon with a wavelength of 3.60×10^{-7} m, and moves to a –2.00 eV energy level. Find the energy value of the original energy level of the electron.

Q4 Laser light with wavelength of 4.7×10^{-7} m is passed through a diffraction grating. It produces an interference pattern on a screen. The third order fringe of the diffraction pattern is at an angle of 0.020 radians to the incident light. Calculate the grating spacing of the diffraction grating.

Practice Questions — Fact Recall

Q1 What is the zero order maximum of an interference pattern produced by a diffraction grating?

Q2 Why are spectra formed when white light passes through a diffraction grating?

Q3 What is emitted by an electron when it moves from a higher energy level to a lower energy level? How does its energy relate to the energy change of the electron?

Q4 Why do absorption lines only appear at particular wavelengths in the spectra of stars?

6. The Big Bang Theory

While astronomers examine space as it is now, cosmologists try to use all the evidence we've got to discover more about the origin, evolution and fate of the universe. And right now, the best idea they've got is the Big Bang theory.

Learning Objectives:

- Know that the cosmological principle states that the universe is homogeneous and isotropic, and that the laws of physics are universal.
- Understand what is meant by the Doppler effect.
- Understand the effects of Doppler shift on electromagnetic radiation.
- Be able to use the Doppler equation $\frac{\Delta\lambda}{\lambda} \approx \frac{\Delta f}{f} \approx \frac{v}{c}$ to quantify the effect of Doppler shift.
- Understand how galactic red shift supports the idea of an expanding universe.
- Understand Hubble's law, $\boldsymbol{v} \approx H_0 d$, for receding galaxies.
- Know that the Hubble constant, H_0, can be measured in both $kms^{-1}Mpc^{-1}$ and s^{-1}.
- Know the Big Bang theory, and understand that the Big Bang gave rise to the expansion of space-time.
- Understand how the existence of microwave background radiation at a temperature of 2.7 K provides experimental evidence for the Big Bang theory.

Specification Reference 5.5.3

The cosmological principle

It's easy to imagine that the Earth is at the centre of the universe, or that there's something really special about it. Earth is special to us because we live here — but on a universal scale, it's just like any other lump of rock.

The idea that no part of the universe is any more special than any other is summarised by the **cosmological principle**.

The Cosmological Principle:

The laws of physics are universal (they're the same everywhere), and on a large scale the universe is:

- **homogeneous** (every part is the same as every other part) and
- **isotropic** (everything looks the same in every direction)

This is a powerful idea — it means we can apply what we know about physics on Earth and in our Solar System to the rest of the universe.

The Doppler effect

You'll have experienced the **Doppler effect** loads of times with sound waves. Imagine a police car driving past you. As it moves towards you its siren sounds higher-pitched, but as it moves away, its pitch is lower. This change in frequency and wavelength is called the Doppler effect.

The frequency and the wavelength change because the waves bunch together in front of the source and stretch out behind it. The amount of stretching or bunching together depends on the speed of the source.

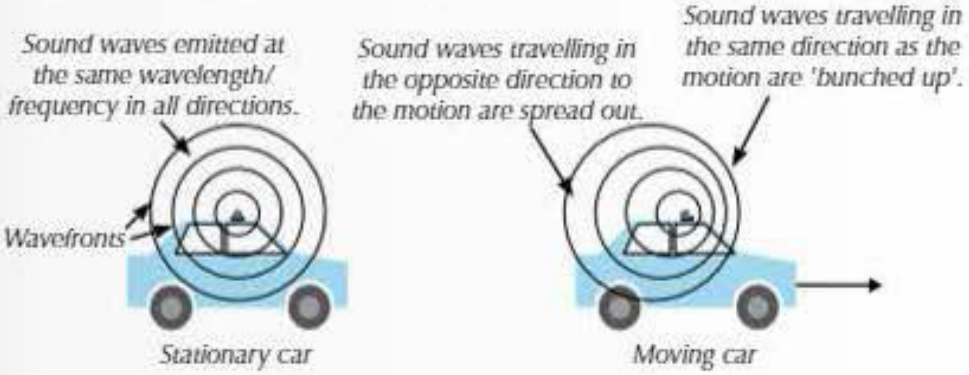

Sound waves emitted at the same wavelength/ frequency in all directions.
Sound waves travelling in the opposite direction to the motion are spread out.
Sound waves travelling in the same direction as the motion are 'bunched up'.
Wavefronts
Stationary car
Moving car

Figure 1: *The Doppler effect of sound waves from a moving police car siren.*

Doppler shift

The Doppler effect happens with all waves, including electromagnetic radiation. When a light source moves away from us, the wavelength of the light reaching us becomes longer and the frequencies become lower. This shifts the light that we receive towards the red end of the electromagnetic spectrum and is called a **red shift**.

Tip: Remember, red light is the lowest frequency of light in the visible spectrum.

The light that we receive from a star moving away from us is redder than the actual light emitted by the star.

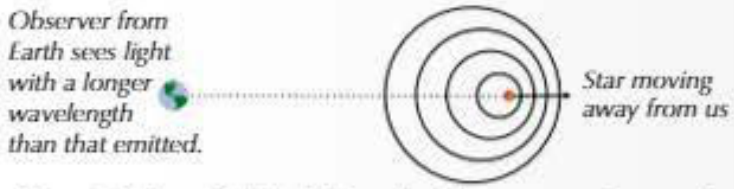


Figure 2: *The red shift of light emitted by a star moving away from us.*

Tip: It's important to realise that the frequency of the source doesn't change, just the frequency of the radiation reaching us.

When a light source moves towards us, the opposite happens. The light from the star undergoes a **blue shift** and looks bluer than it actually is.

Tip: Blue light is at the higher frequency end of the visible spectrum. In blue shift, light gets shunted towards (or beyond) the blue end of the visible spectrum.

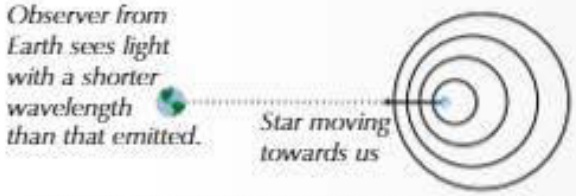


Figure 3: *The blue shift of light emitted by a star moving towards us.*

The amount of Doppler shift (red or blue) depends on how fast the star is moving relative to us (either away from or towards us). The higher the speed, the more the waves are shifted.

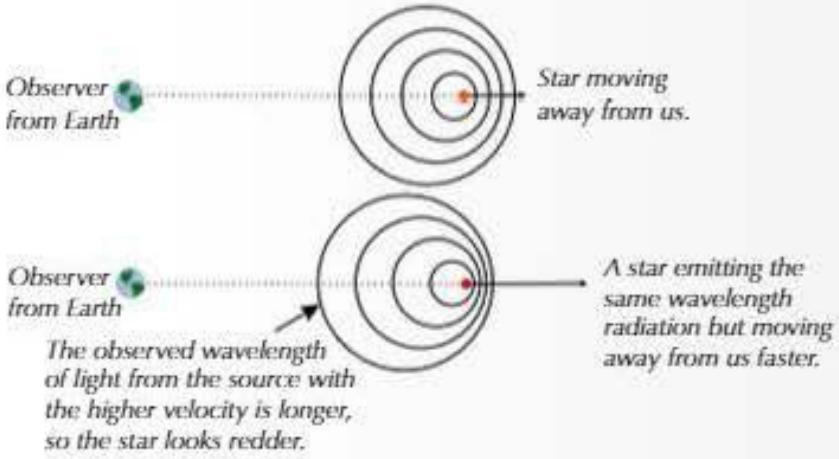

Observer from Earth
Star moving away from us.
Observer from Earth
A star emitting the same wavelength radiation but moving away from us faster.
The observed wavelength of light from the source with the higher velocity is longer, so the star looks redder.

Figure 4: *Red (or blue) shift increases with the velocity of the source.*

You can quantify the amount of Doppler shift using the equation:

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta f}{f} \approx \frac{v}{c}$$

Tip: You can use any units for v and c in this equation, as long as they are the same.

Tip: This equation only works when the speed of the source is much less than the speed of light.

Where:

- $\Delta\lambda$ is the difference between the observed and emitted wavelengths, and λ is the emitted wavelength,
- Δf is the difference between the observed and emitted frequencies, and f is the emitted frequency,
- v is the speed of the source relative to the observer and c is the speed of light.

Examples — **Maths Skills**

A star emitting light with a peak wavelength of 617 nm is moving away from the Earth. When it is detected in the lab, this wavelength of light is Doppler shifted by 5 nm. Calculate the speed at which the star is receding from Earth.

$\frac{v}{c} = \frac{\Delta\lambda}{\lambda}$ so $v = \frac{\Delta\lambda}{\lambda} \times c$

$v = \frac{5}{617} \times (3.00 \times 10^8) = 2.431... \times 10^6 \text{ ms}^{-1}$

$= 2.43 \times 10^6 \text{ ms}^{-1}$ (to 3 s.f.)

The frequency of this wavelength of light is 4.86×10^{14} Hz. Calculate the size of the Doppler shift in light of this frequency from the star.

$\frac{\Delta f}{f} = \frac{\Delta\lambda}{\lambda}$ so $\Delta f = \frac{\Delta\lambda}{\lambda} \times f$

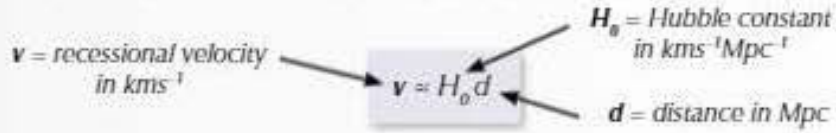
$\Delta f = \frac{5}{617} \times (4.86 \times 10^{14}) = 3.938... \times 10^{12}$ Hz

$= 3.94 \times 10^{12}$ Hz (to 3 s.f.)

The expansion of the universe

Until the early 20th century, cosmologists believed that the Universe was infinite in both space and time (that is, it had always existed) and static. This seemed the only way it could be stable. This changed when Edwin Hubble realised that the spectra from galaxies (apart from a few very close ones) all show red shift — so they're all moving away from us. The amount of galactic red shift gives the recessional velocity — how fast the galaxy is moving away.

Plotting recessional velocity against the distance of the object from the Earth shows that they're proportional — i.e. the speed that galaxies move away from us depends on how far away they are. This suggests that the universe is expanding, and gives rise to Hubble's law:



Since distance is very difficult to measure, astronomers used to disagree greatly on the value of H_0, with measurements ranging from 50 to 100 $\text{kms}^{-1}\text{Mpc}^{-1}$. It's now generally accepted that H_0 lies between 65 and 80 $\text{kms}^{-1}\text{Mpc}^{-1}$ and most agree it's around the low to mid 70s. You'll be given a value to use in the exam. The SI unit for H_0 is s^{-1}. To get H_0 in SI units, you need v in ms^{-1} and d in m.

Example — **Maths Skills**

The Perseus Cluster is located at a distance of around 2.28×10^{24} m from Earth. Assuming that the Hubble constant is equal to 71.9 $\text{kms}^{-1}\text{Mpc}^{-1}$, estimate the recessional velocity of the Perseus Cluster.

First convert H_0 to s^{-1}:

$H_0 = 71.9 \times 1000 = 71\,900 \text{ ms}^{-1}\text{Mpc}^{-1}$

$= 71\,900 \div 3.1 \times 10^{22} = 2.319... \times 10^{-18} \text{ s}^{-1}$

Then plug the values you're given into Hubble's Law:

$v = H_0 d$

$= 2.319... \times 10^{-18} \times 2.28 \times 10^{24}$

$= 5.2881... \times 10^6 \text{ ms}^{-1} = 5.29 \times 10^6 \text{ ms}^{-1}$ (to 3 s.f.)

Tip: Some nearby galaxies are moving towards us due to gravitational attraction. The light from these galaxies shows blue shift.

Tip: To convert a value in $\text{kms}^{-1}\text{Mpc}^{-1}$ to s^{-1}, first multiply it by 1000 to get it in units of $\text{ms}^{-1}\text{Mpc}^{-1}$. Then divide it by the number of metres in an Mpc, 3.1×10^{22}. This will give you units of $\text{ms}^{-1}\text{m}^{-1}$, which is the same as s^{-1}.

The red shift of light from other galaxies isn't caused by them flying away from us through space — they are moving away from us because space itself is expanding. Since the universe is expanding uniformly away from us it seems as though we're at the centre of the universe, but this is an illusion. You would observe the same thing at any point in the universe.

A good way to visualise this is to think of the universe as the surface of a balloon with lots of dots on it, representing galaxies. As you blow up the balloon, the space between all the galaxies (dots) gets bigger. Each galaxy sees all the other galaxies moving away from it, even though it's not at the centre of the motion.

The Big Bang theory

Tip: A closed system just means that energy and momentum are conserved. If there's only so much energy in the universe, as it expands, it must also be cooling.

So the universe is expanding and cooling down (because it's a closed system). So further back in time it must have been smaller, denser and hotter. If you trace time back far enough, you get a Big Bang:

The Big Bang theory:
The universe started off very hot and very dense (perhaps as an infinitely hot, infinitely dense singularity) and has been expanding ever since.

Tip: If the universe began at a specific point in time, i.e. with the Big Bang, then it has a finite age — see p.338.

According to the Big Bang theory, before the Big Bang, there was no space or time — space-time began with the Big Bang, (when time = 0 and the radius of the universe = 0) and has been expanding ever since.

There is a lot of evidence to support the Big Bang theory for the creation of the universe, such as the red shift of galaxies, and the existence of the cosmic microwave background radiation.

Cosmic microwave background radiation

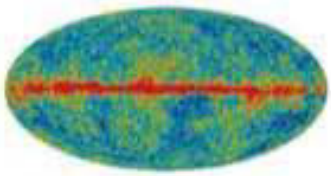


Figure 5: *A WMAP image of the CMBR. You can see that, except for the red horizontal band, which shows Milky Way emissions, the CMBR is mostly homogeneous and isotropic, but shows small variations.*

The Big Bang model predicts that loads of gamma radiation was produced in the very early universe. This radiation should still be observed today (it hasn't had anywhere else to go). Because the universe has expanded, the wavelengths of this cosmic background radiation have been stretched and are now in the microwave region.

Cosmic microwave background radiation (CMBR) was discovered accidentally by Penzias and Wilson in the 1960s. In the late 1980s a satellite called the Cosmic Background Explorer (COBE) was sent up to have a detailed look at the radiation. It found a continuous spectrum corresponding to a temperature of about 2.7 K. The radiation is largely the same everywhere (homogeneous) and in all directions (isotropic), in line with the cosmological principle.

There are very tiny fluctuations in temperature, which were at the limit of COBE's detection. These are due to tiny energy-density variations in the early universe, and are needed for the initial 'seeding' of a star or galaxy formation.

The background radiation also shows a Doppler shift, indicating the Earth's motion through space. It turns out that the Milky Way is rushing towards an unknown mass (the Great Attractor) at over a million miles an hour.

Practice Questions — Application

Q1 The Antennae galaxies are a pair of colliding galaxies at a distance of 1.4×10^7 pc from Earth. Calculate the recessional velocity of the Antennae galaxies using $H_0 = 70$ kms^{-1} Mpc^{-1}.
Give your answer in ms^{-1}.

Q2 The Whirlpool galaxy is receding from us at a speed of 463 kms^{-1}. A frequency of radiation emitted by atomic hydrogen has a wavelength of 0.211 m in the lab. The same radiation, when emitted from the Whirlpool galaxy, is observed to have been red shifted when it is observed from Earth. Calculate the change in the wavelength of this radiation due to red shift.

Practice Questions — Fact Recall

Q1 State the cosmological principle.

Q2 Explain how the Doppler effect makes police car sirens sound higher pitched as they travel towards us.

Q3 What is red shift? What is blue shift?

Q4 Write down Hubble's law. Define all symbols used, and give the units of each quantity.

Q5 What is the Big Bang theory?

Q6 Explain what cosmic microwave background radiation is.

Learning Objectives:

- Be able to estimate the age of the universe from Hubble's law, using $t \approx H_0^{-1}$.
- Know how the universe has evolved from the Big Bang to its present state.
- Know some current ideas about cosmology, including dark matter and dark energy.
- Know that the universe is made up of dark energy, dark matter and a small percentage of ordinary matter.

Specification Reference 5.5.3

7. The Evolution of the Universe

So we have a pretty good idea of how the universe started. But how we got from there to our current state is another question. And there are still a lot of things we don't have a concrete answer for.

The age and size of the universe

If the universe has been expanding at the same rate for its whole life, the age of the universe can be estimated by plugging Hubble's Law into velocity = distance ÷ time. This gives us the equation:

$$t \approx \frac{\text{distance}}{\text{velocity}} \approx H_0^{-1}$$

This is only an estimate, since the universe probably hasn't always been expanding at the same rate. And, unfortunately, since no one knows the exact value of H_0 we can only guess the universe's age.

Example — **Maths Skills**

If $H_0 = 72$ kms^{-1}Mpc^{-1}, calculate the age of the universe.

The Hubble constant is in kms^{-1}Mpc^{-1}, but you need it in s^{-1} to get the time in s.

Multiplying by 10^3 gives it in ms^{-1}Mpc^{-1}. Then, since 1 Mpc = 3.1×10^{22} m, dividing by 3.1×10^{22} gives it in ms^{-1}m^{-1}, which is just s^{-1}.

72 kms^{-1}Mpc^{-1} $= (72 \times 10^3) \div (3.1 \times 10^{22})$ s^{-1} $= 2.3... \times 10^{-18}$ s^{-1}

$$\text{So } t \approx H_0^{-1} \approx \frac{1}{2.3... \times 10^{-18}} = 4.305... \times 10^{17} \text{ s}$$

$$= 4.31 \times 10^{17} \text{ s}$$

$$= 14 \text{ billion years (to 2 s.f.)}.$$

Tip: The universe is bigger than this sphere, but it isn't observable to us, because we can only see light that has had time to travel to us.

The absolute size of the universe is unknown but there is a limit on the size of the observable universe. This is simply a sphere (with the Earth at its centre) with a radius equal to the maximum distance that light can travel during the universe's existence. So if $H_0 = 72$ kms^{-1}Mpc^{-1} then this sphere will have a radius of 14 billion light years. However, when scientists take into account the expansion of the universe (p.335), they find that it is likely to be more like 46-47 billion light years.

The history of the universe

Before 10^{-4} seconds after the Big Bang, this is mainly guesswork. There are plenty of theories around, but not much experimental evidence to back them up. The general consensus at the moment goes something like this:

- **Big Bang to 10^{-43} seconds** — Well, it's anybody's guess, really. At this sort of size and energy, even general relativity stops working properly. This is the "infinitely hot, infinitely small, infinitely dense" bit.
- **From 10^{-43} seconds to 10^{-4} seconds after the Big Bang** — At the start of this period, there's no distinction between different types of force — there's just one grand unified force. Then the universe expands and cools, and the unified force splits into gravity, strong nuclear, weak nuclear and electromagnetic forces. Many cosmologists believe the universe went through a rapid period of expansion called inflation at about 10^{-34} s.

During this period, the universe is a sea of quarks, antiquarks, leptons and photons. The quarks aren't bound up in particles like protons and neutrons, because there's too much energy around. At some point, matter-antimatter symmetry gets broken, so slightly more matter is made than antimatter. Nobody knows exactly how or when this happened, but most cosmologists like to put it as early as possible in the history of the universe.

Once we reach 10^{-4} s after the Big Bang, we're a bit more confident with what went on.

- **10^{-4} seconds after the Big Bang** — This corresponds to a temperature of about 10^{12} K. The universe is cool enough for quarks to join up to form particles like protons and neutrons. They can never exist separately again. Matter and antimatter annihilate each other, leaving a small excess of matter and huge numbers of photons (resulting in the cosmic background radiation that we observe today).
- **About 100 seconds after the Big Bang** — Temperature has cooled to 10^9 K. The universe is similar to the interior of a star. Protons are cool enough to fuse to form helium nuclei.
- **About 300 000 years after the Big Bang** — Temperature has cooled to about 3000 K. The universe is cool enough for electrons (that were produced in the first millisecond) to combine with helium and hydrogen nuclei to form atoms. The universe becomes transparent since there are no free charges for the photons to interact with. This process is called recombination.
- **About 14 billion years after the Big Bang (now)** — Temperature has cooled to about 2.7 K. Slight density fluctuations in the universe mean that, over time, clumps of matter have been condensed by gravity into galactic clusters, galaxies and individual stars.

This covers the basic milestones that cosmologists think the universe must have gone through in order to become the universe we see today. But a lot of it is still a mystery. We can't be fully certain about the timing or mechanism of any of this yet, as there are still parts of the universe whose behaviour we don't understand. Chief among these unknown parts of the universe are dark matter and dark energy.

Dark matter

In the 1930s, the Swiss astronomer Fritz Zwicky calculated the mass of a cluster of galaxies (the COMA cluster) based on the velocity of its outer galaxies and compared this figure to the mass of the cluster as estimated from its luminosity. The mass calculated from the velocity was much bigger, suggesting there was 'extra' mass in the cluster that couldn't be seen.

In the 1970s, Vera Rubin observed that stars at the edges of galaxies were moving faster than they should given the mass and distribution of stars in the galaxy. For Newton's laws to hold, there needed to be extra matter in the galaxies that hadn't been accounted for.

These observations suggest there is something extra in the universe, giving mass to galaxies, that we can't see.

This theoretical substance has been called 'dark matter'. Astrophysicists now estimate that there is about five times as much dark matter as ordinary matter in the universe, and that dark matter makes up about 25% of the universe in total.

Figure 1: *The astronomer Vera Rubin, whose discovery of the 'galaxy rotation problem' revolutionised modern astronomy and cosmology.*

What is dark matter?

Figure 2: *Some equipment being used in an experiment that aims to detect signs of WIMP dark matter.*

Physicists don't know for sure what dark matter is yet. But there are a number of theories about what it could be, and experiments are constantly being run to test them.

One explanation is that dark matter is made up of MACHOs (Massive Compact Halo Objects). These are objects made of normal matter in a very dense form that don't give off light and so are hard to detect, e.g. black holes (p.323) and brown dwarfs (stars that aren't massive enough for nuclear fusion to take place). Astronomers looking for evidence of these kinds of objects have had some success, but it's unlikely that MACHOs made of normal matter account for all the dark matter in the universe, as this would require more protons and neutrons to exist than is compatible with our current understanding of the Big Bang.

Another idea is that dark matter is made of WIMPs (Weakly Interacting Massive Particles). These are exotic particles that don't interact with the electromagnetic force, but do interact with gravity. As yet, though, no particle like this has ever been detected, and WIMPs are currently purely theoretical.

There's also the possibility that dark matter doesn't really exist at all, and is an illusion caused by mistakes in other theories. But most scientists agree that it is there, even if we don't know what it is yet.

Dark energy

Everything in the universe is attracted to everything else by gravity. This means the expansion of the universe should be slowing down. Historically, astronomers debated whether this would slow the expansion of the universe enough to cause it to contract back in on itself (in a so called 'Big Crunch'), or if the universe would go on expanding forever.

In the late 1990s, astronomers discovered something entirely unexpected. Rather than slowing down, the expansion of the universe appears to be accelerating. Astronomers are trying to explain this acceleration using dark energy — a type of energy that fills the whole of space.

Tip: There has been a lot of controversy surrounding dark energy in the astrophysics community — nobody really knows what it is, or if it's even there — but it is now generally accepted that it does exist.

There are various theories of what this dark energy is, but it's really hard to test them. So like dark matter, it's currently a mystery.

Based on current observations, dark energy makes up about 70% of the universe. As dark matter makes up another 25%, this means that only about 5% of the universe is made up of ordinary matter. Or to put it another way, we have very little idea what 95% of the universe is made up of.

Practice Question — Application

Q1 A student uses the equation $t = H_0^{-1}$ to calculate that the universe is approximately 13.7 billion years old.

a) Calculate the value of H_0 used to calculate this age of the universe. Give your answer in $kms^{-1}Mpc^{-1}$.

b) State the assumption that the student must have made to make this calculation.

Practice Questions — Fact Recall

Q1 How can we estimate the age of the universe from the Hubble constant?

Q2 Why can't we measure the absolute size of the universe?

Q3 State the temperature of the universe and describe one change happening in it at each of the following times after the Big Bang:

a) 10^{-4} s

b) 300 000 years

c) 14 billion years

Q4 State two observations which could be explained by the existence of dark matter.

Q5 What does the acronym MACHO stand for?

Q6 What does the acronym WIMP stand for?

Q7 What observations might dark energy explain?

Section Summary

Make sure you know...

- What is meant by the universe, a galaxy, a solar system, a planet, a planetary satellite and a comet.
- How to use distances measured in astronomical units (AU), parsecs (pc) and light years (ly).
- That stellar parallax is used to measure distances to nearby stars due to their apparent motion in the night sky against the distant background as the Earth orbits the Sun.
- How to use the equation $p = \frac{1}{d}$ to make calculations of parallax angle and distance.
- How stars are formed from interstellar dust and gas drawn together by gravity.
- That stars are kept stable by a balance between outwards pressure (from radiation and gas) and gravitational attraction.
- The evolutionary stages of a low-mass star like our Sun, including the terms red giant star, white dwarf star and planetary nebula.
- That a star may only form a white dwarf star if the mass of the star's core is less than the Chandrasekhar limit (approximately 1.4 solar masses).
- That a white dwarf star is prevented from collapsing in on itself by the electron degeneracy pressure within the star.
- The evolutionary stages of stars much more massive than the Sun, including the terms red super giant and supernova, and that the star will become either a neutron star or a black hole.
- That a neutron star is an incredibly dense, very small star which rotates very fast, and emits beams of radio waves.
- That a black hole is an infinitely dense point, with such a strong gravitational field that past a certain point in the field the escape velocity exceeds the speed of light.
- That Wien's displacement law states that $\lambda_{max} \propto \frac{1}{T}$, and how to use this to estimate the peak surface temperature of a star.
- That the luminosity of a star is a measure of the total energy it emits per second and is given by Stefan's law, $L = 4\pi r^2 \sigma T^4$, where σ is Stefan's constant.

cont...

- How to use Wien's displacement law and Stefan's law to estimate the radius of a star.
- That the Hertzsprung-Russell (HR) diagram is a luminosity-temperature plot of a range of stars.
- That there are distinct areas on the HR diagram which correspond to main sequence stars, red giant stars, super red giant stars and white dwarf stars.
- How a transmission diffraction grating can be used to determine the wavelength of light.
- That the fringes of the interference pattern produced by a diffraction grating are defined by the equation $d\sin\theta = n\lambda$.
- That electrons in isolated gas atoms exist in distinct energy levels.
- That electron energy levels are considered to have negative energy values.
- That electrons can emit and absorb photons with specific energies by moving between discrete energy levels and how this gives rise to line emission spectra and line absorption spectra.
- That the energy change in an atomic electron is related to the frequency and wavelength of the emitted/absorbed photon by the equations $\Delta E = hf$ and $\Delta E = \frac{hc}{\lambda}$.
- That the fact that different elements produce unique spectral lines in emission and absorption spectra allows us to use analysis of spectra to identify elements present in stars.
- That the cosmological principle states that the universe is homogeneous and isotropic, and that the laws of physics are the same everywhere in the universe.
- That the Doppler effect is a change in perceived frequency (and so wavelength) of a wave due to the relative motion between the source and the observer.
- That the Doppler effect of electromagnetic waves can result in red shift or blue shift.
- That the observed Doppler shift of an electromagnetic wave can be quantified using the Doppler equation $\frac{\Delta\lambda}{\lambda} \approx \frac{\Delta f}{f} \approx \frac{v}{c}$, where v is the speed of the source relative to the observer.
- That light from nearly all other galaxies is observed to have been red shifted, indicating that galaxies are moving away from each other, and that this supports the idea of an expanding universe.
- That Hubble's law states that the further away a galaxy is, the faster it is receding, and is expressed as $\boldsymbol{v} = H_0 d$, where H_0 is the Hubble constant.
- That the Hubble constant is a measure of the rate of expansion of the universe, and can be measured in either $\text{kms}^{-1}\text{Mpc}^{-1}$ or s^{-1}.
- That the Big Bang theory states that the universe started off very hot and very dense and has been expanding ever since.
- That the Big Bang gave rise to the expansion of space-time.
- That the cosmic microwave background radiation is a continuous spectrum of EM radiation corresponding to a temperature of 2.7 K that pervades throughout the universe and how this is evidence in support of the Big Bang theory.
- That the age of the universe can be estimated using the Hubble constant with the equation $t = H_0^{-1}$.
- How the universe has evolved from the Big Bang to its present state.
- That dark matter is a theoretical form of matter that cannot be seen directly, but which could explain inconsistencies between the velocities of galaxies in clusters, or of stars in galaxies, and their apparent masses.
- That dark energy is a theoretical type of energy that fills the universe, and drives the accelerated expansion of the universe.
- That the universe appears to be made up of dark energy, dark matter, and a small percentage of ordinary, observable matter.

Exam-style Questions

1 A star is reaching the end of the red giant stage of its life. The core of the star has a mass of 2.17×10^{30} kg. Given that the mass of the Sun is 1.99×10^{30} kg, what is the next stage in this star's life cycle?

A neutron star

B super red giant

C white dwarf

D black hole

(1 mark)

2 As a main sequence star evolves into a red giant star, it goes through four phases of nuclear fusion:

1 shell helium fusion
2 core hydrogen fusion
3 core helium fusion
4 shell hydrogen fusion

Which is the correct order in which these phases of fusion begin?

A 4, 2, 3, 1

B 3, 1, 2, 4

C 2, 3, 4, 1

D 2, 4, 3, 1

(1 mark)

3 An electron in a hydrogen atom is in the –13.6 eV energy level. It absorbs a photon with frequency 3.151×10^{15} Hz, and moves to a higher energy level. What is the energy value of this energy level?

A 13.1 eV

B –0.54 eV

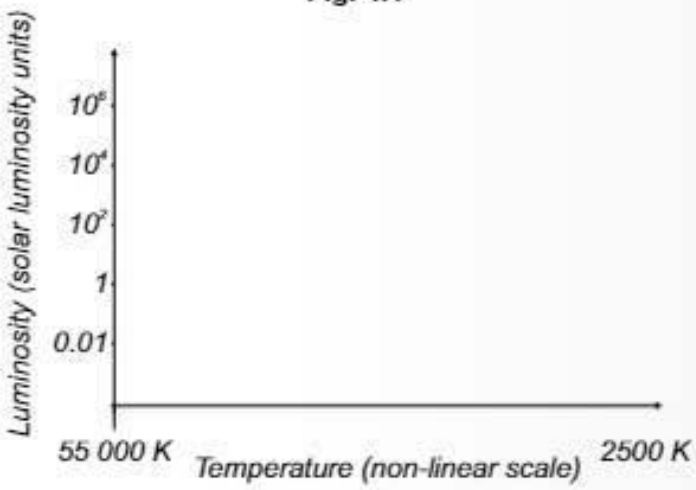
C –26.6 eV

D 13.6 eV

(1 mark)

4 **Fig. 4.1** shows a set of axes for a Hertzsprung-Russell (HR) diagram.

Fig. 4.1



(a) Copy and complete the diagram, marking the main sequence stars, white dwarfs, red giants and super red giants.

(3 marks)

(b) Star X has a surface temperature of around 39 000 K and a luminosity of 200 000 solar luminosity units. Mark the position of Star X on your HR diagram.

(1 mark)

The mass of Star X is around 35 times that of the Sun.
It will eventually become a black hole.

(c) Explain how Star X will evolve from its red super giant phase to become a black hole.

(3 marks)

5 Mu Cephei is a red super giant star in the constellation Cepheus.
Analysis of the radiation emitted by Mu Cephei gives the value of its peak wavelength as λ_{max} = 828.6 nm.

(a) Given that the Sun has a surface temperature of 5800 K and a peak wavelength of approximately 500.0 nm, estimate the surface temperature of Mu Cephei in kelvin.

(2 marks)

A scientist measures the luminosity of Mu Cephei as 1.083×10^{32} W.

(b) Estimate the radius of Mu Cephei in metres.

(2 marks)

(c) The scientist measures the angle of parallax of Mu Cephei as 5.5×10^{-4} arcseconds to the Earth.
Calculate the distance to Mu Cephei in light years.

(3 marks)

(d) Mu Cephei is near the end of its life. It will soon explode in a supernova.
Describe what occurs when a star explodes in a supernova.

(2 marks)

6 The static theory of the universe states that the universe is infinite in both space and time and it has always been that way. This theory is now widely discredited in favour of the Big Bang theory.

(a) Describe how the universe began according to the Big Bang theory.

(1 mark)

Hubble observed that the recessional velocity of distant objects was proportional to their distance from Earth. This observation led to Hubble's law:

$$v \approx H_0 d$$

(b) Explain how Hubble's law supports the Big Bang theory.

(1 mark)

(c) Assuming that $H_0 = 65$ kms^{-1}Mpc^{-1}, estimate the age of the universe in years using Hubble's law. (1 Mpc = 3.1×10^{22} m)

(3 marks)

(d) Red-shift data is only one piece of evidence supporting the Big Bang theory. Describe another piece of evidence that supports the Big Bang theory and explain how it supports the theory.

(2 marks)

7 A scientist is analysing the light from stars using diffraction gratings.

(a) The scientist passes the light produced by a heated sample of hydrogen gas through a diffraction grating with a slit separation of 5.4×10^{-6} m. She uses the second order fringe to produce an emission line spectrum of hydrogen. One of the lines from the emission spectrum makes an angle of 0.18 radians to the incident light. Calculate the wavelength of the light which produced this line.

(2 marks)

(b) Calculate the energy carried by a photon of light with this wavelength. Give your answer in eV.

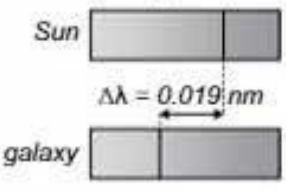
(3 marks)

(c) The scientist passes light from the Sun through the same diffraction grating. When she analyses the spectrum produced by this, she notices dark lines in the spectrum that match the emission lines produced by hydrogen. Explain why this indicates that hydrogen is present in the Sun.

(3 marks)

(d) The scientist compares this spectrum from the Sun to one from a distant galaxy. Parts of these spectra are shown in **Fig. 7.1**. The dark line corresponding to the hydrogen line from part (a) in the galaxy's spectrum is displaced towards the red end of the spectrum by an amount equal to a change in wavelength of 0.019 nm. Using your answer to part (a), calculate the recessional velocity of the galaxy.

Fig. 7.1



(2 marks)

Learning Objectives:

- Know what is meant by the term 'capacitance'.
- Be able to calculate capacitance using the equation $C = \frac{Q}{V}$.
- Know that the unit of capacitance is the farad, F.
- Understand how capacitors are charged and discharged, in terms of electron flow.
- Understand a p.d.-charge graph of a capacitor.
- Know that the area under a p.d.-charge graph gives the energy stored by a capacitor.
- Be able to calculate the energy stored by a capacitor using the equations: $W = \frac{1}{2}QV$, $W = \frac{1}{2}\frac{Q^2}{C}$ and $W = \frac{1}{2}V^2C$
- Know some uses of capacitors for storing energy.

Specification References 6.1.1 and 6.1.2

1. Capacitors

Capacitors are devices that can store up charge and energy. The capacitance of a capacitor tells you how much charge it can hold. Almost all electrical products will contain one somewhere...

What is a capacitor?

Capacitors are electrical components that can store electrical charge. They are made up of two electrical conducting plates separated by a gap or a dielectric (an electrically insulating material). The circuit symbol is two parallel lines — see Figure 1.

Figure 1: *Circuit symbol for a capacitor.*

When a capacitor is connected to a direct current (d.c.) power source, charge builds up on its plates — one plate becomes negatively charged and one becomes positively charged (there's more on this on the next page). The plates are separated by an electrical insulator (which could just be air), so no charge can move between them. This means that a potential difference builds up between the plates of the capacitor. This creates a uniform electric field (p.372) between the two plates — see Figure 2.

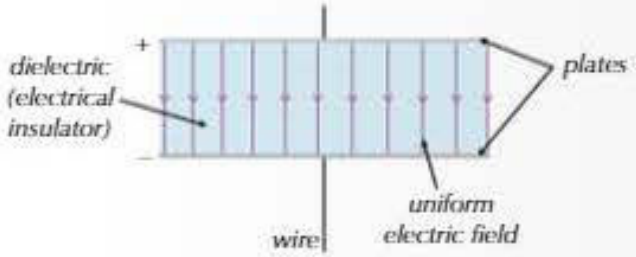
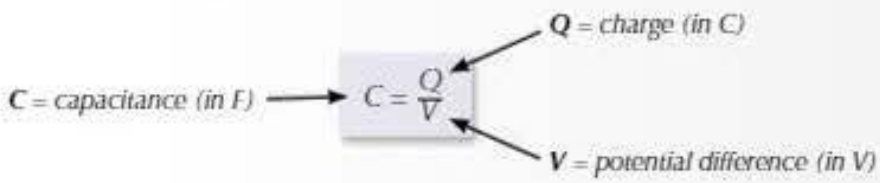


Figure 2: *The uniform electric field formed between the plates of a capacitor.*

The ability of a capacitor to store charge is measured by its capacitance.

The **capacitance** of a capacitor is the amount of charge it is able to store per unit potential difference (p.d.) across it.



Tip: In the capacitance equation, Q is the charge stored on each capacitor plate — the charge is just negative on one plate and positive on the other.

Capacitance is measured in farads. 1 farad (F) is equal to 1 coulomb per volt (CV^{-1}). This is a huge amount of capacitance, so you'll usually see capacitances given in microfarads, μF (1×10^{-6} F), nanofarads, nF (1×10^{-9} F), or picofarads, pf (1×10^{-12} F).

Example — **Maths Skills**

A 100 μF capacitor is charged to a potential difference of 12 V. How much charge is stored by the capacitor?

$C = \frac{Q}{V}$ so by rearranging, $Q = C \times V = (100 \times 10^{-6}) \times 12 = 1.2 \times 10^{-3}$ C

Charging and discharging

When a capacitor is connected to a d.c. power supply (e.g. a battery), a current flows in the circuit until the capacitor is fully charged, then stops.

The electrons flow from the negative terminal of the supply onto the plate connected to it, so a negative charge builds up on that plate.

At the same time, electrons flow from the other plate to the positive terminal of the supply, making that plate positive. These electrons are repelled by the negative charge on the negative plate and attracted to the positive terminal of the supply.

The same number of electrons are repelled from the positive plate as are built up on the negative plate. This means an equal but opposite charge builds up on each plate, causing the potential difference between the plates.

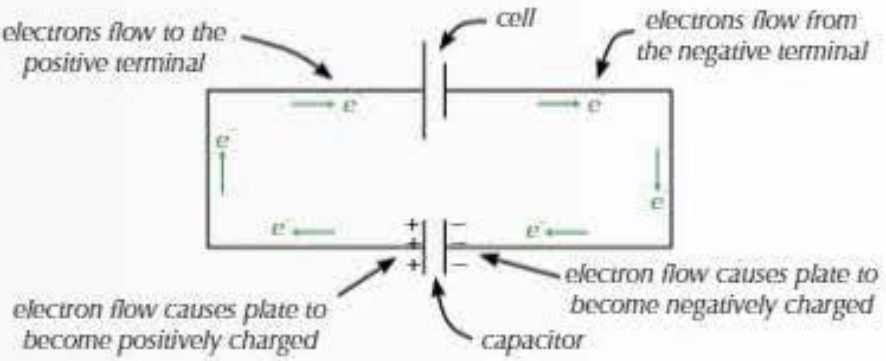


Figure 4: *Flow of electrons as a capacitor charges.*

Initially the current through the circuit is high. But, as charge builds up on the plates, electrostatic repulsion makes it harder and harder for more electrons to be deposited. When the p.d. across the capacitor is equal to the p.d. across the supply, the current falls to zero. The capacitor is fully charged.

If the source of e.m.f. (in this case, a cell) is removed, and the circuit is reconnected without it, the capacitor will begin to discharge. The capacitor then becomes the circuit's source of e.m.f. A current flows around the circuit in the opposite direction to the charging current, as the electrons from the negatively charged plate drift towards the positively charged plate through the circuit.

Figure 3: *A disassembled capacitor, showing two sheets of aluminium foil with paper between them. The foil and paper are rolled up like a big Swiss roll.*

Tip: You can make your own simple capacitor using two pieces of aluminium separated by a piece of paper.

Tip: After removing the power supply, you need to reconnect the circuit to discharge a capacitor. If the capacitor isn't connected in a full circuit it will hold its charge.

Initially, the current is high, due to the strong forces between charges. There is a strong force of repulsion between the electrons on the negative plate, which pushes them away from the negative plate. There is also a strong force of attraction from the positive plate, pulling electrons through the circuit towards the positive plate. But as the electrons reach the positive plate, the potential difference decreases, and so the current decreases. The capacitor is completely discharged when the potential difference between the plates, and so the charge on each plate, becomes zero.

Energy stored by capacitors

Remember that when a capacitor charges, one plate becomes negatively charged while the other becomes positively charged. Like charges repel, so when each plate of a capacitor becomes charged, the charges on that plate are being forced together 'against their will'. This requires energy, which is supplied by the power source and stored as electric potential energy for as long as the charges are held. When the charges are released, the electric potential energy is released.

Deriving the energy-stored equations

You can find the energy stored by a capacitor by using the graph of potential difference against charge for the capacitor (see Figure 6).

The p.d. across a capacitor, of capacitance C, is directly proportional to the charge stored on it, so the graph is a straight line through the origin (since $C = \frac{Q}{V}$, page 346).

Consider a tiny increase in the charge on the plates during the charging process. The electric potential energy stored is the work done to move the extra charge onto the plates against the potential difference across the plates, given by $W = VQ$ (from Module 4: Section 1). Let the small charge being moved be q. The average p.d. over that step is v. So in that small step, the energy stored is $W = qv$, which is given by the area of the red rectangle in Figure 6. The area of the green trapezium in Figure 6 is the area under the graph over the charge difference q, and it is the same as the area of the red rectangle.

Figure 5: *Storm clouds store massive amounts of energy — forming huge natural capacitor plates. Particle collisions cause electrons to be knocked off atoms, giving the bottom of the cloud a charge. This induces a charge in the surface of the Earth. The air in between acts as the dielectric, and if the potential difference gets big enough, the dielectric breaks down and lightning bolts travel between the cloud and the Earth.*

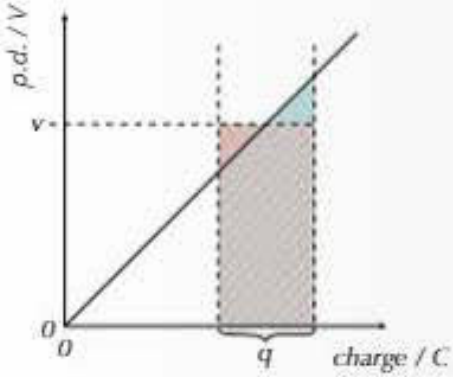


Figure 6: *A potential difference-charge graph showing the energy stored by a small increase in charge on the plates of a capacitor.*

The total energy stored by the capacitor is the sum of all the energies stored in each small step increase in charge, until the capacitor is fully charged. So it's just the area under the graph of p.d. against charge.

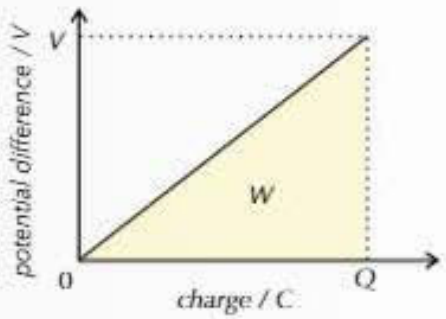


Figure 7: *A plot of potential difference against charge stored for a capacitor. The shaded area gives the energy stored by the capacitor.*

The energy stored is given by the yellow triangle. The area of a triangle is equal to ½ × base × height, so the energy stored by the capacitor is:

W *= energy stored by capacitor (in J)* → $W = \frac{1}{2}QV$

V *= potential difference across capacitor (in V)*

Q *= charge on capacitor (in C)*

Exam Tip
This formula is given in the data and formulae booklet, but you need to understand where it comes from — remember it's the area under a *V*-*Q* graph.

Example — **Maths Skills**

A capacitor is fully charged to a potential difference of 12 V. When it is fully charged, it stores 0.15 C of charge. Calculate the energy stored by it.

$W = \frac{1}{2}QV = \frac{1}{2} \times 0.15 \times 12 = 0.9$ J

Using the capacitance equation $C = \frac{Q}{V}$, you can derive two more equations for the energy stored by a capacitor. Simply rearrange the capacitance equation for Q and V, and substitute them into the energy equation:

Using $Q = CV$:

$$W = \frac{1}{2}V^2C$$

Using $V = \frac{Q}{C}$:

$$W = \frac{1Q^2}{2C}$$

Figure 8: *Benjamin Franklin was one of the first people to store electrical energy, in the mid 1700s. He invented what he called a 'battery' by grouping together many Leyden jars (simple capacitors).*

Examples — **Maths Skills**

A 900 μF capacitor is charged up to a potential difference of 240 V. Calculate the energy stored by the capacitor.

First, choose the best equation to use — you've been given V and C, so you need $W = \frac{1}{2}V^2C$.

Substitute the values in:

$W = \frac{1}{2}V^2C = \frac{1}{2} \times 240^2 \times (900 \times 10^{-6}) = 25.9$ J (to 3 s.f.)

When capacitor X is fully charged, it stores 63 mC of charge and 15 J of electrical energy. Calculate the capacitance of capacitor X.

You've been given W and Q, and need to find C, so use the equation:

$W = \frac{1Q^2}{2C}$

Rearrange for C, and substitute in the values given:

$C = \frac{Q^2}{2W} = \frac{(63 \times 10^{-3})^2}{2 \times 15} = 1.323 \times 10^{-4}$ F $= 130$ μF (to 2 s.f.)

Uses of capacitors

Figure 9: *Some touch screens use capacitors. The screen contains a layer of capacitive material that holds an electrical charge. Since your body is an electrical conductor, when your finger touches the capacitive layer, it changes the charge at that specific point, so the touch screen can work out exactly where you made contact.*

Capacitors are found in loads of electronic devices. They don't store much charge, so can't replace batteries, but they can discharge quicker than batteries, which makes them very useful. What's more, the amount of charge that can be stored and the rate at which it's released can be controlled by selecting different types of capacitor. Some uses for them are:

- Flash photography — when you take a picture, the capacitor has to discharge really quickly to give a short pulse of high current to create a brief, bright flash.
- Back-up power supplies — these often use lots of large capacitors that can release charge for a short period if the power supply goes off — e.g. for keeping computer systems running if there's a brief power outage.
- Smoothing out p.d. — when converting an a.c. power supply to d.c. power, capacitors charge up during the peaks and discharge during the troughs, helping to maintain a constant output.

Practice Questions — Application

Q1 A 0.10 F capacitor is used in a circuit as a back-up in case of a short interruption in the mains power supply. The p.d. supplied to the circuit is 230 V. How much charge can the capacitor store?

Q2 A 40 mF capacitor is connected to a 230 V power source. When fully charged, how much energy will be stored by the capacitor?

Q3 Explain why a capacitor would not be a good source of power for a portable media player.

Q4 A capacitor is charged with 2.25 mC of charge. It stores 1.30 J of energy while holding this charge. Calculate its capacitance.

Practice Questions — Fact Recall

Q1 Write down the definition of capacitance.

Q2 Explain how a charge builds up on each plate of a capacitor when it is connected in a circuit.

Q3 How would you find the energy stored in a capacitor from a graph of potential difference against charge?

Q4 Write down three equations that can be used to calculate the energy stored in a capacitor. Define all symbols used.

2. Capacitors in Circuits

Capacitors are really useful in circuits, but the effect they have in the circuit depends on how they are connected. You'll get a larger total capacitance when they're connected in parallel than when they're in series.

Learning Objectives:

- Know that the total capacitance of two or more capacitors in parallel is given by: $C = C_1 + C_2 + ...$
- Know that the total capacitance of two or more capacitors in series is given by: $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + ...$
- Be able to analyse circuits containing capacitors.
- Understand techniques and procedures used to investigate capacitors in both series and parallel combinations using ammeters and voltmeters (PAG9).

Specification Reference 6.1.1

Capacitors in parallel

If you connect two or more capacitors in parallel with each other, the potential difference across each one is the same. So each capacitor can store the same amount of charge as it would if it was the only capacitor in the circuit. The total charge stored in the circuit is split between the two capacitors, since the charges can take either path.

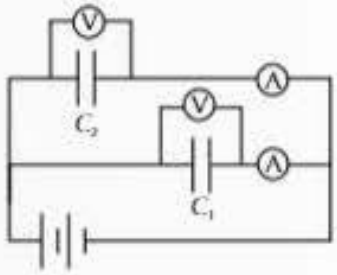


Figure 1: *An example of a circuit containing capacitors connected in parallel*

So the total capacitance of the circuit, C_{total} is equal to:

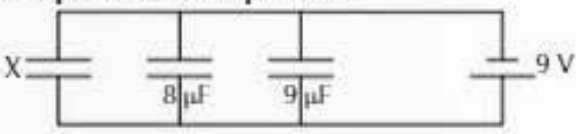
$$C_{total} = \frac{Q_{total}}{V}$$
$$\Rightarrow C_{total} = \frac{Q_1 + Q_2}{V}$$
$$\Rightarrow C_{total} = \frac{Q_1}{V} + \frac{Q_2}{V}$$
$$\Rightarrow C_{total} = C_1 + C_2$$

So the total capacitance is just the sum of the individual capacitances:

$$C_{total} = C_1 + C_2 + ...$$

Examples — **Maths Skills**

The total capacitance of the circuit shown below is 27 μF. Calculate the capacitance of capacitor X.



$C_{total} = C_1 + C_2 + C_X$ so $C_3 = C_{total} - (C_1 + C_2)$.

So, $C_x = 27 - (8 + 9) = 10$ μF

A 12 μF capacitor is connected in parallel with the other capacitors in the circuit above. Calculate the new total capacitance.

$C_{total} = C_1 + C_2 + C_3 + C_4 = 27 + 12 = 39$ μF

Capacitors in series

When you connect capacitors in series, like in Figure 2, the potential difference is shared between them, but each capacitor stores the same charge.

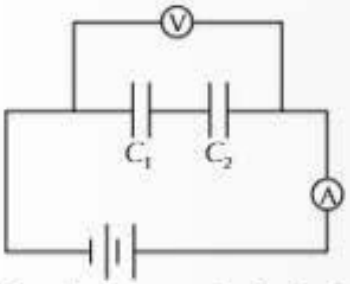


Figure 2: *An example of a circuit containing capacitors connected in series.*

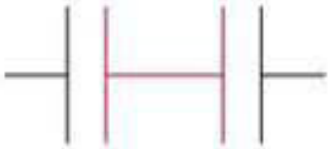


Figure 3: *The red part of the circuit is disconnected from the rest of the circuit by the dielectrics, so the charge can't leave it.*

This is because, as a negative charge builds up on the left plate of capacitor C_1, an equal positive charge builds up on the right plate of C_1. This causes a negative charge to build up on the left plate of C_2 that is equal in size to the charge on the right plate of C_1. This happens because these two plates are disconnected from the rest of the circuit (see Figure 3), so the electrons repelled from the right plate of C_1 have to end up on the left plate of C_2. This then produces an equal positive charge on the right plate of C_2. All the plates have the same charge, so each capacitor stores the same charge.

So the total capacitance of the circuit is:

$$C_{total} = \frac{Q}{V_{total}} = \frac{Q}{V_1 + V_2}$$

$$\text{So, } \frac{1}{C_{total}} = \frac{V_1 + V_2}{Q}$$

$$\Rightarrow \frac{1}{C_{total}} = \frac{V_1}{Q} + \frac{V_2}{Q}$$

$$\Rightarrow \frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Tip: You'll have met series and parallel relations like this in Module 4 with resistors. Don't get them confused though — total capacitance is bigger when components are connected in parallel than in series, while total resistance is the opposite.

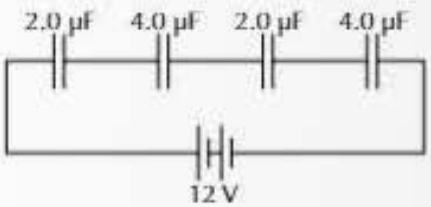
This means that you can find the total capacitance of capacitors connected in series using:

$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$$

Tip: You could be asked to solve circuit problems using the equations for both series and parallel capacitors. They'll work for any number of capacitors.

Example — **Maths Skills**

The diagram on the right shows a circuit containing capacitors. Calculate the total capacitance of the circuit.



$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$, so:

$$\frac{1}{C_{total}} = \frac{1}{2} + \frac{1}{4} + \frac{1}{2} + \frac{1}{4} = \frac{3}{2}$$

so $C_{total} = \frac{2}{3} = 0.666\ldots = 0.67\ \mu F$ (to 2 s.f.)

Investigating capacitors in series and in parallel

PRACTICAL ACTIVITY GROUP 9

You can investigate the properties of capacitors in series and in parallel for yourself. Start by setting up the circuit shown in Figure 4, using three identical capacitors.

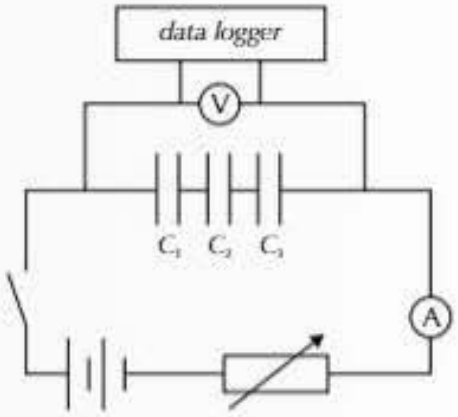


Figure 4: *An example of a circuit that can be used to investigate capacitance in series.*

1. Set the variable resistor to a fairly high resistance value and record it, making sure there is plenty of room for you to decrease the resistance during the experiment.
2. Close the switch. The capacitors will begin charging. Record the initial value of the current in the circuit.
3. The data logger connected to the voltmeter can be used to record the potential difference over time. Constantly adjust the variable resistor to keep the charging current constant for as long as you can (it'll be almost impossible when the capacitor is nearly fully charged).
4. Once the capacitors are fully charged (when the current drops to zero), open the switch.

Next, set up the circuit shown in Figure 5, using another three capacitors, identical to the first three.

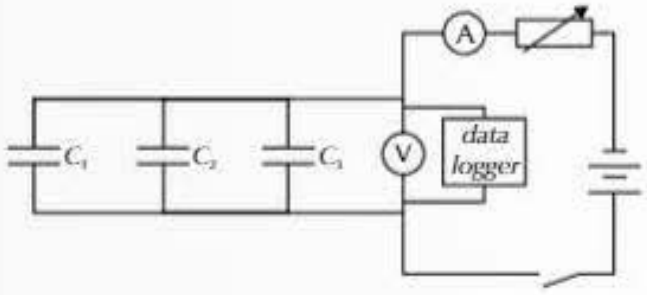


Figure 5: *An example of a circuit that can be used to investigate capacitance in parallel.*

Repeat steps 1-4 for this circuit (making sure the variable resistor starts at the same resistance as it did for the first experiment).

Once you have results for both circuits, plot a graph of current against time for each circuit. Use the constant value of current you kept the circuit at, and use the time value measured by the data logger when the current dropped to zero (the capacitors reached full charge).

Tip: Remember to carry out a risk assessment before you start any experiment.

Tip: You could use the same three capacitors from the first experiment, but if you do, make sure they have all been fully discharged before you start this experiment.

Assuming each of your identical capacitors has a capacitance C and could store a maximum charge of Q_c if they were the only capacitor in the circuit, your results should look something like those in Figure 6.

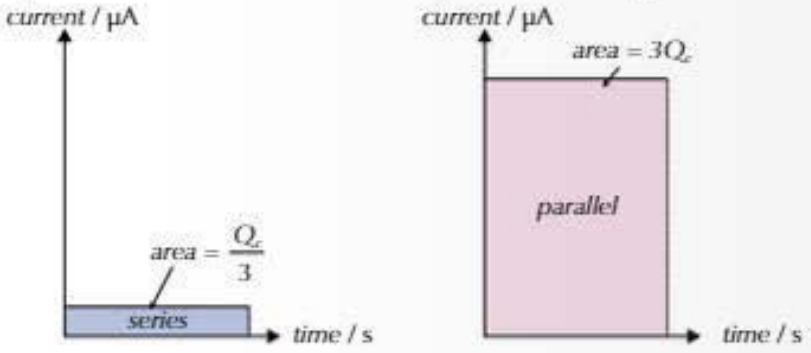


Figure 6: *The current-time graphs of three capacitors connected in series (left) and in parallel (right).*

Figure 7: *Some digital multimeters can be used to measure capacitance directly.*

The area beneath a current-time graph is equal to $I \times \Delta t = \Delta Q$, so the area gives the total charge stored by the capacitors.

You can also plot a graph of charge against potential difference for each circuit. Calculate the charge stored by the capacitors at each time reading taken by the data logger, using the equation $\Delta Q = I\Delta t$. Your graphs should look something like those in Figure 8.

The graphs are straight lines through the origin, so the gradient of the graphs is equal to charge divided by potential difference at any point on the line. So the gradient gives the total capacitance of the capacitors in the circuit. You should see that the gradient of your graph gives the same answer for the total capacitance of your circuit as the equations on pages 351-352.

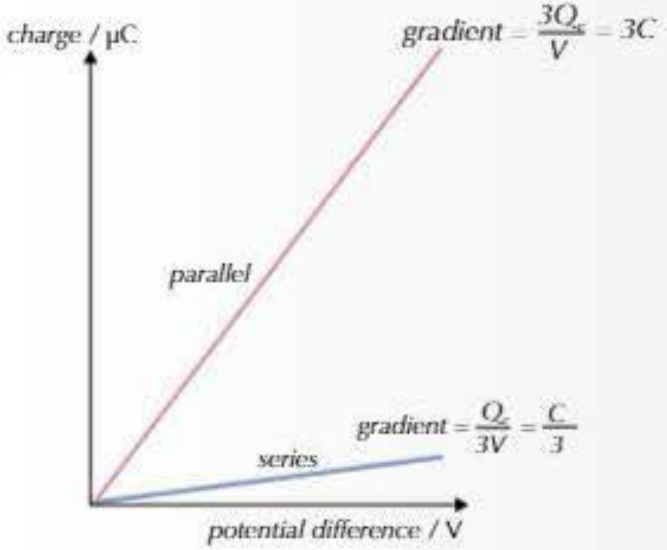
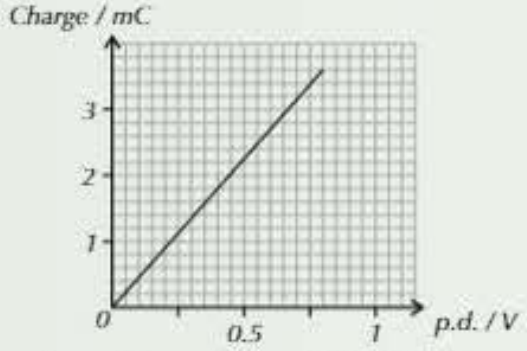


Figure 8: *A graph for charge stored against p.d. for three capacitors connected in series and in parallel. The gradient of the graph gives the total capacitance.*

Practice Questions — Application

Q1 Two identical capacitors are connected in series in a circuit. It takes 2 minutes for them to become fully charged when they are supplied with a constant current. Given that one of the capacitors connected on its own in the same circuit can store a charge of 1.2×10^{-3} C, calculate the current used to charge the two capacitors.

Q2 The diagram below shows the graph of charge against p.d. for a number of identical capacitors connected in parallel in a circuit. The capacitance of one of these capacitors is 1.5×10^{-3} F.



a) Calculate the total capacitance of the circuit.

b) How many capacitors were connected in the circuit?

Practice Questions — Fact Recall

Q1 Write down the formula you would use to find the total capacitance of a circuit containing a number of capacitors connected in series.

Q2 What value is given by the gradient of a straight line graph of charge against potential difference across a set of capacitors?

Learning Objectives:

- Understand the effect of charging and discharging a capacitor through a resistor.
- Understand techniques and procedures to investigate the charge and discharge of a capacitor using both meters and data loggers (PAG9).
- Understand graphical methods and spreadsheet modelling of the equation $\frac{\Delta Q}{\Delta t} = -\frac{Q}{CR}$ for a discharging capacitor.

Specification Reference 6.1.3

Tip: As always, be sure to carry out a risk assessment before you start any of the investigations in this section.

Tip: Data loggers can be connected to a computer which will collect all your data for you and plot all sorts of graphs of it.

Tip: In Figure 2, I_0 is the initial current in the circuit. V_0 and Q_0 are the p.d. across and charge stored by the capacitor when it is fully charged.

Tip: Current is the odd one out here — while everything else increases, current decreases as time passes.

3. Investigating Charging and Discharging Capacitors

Charging or discharging a capacitor 'through' a fixed resistor makes it easier to investigate.

Investigating charging a capacitor

PRACTICAL ACTIVITY GROUP 9

As you'll have seen when trying to keep the current constant in the experiment on page 353, capacitors have an effect on the properties of a circuit (e.g. current in the circuit, potential difference across other components, etc). You need to know how to measure these effects, and explain how the capacitor causes them.

A capacitor can charge (and discharge) fairly quickly, making the process difficult to investigate. Including a fixed resistor in your test circuit and charging the capacitor 'through' the resistor is a good idea. The resistance in the circuit will slow down the rate at which the capacitor charges or discharges, which makes the process much easier to investigate.

To investigate a charging capacitor, set up the test circuit in Figure 1.

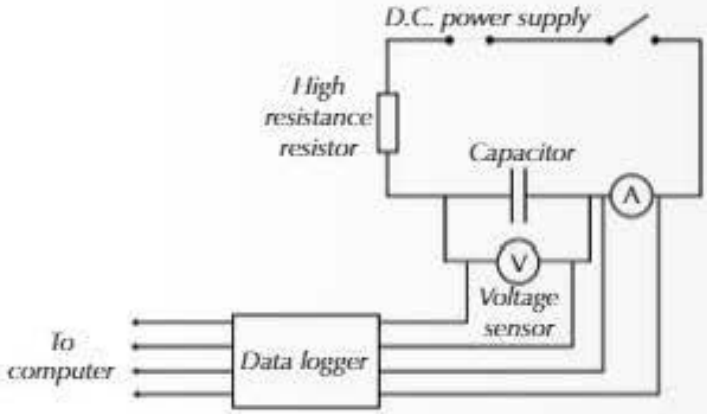


Figure 1: *A test circuit which can be used to investigate a circuit containing a charging capacitor.*

- Close the switch to allow the capacitor to begin charging.
- Let the capacitor charge whilst the data logger records both the p.d. (from the voltmeter) and the current (from the ammeter) over time.
- When the current through the ammeter is 0, the capacitor is fully charged.
- You can then use a computer to plot a graph of current, p.d. or charge against time (remember $\Delta Q = I\Delta t$) — see Figure 2.

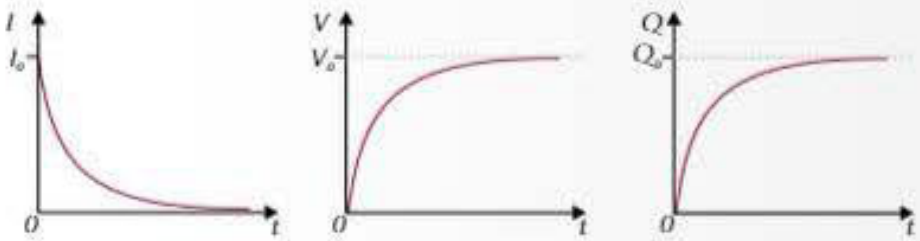


Figure 2: *Graphs to show charging current against time (left), potential difference against time (middle) and charge against time (right) for a capacitor being charged through a fixed resistor.*

As soon as the switch is closed, a current starts to flow. The potential difference across the capacitor is zero at first, so there is no p.d. opposing the current. The potential difference of the power supply causes an initial relatively high current of $\frac{V}{R}$ to flow (where V is the voltage of the power supply and R is the resistance of the resistor).

As the capacitor charges, the p.d. across the capacitor gets bigger, the p.d. across the resistor gets smaller, and the current drops. The charge (Q) on the capacitor is proportional to the potential difference across it, so the Q-t graph is the same shape as the V-t graph.

Investigating discharging a capacitor

You can carry out a similar experiment to investigate a discharging capacitor. Simply open the switch, remove the power supply, and then reconnect the circuit as shown in Figure 3.

PRACTICAL ACTIVITY GROUP 9

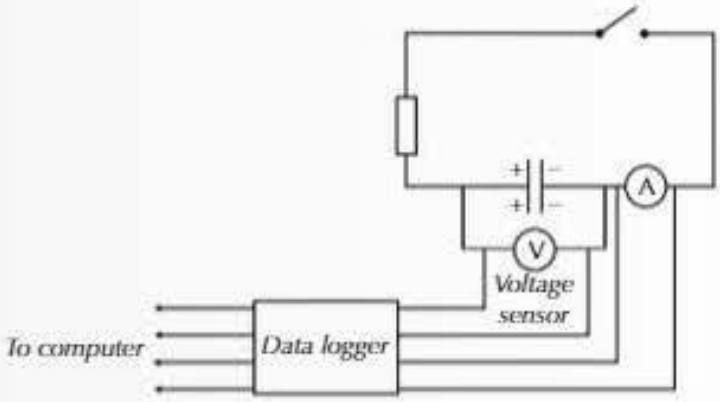


Figure 3: *A test circuit which can be used to investigate a circuit containing a discharging capacitor.*

- Close the switch to allow the capacitor to begin discharging.
- Let the capacitor discharge whilst the data logger records potential difference and current over time.
- When the current through the ammeter and the potential difference across the plates fall to zero, the capacitor is fully discharged.
- You can once more plot graphs of p.d., charge and current against time (see Figure 5).

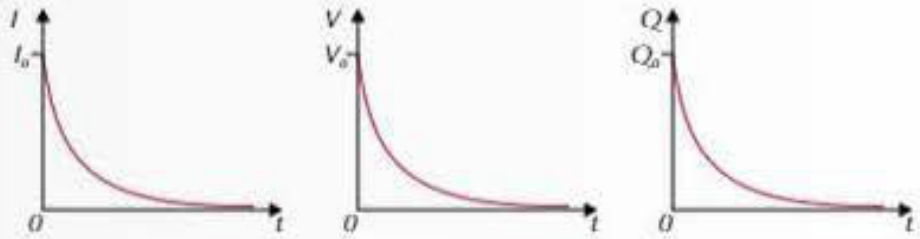


Figure 5: *Graphs to show discharging current against time (left), potential difference against time (middle) and charge against time (right) for a capacitor discharging through a fixed resistor.*

The electrons (current) flow from the negative plate to the positive plate. Initially, the current is high, but as the charge leaves the plates, the potential difference across the plates decreases. So the electrostatic repulsion decreases, reducing the flow of current.

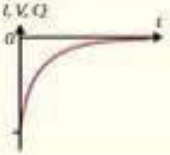
Tip: Remember, $V = IR$ so $I = V/R$.

Tip: You could also do these experiments without the data logger, by manually taking readings from the ammeter and voltmeter at fixed time intervals. But you'll need to use a fixed resistor with a very high resistance to make sure the capacitor charges and discharges slowly enough for you to take enough measurements.

Figure 4: *Some capacitors must be connected the right way round in the circuit. In this photo, the strip of minus signs indicates which pin should be connected to the negative terminal.*

Tip: V is proportional to Q (see page 346), so you always get the same shape graph whether you use V or Q on the vertical axis.

Tip: Depending on your apparatus, you may get graphs like this:



This is because the charge is flowing in the opposite direction to the charging current, so the reading on your meters will have changed sign.

Modelling a discharging capacitor

You can predict roughly how the stored charge will change over time for any discharging capacitor with capacitance C and initial charge Q, in a circuit with resistance R. You need to find the change in charge over a tiny time interval and repeat this process for a long period of time. To do this, you'll need an equation which relates ΔQ and Δt.

You know that $I = \frac{\Delta Q}{\Delta t}$ and also that $I = \frac{V}{R}$, so:

$$\frac{\Delta Q}{\Delta t} = \frac{V}{R}$$

For capacitors, $Q = CV$ so $V = \frac{Q}{C}$. Combine these and you get:

$$\frac{\Delta Q}{\Delta t} = -\frac{Q}{CR}$$

Tip: There's a minus sign here because the charge is decreasing over time.

Finally, multiply both sides by Δt to get:

$$\Delta Q = -\frac{Q}{CR}\Delta t$$

You can use this formula to plot a graph of Q against t, by working out how Q changes for each small change in t. Graphical methods like this are really useful for modelling relationships, but they can take a long time to do by hand. So using a spreadsheet can be very helpful.

Using a spreadsheet to model discharge

You can use this equation in a spreadsheet to demonstrate how a capacitor gradually discharges over time.

Tip: There's another example of modelling exponential relationships with spreadsheets on p.434.

1. Create a new spreadsheet, with columns for the time elapsed since the capacitor began discharging, t, the change in charge, ΔQ, and the charge remaining on the capacitor, Q.
2. Choose an initial starting charge, Q_{init}, for the capacitor along with a value for the capacitance C and the resistance R of the resistor.
3. Choose a sensible time interval Δt that is significantly less than CR. This will let you plot more precise graphs showing the relationship between charge and time.
4. In the initial row, $t_0 = 0$ and $Q_0 = Q_{init}$. Leave ΔQ blank in this row.
5. In the next row:
   - The new time $t_1 = t_0 + \Delta t$.
   - You can write a formula for the change in charge, ΔQ, over the time interval by using the equation $\Delta Q = -\frac{Q}{CR}\Delta t$ above. Let's call it $(\Delta Q)_1$.
   - The new charge is $Q_1 = Q_0 + (\Delta Q)_1$ so write a formula for that too.
6. Repeat this process of calculating the values in each row from Q and t in the row above. If you write the formulas correctly, you can use the spreadsheet program to automatically fill in as many rows as you want.
7. Once you have enough points, plot a graph of charge against time — see Figure 8. It should be similar to the graph for a discharging capacitor shown on page 357.

Tip: Figure 6 on the next page shows how all this can be laid out.

Tip: This process is called iteration — where you apply an equation repeatedly, using the previous result as the input for the next calculation. A spreadsheet that does this is known as an iterative spreadsheet, and its data is known as iterative data.

Data input cells		
	Δt	E.g. 1×10^{-12} s
	C	E.g. 3×10^{-12} F
	R	E.g. 12 Ω
	$Q_{initial}$	E.g. 6×10^{-7} C

t (s)	ΔQ (C)	Q (C)
$t_0 = 0$		$Q_0 = Q_{initial}$
$t_1 = t_0 + \Delta t$	$(\Delta Q)_1 = -\frac{Q_0}{CR}\Delta t$	$Q_1 = Q_0 + (\Delta Q)_1$
$t_2 = t_1 + \Delta t$	$(\Delta Q)_2 = -\frac{Q_1}{CR}\Delta t$	$Q_2 = Q_1 + (\Delta Q)_2$

Figure 6: *An example of the formulas that can be used to create an iterative spreadsheet of charge over time for a discharging capacitor.*

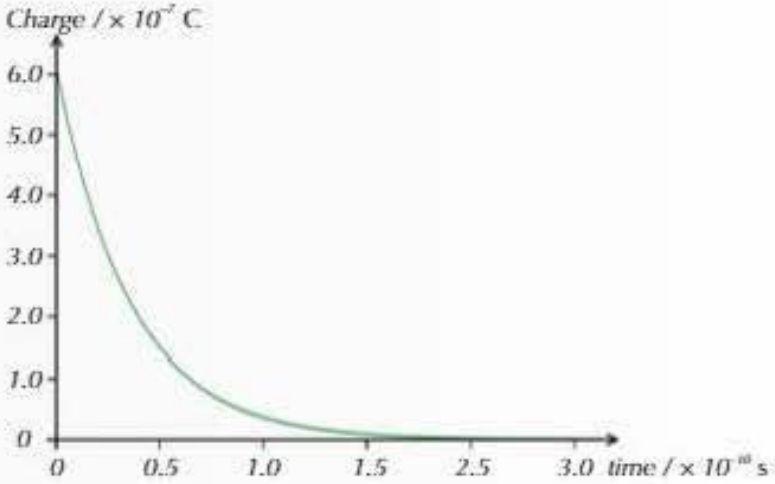


Figure 8: *An example of a charge-time graph produced from a spreadsheet model of a discharging capacitor, using the values in Figures 6 and 7.*

Tip: The graph you would get from these initial values is plotted in Figure 8.

	A	B	C
1	Δt in s	1E-12	
2	C in F	3E-12	
3	R in Ω	12	
4	$Q_{initial}$ in C	6E-07	
5			
6	t in s	ΔQ in C	Q in C
7	0.00E+00		6.00E-07
8	1.00E-12	-1.67E-08	5.83E-07
9	2.00E-12	-1.62E-08	5.67E-07
10	3.00E-12	-1.58E-08	5.51E-07
11	4.00E-12	-1.53E-08	5.36E-07
12	5.00E-12	-1.49E-08	5.21E-07
13	6.00E-12	-1.45E-08	5.07E-07
14	7.00E-12	-1.41E-08	4.93E-07
15	8.00E-12	-1.37E-08	4.79E-07

Figure 7: *A sample of the spreadsheet produced by the example values in Figure 6. Once you've filled in rows 7 and 8, you can automatically fill the rest of the rows using the formulas in row 8, if you've written the formulas correctly.*

Tip: To produce the graph in Figure 8, from the data in Figure 7, over 200 rows of iterative data were used.

Practice Question — Application

Q1 A fully-charged 4.2 mF capacitor is connected in a circuit in series with a 5.5×10^3 Ω resistor. It discharges 85 nC of charge in the first 4.5 μs. Calculate the charge on the capacitor when it is fully charged.

Practice Questions — Fact Recall

Q1 Draw curves showing how the p.d. across the capacitor and current through the circuit vary with time when a capacitor is charged through a fixed resistor.

Q2 How can you alter the circuit used to charge a capacitor so that the capacitor discharges?

Q3 Describe an experiment you could do to investigate a discharging capacitor and plot graphs of current in the circuit and potential difference across the capacitor against time.

Learning Objectives:

- Be able to analyse capacitor-resistor circuits.
- Know and understand equations in the form of $x = x_0 e^{-\frac{t}{CR}}$ and $x = x_0(1 - e^{-\frac{t}{CR}})$ for capacitor-resistor circuits.
- Understand the constant-ratio property of exponential decay graphs.
- Know what is meant by the time constant of a capacitor-resistor circuit, $\tau = CR$.

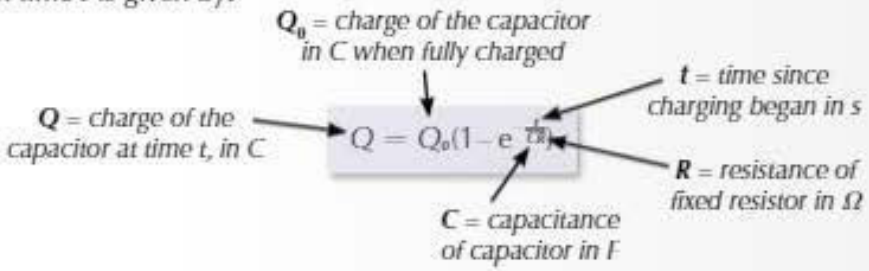
Specification References 6.1.1 and 6.1.3

4. Charging and Discharging Calculations

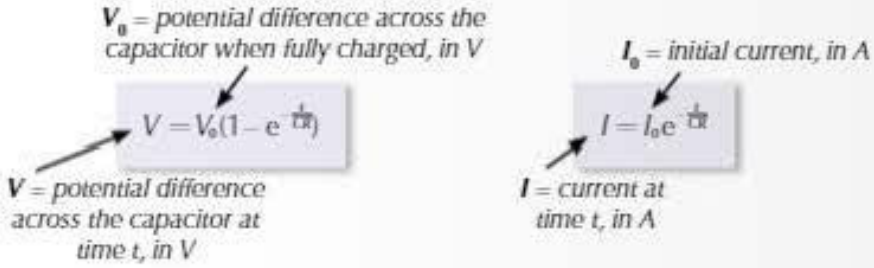
The shapes of the graphs that you plotted in the previous section are actually exponential graphs. There are exponential relationships between V, Q or I and time for charging and discharging capacitors.

Equations for a charging capacitor

The charge on a charging capacitor in a capacitor-resistor circuit after a given time t is given by:


Q_0 = charge of the capacitor in C when fully charged
t = time since charging began in s
Q = charge of the capacitor at time t, in C
$Q = Q_0(1 - e^{-\frac{t}{CR}})$
R = resistance of fixed resistor in Ω
C = capacitance of capacitor in F

The formula for calculating the voltage across a charging capacitor is of the same form, but the formula for the charging current is different — it decreases exponentially. The formulas for both are:


V_0 = potential difference across the capacitor when fully charged, in V
$V = V_0(1 - e^{-\frac{t}{CR}})$
V = potential difference across the capacitor at time t, in V
I_0 = initial current, in A
$I = I_0 e^{-\frac{t}{CR}}$
I = current at time t, in A

Example — **Maths Skills**

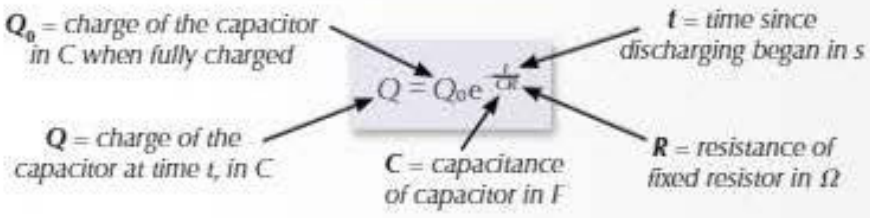
When fully charged, a 3.00 μF capacitor holds 36.0 μC of charge. It is connected in series with a 56.0 kΩ resistor. Calculate the charge on the capacitor 0.500 seconds after it begins charging.

$$Q = Q_0(1 - e^{-\frac{t}{CR}}) = 36.0 \times 10^{-6}(1 - e^{-\frac{0.500}{3.00 \times 10^{-6} \times 56.0 \times 10^{3}}})$$
$$= 3.416... \times 10^{-5} = 34.2\,\mu C \text{ (to 3 s.f.)}$$

Tip: Make sure you don't miss a prefix — capacitance is often given in micro-, nano- or picofarads.

Equations for a discharging capacitor

The charge left on the plates of a capacitor discharging from full in a capacitor-resistor circuit is given by the equation:


Q_0 = charge of the capacitor in C when fully charged
t = time since discharging began in s
$Q = Q_0 e^{-\frac{t}{CR}}$
Q = charge of the capacitor at time t, in C
C = capacitance of capacitor in F
R = resistance of fixed resistor in Ω

The same is true for the potential difference and current:

$$V = V_0 e^{-\frac{t}{CR}} \qquad I = I_0 e^{-\frac{t}{CR}}$$

Tip: V_0 is the potential difference across the capacitor when it is fully charged, and I_0 is the initial current flowing through the circuit.

Example — **Maths Skills**

A 0.20 mF capacitor was charged to a potential difference of 12 V and then discharged through a fixed 50 kΩ resistor. Calculate the charge on the capacitor after 1 second.

The question tells you $C = 0.20 \times 10^{-3}$ F, $V = 12$ V, $R = 50 \times 10^{3}$ Ω and $t = 1$.

First you need to calculate the initial charge.

$C = \frac{Q}{V}$ so $Q = CV$, so the initial charge is:

$Q_0 = CV = 0.20 \times 10^{-3} \times 12 = 2.4 \times 10^{-3}$ C

Then use the equation:

$$Q = Q_0 e^{-\frac{t}{CR}}$$
$$= 2.4 \times 10^{-3} \times e^{-\frac{1}{(0.20 \times 10^{-3}) \times (50 \times 10^{3})}}$$
$$= 2.4 \times 10^{-3} \times e^{-0.1}$$
$$= 2.171... \times 10^{-3} = 2 \times 10^{-3} \text{ C (to 1 s.f.)}$$

Tip: There's some help with logarithms and exponentials on pages 472-473. There should be a button on your calculator for 'e' that you can use for this calculation.

Charging and discharging times

The time taken for a capacitor to charge and discharge depends on two factors:

- The capacitance of the capacitor (C). This affects the amount of charge that can be transferred at a given voltage.
- The resistance of the circuit (R). This affects the current in the circuit.

For given values of C and R, the quantity x in an exponential relationship of the form $x = x_0 e^{-\frac{t}{CR}}$ always takes the same time to halve, no matter what the initial value of x is.

This is because, when x is halved, $x = \frac{1}{2}x_0$.

So:

$$\frac{1}{2}x_0 = x_0 e^{-\frac{t}{CR}}$$
$$\Rightarrow \frac{1}{2} = e^{-\frac{t}{CR}}$$

Taking logs of both sides:

$$\ln\left(\frac{1}{2}\right) = \ln\left(e^{-\frac{t}{CR}}\right)$$

$\frac{1}{2} = 2^{-1}$, and $\ln(A^B) = B\ln(A)$, so:

$$-\ln(2) = \frac{-t}{CR}$$
$$\Rightarrow t = \ln(2)CR$$

$\ln(2)$ is just a number, and the capacitance, C, and resistance, R, are fixed for a given capacitor-resistor circuit, so:

$$t = \text{constant}$$

So the time taken to halve is always the same. In fact, for a given proportion it always takes the same time for that proportion of the charge to be lost — it's known as the constant-ratio property of exponential relationships.

Tip: The same applies to potential difference and current. In a discharging capacitor it will always take the same time for the p.d. or current to decrease by a given proportion.

Time constant

When the discharge time t is equal to CR the equation for the charge left on a discharging capacitor becomes:

$$Q = Q_0 e^{-1}.$$

So when $t = CR$:

$$\frac{Q}{Q_0} = \frac{1}{e}, \text{ where } \frac{1}{e} \approx \frac{1}{2.718} \approx 0.37$$

Tip: τ is the Greek letter 'tau'.

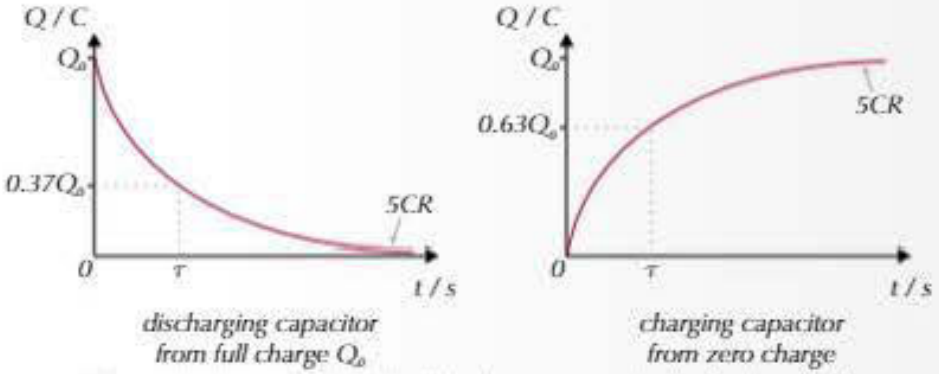
The time $t = CR$ is known as the **time constant**, τ, and is the time taken for the charge on a discharging capacitor (Q) to fall to about 37% of Q_0. It's also the time taken for the charge of a charging capacitor to rise to about 63% of Q_0 (Figure 1).

Tip: For a charging capacitor, no matter what charge it starts at, after the time constant has elapsed the charge on the capacitor will be 63% closer to fully charged.

It doesn't matter when you start looking at the charge on the capacitor after it has begun discharging (i.e. what value you take as Q_0), it will always have dropped to 37% of that value after a period of time equal to the time constant because of the constant-ratio property of exponential decay.

The larger the resistance in series with the capacitor, the longer it takes to charge or discharge. In practice, the time taken for a capacitor to charge or discharge fully is taken to be about $5CR$ or 5τ.

Tip: Q is proportional to V, so the time constant is also the time taken for the voltage to decrease to about 37% of the initial voltage (or increase to about 63% of the source voltage while charging).



discharging capacitor from full charge Q_0

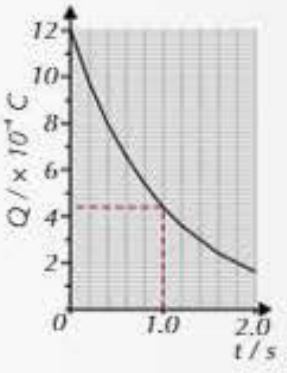
charging capacitor from zero charge

Figure 1: *Q-t graphs showing the time constant for discharging (left) and charging (right) a capacitor through a fixed resistor.*

You can calculate the time constant directly using $\tau = CR$ or you can find it from a graph. These may be graphs of Q against t (as shown above and in the next example) or log-plots of $\ln(Q)$ against t (see next page).

Example — Maths Skills

A capacitor was discharged from full through a 10 kΩ resistor (to 2 s.f.). The graph below shows how the charge on the capacitor changed over the first 2.0 seconds. Calculate the time constant and the capacitance.



The initial charge is 12×10^{-4} C, and you want to find the time taken for the charge to decrease to 37% of that.

So $0.37 \times (12 \times 10^{-4}) = 4.4 \times 10^{-4}$ C.

Using the graph, the charge is 4.4×10^{-4} C when the time is 1.0 s, so the time constant, τ, is 1.0 s.

$\tau = CR$, so, $C = \frac{\tau}{R}$

$$C = \frac{1.0}{10 \times 10^3}$$

$$= 1.0 \times 10^{-4} \text{ F} = 0.10 \text{ mF (to 2 s.f.)}$$

Tip: You might be asked to find the time it takes for a capacitor to discharge (or charge) to a certain percentage of its total charge.

Tip: Make sure that you remember that Q / Q_0 is the proportion of the charge <u>on the plates</u>, not the proportion of the charge that's been lost.

Log-linear graphs

Log-linear graphs are plots where one of the axes is logarithmic. Log-linear and log-log plots are useful as they can often be used to produce a graph which is a straight line when linear axes would give a curve. So they're another good way of graphically displaying how charge, potential difference and current vary over time for a discharging capacitor.

Tip: Log-log plots are where both axes are logarithmic.

Starting from the equation for charge on a discharging capacitor, the natural logarithm is taken of both sides:

$$Q = Q_0 e^{-\frac{t}{CR}}$$

$$\ln(Q) = \ln\left(Q_0 e^{-\frac{t}{CR}}\right)$$

$\ln(A \times B) = \ln(A) + \ln(B)$, so this can be written as:

$$\ln(Q) = \ln(Q_0) + \ln(e^{-\frac{t}{CR}})$$

Another log rule is $\ln(e^{\wedge}) = A$, so:

$$\ln(Q) = \left(-\frac{1}{CR}\right)t + \ln(Q_0)$$

Tip: Log graphs come up a few times in physics — see p.484 for how to deal with them.

If you then plot a graph of $\ln(Q)$ against t, you will see it is a straight line (Figure 2). By comparing the equation for $\ln(Q)$ to the equation for a straight line, you can see that the gradient is equal to $-\frac{1}{CR} = -\frac{1}{\tau}$.

Tip: This equation is in the form $y = mx + c$ — the equation of a straight line.

So you can find the time constant from the log-linear graph by dividing –1 by the gradient of the line.

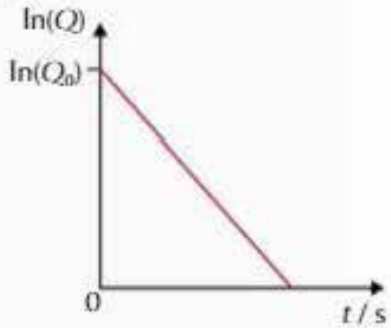


Figure 2: *A plot of ln(Q) against t.*

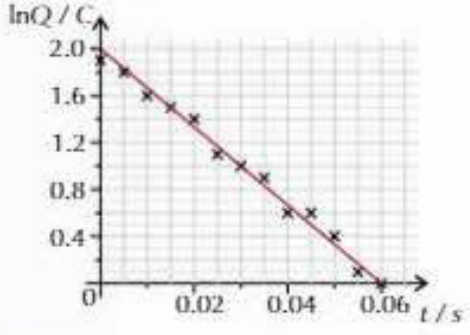
Tip: The graph's intercept with the vertical axis is $\ln(Q_0)$ — it's the c in $y = mx + c$.

You could also plot $\ln(V)$ or $\ln(I)$ against time and find the time constant in the same way, as V and I both have the same dependence on C and R.

Tip: You can also plot a graph like this for the current in a charging capacitor. It's a bit more complicated for charge and potential difference though.

Example — **Maths Skills**

A student has done an experiment to find out how the charge on a discharging capacitor varies with time. She plots her results on a ln$Q$-t graph and draws a line of best fit. The graph is shown below. Use the graph to find the time constant of the circuit.



The gradient of a ln$Q$-t graph is equal to $-\frac{1}{CR}$.

To find the time constant, $\tau = CR$, you divide –1 by the gradient.

$$\text{Gradient} = \frac{2.0 - 0}{0.06 - 0} = -33.333...$$

$$\tau = \frac{-1}{-33.333...} = 0.030 \text{ s (to 2 s.f.)}$$

Tip: You'll need $Q = CV$ for this, which is given in the data and formulae book.

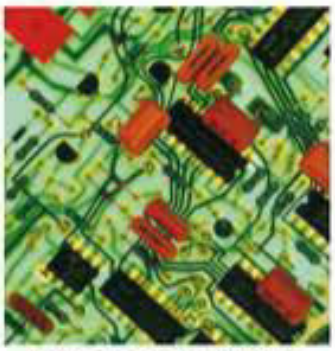


Figure 3: *A circuit from a burglar alarm containing capacitors. They can be used with big resistors to create a time delay. When the door is opened the capacitor begins to discharge and when it has lost a certain amount of its charge the alarm goes off, giving you time beforehand to enter the code.*

Practice Questions — Application

Q1 A capacitor with capacitance 0.60 μF is initially charged to 15.0 V. It is then connected across a resistor with resistance 2.6 kΩ. Calculate the potential difference across the capacitor 1.5 ms after it has begun to discharge.

Q2 Calculate the charge stored on a 15 nF capacitor, originally charged to a potential difference of 230 V and then discharged for 0.01 seconds through a 50 kΩ resistor.

Q3 A capacitor discharged in series with a resistor can be used to create a time delay function in electronics. In a particular burglar alarm, the alarm goes off when a 15 μF capacitor discharged through a 400 kΩ resistor has lost 63% of its initial charge.

a) Calculate the time delay on the alarm.

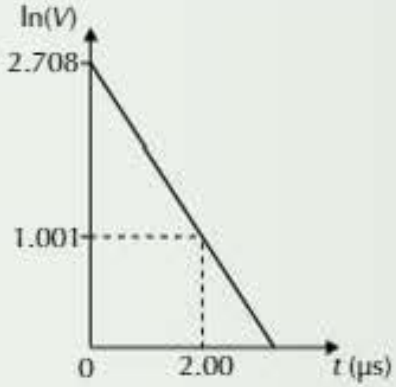
The manufacturer wants to increase the capacitance of the capacitor in order to make the time delay 60 seconds.

b) What capacitance would be needed for this time delay?

c) Explain why this might not be practical, and suggest what other component they could replace instead.

Q4 A discharging capacitor loses 70% of its initial charge, Q_0, in 20.0 seconds. Find the time constant of the capacitor-resistor circuit.

Q5 A plot of ln(V) against t is created for a discharging capacitor.



a) What does the vertical intercept of the graph represent?

b) Calculate the initial voltage.

c) Calculate the time constant for the capacitor.

Practice Questions — Fact Recall

Q1 What is the equation for calculating the charge on a capacitor charging through a fixed resistor R, after a certain time t?

Q2 Write down the equation for the p.d. across a discharging capacitor.

Q3 What two factors do the charging and discharging time of a capacitor depend on?

Q4 What is meant by the time constant when discharging a capacitor?

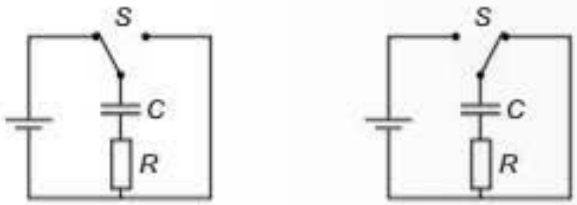
Section Summary

Make sure you know...

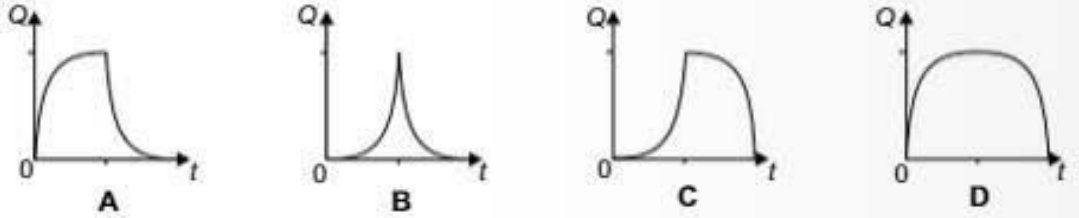
- That a capacitor is an electrical component that can store electric charge.
- That capacitance is defined as the amount of charge stored per unit potential difference, and it can be calculated using the equation $C = \frac{Q}{V}$.
- That capacitance is measured in farads, F.
- How the flow of electrons causes an equal and opposite charge to build up on each plate of a capacitor when the capacitor is being charged.
- How the flow of electrons causes the charge on each plate of a capacitor to decrease when it is being discharged.
- That the area underneath a graph of charge against p.d. is the energy stored by the capacitor.
- The equations for energy stored by a capacitor: $W = \frac{1}{2}QV = \frac{1}{2}V^2C = \frac{1}{2}\frac{Q^2}{C}$.
- Some uses of capacitors in the storage of electrical energy.
- That the total capacitance of two or more capacitors in parallel is given by: $C = C_1 + C_2 + ...$
- That the total capacitance of two or more capacitors in series is given by: $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + ...$
- How to perform an experiment to investigate the effect of connecting capacitors in series and in parallel.
- Understand that the rate at which a capacitor charges or discharges in a circuit can be slowed by including a resistor in the circuit.
- How to perform an experiment using a capacitor-resistor circuit to investigate the charging and discharging of a capacitor, using both meters and data loggers to record data.
- How the current in a circuit changes as a capacitor charges and discharges, including a graph of current against time.
- How the potential difference across a capacitor changes as it charges and discharges, including a graph of p.d. against time.
- How the charge on a capacitor changes as it charges and discharges, including a graph of charge against time.
- That the equation $\frac{\Delta Q}{\Delta t} = -\frac{Q}{CR}$ can be used to predict the remaining charge on a discharging capacitor after a given small period of time.
- How to use this equation to model a discharging capacitor using a spreadsheet.
- For a charging capacitor in a capacitor-resistance circuit, the charge on the capacitor at time t is given by $Q = Q_0(1 - e^{-\frac{t}{CR}})$ and the potential difference across the capacitor at time t is given by $V = V_0(1 - e^{-\frac{t}{CR}})$.
- For a discharging capacitor in a capacitor-resistance circuit, the charge on the capacitor at time t is given by $Q = Q_0 e^{-\frac{t}{CR}}$ and the potential difference across the capacitor at time t is given by $V = V_0 e^{-\frac{t}{CR}}$.
- For a charging or discharging capacitor in a capacitor-resistance circuit, the current at time t is given by $I = I_0 e^{-\frac{t}{CR}}$.
- The constant-ratio property of exponential functions, and how this applies to charging and discharging capacitors.
- That the time constant is given by the product of the capacitance and resistance of the circuit, $\tau = CR$.
- That the time constant, τ, of a capacitor-resistor circuit is the time taken for a capacitor to discharge to 37% of its initial value.

Exam-style Questions

1 When the switch S in the circuit shown below is in position 1, the capacitor C is fully charged by the battery through resistor R. The switch is then moved to position 2 and the capacitor is allowed to discharge fully through the resistor.



Which graph correctly shows how the charge, Q, on the capacitor varies with time, t, during this process?



(1 mark)

2 A capacitor discharging through a fixed 100 kΩ resistor loses 35% of its charge in 1 s. What is the capacitance of the capacitor?

A 9.5×10^{-6} F

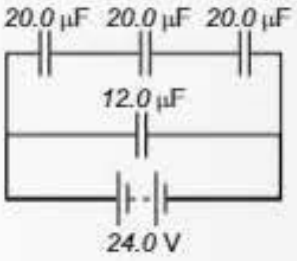
B 5.3×10^{-5} F

C 2.3×10^{-5} F

D 2.2×10^{-2} F

(1 mark)

3 The diagram on the right shows a circuit containing capacitors. Three 20 μF capacitors are connected in series in the circuit. A 12 μF is connected in parallel across these three capacitors. What is the total capacitance of the circuit?



A 10.0 μF

B 18.7 μF

C 12.2 μF

D 72.0 μF

(1 mark)

4 A student investigating capacitance charges a capacitor from a 12 V d.c. supply and uses a voltage sensor and a charge sensor connected to a data logger to measure the potential difference across the capacitor at regular intervals of charge stored by the capacitor. A computer is used to plot a graph of p.d. against charge.

(a) (i) Copy the axes below and sketch the graph obtained by the computer on the axes.

(1 mark)

(ii) State what quantity the area enclosed by the line and the horizontal axis represents.

(1 mark)

(b) When the capacitor is fully charged, it has a charge of 18 μC. Calculate the capacitance of the capacitor.

(2 marks)

The student then discharges the capacitor through a 29 kΩ resistor.

(c) What is the potential difference across the capacitor plates after 0.030 s?

(2 marks)

(d) Sketch a graph showing how the current supplied to the resistor varies over time as the capacitor discharges.

(1 mark)

(e) Calculate the time taken for the current in the circuit to fall to 10% of its initial value.

(2 marks)

5 A 3.0×10^{-3} F capacitor was fully charged to 50.0 V through a 2.0 kΩ resistor.

(a) Calculate how much energy was stored by the capacitor.

(2 marks)

(b) Show that the time constant of the circuit is 6 seconds.

(1 mark)

(c) Calculate the charge gained by the capacitor in 14 seconds.

(2 marks)

(d) Sketch a graph of charge against time for the first 14 seconds of charging.

(1 mark)

Learning Objectives:

- Understand the concept of electric fields as being one of a number of forms of field giving rise to a force.
- Know that electric fields are due to charges.
- Understand how electric field lines are used to map electric fields.
- Know what electric field strength is.
- Know how to use the equation $E = \frac{F}{Q}$.
- Know that a uniformly charged sphere can be modelled as a point charge at its centre.
- Know how to calculate the force between two point charges using Coulomb's law, $F = \frac{Qq}{4\pi\varepsilon_0 r^2}$.
- Know how to calculate electric field strength using $E = \frac{Q}{4\pi\varepsilon_0 r^2}$ for a point charge.

Specification References 6.2.1 and 6.2.2

Tip: Remember, vector means it has a magnitude and a direction.

Tip: Electric field strength can be measured in Vm^{-1} as well as NC^{-1}.

1. Electric Fields

Electric fields are a lot like gravitational fields. But although the concepts are similar, there are still some subtle differences, so make sure you follow carefully.

What are electric fields?

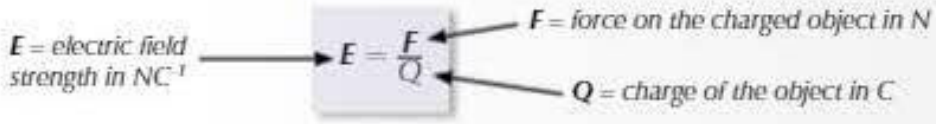
There are several fields that can cause forces — for example you met gravitational fields on p.297. Electric fields can also give rise to a force, but electric fields can be attractive or repulsive, unlike gravitational fields (which are always attractive). It's all to do with charge. Any object with charge has an electric field around it — the region where it can attract or repel other charges.

Electric charge is measured in coulombs (C) and can be positive or negative. Oppositely charged particles attract each other, and like charges repel each other. If a charged object is placed in an electric field, then it will experience a force.

Just like with gravitational fields, electric fields can be represented by field lines. Electric field lines are drawn to show the direction of the force that would act on a positive charge — see p.369 and p.372.

Electric field strength

Electric field strength, E, is defined as the force per unit positive charge. It's the force that a charge of +1 C would experience if it was placed in an electric field.



E is a vector pointing in the direction that a positive charge would move. The units of E are newtons per coulomb (NC^{-1}). The field strength may depend on where you are in the field (see radial fields on page 370).

Be careful — in this equation F is the force acting on a charge Q which is in the electric field. Q is not causing the electric field. Don't confuse it with the Q in the electric field strength equation on page 370.

Example **Maths Skills**

Find the force acting on an electron in an electric field with a field strength of 5000 NC^{-1}. The charge on an electron is -1.60×10^{-19} C.

Just rearrange the equation for electric field strength and put in the numbers:

$$E = \frac{F}{Q} \Rightarrow F = E \times Q = 5000 \times (-1.60 \times 10^{-19}) = -8 \times 10^{-16} \text{ N}$$

Radial fields

Point charges have a **radial electric field** (see Figure 1). Remember, the field lines show the direction in which a positive charge (shown by $+q$ in Figure 1) would feel a force when placed in the electric field. So for a positive point charge, $+Q$, the field lines point away from the point charge, and for a negative point charge, $-Q$, they point towards it.

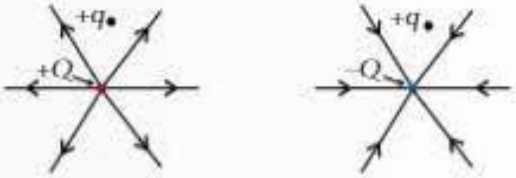


Figure 1: *Electric field lines for a positive point charge and a negative point charge.*

Tip: The charge $+q$ in Figure 1 also has an electric field — it just hasn't been drawn here.

Notice how the field lines get further apart as you move further from the charge — this shows that the electric field strength is getting weaker.

A uniformly charged sphere has its charge evenly distributed across its surface. You can treat this as if it is a point charge with all of its charge concentrated at the centre of the sphere. Any body that behaves as if all its charge is concentrated at the centre has a radial field.

Figure 2: *You can charge a balloon by rubbing it against a material. The charge of the balloon can then be treated as a point charge at the balloon's centre (assuming the balloon is spherical and the charge is evenly distributed over the balloon's surface).*

Coulomb's law

You can calculate the force on a charged object in a radial electric field using Coulomb's law. It gives the force of attraction or repulsion between two point charges, Q and q, in a vacuum:

$$F = \frac{Qq}{4\pi\varepsilon_0 r^2}$$

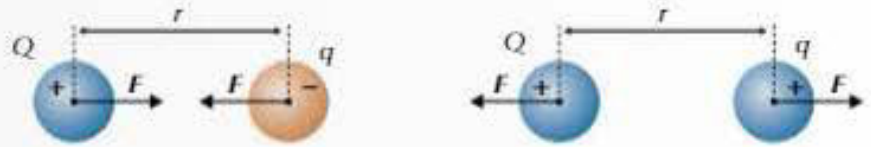
F = force on the object in N

Q and q = charges of the two objects in C

ε_0 = "epsilon-nought", the permittivity of free space $= 8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}$

r = distance between Q and q in m

The force on Q is always equal and opposite to the force on q — the direction depends on the charges.



*If the charges are opposite then the force is attractive. **F** will be negative.*

*If the charges are alike then the force is repulsive. **F** will be positive.*

Figure 3: *The direction of the forces on two charged objects.*

Tip: It's the interaction between the electric fields of Q and q that causes the force between them.

Coulomb's law is another case of an inverse square law, so $F \propto \frac{1}{r^2}$ (page 299). The further apart the charges, the weaker the force between them. The size of the force F also depends on the permittivity, ε, of the material between the two charges. For free space (a vacuum), the permittivity is $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}$. If the two charges weren't in a vacuum, you would replace ε_0 with the ε for the material they're in (see page 372 for more on permittivity).

Tip: You can also give the units of ε_0 as Fm^{-1}.

Exam Tip
Always assume the charges are in a vacuum if the question doesn't say otherwise.

Example — **Maths Skills**

Find the acceleration experienced by a free electron 2.83 mm from the centre of a sphere carrying a charge of +0.510 μC. The charge of an electron is -1.60×10^{-19} C and the mass of an electron is 9.11×10^{-31} kg. (You may assume the sphere has a radius less than 2.83 mm.)

The acceleration of an object is given by $F = ma \Rightarrow a = \frac{F}{m}$.
You know the mass, so find the force using Coulomb's law:

$$F = \frac{Qq}{4\pi\varepsilon_0 r^2} = \frac{(0.510 \times 10^{-6}) \times (-1.60 \times 10^{-19})}{4\pi\varepsilon_0 \times (2.83 \times 10^{-3})^2}$$
$$= -9.161... \times 10^{-11} \text{ N}$$

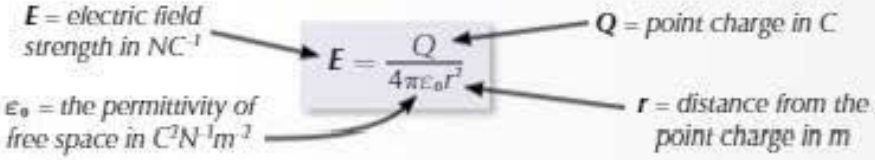
Then use this to find the acceleration:

$$a = \frac{F}{m} = \frac{-9.161... \times 10^{-11}}{9.11 \times 10^{-31}} = -1.00564... \times 10^{20}$$
$$= -1.01 \times 10^{20} \text{ ms}^{-2} \text{ (to 3 s.f.)}$$

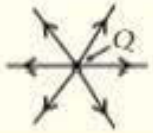
Tip: The acceleration is negative because it's towards the sphere.

Electric field strength in radial fields

When the electric field is being generated by a point charge, we call the charge generating the field Q and redefine the charge experiencing the force as q (as in Figure 1 on p.369). In a radial field, E is equal to:


E = electric field strength in NC⁻¹
E = Q / 4πε₀r²
Q = point charge in C
ε₀ = the permittivity of free space in C²N⁻¹m⁻²
r = distance from the point charge in m

Tip: Q is the point charge creating the radial field, not the charge experiencing a force inside the field.



Example — **Maths Skills**

The electric field strength 0.15 m away from the centre of a charged sphere is 44 000 NC⁻¹. What's the charge on the sphere? (You may assume the sphere has a radius less than 0.15 m.)

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \Rightarrow Q = (4\pi\varepsilon_0 r^2)E = 4\pi \times (8.85 \times 10^{-12}) \times 0.15^2 \times 44\,000$$
$$= 1.10100... \times 10^{-7}$$
$$= 1.1 \times 10^{-7} \text{ C (to 2 s.f.)}$$

Tip: Remember — a uniformly charged sphere can be treated as if all its charge acts at its centre (p.369).

This is another case of the inverse square law — $E \propto \frac{1}{r^2}$. Field strength decreases as you go further away from Q. On a diagram, the field lines of a radial field get further apart (see Figure 1 on p.369), and if you plot the electric field strength against r you get the same shape as for gravitational field strength on page 301. If the charge isn't a point charge (e.g. if it's a charged metal sphere), then the electric field strength inside the object doesn't have the same $E \propto \frac{1}{r^2}$ relation (you don't need to worry what the electric field is inside a charged object, it just doesn't follow an inverse square law).

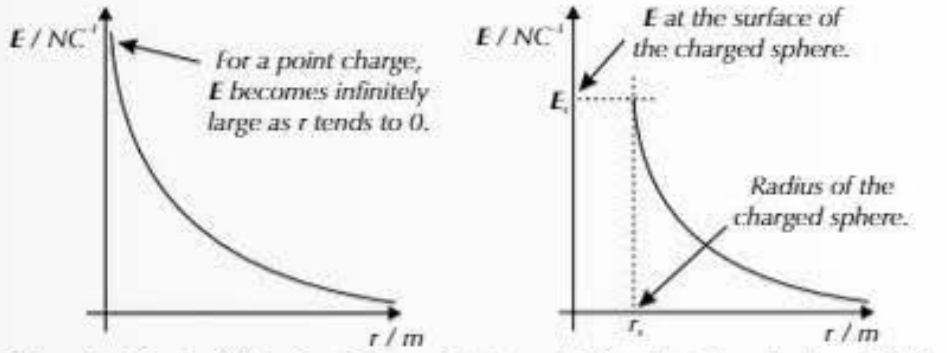

E / NC⁻¹
For a point charge, E becomes infinitely large as r tends to 0.
r / m
E / NC⁻¹
E at the surface of the charged sphere.
E_s
Radius of the charged sphere.
r_s
r / m

Figure 4: *Graphs of **E** against r for a point charge (left) and a charged sphere (right).*

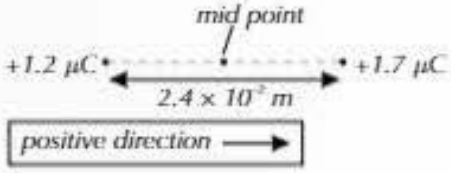
You can also work out the combined electric field strength for a point that lies between two different charges, Q_1 and Q_2. Remember, **E** is a vector, so you need to pick a positive direction.

Example — **Maths Skills**

Find the electric field strength at a point halfway between point charges of $Q_1 = +1.7\ \mu C$ and $Q_2 = +1.2\ \mu C$, separated by 2.4×10^{-2} m.

You're working with vectors, so choose a positive direction.

Let's say towards Q_1 is positive.



Both charges would repel a positive charge, so **E** due to Q_2 is positive and **E** due to Q_1 is negative. So the electric field strength, **E**, is equal to **E** due to Q_2, minus **E** due to Q_1:

$$E = \frac{1.2\times10^{-6}}{4\pi\times(8.85\times10^{-12})\times(1.2\times10^{-2})^2} - \frac{1.7\times10^{-6}}{4\pi\times(8.85\times10^{-12})\times(1.2\times10^{-2})^2}$$

$$= \frac{1.2\times10^{-6} - 1.7\times10^{-6}}{4\pi\times(8.85\times10^{-12})\times(1.2\times10^{-2})^2} = \frac{-0.5\times10^{-6}}{4\pi\times(8.85\times10^{-12})\times(1.2\times10^{-2})^2}$$

$$= -3.122...\times10^{7} = -3.1\times10^{7}\ \text{NC}^{-1}\ \text{(to 2 s.f.)}$$

Practice Questions — Application

Q1 Two electrons are fired towards each other and reach a separation of 5.22×10^{-13} m. What's the force on each electron at this point?

Q2 The electric field generated by a charged sphere with charge 4.15 μC is measured as 15 000 NC^{-1}. How far from the centre of the sphere must the measuring instrument be?

Q3 A particle with charge 5.0×10^{-5} C is in an electric field and experiences a force of 0.080 N. Calculate the magnitude of the electric field strength.

Practice Questions — Fact Recall

Q1 What's **E** a measure of?

Q2 Draw the electric field generated by a positive point charge.

Q3 Give the equation of Coulomb's law.

Q4 Sketch a graph of **E** and r for a positive point charge, where r is the distance from the charge.

Tip: Graphs of **E** against r usually only show the magnitude of **E** (so they're always positive). But if **E** was negative, the graph would just be reflected in the horizontal axis.

Tip: The graph of **g** against r is the same shape, but it's always under the r axis as **g** is always negative (p.301).

Tip: This only works if the point you're interested in lies on the same line as the two charges. Anywhere else and you have to start finding resultant vectors.

Tip: You can also find the total force, **F**, on a charge that's between two charges. It's very similar to this method, since $F = Eq$.

Tip: You could also have said that towards Q_2 was positive. You'd get a positive answer, showing that a positive charge at the mid point would move towards Q_2.

Tip: The charge on an electron, $-e$, is -1.60×10^{-19} C.

Learning Objectives:

- Know what a parallel plate capacitor is.
- Know how to calculate uniform electric field strength using $E = \frac{V}{d}$.
- Know what is meant by the permittivity of a material.
- Know and be able to use the equations $C = \frac{\varepsilon_0 A}{d}$ and $C = \frac{\varepsilon A}{d}$ where $\varepsilon = \varepsilon_r \varepsilon_0$.
- Be able to describe the motion of charged particles in a uniform electric field.

Specification Reference 6.2.3

Tip: E can be in Vm^{-1} or NC^{-1}.

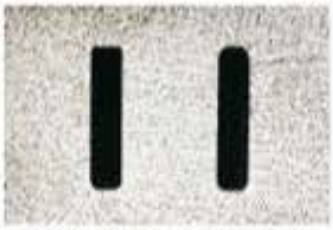


Figure 2: *Electric field lines between two plates shown by the alignment of pepper flakes in oil. The straight electric field lines between the plates show the area with the uniform electric field.*

Tip: Remember, capacitance is measured in farads (F). This is a large unit though, so you'll often see nano- or picofarad capacitors.

Tip: ε_r doesn't have any units because it's a ratio of the size of the electric field generated in the material, compared to if it was generated in a vacuum.

2. Uniform Electric Fields

You've met radial electric fields, now it's time for uniform electric fields. You get uniform fields in parallel plate capacitors, which you met on p.346.

Electric field strength in uniform fields

A **uniform electric field** is one with the same electric field strength everywhere. It can be produced by connecting two parallel plates to the opposite poles of a battery — see Figure 1. This is a parallel plate capacitor. The field lines point from the plate with the more positive potential to the plate with the less positive potential.

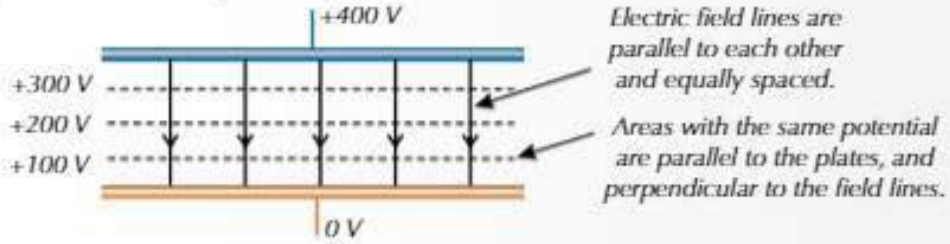

+400 V
+300 V
+200 V
+100 V
0 V
Electric field lines are parallel to each other and equally spaced.
Areas with the same potential are parallel to the plates, and perpendicular to the field lines.

Figure 1: *Electric field lines between parallel plates.*

The field strength E is the same at all points between the two plates and is given by:

E = electric field strength in Vm^{-1} → $E = \frac{V}{d}$ ← V = potential difference between the plates in V; ← d = distance between the plates in m

Note that here the potential difference between the plates is the same as the potential of the top plate. This is because for this example the potential of the bottom plate is 0 V. This won't always be the case, so you might find it easier to think of it as ΔV.

Example — **Maths Skills**

What's the electric field strength between two parallel plates 0.15 m apart with a potential of +650 V and +200 V respectively?

$$E = \frac{\Delta V}{d} = \frac{650 - 200}{0.15} = 3000\ \text{Vm}^{-1}$$

Capacitance of parallel plate capacitors

The capacitance (p.346) of a capacitor depends on the **permittivity** of the material between the plates — how easy it is to generate an electric field in the material. It also depends on the dimensions of the capacitor. When there's a vacuum between the plates, capacitance can be calculated by using:

C = capacitance of a capacitor in F → $C = \frac{\varepsilon_0 A}{d}$ ← ε_0 = permittivity of free space in Fm^{-1}; ← A = area of the plates in m^2; ← d = separation of the plates in m

If the plates have a material in between them instead of a vacuum, ε_0 is replaced with permittivity, ε:

ε = permittivity of material in Fm^{-1} → $C = \frac{\varepsilon A}{d}$ where ε is given by: $\varepsilon = \varepsilon_r \varepsilon_0$ ← ε_r = relative permittivity

Example — **Maths Skills**

A capacitor is made up of two parallel plates, each with an area of 12 mm². The plates are separated by a 0.10 mm thick piece of paper which has a relative permittivity of 2.7. Calculate the capacitance of this capacitor.

Permittivity of paper = $\varepsilon = \varepsilon_r\varepsilon_0 = 2.7 \times 8.85 \times 10^{-12} = 2.3895 \times 10^{-11}$ Fm^{-1}

Then calculate the capacitance:

$$C = \frac{\varepsilon A}{d} = \frac{2.3895 \times 10^{-11} \times 12 \times 10^{-6}}{0.10 \times 10^{-3}} = 2.8674 \times 10^{-12} = 2.9 \times 10^{-12} \text{ F (to 2 s.f.)}$$

Tip: You can assume that the paper fully fills the space between the plates.

Tip: You could also give the answer as 2.9 pF.

Charged particles in a uniform electric field

A uniform electric field can be used to determine whether a particle is charged or not. The path of a charged particle moving through an electric field (at an angle to the field lines) will bend — the direction depends on whether it's a positive or negative charge.

A charged particle entering an electric field at right angles to the field (as in Figure 3) feels a constant force, equal to $\boldsymbol{F} = \boldsymbol{E}Q$, parallel to the electric field lines. If the particle is positively charged then the force acts on it in the same direction as the field lines. If it's negatively charged, the force is in the opposite direction to the field lines. The work done on the particle by this force ($W = \boldsymbol{F}d$) increases its kinetic energy and causes it to accelerate at a constant rate in the direction of the force (Newton's second law). If the particle's velocity initially has a component at right angles to the field lines, this component will remain unchanged and the velocity in this direction will be uniform. That's Newton's first law. The combined effect of constant acceleration and constant velocity at right angles to one another is a curved path.

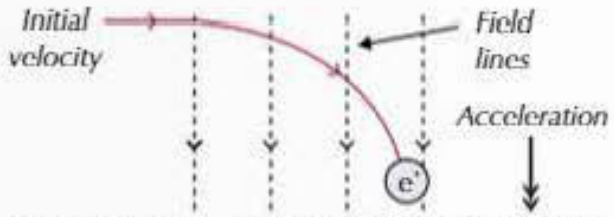


Figure 3: *Motion of a positron (which is positively charged) in a uniform electric field.*

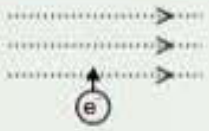
Tip: Remember, the equation for the kinetic energy of an object is $E_k = \frac{1}{2}mv^2$.

Tip: This motion is similar to a projectile in a uniform gravitational field, which you met in Module 3.

Tip: An electron (which is negatively charged) would accelerate upwards rather than downwards in the electric field in Figure 3.

Practice Questions — Application

Q1 Copy the diagram on the right and complete the path of the electron as it travels through the electric field.



Q2 An alpha particle with a charge of +2e and a mass of 6.64×10^{-27} kg is suspended freely between two parallel plates. The top plate has no charge and the bottom plate is charged to +5.00 nV. How far apart are the plates if the alpha particle isn't moving?

Q3 A capacitor is made up of two parallel plates, each with a 7.0×10^{-4} m² area. The vacuum gap between the plates is 8.5×10^{-6} m wide. Calculate the capacitance of the capacitor.

Tip: For Q2, if the particle isn't moving, the upwards force from the electric field must equal the downwards force from gravity (the weight, $\boldsymbol{W} = mg$).

Tip: 1 nV = 1×10^{-9} V.

Practice Question — Fact Recall

Q1 Give the equation that relates the permittivity of a material to the permittivity of free space. Define all symbols.

Learning Objectives:

- Know that the electric potential at a point in an electric field is the work done in bringing a unit positive charge from infinity to the point.
- Know that electric potential is zero at infinity.
- Be able to use the electric potential equation $V = \frac{Q}{4\pi\varepsilon_0 r}$ at a distance r from a point charge.
- Understand changes in electric potential.
- Be able to use $E = Vq = \frac{Qq}{4\pi\varepsilon_0 r}$ to calculate electric potential energy of a point charge q at a distance r from a point charge Q.
- Be able to sketch a force-distance graph for a point or spherical charge and know that work done to move a charge is equal to the area under the graph.
- Know and be able to use the equation $C = 4\pi\varepsilon_0 R$ for the capacitance of an isolated sphere, and be able to derive it.

Specification Reference 6.2.4

Tip: If you have more than one charge creating an electric field, you can find the total electric potential at a single point by adding the electric potentials at that point due to each charge.

Tip: You can give electric potential in V or JC^{-1}.

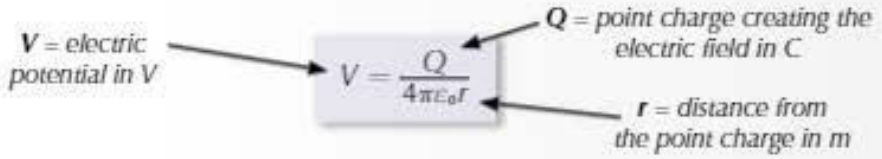
3. Electric Potential

Just like the gravitational potential of a mass in a gravitational field, you can find the electric potential of a charge in an electric field. As electric forces can be attractive or repulsive, electric potential can be positive or negative.

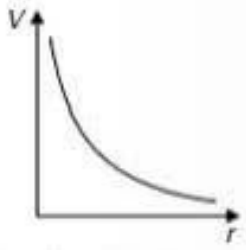
What is electric potential?

All points in an electric field have an **electric potential**, V. This is equal to the work done bringing a unit positive charge (+1 C) from a point infinitely far away to that point in the electric field. This means that at infinity, the electric potential will be zero.

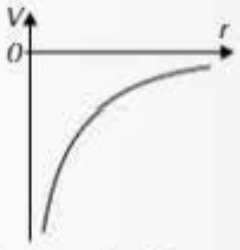
So electric potential is the potential energy per unit charge. In a radial field around a point charge, electric potential is given by:


V = electric potential in V
$V = \frac{Q}{4\pi\varepsilon_0 r}$
Q = point charge creating the electric field in C
r = distance from the point charge in m

The sign of V depends on the charge Q — V is positive when Q is positive and the force is repulsive, and negative when Q is negative and the force is attractive. The absolute magnitude of V is greatest on the surface of the charge, and decreases as the distance from the charge increases.



V changing with r for a positive charge (repulsive force). V is initially positive and tends to zero as r increases towards infinity.



V changing with r for a negative charge (attractive force). V is initially negative and tends to zero as r increases towards infinity.

Figure 1: *V changing with r for a positive charge (left) and for a negative charge (right).*

If a charge is moved within an electric field, it can experience a change in electric potential. For example, if a point charge is moved away from or towards a spherical charge, or if a point charge is moved from one plate of a parallel plate capacitor to the other plate.

Example — **Maths Skills**

A positively charged particle is placed 0.035 m from the centre of a sphere with a charge of +3.1 μC and radius 0.02 m. If the particle is then repelled by 0.19 m, what change in potential does it experience?

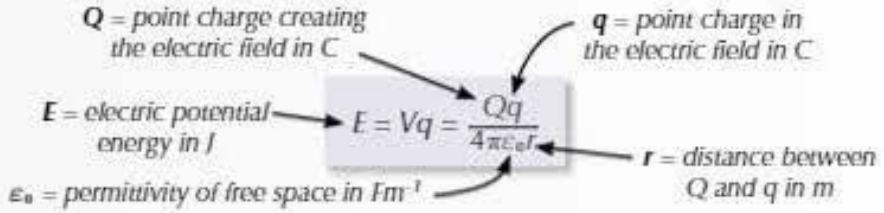
Change in potential = final potential − initial potential

$$= \frac{(3.1 \times 10^{-6})}{4\pi\varepsilon_0(0.035 + 0.19)} - \frac{(3.1 \times 10^{-6})}{4\pi\varepsilon_0 \times 0.035}$$

$$= -6.72529... \times 10^5 = -6.7 \times 10^5 \text{ V (to 2 s.f.)}$$

Electric potential energy

You saw on the previous page that electric potential is the electric potential energy that a unit positive charge (+1 C) would have at a certain point. This means you can find the **electric potential energy** for any charge, q, at that point in the electric field by multiplying the electric potential, V, by the value of the charge. So electric potential energy = Vq. By substituting in the equation for V from the previous page, you get:


Q = point charge creating the electric field in C
q = point charge in the electric field in C
E = electric potential energy in J
$E = Vq = \frac{Qq}{4\pi\varepsilon_0 r}$
r = distance between Q and q in m
ε_0 = permittivity of free space in Fm^{-1}

If you move a unit charge in an electric field and change its electric potential energy, then work has to be done by a force. The work done is equal to the change in the charge's electric potential energy. Both energy and work done are measured in joules (J).

For a radial field (e.g. of a point or spherical charge) you can plot the force applied to the unit charge being moved, F, against the distance, r, from the charge producing the electric field. ($F \propto \frac{1}{r^2}$ — see p.369.) The area under the graph between two points gives the work done, W, to move a unit charge between those two points — see Figure 3.

Figure 3: *The area under a graph of F against r is used to find the work done, W, to move a unit charge between r_1 and r_2 in the electric field of a point charge (left) and of a uniformly charged sphere with radius R (right).*

Example — **Maths Skills**

A +1.6 μC point charge is in a vacuum.
A −0.80 μC point charge is 0.072 m from the positive charge.
a) Calculate the electric potential energy of the negative charge.

$$\text{Electric potential energy} = \frac{Qq}{4\pi\varepsilon_0 r} = \frac{(1.6\times10^{-6})\times(-0.80\times10^{-6})}{4\pi\times8.85\times10^{-12}\times0.072}$$
$$= -0.15985... = -0.16 \text{ J (to 2 s.f.)}$$

b) Calculate the work done to move the negative charge to be 0.015 m away from the positive charge.

Work done = change in electric potential energy
= final electric potential energy − initial electric potential energy
$$= \frac{(1.6\times10^{-6})\times(-0.80\times10^{-6})}{4\pi\times8.85\times10^{-12}\times0.015} - (-0.15985...)$$
$$= -0.6074... = -0.61 \text{ J (to 2 s.f.)}$$

Tip: You might see electric potential energy labelled as E — don't confuse this E with the E for electric field strength on p.368.

Tip: If the charges weren't in a vacuum, then ε_0 would just be replaced with ε (see p.372).

Figure 2: *Static charge on a comb doing work against gravity to lift a trickle of water.*

Tip: If you're asked to estimate W from a F–r graph, you'll probably have to work out the area under the graph by counting the number of squares on the graph paper or by splitting the area up into trapeziums.

Tip: As with the E-r graphs on p.371, you don't need to worry about the force, F, inside a charged sphere, just that it doesn't follow an inverse square law.

Tip: There's a force of attraction between the two charges in this example, which is why the work done to move them closer together is negative.

Capacitance of a charged sphere

Tip: For an 'isolated' charged sphere, you can assume the sphere is infinitely far away from all other charges.

You can use the formula for the electric potential of a radial field and the fact that $Q = CV$ (p.346) for capacitors to derive an expression for the capacitance of an isolated charged sphere, assuming the charge is evenly distributed.

As it is a charged sphere, you can assume all of its charge is at its centre and treat it like a point charge.

Tip: We put the radius R for the distance, because we want an equation for the electric potential at the surface of the sphere.

Substitute $V = \frac{Q}{C}$ into $V = \frac{Q}{4\pi\varepsilon_0 R}$, where R is the radius of the sphere:

$$\frac{Q}{C} = \frac{Q}{4\pi\varepsilon_0 R}$$

Cancel out the Q's:

$$\frac{1}{C} = \frac{1}{4\pi\varepsilon_0 R}$$

Tip: You need to be able to derive this equation from $Q = CV$ and $V = \frac{Q}{4\pi\varepsilon_0 R}$.

Rearrange for C, which gives:

$$C = 4\pi\varepsilon_0 R$$

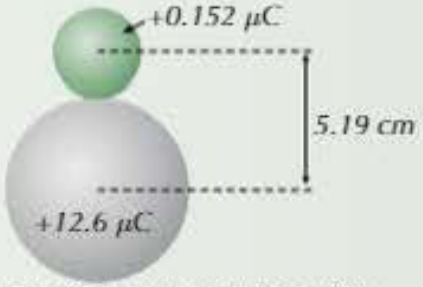
ε_0 = *permittivity of free space in Fm^{-1}*

C = *capacitance of charged sphere in F*

R = *radius of the charged sphere in m*

Practice Question — Application

Q1 A small metal sphere is being held stationary directly next to a charged metal sphere, as shown on the right.



a) What's the electric potential due to the larger sphere at the centre of the smaller metal sphere?

Tip: The value of ε_0 is 8.85×10^{-12} Fm^{-1}.

b) The smaller sphere is released so that it can move freely, and is repelled by 12.9 cm before being stopped again. How much work is done in moving the sphere by this distance?

c) The smaller sphere has a radius of 2.11 cm. Calculate its capacitance when it's in isolation.

Practice Questions — Fact Recall

Q1 What is meant by the electric potential of a point?

Q2 Sketch the graph of electric potential V against distance r in a radial field for:

a) a positive point charge.

b) a negative point charge.

Q3 State the two equations needed to derive an equation for the capacitance of a charged sphere in terms of the radius of the sphere.

4. Comparing Electric and Gravitational Fields

Learning Objective:
- Know and understand the similarities and differences between the gravitational field of a point mass and the electric field of a point charge.

Specification Reference 6.2.2

You might have thought a lot of the formulas from the last topic looked familiar — electric and gravitational fields are more similar than you might think...

The main similarities

A lot of the formulas used for electric fields are the same as those used for gravitational fields but with Q (or q) instead of M (or m) and $\frac{1}{4\pi\varepsilon_0}$ instead of G:

Gravitational	Electric
Gravitational field strength: $g = \frac{F}{m}$	Electric field strength: $E = \frac{F}{Q}$
Newton's law of gravitation: $F = -\frac{GMm}{r^2}$	Coulomb's law: $F = \frac{Qq}{4\pi\varepsilon_0 r^2}$
Gravitational field strength for a radial field: $g = -\frac{GM}{r^2}$	Electric field strength for a radial field: $E = \frac{Q}{4\pi\varepsilon_0 r^2}$
Gravitational potential: $V_g = -\frac{GM}{r}$	Electric potential: $V = \frac{Q}{4\pi\varepsilon_0 r}$

Tip: The graphs of g against r (p.301) and V_g against r for gravitational fields (p.303) are also very similar to the graphs of E against r (p.371) and V against r for electric fields (p.374).

Other similarities that are useful to know include:

Gravitational field strength, g, is force per unit mass.	Electric field strength, E, is force per unit positive charge.
Newton's law of gravitation for the force between two point masses is an inverse square law. $F \propto \frac{1}{r^2}$	Coulomb's law for the electric force between two point charges is also an inverse square law. $F \propto \frac{1}{r^2}$
The gravitational field lines for a point mass...	The electric field lines for a negative point charge...
Gravitational potential, V_g, is potential energy per unit mass and is zero at infinity.	Electric potential, V, is potential energy per unit positive charge and is zero at infinity.

Tip: Don't forget — a spherical mass acts as a point mass and a spherical charge acts as a point charge.

The main differences

Although gravitational and electric fields are similar, they're not the same:

- Gravitational forces are always attractive. Electric forces can be either attractive or repulsive.
- Objects can be shielded from electric fields, but not gravitational fields.
- The size of an electric force depends on the medium between the charges, e.g. plastic or air. For gravitational forces, this makes no difference.

Practice Questions — Fact Recall

Q1 List three similarities between electric and gravitational fields.

Q2 Give two differences between electric and gravitational fields.

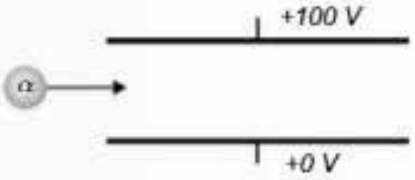
Section Summary

Make sure you know...

- That an electric field is one type of field that gives rise to a force.
- That you get electric fields around charges.
- That the electric field strength, $\boldsymbol{E}$, is defined as the force per unit positive charge in an electric field.
- How to use the equation $\boldsymbol{E} = \frac{\boldsymbol{F}}{Q}$ to calculate electric field strength.
- That an electric field around a point charge is radial.
- How to represent radial electric fields using field lines, where the direction of the field lines show the direction in which a positive charge would move if it was placed in the field.
- That the further apart electric field lines are, the weaker the electric field at that point.
- That a charged sphere can be treated as if all of its charge is concentrated at the centre of the sphere, and so has a radial electric field.
- How to use Coulomb's law, $\boldsymbol{F} = \frac{Qq}{4\pi\varepsilon_0 r^2}$, to find the force between two point charges in a vacuum.
- That ε_0 is the permittivity of free space.
- How to use the equation $\boldsymbol{E} = \frac{Q}{4\pi\varepsilon_0 r^2}$ to calculate the magnitude of $\boldsymbol{E}$ in a radial field.
- What a graph of $\boldsymbol{E}$ against r for a radial field looks like.
- What is meant by a uniform electric field.
- What a parallel plate capacitor is, and that it contains a uniform electric field when there is a potential difference across the plates.
- How to use the equation $\boldsymbol{E} = \frac{V}{d}$ to calculate the electric field strength of a uniform electric field.
- That the equation used to calculate the capacitance of a parallel plate capacitor is $C = \frac{\varepsilon_0 A}{d}$ when there is a vacuum between the two plates, and that the equation becomes $C = \frac{\varepsilon A}{d}$ when there is another material between the two plates.
- The equation $\varepsilon = \varepsilon_r \varepsilon_0$ and how to use it to calculate the permittivity of a material.
- That a charged particle follows a curved path when entering an electric field at right angles.
- What is meant by the electric potential at a point, and that electric potential becomes zero at an infinite distance from a point charge.
- How to use the equation $V = \frac{Q}{4\pi\varepsilon_0 r}$ to calculate the electric potential at a given point in a radial electric field.
- That a charge can experience a change in electric potential when moving through an electric field.
- How to use the formula $\frac{Qq}{4\pi\varepsilon_0 r}$ to calculate the electric potential energy at any distance from a point charge.
- How to sketch a force-distance graph for both a point and a spherical charge, and that the area under this graph between two distances will give the work done to move a unit charge between those two distances.
- How to derive the equation $C = 4\pi\varepsilon_0 R$ and how to use it to calculate the capacitance of an isolated charged sphere.
- The similarities and differences between gravitational and electric fields.

Exam-style Questions

1 The diagram shows a (positively charged) alpha particle being fired in a straight line between charged parallel plates, travelling parallel to the plates.
Which of the following statements is correct?



A The particle's path will be deflected downwards.

B The particle's path will be deflected upwards.

C The particle's path will not be deflected.

D The particle will be forced back in the direction it came.

(1 mark)

2 Which of the following is true for an isolated charged sphere in a vacuum?

A The capacitance of the sphere is only dependent on its mass.

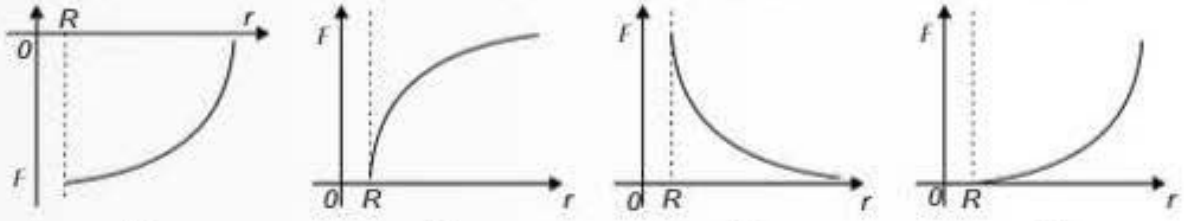
B The capacitance of the sphere is only dependent on its radius.

C The capacitance of the sphere is only dependent on its charge.

D The capacitance of the sphere is only dependent on its charge and radius.

(1 mark)

3 Which of the following graphs shows how the magnitude of the force, F, needed to move a point charge away from a charged sphere with radius R, varies with distance, r.



(1 mark)

4 Point A is exactly halfway between a lithium nucleus (containing 3 protons and 2 neutrons) and an electron. The electric field strength at point A is $\boldsymbol{E}$.

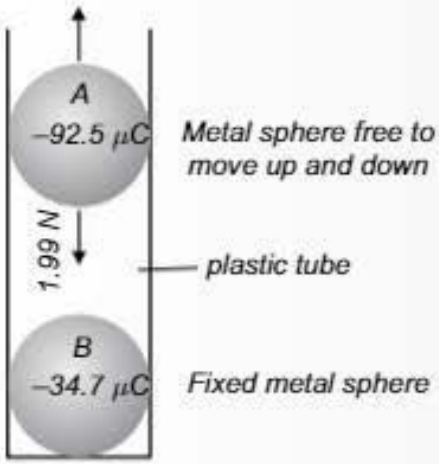
One of the protons is then removed from the lithium nucleus, making it a helium nucleus.
What is the new electric field strength at point A?

A $\frac{3E}{4}$ **B** $\frac{E}{2}$ **C** $\frac{2E}{3}$ **D** $2E$

(1 mark)

5 A charged sphere carrying a charge of –34.7 μC is fixed to the bottom of a plastic tube. Another sphere, with a charge of –92.5 μC, is dropped into the tube and allowed to move freely until it comes to a rest above the fixed sphere. The force due to gravity acting on the top sphere is 1.99 N. This is shown in **Fig 5.1**.

Fig 5.1

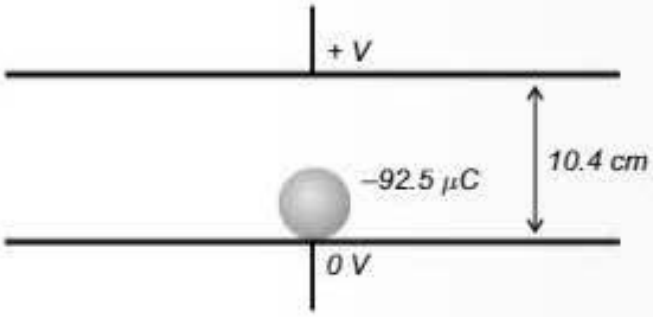


(a) Calculate the distance between the centres of the spheres when sphere *A* comes to rest. You can assume that they do not touch.

(2 marks)

The top sphere is now placed on a smooth, horizontal plane between two parallel plates, as shown (from above) in **Fig 5.2**.

Fig 5.2



(b) Describe the electric field produced between the two parallel plates.

(2 marks)

(c) The electric field causes a force of 0.18 N to act on the sphere. Calculate the electric potential of the top plate, *V*.

(3 marks)

(d) The capacitance of the two parallel plates is 34.5 pF, and the area of each plate is 0.31 m^2. State whether the space between the plates is a vacuum or not.

(2 marks)

1. Magnetic Fields

You've met gravitational fields, you've met electric fields, and now it's time for the final sort — magnetic fields.

Learning Objectives:

- Know that magnetic fields are due to moving charges or permanent magnets.
- Be able to use magnetic field lines to map magnetic fields.
- Be able to sketch the magnetic field patterns for a long straight current-carrying conductor, a flat coil and a long solenoid.
- Be able to use Fleming's left-hand rule.

Specification Reference 6.3.1

What is a magnetic field?

A **magnetic field** is similar to a gravitational field (page 297) and an electric field (page 368) — it's a region in which a force acts. In a magnetic field, a force is exerted on magnetic materials (e.g. iron).

Magnetic fields exist around permanent magnets and moving charges. Magnetic fields can be represented (mapped) by **field lines**. Field lines go from north to south. The closer the lines, the stronger the field. If the field lines are equally spaced and in the same direction the field is uniform (i.e. the same everywhere).

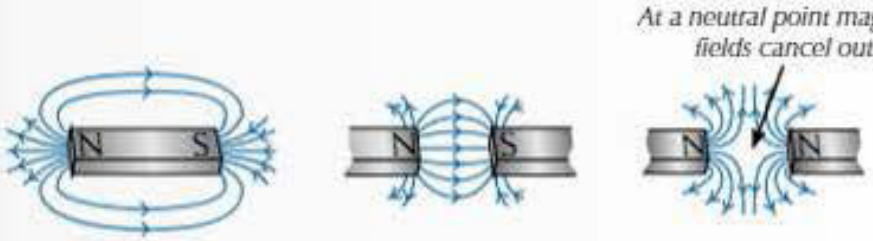


Figure 1: *The magnetic fields created by bar magnets.*

Magnetic fields around a wire

When current flows in a wire or any other long straight conductor, a magnetic field is induced around the wire. The field lines are concentric circles centred on the wire. The direction of a magnetic field around a current-carrying wire can be worked out with the right-hand rule:

- Curl your right hand into a fist and stick your thumb up.
- Point your thumb in the direction of the current through the wire.
- Your curled fingers will then show the direction of the field.

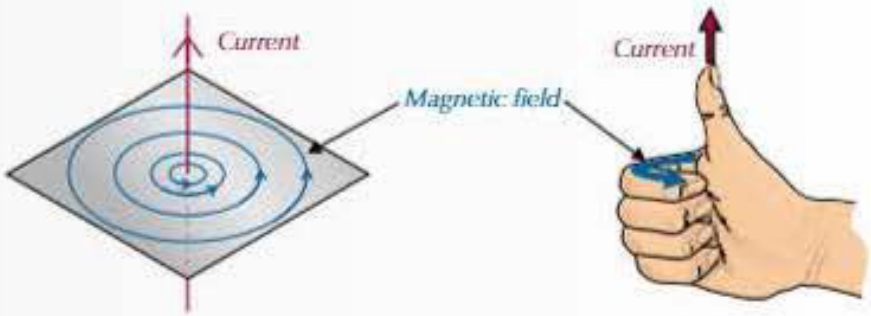


Figure 3: *Diagram to show how your right hand can be used to show the direction of magnetic field lines around a current-carrying wire.*

Figure 2: *A current-carrying wire induces a circular magnetic field around it — the needles of the small compasses follow a circle around the wire.*

Flat coils and solenoids

You can loop a current-carrying wire into a coil to get different magnetic field shapes. There are three types of coil you need to know: the single turn coil, the flat coil and the long **solenoid** (see Figure 4).

Tip: A solenoid is just a coil with length.

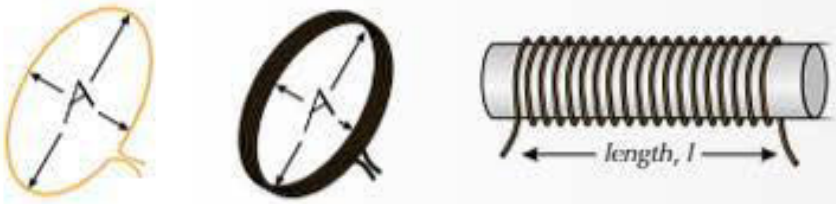
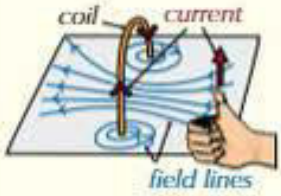


Figure 4: *A single turn coil of area A (left), a flat coil of area A and N turns where N >1 (middle), and a long solenoid of N turns and length l* (right).

The magnetic field surrounding a single turn coil and a flat coil is doughnut shaped, while a solenoid forms a field like a bar magnet (see Figure 5).

Tip: The right-hand rule also works for a coil. Point your thumb in the direction of the current and you'll see that the induced magnetic field curls around the coil like a doughnut.



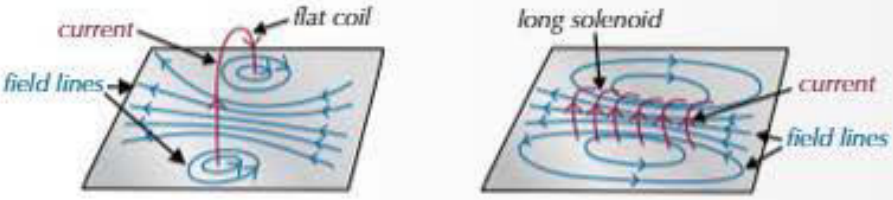


Figure 5: *The magnetic fields created by a current-carrying wire in a flat coil (left) and a solenoid (right).*

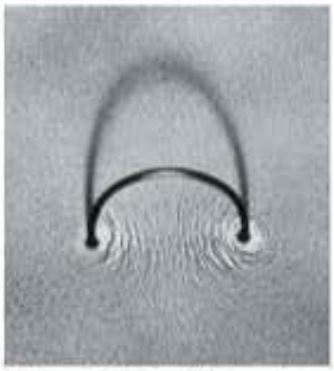


Figure 6: *Steel filings show the magnetic field around a single-turn coil.*

Force on a current-carrying wire

If you put a current-carrying wire into an external magnetic field (e.g. between two magnets), the field around the wire and the field from the magnets are added together. This causes a resultant field — lines closer together show where the magnetic field is stronger. These bunched lines cause a 'pushing' force on the wire. Figure 7 shows the resultant field and force when a current-carrying wire is perpendicular to an external magnetic field.

Tip: A solid dot like in Figure 7 (●) shows current flowing out of the page (or towards the reader). A circle with a cross (⊗) shows current going into the page (or away from the reader). This notation can also be used to show field lines going into or coming out of the page.

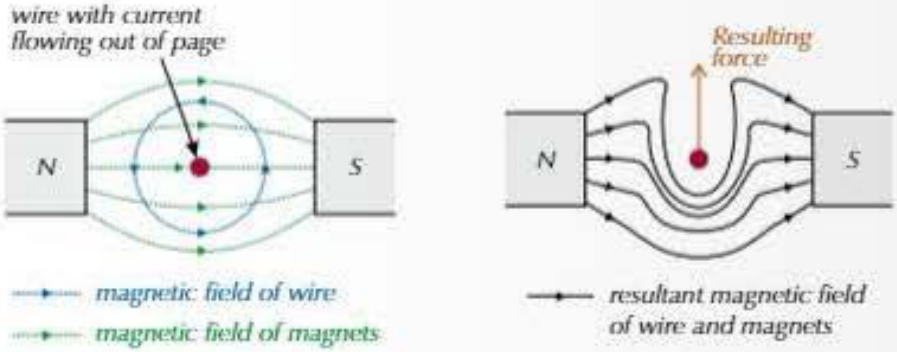


Figure 7: *Diagrams showing the magnetic field of a wire in an external magnetic field (left) and the resultant magnetic field and force produced (right).*

The size of the force depends on the component of the magnetic field that is perpendicular to the current. The direction of the force is always perpendicular to both the current direction and the magnetic field — it's given by Fleming's left-hand rule (see the next page). If the current is parallel to the field lines the size of the force is 0 N — there is no component of the magnetic field perpendicular to the current.

Fleming's left-hand rule

You can use your left hand to find the direction of the current, the direction of the external magnetic field or the direction of the force on the wire (as long as you know the other two). Stretch your thumb, forefinger and middle finger out, as shown in Figure 8, and use the following rules:

- The First finger points in the direction of the uniform magnetic Field.
- The seCond finger points in the direction of the conventional Current.
- The thuMb points in the direction of the force (the direction of Motion).

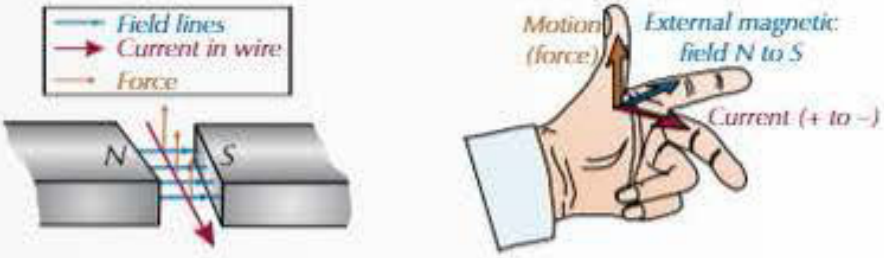


Figure 8: *Fleming's left-hand rule for a current-carrying wire in a magnetic field.*

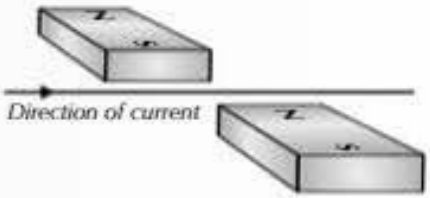
Tip: The external magnetic field in Fleming's left-hand rule doesn't take into account the field around the wire as in Figure 7. It's just the field before the current was there.

Tip: Conventional current flows in the direction that a positive charge would flow, i.e. from positive to negative.

Tip: As long as the magnetic poles are wide enough, you can assume the magnetic field between them is uniform.

Example

A current-carrying wire runs between two magnets, as shown on the right. What direction will the force on the wire be?



Just use Fleming's left-hand rule to find the direction of the force. Remember, the magnetic field goes from the north pole to the south pole, so the force on the wire acts upwards.

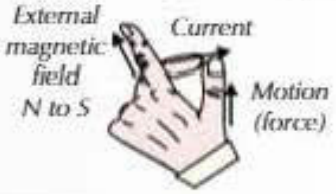
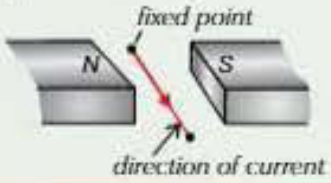


Figure 9: *British physicist John Ambrose Fleming.*

Practice Questions — Application

Q1 A current-carrying wire is shown on the right with the current travelling into the page. Copy the diagram and sketch the magnetic field generated by the current.

Q2 A section of wire with a direct current running through it is fixed at two points and placed in a uniform magnetic field as shown. In what direction will the force on the wire act?



Practice Questions — Fact Recall

Q1 Where are magnetic fields found?

Q2 In which direction do magnetic field lines point?

Q3 State which aspect of a magnetic field is represented by the closeness of magnetic field lines.

Q4 Which hand can you use to find the direction of the force acting on a current-carrying wire at a right angle to a magnetic field? State what each finger represents on this hand.

Learning Objectives:

- Know what magnetic flux density is, and that its unit is the tesla.
- Know how to calculate the force acting on a current-carrying conductor using $F = BIL\sin\theta$.
- Know techniques and procedures used to determine the uniform magnetic flux density between the poles of a magnet using a current-carrying wire and a digital balance.

Specification Reference 6.3.1

2. Magnetic Flux Density

Magnetic fields, as you might expect, can have different field strengths. The strength of a magnetic field is known as the magnetic flux density.

What is magnetic flux density?

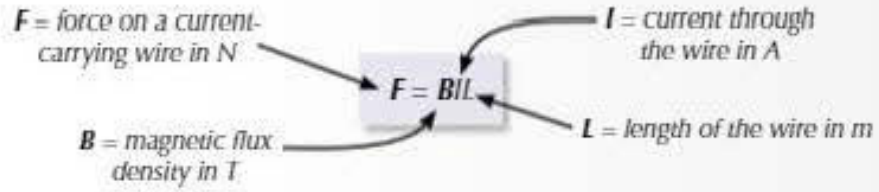
The force on a current-carrying wire at a right angle to an external magnetic field is proportional to the **magnetic flux density**, B. Magnetic flux density is sometimes called the strength of the magnetic field. It is defined as:

> The force on one metre of wire carrying a current of one amp at right angles to the magnetic field.

Magnetic flux density is a vector quantity with both a direction and a magnitude. It is measured in teslas, T. One tesla is equal to one newton per amp per metre:

$$1 \text{ tesla} = 1 \frac{\text{N}}{\text{Am}}$$

When a current-carrying wire is at 90° to a magnetic field, the size of the force on the wire, F, is proportional to the current, I, the length of wire in the field, L, and the flux density, B, of the external magnetic field. This gives the equation:


F = force on a current-carrying wire in N
I = current through the wire in A
F = BIL
L = length of the wire in m
B = magnetic flux density in T

Example — **Maths Skills**

A section of wire carrying a current of 5.2 A is placed at right angles to a uniform magnetic field with a flux density of 19 mT. If the wire experiences a force of 1.2×10^{-2} N, what length of wire is inside the magnetic field?

First rearrange the formula $F = BIL$ to make L the subject:

$$F = BIL \Rightarrow L = \frac{F}{BI}$$

Put in the numbers:

$$L = \frac{1.2 \times 10^{-2}}{(19 \times 10^{-3}) \times 5.2} = 0.121... = 0.12 \text{ m (to 2 s.f.)}$$

When a current-carrying wire is at any angle to the magnetic field, the force acting on the wire is caused by the component of the magnetic field which is perpendicular to the wire — $B\sin\theta$, see Figure 1.

Tip: It can help to think of flux density as the number of magnetic field lines per unit area.

Tip: One tesla is also equivalent to 1 weber per square metre. Webers (Wb) are the unit of magnetic flux (page 394).

Tip: 1 mT (millitesla) is equal to 1×10^{-3} T.

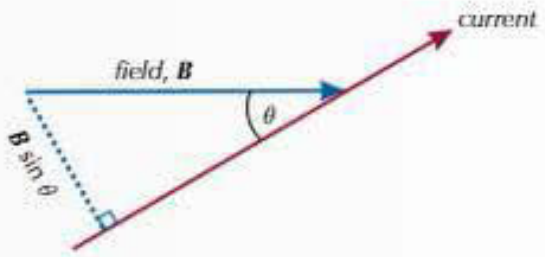
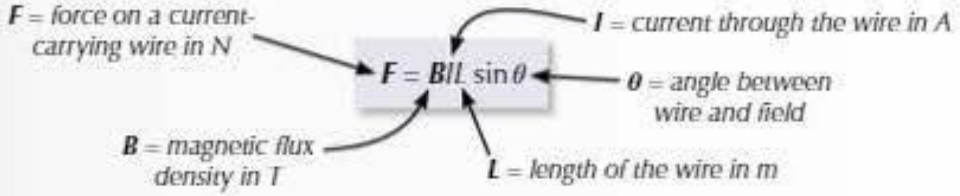


Figure 1: *Diagram showing a magnetic field, **B**, at an angle, θ, to a current-carrying wire. The component of the magnetic field which is perpendicular to the current-carrying wire is also shown (= **B**sinθ).*

So, for a wire at an angle θ to the field, the force acting on the wire is given by:


F = force on a current-carrying wire in N
I = current through the wire in A
F = BIL sin θ
θ = angle between wire and field
B = magnetic flux density in T
L = length of the wire in m

The $\sin\theta$ in the equation means that when the wire and magnetic field are perpendicular to one another ($\theta = 90°$), the force acting on the wire is at a maximum ($\sin\theta = 1$). When the current and field are parallel ($\theta = 0°$) there's no force acting on the wire ($\sin\theta = 0$) — see Figure 3.

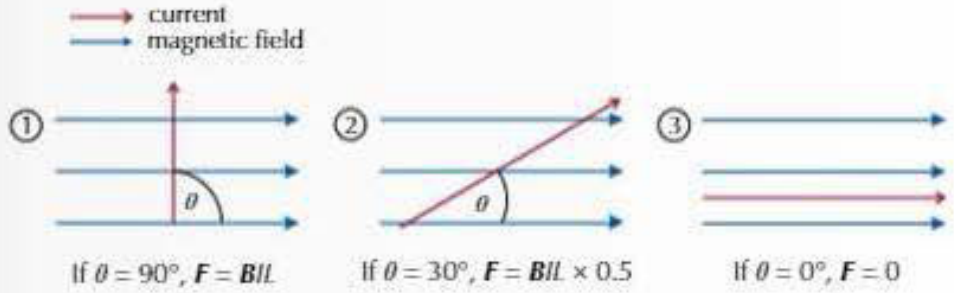


Figure 3: *The force acting on a current-carrying wire in a uniform magnetic field at 90°, 30° and 0°.*

Figure 2: *Serbian-American physicist Nikola Tesla, after whom the SI unit of magnetic flux density is named.*

Tip: Remember, $\sin 90° = 1$ and $\sin 0° = 0$.

Exam Tip
You'll be given this equation in the exam data and formulae booklet, but not $F = BIL$. Remember, $F = BIL$ is just a special case where $\sin\theta = \sin 90° = 1$ (see Figure 3).

Example **Maths Skills**

A 0.71 m wire carrying a current of 4.1 A lies at 32° to a uniform magnetic field. The magnetic field exerts a force of 0.17 N on the wire. Calculate the magnetic flux density of the magnetic field.

Rearrange $F = BIL\sin\theta$ to make B the subject, then plug in the numbers:

$$F = BIL\sin\theta \Rightarrow B = \frac{F}{IL\sin\theta}$$

$$B = \frac{0.17}{4.1 \times 0.71 \times \sin 32°} = 0.110...$$

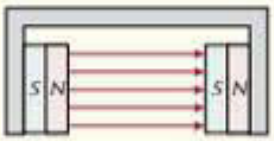
$$= 0.11\ \text{T (to 2 s.f.)}$$

Measuring magnetic flux density

Tip: Remember to carry out a full risk assessment before you do this experiment.

You can use a digital balance and the set-up shown in Figure 4 to investigate the uniform magnetic field between the poles of a magnet and obtain a value for flux density, B, using $F = BIL$.

Tip: The 'slab' magnets used in this experiment have poles on their largest faces.



This sort of magnet is sometimes called a 'magnadur magnet'.

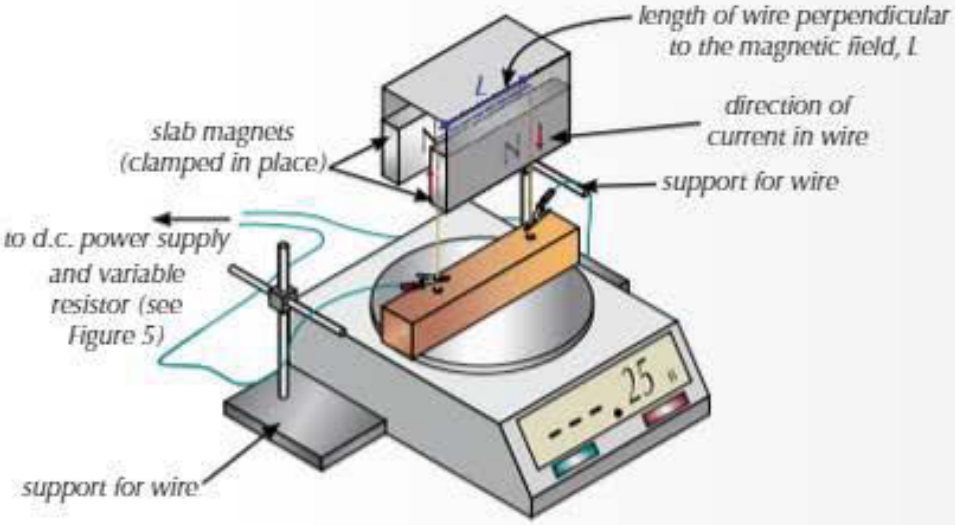

length of wire perpendicular to the magnetic field, L
direction of current in wire
slab magnets (clamped in place)
support for wire
to d.c. power supply and variable resistor (see Figure 5)
support for wire
L
N
N

Figure 4: *Set-up for an experiment to measure magnetic flux density.*

Tip: You only need to measure the length of wire shown on the diagram. Even though the vertical parts of the hoop are perpendicular to the field, the forces they feel act horizontally, and so don't affect the mass reading on the balance.

Set up the experiment shown in Figure 4. A square hoop of metal wire is positioned so that the top of the hoop, length L, passes through the magnetic field, and is perpendicular to it. When a current flows, the length of wire in the magnetic field will experience a downwards force (Fleming's left-hand rule), which will cause a reading on the mass balance.

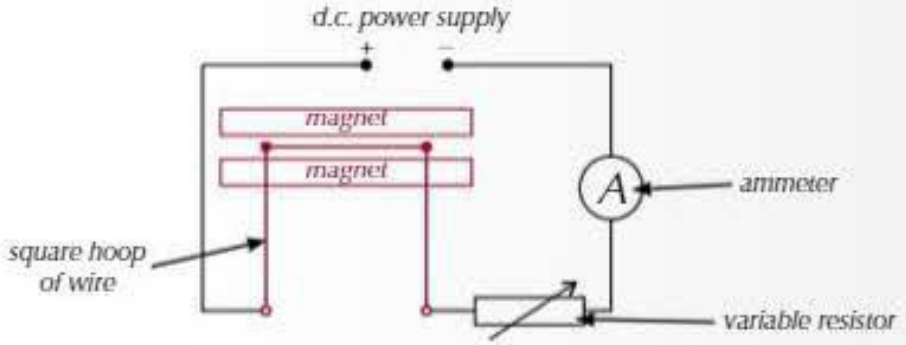


Figure 5: *Circuit diagram for experiment to measure magnetic flux density. The section in red is shown in Figure 4.*

Tip: It's important to zero the balance when no current is flowing so that the mass reading is only due to the force caused by the current in the magnetic field (and not due to the mass of the equipment).

The d.c. power supply should be connected to a variable resistor so that you can alter the current. Zero the digital balance when there is no current through the wire. Turn on the d.c. power supply — if the mass reading is negative, turn off the d.c. power supply and swap the crocodile clips over so that the mass is positive.

Tip: A d.c. power supply is used so that the direction of the force is constant — if an a.c. supply was used, the direction of the force would keep changing.

Note the mass showing on the digital balance and the current. Then use the variable resistor to change the current and record the new mass reading. Repeat this until you have tested a large range of currents, then do the whole thing twice more so that you have 3 mass readings for each current. Calculate the mean for each mass reading to improve the precision of the results.

Convert your mass readings into force using $F = mg$, then plot your data on a graph of force F against current I. Draw a line of best fit. You should find that you get a graph through the origin showing that force is proportional to current. Because $F = BIL$, the gradient of the line of best fit is equal to BL. Because the length is constant, you can divide the gradient by your value for L to get the magnetic flux density, B.

Example — **Maths Skills**

A student carries out the experiment described above to find the magnetic flux density, B. Complete the data table below and plot the results on a graph. Describe the relationship between force and current, and estimate a value for B.

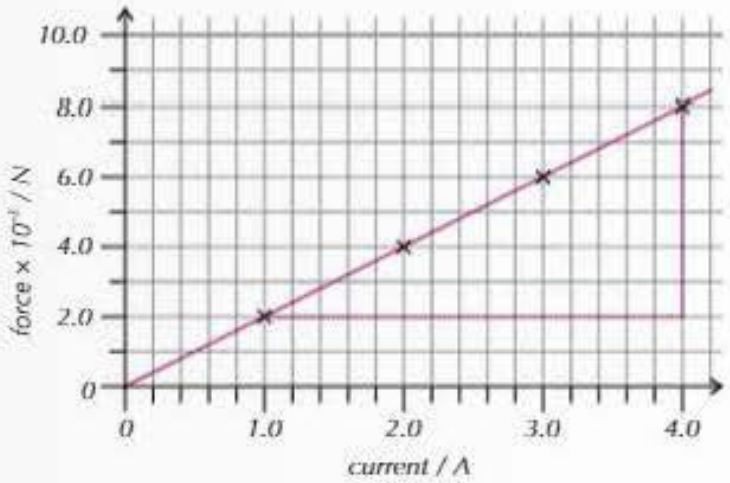
Convert the masses in g to kg by dividing them by 1000.
Then use $F = mg$ to turn the masses into forces and draw the graph:

Tip: Remember $g = 9.81\ \text{Nkg}^{-1}$.

Tip: Give the forces to the least number of s.f. that the data is given to.

current / A	*mean mass recorded on mass balance / g*	*force acting on current-carrying wire / N*
1.0	0.20	0.0020
2.0	0.41	0.0040
3.0	0.61	0.0060
4.0	0.82	0.0080

Length $L = 0.050$ m throughout.



The graph is a straight line through the origin, so force is directly proportional to current.

$$BL = \text{gradient} = \frac{\Delta y}{\Delta x} = \frac{(8.0 \times 10^{-3}) - (2.0 \times 10^{-3})}{4.0 - 1.0}$$
$$= 2.0 \times 10^{-3}$$

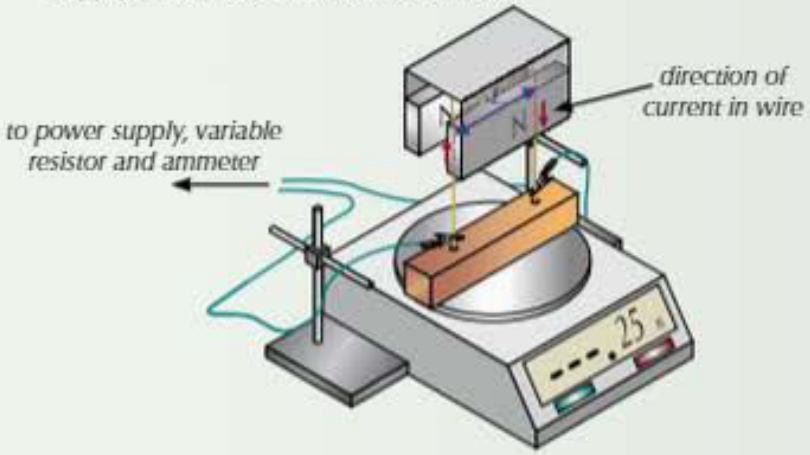
$$B = \text{gradient} \div L = 2.0 \times 10^{-3} \div 0.050$$
$$= 0.040\ \text{T}$$

Practice Questions — Application

Q1 a) A 25 cm straight wire carrying a current of 4.0 A is placed into a magnetic field of magnetic flux density 0.20 T. The angle between the magnetic field lines and the wire is 53°. Calculate the force acting on the wire due to the field.

b) The wire is rotated so that it's parallel to the magnetic field lines. What is the force acting on the wire now?

Q2 The diagram below shows an experiment set-up to determine the magnetic flux density of a uniform magnetic field between two slab magnets. A square wire hoop is placed such that the horizontal length L is perpendicular to the field between the magnets. The wire carries a current of 3.0 A. The hoop is connected to a d.c. power supply, a variable resistor and ammeter.



a) Draw a circuit diagram of the set-up described above.

b) Explain why a variable resistor is used in the circuit.

c) Explain why the vertical lengths of wire are unimportant and only length L is measured.

d) The mass shown on the balance is recorded for a range of currents. Explain how this information can be used to determine the magnetic flux density.

Practice Questions — Fact Recall

Q1 Give the condition needed to use the equation $\boldsymbol{F} = \boldsymbol{B}IL$ to calculate the force on a current-carrying wire in a magnetic field.

Q2 Explain why the force acting on a wire due to an external magnetic field is at a maximum when the wire and the magnetic field are perpendicular to one another.

3. Forces on Charged Particles

Any charged particle in a magnetic field feels a force as long as its moving — you need to know how to calculate the size and direction of the force.

Learning Objectives:

- Be able to use $F = BQv$ to calculate the force on a charged particle travelling at right angles to a uniform magnetic field.
- Understand the effect on a charged particle moving in a magnetic field, and how this can lead to circular orbits of charged particles.
- Know what happens to charged particles moving in a region occupied by both electric and magnetic fields, and how this is used in a velocity selector.

Specification Reference 6.3.2

Charged particles in a magnetic field

A force acts on a charged particle moving in a magnetic field. This is why a current-carrying wire experiences a force in a magnetic field (page 382) — electric current in a wire is the flow of negatively charged electrons.

- The force on a current-carrying wire in a magnetic field perpendicular to the current is given by $F = BIL$ (page 384).
- Electric current, I, is the flow of charge, Q, per unit time, t. So $I = \frac{Q}{T}$.
- A charged particle which moves a distance L in time t has a velocity, $v = \frac{L}{t}$. So $L = vt$.

Putting all these equations together gives the force acting on a single charged particle moving through a magnetic field, where its velocity is perpendicular to the magnetic field:

$$F = BIL = B\frac{Q}{t}vt$$

$$F = BQv$$

F = force in N

B = magnetic flux density in T

Q = charge on the particle in C

v = velocity of the particle in ms^{-1}

Exam Tip

In many exam questions, Q is the size of the charge on the electron, which is 1.60×10^{-19} C.

Example — **Maths Skills**

An electron travels at a velocity of 2.00×10^4 ms^{-1} perpendicular to a uniform magnetic field of strength 2.00 T. What is the magnitude of the force acting on the electron? (The magnitude of the charge on an electron is 1.60×10^{-19} C.)

Just use the equation $F = BQv$ and put the correct numbers in:

$$F = BQv$$
$$= 2.00 \times (1.60 \times 10^{-19}) \times (2.00 \times 10^4)$$
$$= 6.40 \times 10^{-15} \text{ N}$$

The circular path of particles

Charged particles travelling perpendicular to a magnetic field travel in a circular path — see Figure 2. By Fleming's left-hand rule the force on a moving charge travelling perpendicular to a magnetic field is always perpendicular to its direction of travel. Mathematically, that is the condition for circular motion (page 277).

To use Fleming's left-hand rule (page 383) for charged particles, use your second finger (normally current) as the direction of motion for a positive charge. If the particle carries a negative charge (i.e. an electron), point your second finger in the opposite direction to its motion.

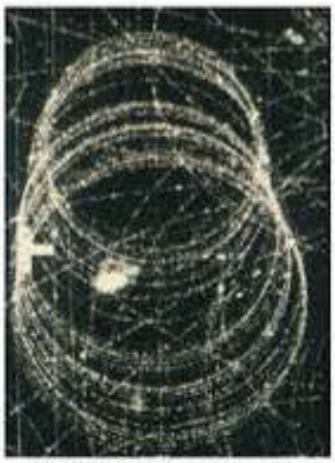


Figure 1: *Circular tracks made by charged particles in a cloud chamber with an applied magnetic field.*

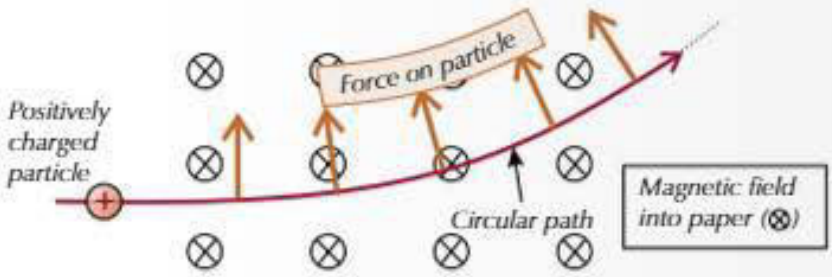


Figure 2: *A charged particle moving perpendicular to a magnetic field follows a circular path.*

Tip: You'll often need to draw 3D situations in 2D like this (or interpret 2D diagrams as a 3D situation). Just remember that a cross always means "into the page" and a dot means "out of the page".

This effect is used in particle accelerators such as cyclotrons and synchrotrons, which use magnetic fields to accelerate particles to very high energies along circular paths.

It's also used in mass spectrometers to analyse chemical samples. Ions (charged particles) with the same velocity are made to enter a magnetic field which deflects them in a curved path towards a detector. The radius of curvature depends on the charge and mass of the particles (see equation below). The identity of the ions reaching the detector can be deduced from their mass to charge ratio.

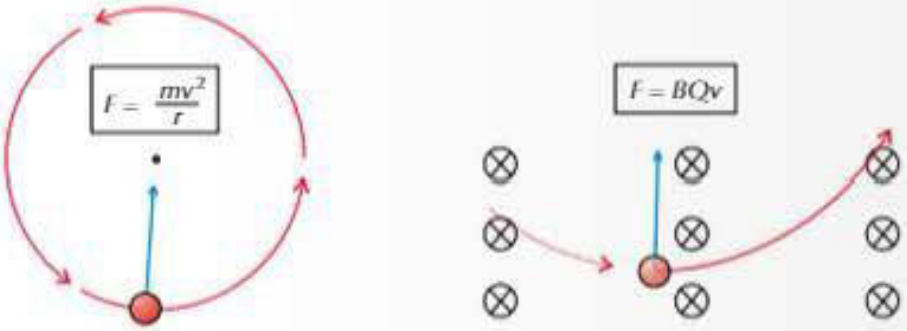


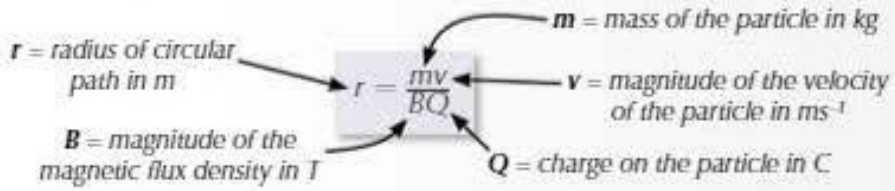
Figure 3: *Diagrams showing the force acting on a particle in a circular orbit (left) and the force acting on a particle in a magnetic field (right).*

Tip: For more on circular motion, see pages 273-279.

The radius of the circular path followed by charged particles in a magnetic field can be found by combining the equations for the force on a charged particle in a magnetic field and for the force on a particle in a circular orbit.

$$F = \frac{mv^2}{r} \quad \text{and} \quad F = BQv \quad \Rightarrow \quad \frac{mv^2}{r} = BQv$$

This rearranges to:


$r = \frac{mv}{BQ}$
r = radius of circular path in m
m = mass of the particle in kg
v = magnitude of the velocity of the particle in ms⁻¹
B = magnitude of the magnetic flux density in T
Q = charge on the particle in C

Tip: A 'v' is cancelled from the top and bottom in the final rearrangement here.

This means that:

- The radius of curvature increases (i.e. the particle is deflected less) if the mass or velocity of the particle increase.
- The radius of curvature decreases (i.e. the particle is deflected more) if the strength of the magnetic field or the charge on the particle increase.

Example — **Maths Skills**

A proton travels through a magnetic field of flux density 6.0 mT at a velocity of 110 ms^{-1}. The direction of travel of the proton is perpendicular to the magnetic field. Calculate the radius of the circular path of the proton.

$$r = \frac{mv}{BQ} = \frac{(1.673\times10^{-27})\times110}{(6.0\times10^{-3})\times(1.60\times10^{-19})} = 1.916...\times10^{-4} = 0.19\,\text{mm (to 2 s.f.)}$$

Tip: Protons have a mass of 1.673×10^{-27} kg and a charge equal to the elementary charge $e = 1.60 \times 10^{-19}$ C. You'll be given these in your data and formulae booklet in the exam.

Examples — **Maths Skills**

For each of the following, say which of the two particles would follow a circular path with the smaller radius in a magnetic field of flux density *B*.

a) A carbon-12 nucleus with velocity *v*, relative mass of 12 and relative charge of +6, and a carbon-14 nucleus with velocity *v*, relative mass of 14 and relative charge of +6.

The radius of the circular path followed by the particles is given by:

$$r = \frac{mv}{BQ}$$

In this case v, B and Q are identical for both particles but m is larger for the carbon-14 nucleus. As r is directly proportional to m, the carbon-12 nucleus will follow the circular path with the smaller radius.

b) A carbon-14 nucleus with velocity *v*, relative mass of 14 and relative charge of +6, and a nitrogen-14 nucleus with velocity *v*, relative mass of 14 and relative charge of +7.

m, v and B are identical for both particles but Q is larger for the nitrogen nucleus. As r is inversely proportional to Q, the nitrogen nucleus will follow the circular path with the smaller radius.

Figure 4: *False-colour cloud chamber image showing the different paths of a proton (red) and an alpha particle (yellow) in the chamber's magnetic field.*

Velocity selectors

Velocity selectors are used to separate out charged particles of a certain velocity from a stream of accelerated charged particles moving at a range of speeds. They do this by applying both a magnetic and an electric field at the same time perpendicular to each other, while a stream of particles is fired perpendicularly to both fields at a device with a narrow gap called a collimator.

Particles fired into the velocity selector experience opposing forces from the electric and magnetic fields (see Figure 5). So, for positively-charged particles travelling through an electric field that goes from top to bottom and a magnetic field that goes straight into the page:

- The magnetic field tries to deflect the particles upwards — check this with Fleming's left-hand rule. The force on each particle is $\boldsymbol{F} = \boldsymbol{BQv}$ (see page 389).
- The electric field tries to deflect particles downwards (opposite charges attract, like charges repel). The force on the particle is $\boldsymbol{F} = \boldsymbol{EQ}$ (see p.373).

Tip: Negatively-charged particles would experience these forces in the opposite directions. Here, the magnetic field would try to deflect the particles downwards and the electric field would try to deflect the particles upwards.

Tip: For the examples on this page, the electric field, E, goes top to bottom (i.e. down the page), and the magnetic field, B, goes straight into the page. The particles are positively-charged.

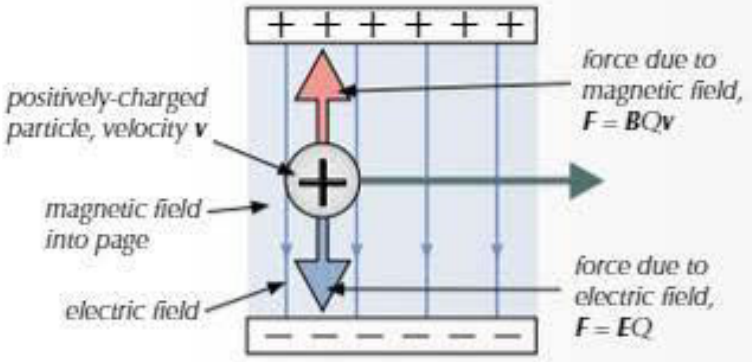
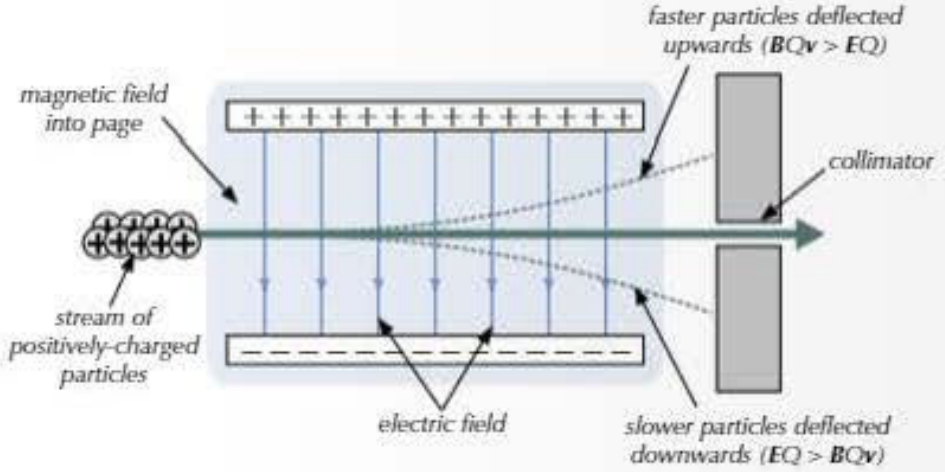

positively-charged particle, velocity v
magnetic field into page
electric field
force due to magnetic field, F = BQv
force due to electric field, F = EQ

Figure 5: *A charged particle travelling through an electric field that goes from top to bottom and a magnetic field that goes straight into the page. The particle experiences opposing forces from the two fields.*

Particles will be deflected unless the forces balance (i.e. $BQv = EQ$). Cancelling Qs and rearranging gives:

v = velocity of the particle in ms^{-1} → $v = \frac{E}{B}$ ← E = electric field strength in NC^{-1}; B = magnetic flux density in T

So only particles with velocity $v = \frac{E}{B}$ as given above will travel in a straight line to pass through the gap in the collimator.


faster particles deflected upwards (BQv > EQ)
magnetic field into page
collimator
stream of positively-charged particles
electric field
slower particles deflected downwards (EQ > BQv)

Tip: The directions of deflection are reversed for negatively-charged particles.

Figure 6: *Diagram showing the possible paths of positively-charged particles through a velocity selector.*

You can select and vary the velocity of the particles that get through the collimator by changing the strength of the magnetic or electric fields. Velocity selectors are often used in mass spectrometers to ensure that the accelerated particles entering the magnetic field have the same velocity.

Tip: A mass spectrometer is a device that identifies particles by their mass to charge ratio (see p.390).

Example — **Maths Skills**

Calculate the magnetic flux density required in a velocity selector to isolate charged particles travelling through a 3.4×10^4 NC^{-1} electric field at 2.1×10^5 ms^{-1}.

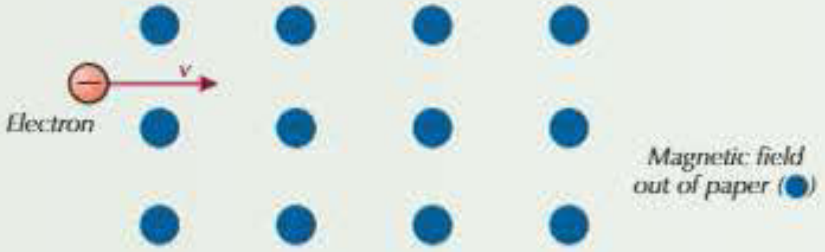
Rearrange $v = \frac{E}{B}$ and put the correct numbers in:

$$v = \frac{E}{B} \Rightarrow B = \frac{E}{v} = \frac{3.4 \times 10^4}{2.1 \times 10^5} = 0.1619... = 0.16 \text{ T (to 2 s.f.)}$$

Practice Questions — Application

Q1 Why would a neutron moving through a magnetic field perpendicular to its direction of motion not experience a force?

Q2 In which direction will the force due to the magnetic field act on the electron in the diagram below?



Q3 Find the force that acts on a particle with a charge of 3.2×10^{-19} C travelling at a velocity of 5.5×10^{3} ms^{-1} perpendicular to a magnetic field with a flux density of 640 mT.

Q4 A charged particle is moving at right angles to both an electric field and a magnetic field, which are also perpendicular to each other. The flux density of the magnetic field is 20.0 mT. Calculate the magnitude of the electric field if the particle is moving at 5.2×10^{6} ms^{-1} in a straight line.

Q5 a) The size of the force needed to keep an object in circular motion is given by $F = \frac{mv^2}{r}$. Combine this with $F = BQv$ to find an expression for the magnetic flux density B needed to keep a charged particle in circular motion for a given radius r and orbital speed v.

b) In a circular particle accelerator with a radius of 5.49 m, protons are accelerated to 1.99×10^{7} ms^{-1}. Find the magnetic flux density B that's required to keep the protons following the circular path of the accelerator. $m_p = 1.673 \times 10^{-27}$ kg and $Q_p = 1.60 \times 10^{-19}$ J.

Tip: Neutrons, along with protons, are one of the nucleons found in atoms. They have a mass of 1.675×10^{-27} kg and carry no charge.

Practice Question — Fact Recall

Q1 If an electron is travelling through a uniform magnetic field perpendicular to its velocity, what shape will its path take? (Assume it has infinite space to move into.)

Learning Objectives:

- Know what magnetic flux, ϕ, is and that it's measured in webers.
- Know what magnetic flux linkage is.
- Be able to calculate magnetic flux using $\phi = BA\cos\theta$.

Specification Reference 6.3.3

4. Magnetic Flux and Flux Linkage

Magnetic flux and flux density are used to describe the magnetic field strength (magnetic flux density) in a defined area. If the magnetic flux of a field is cut by a conductor, an e.m.f. is induced in the conductor.

Magnetic flux

You met magnetic flux density, B, on p.384 — it's a measure of the strength of a magnetic field (or you can think of it as the number of field lines per unit area). The total **magnetic flux**, ϕ, passing through an area, A, perpendicular to a magnetic field, B, is defined as:

Tip: You can only use the equation $\phi = BA$ if B is normal to A (see Figure 1) — otherwise there's an extra term in the equation, which you'll see on page 396.

You can think of flux as the number of field lines. But remember that flux is continuous — field lines are just a way of drawing it. Figure 1 shows magnetic flux inside a single loop coil. The flux inside the coil is $\phi = BA$, where A = area of coil.

Figure 2: *German physicist Wilhelm Eduard Weber made major advances in the field of electromagnetism. The unit of magnetic flux is named after him.*

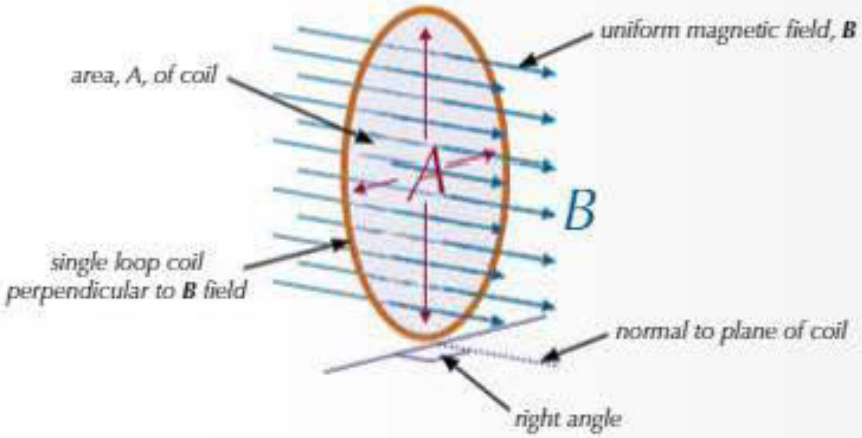


Figure 1: *Diagram showing a single loop coil which has an area A perpendicular to a magnetic field of magnetic flux density B.*

Example **Maths Skills**

A square with sides of length 4.5 cm is placed in a magnetic field, normal to the field's direction. Find the magnetic flux passing through the square if the magnetic flux density is 0.92 T.

Start by finding the area of the square, remembering to convert the units:

$$A = (4.5 \times 10^{-2}) \times (4.5 \times 10^{-2}) = 2.025 \times 10^{-3}\ \text{m}^2$$

Then just put the numbers into the equation above:

$$\phi = BA = 0.92 \times (2.025 \times 10^{-3}) = 1.863 \times 10^{-3}$$
$$= 1.9 \times 10^{-3}\ \text{Wb (to 2 s.f.)}$$

Electromagnetic induction

If there is relative motion between a conducting rod and a magnetic field, the electrons in the rod will experience a force (see p.389), which causes them to accumulate at one end of the rod.

This induces an **electromotive force (e.m.f.)** across the ends of the rod exactly as connecting a battery to it would — this is called **electromagnetic induction**. If the rod is part of a complete circuit, then an induced current will flow through it.

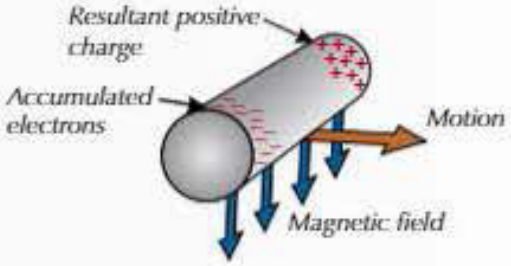


Figure 3: *A conducting rod moving through a magnetic field.*

An e.m.f. is induced when the conductor cuts the magnetic flux. The conductor can move and the magnetic field stay still or the other way round — you get an e.m.f. either way.

You can induce an e.m.f. in a flat coil or solenoid in the same way. In either case, the e.m.f. is caused by the magnetic field (or 'magnetic flux') that passes through the coil changing (see Figure 5). If the coil is part of a complete circuit, an induced current will flow through it.

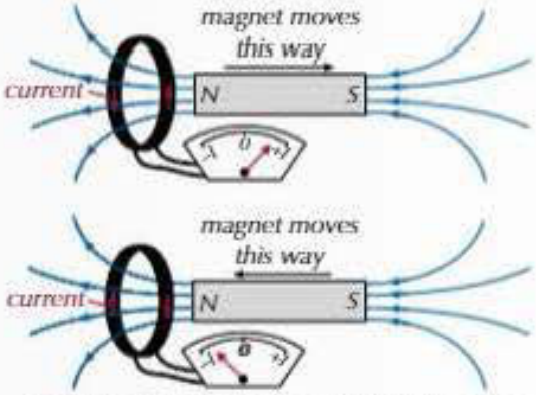


Figure 5: *Current and e.m.f. induced when a magnet is moved towards and away from a coil.*

Tip: E.m.f. is another way of saying voltage — you saw it in Module 4 if you need a reminder. See pages 398-407 for more on induced e.m.f.

Figure 4: *Electromagnetic induction is used to charge electric toothbrushes, avoiding the need for exposed wires.*

Tip: The e.m.f. can either be positive or negative — it depends on the direction of movement and the way you connect the wires.

If the wires are left connected in the same way, the e.m.f. will alternate as the magnet moves back and forth.

Magnetic flux linkage

When a wire coil is moved in a magnetic field, the size of the e.m.f. induced depends on the magnetic flux passing through the coil, ϕ, and the number of turns on the coil cutting the flux (p.398). The product of these is called the **magnetic flux linkage** (or just flux linkage), $N\phi$. For a coil of N turns normal to $\boldsymbol{B}$, the flux linkage is given by:

Flux linkage in Wb → Flux linkage $= N\phi$

ϕ = *magnetic flux in Wb*

$\boldsymbol{N}$ = *number of turns on the coil cutting the flux*

You saw on the last page that magnetic flux, ϕ, is equal to $\boldsymbol{B}A$, so it follows that magnetic flux linkage, $N\phi$, is equal to $\boldsymbol{B}AN$.

Example — **Maths Skills**

The flux linkage of a coil with a cross-sectional area of 0.33 m² normal to a magnetic field of flux density 0.15 T is 4.0 Wb. How many turns are in the coil?

Just rearrange the equation for flux linkage to make N the subject and then put the numbers in:

$$[N\phi] = BAN \Rightarrow N = \frac{[N\phi]}{BA}$$

$$= \frac{4.0}{0.15 \times 0.33} = 80.808... = 81 \text{ turns (to 2 s.f.)}$$

Tip: Don't be tempted to cancel down the '*N*'s here — consider the flux linkage, $N\phi$, as a stand-alone term.

Flux linkage at an angle

When the magnetic flux isn't perpendicular to the area you're interested in (e.g. Figure 6), you need to use trigonometry to resolve the magnetic field vector into components that are parallel and perpendicular to the area.

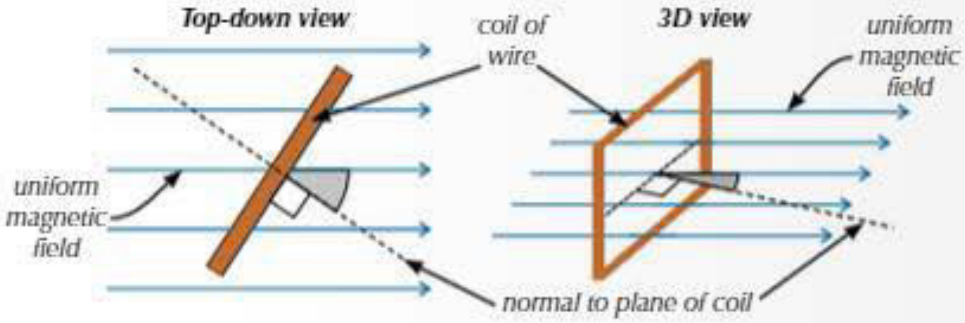


Figure 6: *A coil at an angle to a uniform magnetic field.*

Tip: Pulling the coil in Figure 6 out of the magnetic field would induce an e.m.f.

To find the magnetic flux passing through a coil, you're interested in the component of the magnetic field perpendicular to the area of the coil. By trigonometry, this is equal to $B\cos\theta$ — see Figure 7.

Tip: Remember SOC CAH TOA: $\cos\theta$ = adjacent ÷ hypotenuse

See page 487 for more.

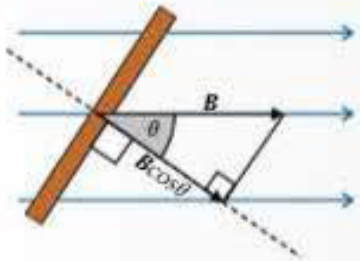


Figure 7: *For a coil at an angle to a magnetic field, the component of the field perpendicular to the area of the coil is $B\cos\theta$, where θ is the angle between the field and the normal to the plane of the coil.*

So for a single loop of wire when B is not perpendicular to area, you can find the magnetic flux using this equation:

$$\phi = BA\cos\theta$$

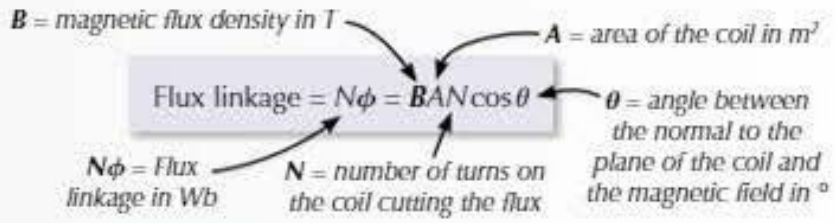
ϕ = magnetic flux in Wb

B = magnetic flux density in T

θ = angle between the normal to the plane of the coil and the magnetic field in °

A = area of the coil in m²

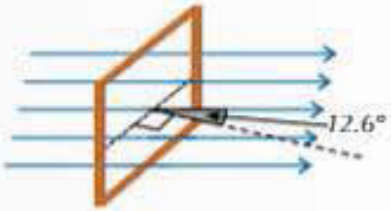
And for a coil with N turns, you can find the flux linkage with the equation:


B = magnetic flux density in T
A = area of the coil in m²
Flux linkage = Nϕ = BAN cos θ
θ = angle between the normal to the plane of the coil and the magnetic field in °
Nϕ = Flux linkage in Wb
N = number of turns on the coil cutting the flux

Exam Tip
You won't be given this in the data and formulae booklet, but you will be given $\phi = BA\cos\theta$, so you just need to remember that flux linkage = $N\phi$.

Example — **Maths Skills**

A rectangular coil of wire with exactly 200 turns and sides of length 5.00 cm and 6.51 cm is rotating in a magnetic field with $B = 8.56 \times 10^{-3}$ T. Find the flux linkage of the coil when the normal to the area of the coil is at 12.6° to the magnetic field, as shown below.



First find the area of the coil:

Area = $(5.00 \times 10^{-2}) \times (6.51 \times 10^{-2}) = 3.255 \times 10^{-3}$ m²

Then just put the numbers into the equation:

$N\phi = BAN\cos\theta$
$= (8.56 \times 10^{-3}) \times (3.255 \times 10^{-3}) \times 200 \times \cos 12.6° = 5.438... \times 10^{-3}$
$= 5.44 \times 10^{-3}$ Wb (to 3 s.f.)

Practice Questions — Application

Q1 A student sets up an experiment to measure the strength of the Earth's magnetic field, B. He does this by measuring the flux linkage of a rectangular wire coil.

a) Explain why the flux linkage of the coil changes when he rotates the coil.

b) The student finds the highest value of flux linkage to be 1.3×10^{-6} Wb. If he used a coil with an area of 25 cm² and exactly 10 turns, find the local value of B.

Q2 A coil of wire with 355 turns and an area of 145 cm² is placed in a 2.06 mT magnetic field. Calculate what angle the normal to the plane of the coil would need to be at to the field in order for the coil to have a magnetic flux linkage of 10.0×10^{-3} Wb.

Practice Questions — Fact Recall

Q1 What are the units of magnetic flux?

Q2 What is induced when a magnetic field through a conductor changes?

Q3 What must ϕ be multiplied by to calculate magnetic flux linkage?

Learning Objectives:

- Know Faraday's law of electromagnetic induction.
- Know that e.m.f. = –rate of change of magnetic flux linkage, and be able to calculate induced e.m.f. using $\varepsilon = -\frac{\Delta(N\phi)}{\Delta t}$.
- Know techniques and procedures used to investigate magnetic flux using search coils.
- Know Lenz's law.

Specification Reference 6.3.3

5. Faraday's Law and Lenz's Law

There's more electromagnetic induction coming up for you to sink your teeth into. In this topic, you'll see how Faraday and Lenz tried to explain the phenomenon better with the inventively named Faraday's law and Lenz's law.

Faraday's law

Faraday's law links the rate of change of flux linkage with e.m.f.:

> Induced e.m.f. is directly proportional to the rate of change of flux linkage.

It can be written as:

$\Delta(N\phi)$ = change in flux linkage in Wb

ε = induced e.m.f. in V

Δt = time taken for flux linkage to change in s

$$\varepsilon = -\frac{\Delta(N\phi)}{\Delta t} = -\frac{N\Delta\phi}{\Delta t}$$

N = number of turns on the coil cutting the flux

$\Delta\phi$ = change in magnetic flux in Wb

For a coil, induced e.m.f. depends on the number of turns and how fast flux through the coil is changing. The unit of flux, the weber (Wb), is defined in terms of the e.m.f. induced:

> A change in flux linkage of one weber per second will induce an electromotive force of 1 volt in a loop of wire.

Tip: The minus sign is Lenz's law – see page 401.

Tip: An electromagnet is an electrical magnet that can be turned on and off, and its field strength can also be varied.

Tip: In this first example, it's the changing magnetic flux density (B) that causes the change in flux linkage. In this second example, it's the changing area (A) that causes the change in flux linkage.

Tip: There's only 1 'turn' in a conducting rod, so you can ignore the N in Faraday's law.

Example — Maths Skills

A coil with 75 turns and an area of 12 cm² is placed between two electromagnets. The flux density of the magnetic field between the electromagnets is then uniformly increased from 0.15 T to 1.5 T over 0.20 seconds. If the plane of the coil is perpendicular to the magnetic field lines of the field, calculate the e.m.f. induced across the coil.

Convert area to m²: $12\ \text{cm}^2 = 12 \times 10^{-4} = 0.0012\ \text{m}^2$

Then start with Faraday's law: $\varepsilon = -\frac{\Delta(N\phi)}{\Delta t}$

You saw on page 395 that $N\phi = BAN$, so:

$$\varepsilon = -\frac{\Delta(BAN)}{\Delta t} = -\frac{(1.5 - 0.15) \times 0.0012 \times 75}{0.20} = -0.6075$$
$$= -0.61\ \text{V (to 2 s.f.)}$$

When using the formula above for a conducting rod, think of flux change as field lines being 'cut' as the rod moves. Remember that magnetic flux $\phi = BA$ (see p.394) — here, think of A as the area of flux cut in a certain time.

Example — Maths Skills

A copper bar in a uniform magnetic field of 0.11 T is moved perpendicular to the field for 0.63 seconds. The total area that is covered by the bar in this time is 42 cm². Calculate the e.m.f. induced across the bar.

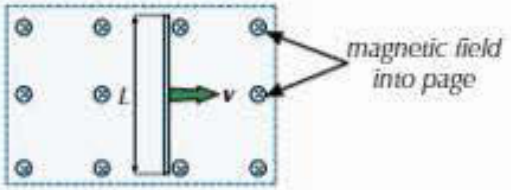
$\varepsilon = -\frac{\Delta(N\phi)}{\Delta t}$ and $\phi = BA$, so:

$$\varepsilon = -\frac{\Delta(BAN)}{\Delta t} = -\frac{0.11 \times (42 \times 10^{-4})}{0.63} = -7.33\ldots \times 10^{-4} = -0.73\ \text{mV (to 2 s.f.)}$$

Faraday's equation can be used to find the e.m.f. in terms of a rod's velocity through a magnetic field.

Example — **Maths Skills**

The diagram below shows a conducting rod of length L moving through a perpendicular uniform magnetic field, B, at a constant velocity, v. Show that e.m.f. induced in the rod is equal to $-BLv$.



Displacement of rod is $s = v\Delta t$ (displacement = velocity × time). Area of flux it cuts, $A = l \times s = lv\Delta t$. Total magnetic flux cut through, $\Delta\phi = BA = Blv\Delta t$

Faraday's law gives $\varepsilon = -\frac{\Delta(N\phi)}{\Delta t} = -\frac{\Delta\phi}{\Delta t}$ (since $N = 1$)

So the induced e.m.f., $\varepsilon = -\frac{\Delta\phi}{\Delta t} = -\frac{Blv\Delta t}{\Delta t} = -Blv$

Exam Tip

You could be asked to find the e.m.f. induced on something other than a rod, e.g. the Earth's magnetic field across the wingspan of a plane. Just think of it as a moving rod and use the equation as usual.

Figure 1: *British physicist Michael Faraday.*

Electromagnetic induction graphs

The e.m.f. induced in a conductor by a changing external magnetic field is equal to the negative of the gradient of a graph of flux linkage ($N\phi$) against time. The area under a graph of e.m.f. against time gives the negative of the change in flux linkage — see Figure 2.

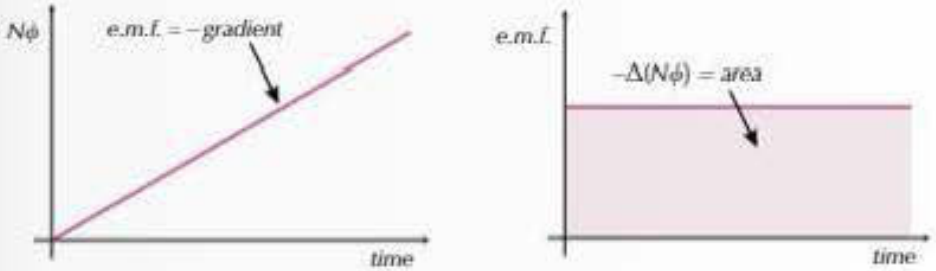
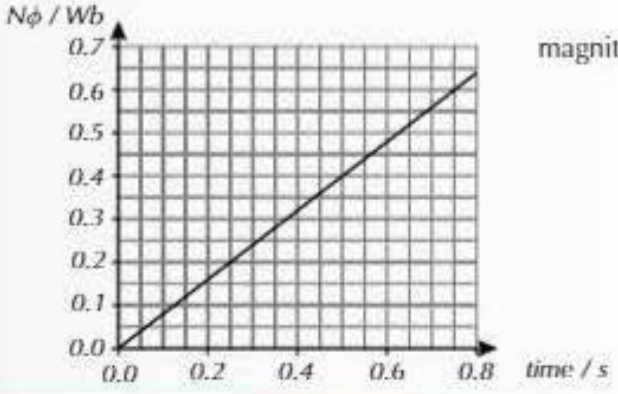


Figure 2: *Graphs of flux linkage against time (left) and e.m.f. against time (right) for a conductor in a changing external magnetic field.*

Example — **Maths Skills**

A coil is placed into a uniform magnetic field. The magnetic field density of the magnetic field is then steadily increased over time. The graph below shows the flux linkage of the coil against time. Calculate the magnitude of the e.m.f. induced in the coil.



magnitude of e.m.f.

$$= \text{magnitude of gradient}$$

$$= \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{(0.6 - 0.0)}{(0.75 - 0.0)}$$

$$= 0.8\ \text{V}$$

Investigating magnetic flux

Tip: Remember to carry out a risk assessment before starting this experiment.

You can find the magnetic flux, ϕ, passing through an area between two bar magnets using the set-up in Figure 3.

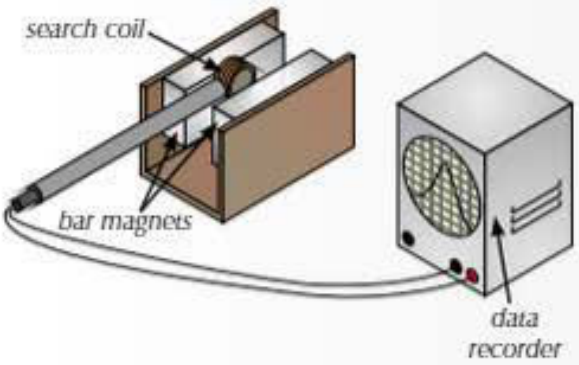


Figure 3: *Set-up for experiment to investigate magnetic flux.*

Tip: This set-up uses magnets with poles on their largest faces again — as on p.386.

Place two bar magnets a small distance apart with opposite poles facing each other — they should be far enough apart not to snap together, but otherwise as close as possible to give a uniform field.

Get a search coil — this is a small coil of wire with a known number of turns (N) and a known area (A). Connect it to a data recorder and set the recorder to measure the induced e.m.f. with a very small time interval between readings.

Place the search coil in the middle of the magnetic field so that the area of the coil is parallel to the surface of the magnets. Start the data recorder. Keeping the coil in the same orientation, immediately move the coil out of the field. An e.m.f. will be induced due to the magnetic flux linkage through the coil changing from maximum, $N\phi$, to zero as you remove the coil from the field.

Tip: You can also use this method to find the magnetic flux density, $\boldsymbol{B}$, between the magnets. Flux linkage = $\boldsymbol{B}AN$ (p.395), so just divide the flux linkage by AN, where A is the area of the search coil.

Use your data or the data recorder to plot a graph of induced e.m.f. against time. Using Faraday's Law, estimating the area under the graph of e.m.f. against time and taking the negative gives you an estimate for the total flux linkage change (p.399). The final flux linkage is zero since the magnetic flux density, $\boldsymbol{B}$, is zero away from the magnets, so the change in flux linkage is equal to the flux linkage in the uniform magnetic field. Flux linkage = $N\phi$, so to find ϕ in area A, divide the total flux linkage change by the number of turns on the search coil (N).

Tip: Remember, you can estimate the area under a graph by counting the number of squares under the graph then multiplying by the value of one square (see p.478).

Repeat this experiment several times and find the mean of your values for ϕ.

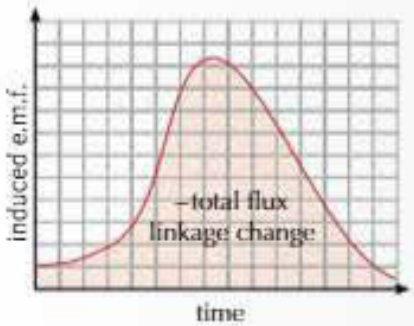


Figure 4: *An example of a graph of e.m.f. against time obtained by moving a coil out of a magnetic field.*

Lenz's law

The direction of an induced e.m.f. (and current) are given by **Lenz's law**:

> The induced e.m.f. is always in such a direction as to oppose the change that caused it.

A changing magnetic field induces an e.m.f. in a coil (see page 395). If the coil is part of a complete circuit, a current is induced in the same direction as the induced e.m.f. The induced current then produces its own magnetic field (p.381). Lenz's law says:

- If the original magnetic field is getting stronger, the current will be induced in the direction that generates a magnetic field in the opposite direction to the external field, to try to weaken it.
- If the original magnetic field is getting weaker (collapsing), the current will be induced in the direction that generates a magnetic field in the same direction as the external field, to try to maintain it.

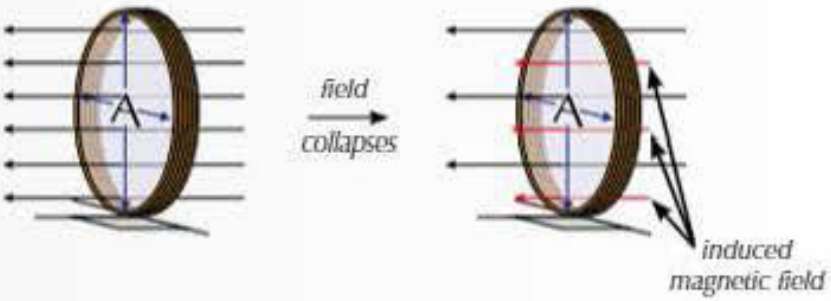
Tip: This is why there's a minus sign in Faraday's Law.

Tip: Remember, a current is only induced if the conductor is part of a complete circuit — if it isn't an e.m.f. will be induced but no current will flow.

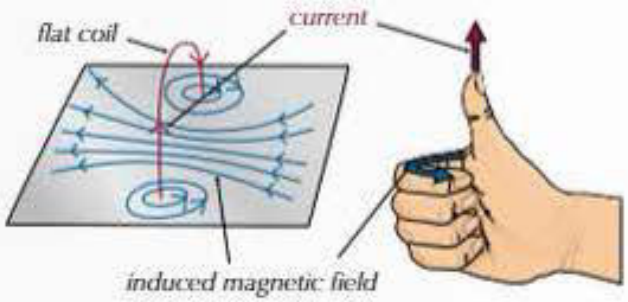
Figure 5: *A magnetic weight (right) takes longer to fall down a copper tube than a non-magnetic weight (left) due to Faraday's and Lenz's laws. The falling magnet induces a current in the tube that generates a magnetic field which opposes the magnetic field of the magnet, slowing it down.*

Example

The area of a flat coil is perpendicular to a magnetic field as it collapses by 50% as shown below. What will be the direction of the current induced in the loop?



A collapsing field means the field is getting weaker and the field lines are getting further apart (p.381). So by Lenz's law the current induced in the coil will induce a magnetic field in the same direction as the collapsing field to try to maintain the original field. Use the right-hand rule (p.381) to find the direction of the induced current.

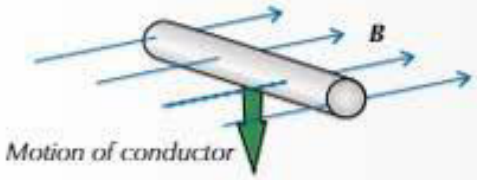


The induced field is to the left, so the induced current is clockwise when viewed from the right.

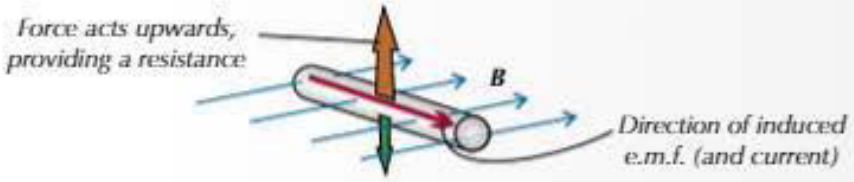
Tip: If you've forgotten which finger is which, here's a reminder:

Example

Lenz's law says that the induced e.m.f. will produce a force that opposes the motion of the conductor — in other words a resistance. Picture a straight conductor being moved down through a perpendicular magnetic field.



Using Fleming's left-hand rule (see p.383), point your thumb in the direction of the force of resistance — which is in the opposite direction to the motion of the conductor. Point your first finger in the direction of the field. Your second finger will now give you the direction of the induced e.m.f.



If the conductor is connected as part of a circuit, a current will be induced in the same direction as the induced e.m.f.

Practice Questions — Application

Q1 Explain how an e.m.f. can be induced across the wings of a plane as it is flown.

Q2 A wire coil with exactly 50 turns and an area of 0.24 m^2 is placed perpendicular to a magnetic field with flux density 1.5 T. A few minutes later the magnetic field is switched off. Calculate the magnitude of the e.m.f. induced in the coil if the magnetic flux density decreased to 0 T in 0.32 s.

Q3 The diagram on the right shows a 1.2 m long copper wire moving upwards at a velocity of 0.50 ms^{-1} through a perpendicular magnetic field of flux density 5.4 mT.

a) The wire is part of a complete circuit. State the direction of the current induced in the wire.

b) Calculate the magnitude of the e.m.f. induced in the wire.

Practice Questions — Fact Recall

Q1 What's Faraday's law?

Q2 What is the gradient of a graph of flux linkage against time equal to?

Q3 A student places a search coil, with area *A*, connected to a data recorder between two bar magnets. Keeping the area of the coil parallel to the surfaces of the magnets, he pulls the coil out of the field. The data recorder plots a graph of the e.m.f. induced in the coil against time. Explain how the student can use this graph to work out the magnetic flux passing through area *A* between the two magnets.

Q4 What's Lenz's law?

6. Uses of Electromagnetic Induction

Electromagnetic induction allows electricity to be produced on a large scale. You'll need to know how it's is used in generators and transformers.

Learning Objectives:

- Know how a simple a.c. generator works.
- Know how a simple laminated iron-cored transformer works.
- Be able to use $\frac{n_s}{n_p} = \frac{V_s}{V_p} = \frac{I_p}{I_s}$ for an ideal transformer.
- Know techniques and procedures used to investigate transformers.

Specification Reference 6.3.3

Induced e.m.f. in a rotating coil

The movement of a conductor in a magnetic field is used to generate electricity. When a coil such as that in Figure 1 rotates uniformly (at a steady speed) in a magnetic field, the coil cuts the flux and an alternating e.m.f. is induced.

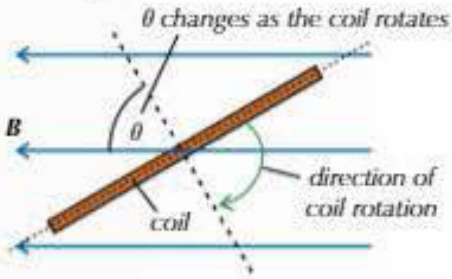


Figure 1: *The value of θ changes as the coil rotates.*

The amount of flux cut by the coil (flux linkage) is given by $N\phi = BAN\cos\theta$ (see page 397).

As the coil rotates, θ changes so the flux linkage varies sinusoidally between $+BAN$ and $-BAN$. Because the flux linkage is varying sinusoidally (see Figure 2), the rate of change of flux linkage is always changing, so the induced e.m.f., ε, is also changing (due to Faraday's law, p.398). You saw on p.399 that the induced e.m.f. is equal to the negative of the gradient of a graph of $N\phi$ against time. So the e.m.f. also varies sinusoidally — it is at a maximum when the plane of the coil is parallel to the lines of flux, and is zero when the coil is perpendicular to the lines of flux — see Figure 2.

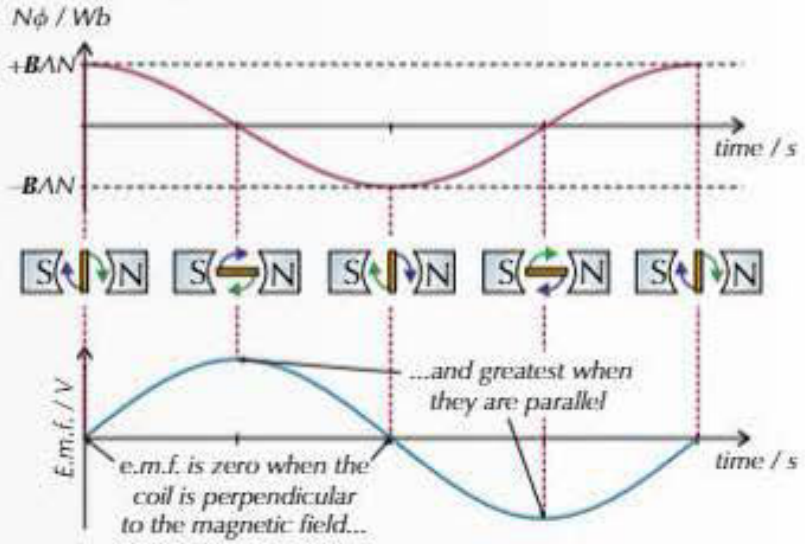


Figure 2: *The sinusoidal graphs produced by plotting flux linkage and induced e.m.f. against time for a rotating coil in a uniform magnetic field.*

The e.m.f. induced can be altered by changing the speed of rotation or the size of the magnetic field:

- Increasing the speed of rotation will increase the frequency of rotation. This in turn causes an increase in the rate of change of flux linkage, and therefore increases the maximum e.m.f..
- Increasing the magnetic flux density $\boldsymbol{B}$ will increase the maximum e.m.f., but will have no effect on the frequency of rotation.

Tip: Note the axis of rotation is perpendicular to the magnetic field.

Tip: Figure 1 is a top-down view of the coil, as in Figure 6 on p.396.

Tip: 'Sinusoidally' means it follows the same pattern as a sin (or cos) curve.

Tip: You don't need to know these graphs, they're just here to help you understand generators (see next page).

Tip: The frequency of the rotation of the coil, f, is related to the period of the coil, T, by the equation $f = \frac{1}{T}$. You saw this on p.275.

Figure 3: *The Gramme dynamo was one of the first electric generators used to deliver power for industry.*

Generators

Generators, or dynamos, convert kinetic energy into electrical energy — they induce an electric current by rotating a coil in a magnetic field.

Figure 4 shows a simple alternator — a generator of alternating current (a.c.). It has slip rings and brushes to connect the coil to an external circuit. The output voltage and current change direction with every half rotation of the coil, producing an alternating current.

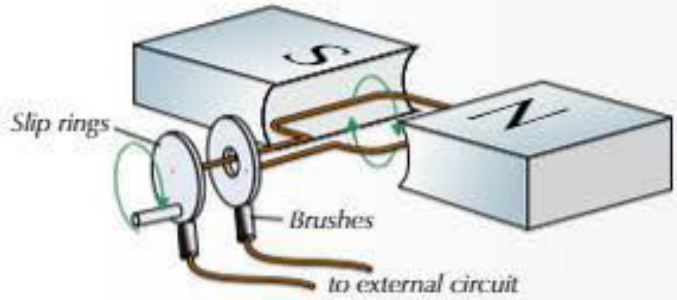


Figure 4: *A simple alternator.*

What's a transformer?

Transformers are devices that make use of electromagnetic induction to change the size of the voltage for an alternating current. They consist of two coils of wire wrapped around an iron core — see Figure 5.

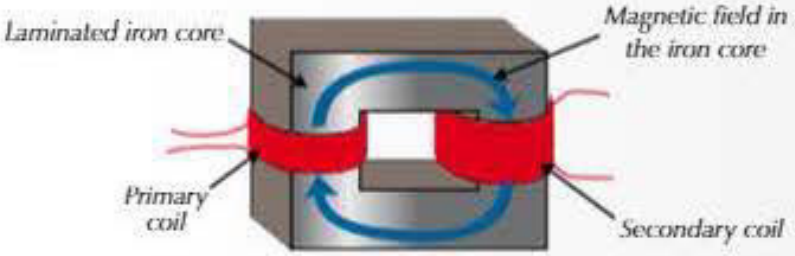


Figure 5: *The basic structure of a (step-up) transformer.*

Figure 6: *An early version of a transformer.*

An alternating current flowing in the primary (or input) coil produces an alternating magnetic field, causing the core to magnetise, demagnetise and remagnetise continuously in opposite directions. The changing magnetic field is passed through the iron core to the secondary (or output) coil, where the rapidly changing flux linkage through the coil induces an alternating voltage (e.m.f.) due to Faraday's law — see page 398. The e.m.f. produced is of the same frequency as the input voltage.

The magnitude of the induced e.m.f. is proportional to the change in flux linkage, $\Delta(N\phi)$, so the size of the e.m.f. induced in the secondary coil is determined by the ratio of the number of turns on each coil along with the voltage across the primary coil. Step-up transformers increase the voltage by having more turns on the secondary coil than the primary. Step-down transformers reduce the voltage by having fewer turns on the secondary coil.

Power loss in transformers

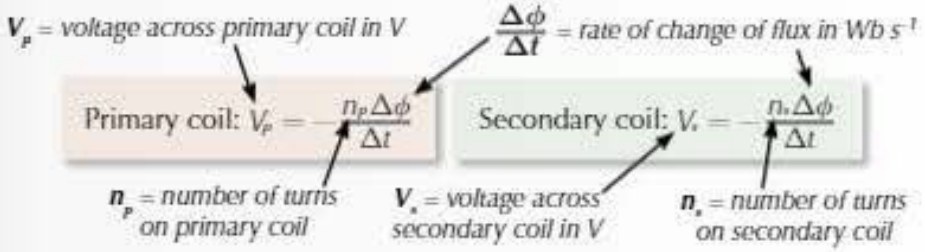
Real-life transformers aren't 100% efficient — some power is always lost. The metallic core is being cut by the continuously changing flux, which induces an e.m.f. in the core. In a continuous core this causes currents called eddy currents, which cause it to heat up and energy to be lost.

Eddy currents are looping currents induced by the changing magnetic flux in the core. They create a magnetic field that acts against the field that induced them, reducing the field strength. They also dissipate energy by generating heat. The effect of eddy currents can be reduced by laminating the core — this involves having layers of the core separated out by thin layers of insulator, so a current can't flow.

Tip: There's more on power loss in transformers on page 407.

Calculating the e.m.f. induced in transformer coils

From Faraday's law (page 398), the induced e.m.f.s in both the primary (p) and secondary (s) coils can be calculated:


V_p = voltage across primary coil in V
$\frac{\Delta\phi}{\Delta t}$ = rate of change of flux in Wb s^{-1}
Primary coil: $V_p = -\frac{n_p \Delta\phi}{\Delta t}$
Secondary coil: $V_s = -\frac{n_s \Delta\phi}{\Delta t}$
n_p = number of turns on primary coil
V_s = voltage across secondary coil in V
n_s = number of turns on secondary coil

Ideal transformers are 100% efficient, so power in equals the power out. Power is current × voltage, so for an ideal transformer $I_p V_p = I_s V_s$, or:

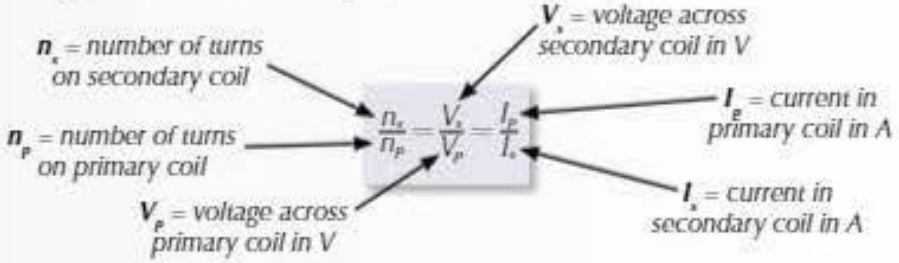
$$\frac{I_p}{I_s} = \frac{V_s}{V_p}$$

Tip: In an ideal transformer, the flux through the secondary coil is the same as the flux through the primary coil and no energy is lost in the transfer.

This can be combined with the equations for induced e.m.f. in each coil:

$$\frac{I_p}{I_s} = \frac{V_s}{V_p} = \frac{-n_s \frac{\Delta\phi}{\Delta t}}{-n_p \frac{\Delta\phi}{\Delta t}}$$

Which gives the transformer equation:


V_s = voltage across secondary coil in V
n_s = number of turns on secondary coil
I_p = current in primary coil in A
n_p = number of turns on primary coil
$\frac{n_s}{n_p} = \frac{V_s}{V_p} = \frac{I_p}{I_s}$
I_s = current in secondary coil in A
V_p = voltage across primary coil in V

Exam Tip
This is how the equation will appear in your data and formulae booklet.

Example — **Maths Skills**

What is the output voltage for a transformer with a primary coil of 120 turns, a secondary coil of 350 turns and an input voltage of 230 V?

Just use the equation for an ideal transformer and rearrange it to make the output voltage (V_s) the subject:

$$\frac{n_s}{n_p} = \frac{V_s}{V_p} \Rightarrow V_s = \frac{V_p n_s}{n_p} = \frac{230 \times 350}{120} = 670.83... = 670\text{ V (to 2 s.f.)}$$

Example — **Maths Skills**

The secondary coil of a step-up transformer has 95 turns. Calculate the number of turns on the primary coil if the current across the transformer decreases from 22 A to 4.0 A.

Rearrange the ideal transformer equation to make n_p the subject:

$$\frac{n_s}{n_p} = \frac{I_p}{I_s} \Rightarrow n_p = \frac{n_s I_s}{I_p} = \frac{95 \times 4.0}{22} = 17.27... = 17 \text{ turns (to 2 s.f.)}$$

Investigating transformers

You can investigate the relationship between the number of turns and the voltages across the coils of a transformer by setting up the equipment as shown in Figure 7.

Tip: Make sure you carry out a risk assessment before carrying out any experiment.

Put two C-cores together and wrap wire around each to make the coils. Begin with 5 turns in the primary coil and 10 in the secondary coil (a ratio of 1:2). Turn on the a.c. supply to the primary coil. Use a low voltage — remember transformers increase voltage, so make sure you keep it at a safe level. Record the voltage across each coil.

Keeping V_p the same so it's a fair test, repeat the experiment with different ratios of turns. Try 1:1 and 2:1. Divide n_s by n_p and V_s by V_p. You should find that for each ratio of turns, $\frac{n_s}{n_p} = \frac{V_s}{V_p}$.

Tip: The resistor is connected to the secondary coil to stop large currents from being generated in the circuit.

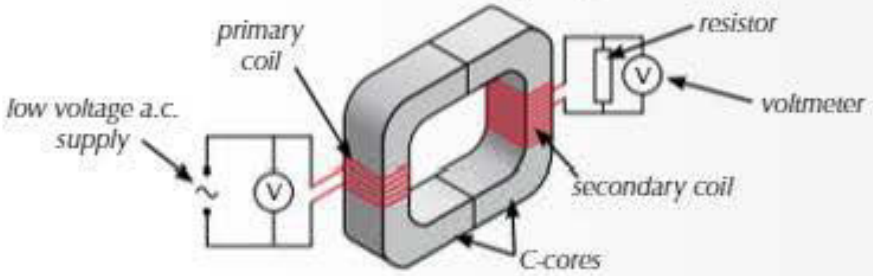


Figure 7: *Experimental set-up for investigating the relationship between the number of turns and voltages in transformers.*

To investigate the relationship between current and voltage of the transformer coils for a given number of turns in the coil you can use the same equipment as before, but add a variable resistor to the primary coil circuit and an ammeter to both circuits as shown in Figure 8.

Turn on the power supply and record the current through and voltage across each coil. Leaving the number of turns constant, adjust the variable resistor to change the input current. Record the current and voltage for each coil, then repeat this process for a range of input currents.

Tip: These formulas won't quite work in your investigation because real transformers aren't 100% efficient.

You should find that for each current, $\frac{n_s}{n_p} = \frac{V_s}{V_p} = \frac{I_p}{I_s}$.

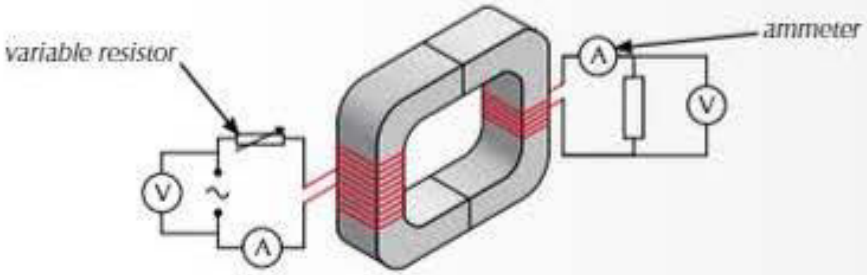


Figure 8: *Set-up for investigating the relationship between current and voltage in a transformer.*

Transformers in the national grid

Transformers are an important part of the national grid. Electricity from power stations is sent round the country in the national grid at the lowest possible current. This is because a high current causes greater energy losses due to heating in the cables. The power losses due to the resistance of the cables is equal to $P = I^2R$ — so if you double the transmitted current, you quadruple the power lost. Using cables with the lowest possible resistance can also reduce energy loss.

Since power = current × voltage, a low current means a high voltage for the same amount of power transmitted. Transformers allow us to step up the voltage to around 400 000 V for transmission through the national grid.

High voltages raise safety and insulation issues, so the voltage has to be stepped back down to a safer 230 V at local substations before it can be used in homes.

Tip: You saw $P = I^2R$ in Module 4: Section 1.

Tip: Although some energy is lost inside the transformers at each end, it's nowhere near as much as the energy that would be lost if the electricity were transmitted at 230 V.

Other uses of transformers

Transformers are used in many household devices. Lots of electronic devices like laptops, mobiles, monitors and speakers can't function using a standard 230 V mains supply — they need a much lower voltage (and usually a d.c. supply too).

The chargers for these devices contain transformers to adjust the voltage — they're contained in the plug or a box in the cable.

Figure 9: *Plugs can contain transformers to reduce the voltage taken from the mains.*

Practice Questions — Application

Q1 a) The primary coil of a transformer has 250 turns. The secondary coil has 420 turns. If the voltage across the primary coil is 190 V, calculate the voltage across the secondary coil.

b) There is a current of 13 A in the secondary coil of a different transformer. The voltage across the primary coil is 120 V and the voltage across the secondary coil is 75 V. Calculate the current in the primary coil.

Q2 Laptop chargers contain a transformer that changes the voltage from the mains supply. Mains electricity in the UK supplies 230 V.

a) Low resistance wires can be used to increase the efficiency of a transformer. State one other way in which the efficiency of a transformer can be improved.

b) The laptop requires a voltage of 19 V.

(i) Does the charger use a step-up or step-down transformer?

(ii) The primary coil in the transformer has 110 turns. Calculate the number of turns in the secondary coil to the nearest whole number.

Practice Questions — Fact Recall

Q1 Describe how a simple generator produces an alternating current.

Q2 How does a transformer change the voltage of an electricity supply?

Q3 Wire can be wrapped around two C-cores to make a transformer. This transformer can then be used to investigate the relationship between the ratio of turns on the primary and secondary coils and the output voltage. This is done by varying the number of secondary coils whilst keeping the number of primary coils the same. State one other factor which should be kept constant in this experiment.

Section Summary

Make sure you know...

- That magnetic fields exist around permanent magnets and moving charges.
- That magnetic field lines point from north to south, and the closer the field lines the stronger the field.
- How to find the direction of a magnetic field around a current-carrying wire with the right-hand rule.
- The shape and direction of the magnetic fields around a long straight current-carrying wire, a flat coil and a long solenoid.
- How to use Fleming's left-hand rule to find the direction of the current, the direction of the external magnetic field or the direction of the force on a wire, provided the other two are given.
- That magnetic flux density is the force on one metre of wire carrying a current of one amp at right angles to a magnetic field.
- That the unit for magnetic flux density is the tesla, T, and that one tesla is equal to one newton per amp per metre.
- How to calculate the force acting on a current-carrying conductor at an angle θ to a magnetic field using $\boldsymbol{F} = \boldsymbol{B}Il\sin\theta$.
- How the uniform magnetic flux density between the poles of a magnet can be measured using a current-carrying wire and a digital balance.
- How to calculate the force acting on a single charged particle moving through a magnetic field, where its velocity is perpendicular to the magnetic field, using $\boldsymbol{F} = \boldsymbol{B}Q\boldsymbol{v}$.
- That the force on a moving charge travelling perpendicular to a magnetic field is always perpendicular to its direction of travel, so charged particles in a uniform magnetic field can move in circular orbits.
- How electric and magnetic fields are used in velocity selectors to separate out particles of a certain velocity from a stream of accelerated charged particles moving at a range of speeds.
- That the total magnetic flux, ϕ, passing through an area that is perpendicular to a magnetic field is equal to the area multiplied by the magnetic flux density.
- That magnetic flux is measured in webers.
- That the magnetic flux linkage of a coil in a magnetic field is the product of the magnetic flux passing through the coil multiplied by the number of turns in the coil.
- How to calculate the magnetic flux through an area that is not perpendicular to a magnetic field using $\phi = \boldsymbol{B}A\cos\theta$.
- That Faraday's law states that induced e.m.f. is directly proportional to the rate of change of flux linkage, and be able to calculate induced e.m.f. using $\varepsilon = -\dfrac{\Delta(N\phi)}{\Delta t}$.
- That the gradient of a graph of flux linkage against time is equal to the negative of the induced e.m.f..
- That the area under a graph of induced e.m.f. against time is equal to $-\Delta(N\phi)$.
- How a search coil connected to a data recorder can be used to investigate magnetic flux between two bar magnets by moving the coil out of the field and plotting a graph of induced e.m.f. against time.
- That Lenz's law states that the induced e.m.f. is always in such a direction as to oppose the change that caused it.
- How a simple a.c. generator converts kinetic energy into electrical energy by rotating a coil in a magnetic field.
- How simple laminated iron-cored transformers work, and how the size of the voltage induced in the secondary coil is determined by the ratio of the number of turns on each coil and the voltage across the primary coil.
- How to use the equation $\dfrac{n_s}{n_p} = \dfrac{V_s}{V_p} = \dfrac{I_p}{I_s}$ for an ideal transformer.
- How to investigate the relationship between voltage, current and the number of turns in a transformer.

Exam-style Questions

1 A 33 cm long conducting rod moves perpendicular to a magnetic field of magnetic flux density 21 mT. An e.m.f. of 4.5 mV is generated across its length. At what velocity is the bar moving?

A 0.071 ms^{-1}

B 14.1 ms^{-1}

C 0.65 ms^{-1}

D 1.5 ms^{-1}

(1 mark)

2 In a velocity selector, electrons are travelling perpendicularly through an electric field and a magnetic field, which are at right angles to each other. The velocity selector is used to select electrons moving with a velocity of 42 kms^{-1} by varying the magnetic flux density, so only electrons travelling at this speed will travel in a straight line and pass through a hole in a collimator. The field strength of the electric field is 37×10^3 NC^{-1}. What is the magnetic flux density of the magnetic field?

A 0.88 T

B 1.1 T

C 8.8×10^2 T

D 1.1×10^{-3} T

(1 mark)

3 An electron is fired into a uniform magnetic field with a flux density of 0.93 T at a speed of 8.1×10^7 m, as **Fig 3.1** shows.

(a) What shape will the electron's path take? You can assume that the electron's velocity is perpendicular to the magnetic field.

(1 mark)

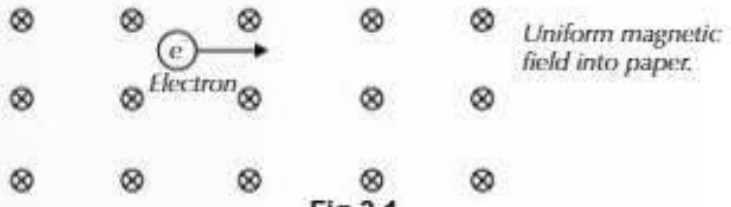


Fig 3.1

(b) Calculate the magnitude of the force the electron will experience, and state its direction.

(2 marks)

(c) An alpha particle of charge +2*e* is fired into the same magnetic field as the electron at the same speed. State the magnitude and direction of the force experienced by the alpha particle.

(2 marks)

4 An experiment is set up as shown in **Fig 4.1** to measure the flux density between two magnets. A stiff metal wire is clamped in place so that it passes through a metal cradle resting on a top pan balance. A magnet is attached to either side of the metal cradle, with opposite sides facing each other so that there is a uniform magnetic field between them. Crocodile clips connect the stiff metal wire to a circuit containing an ammeter and a variable d.c. power supply. The balance is zeroed before the power supply is switched on.

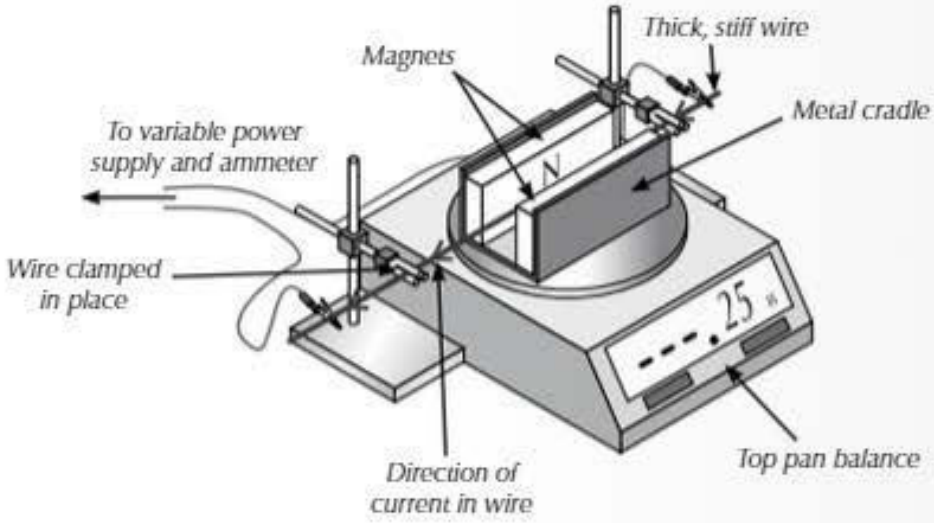


Fig 4.1

(a) State the direction of the force acting on the metal cradle due to the current in the wire.

(1 mark)

(b) Explain how the set-up shown in **Fig 4.1** could be used to determine the magnetic flux density of the magnetic field between the magnets, while keeping the length of wire constant.

(4 marks)

(c) A 52 cm long current-carrying wire is in a uniform magnetic field with a flux density of 44 mT. The wire is perpendicular to the lines of magnetic flux. If the wire experiences a force of 68 mN, calculate the current passing through the wire.

(2 marks)

5 The voltage across the primary coil of a step-down transformer is 230 V and the voltage across the secondary coil is 7 V.

(a) Suggest why the transformer won't work if the current supplied to the primary coil is a direct current.

(2 marks)

(b) The sum of the turns in the primary and secondary coils around the iron core equals 250. Calculate the number of turns of the secondary coil around the core.

(2 marks)

(c)* Describe an experiment that can be used to investigate the relationship between the current and voltage of transformer coils. Include a labelled diagram of the apparatus used and suggest how the accuracy of the experiment could be improved.

(6 marks)

* The quality of your response will be assessed in this question.

1. Atomic Structure

Our ideas about the structure of the atom have changed a lot over the years. But we began to settle on our current model at the start of the last century. It's thanks to some clever folks called Rutherford, Geiger and Marsden...

Learning Objectives:

- Understand the alpha-particle scattering experiment and how this provided evidence of the atom containing a small, charged nucleus.
- Understand the simple nuclear model of the atom, including protons, neutrons and electrons.
- Understand what is meant by the proton number and nucleon number of a nucleus.
- Understand the ${}^{A}_{Z}X$ notation for the representation of nuclei.
- Know what is meant by an isotope.

Specification Reference 6.4.1

The development of the atomic model

Following his discovery of the electron in the late 19th century, J.J. Thomson proposed the Thomson model of the atom, also known as the 'plum pudding' model. This model said that atoms were made up of a globule of positive charge, with negatively charged electrons sprinkled in it, like fruit in a plum pudding. It was widely accepted at the time, until the Rutherford alpha-scattering experiment of 1909.

The Rutherford alpha-scattering experiment

In Rutherford's laboratory, Hans Geiger and Ernest Marsden studied the scattering of alpha particles by thin metal foils — see Figure 1.

A stream of alpha particles from a radioactive source was fired at very thin gold foil, which was surrounded by a circular fluorescent screen. When alpha particles from a radioactive source strike a fluorescent screen, a tiny visible flash of light is produced. Geiger and Marsden recorded these flashes, and counted the number of alpha particles scattered at different angles.

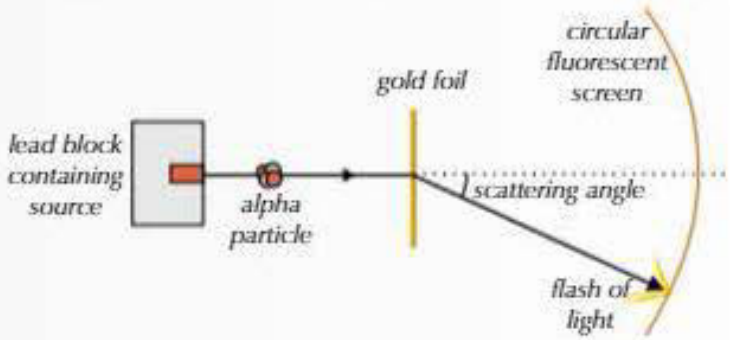


Figure 1: *The experimental set-up for detecting the deflection of alpha particles by thin gold foil.*

If the Thomson model was right, all the flashes should have been seen within a small angle of the beam, as shown in Figure 3. However, this wasn't what they saw.

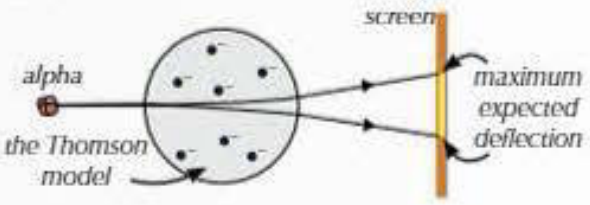


Figure 3: *The expected scattering of alpha particles from the Thomson model of the atom.*

Tip: The circular fluorescent screen extends almost all the way around the other equipment.

Figure 2: *Hans Geiger and Ernest Rutherford, pictured in their lab with some equipment they used to detect alpha particles.*

Geiger and Marsden observed that most alpha particles went straight through the foil, but a few scattered at angles greater than 90°, sending them back the way they came. These results led Rutherford to some important conclusions:

- Most of the alpha particles went straight through the foil. So the atom is mainly empty space.
- Some of the alpha particles were deflected through large angles, so the centre of the atom must have a highly positive charge to repel them. Rutherford named this the nucleus.
- Very few particles were deflected by angles greater than 90°, so the nucleus must be tiny.
- Most of the mass must be in the nucleus, since the fast alpha particles (with high momentum) were deflected backwards by the nucleus.

So most of the mass and the positive charge in an atom must be contained within a tiny, central nucleus. This led to the **nuclear model of the atom**.

Tip: The idea of most of the mass being in the nucleus might be easier to visualise with larger objects. For example, imagine firing a golf ball at a bowling ball — the golf ball would just bounce straight backwards, and the bowling ball would barely move at all, because the mass of the bowling ball is so much bigger than the mass of the golf ball. It's the same with firing an alpha particle at a nucleus — the alpha particle is deflected straight back because the mass of the nucleus is so large.

The simple nuclear model of the atom

Inside every atom, there's a positive nucleus containing **neutrons** (which have no charge) and positively charged **protons**. Protons and neutrons are both known as **nucleons**. Orbiting this core are the negatively charged electrons.

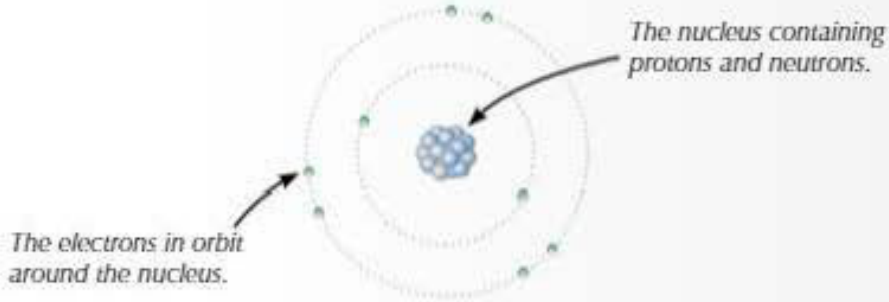


Figure 4: *An oxygen atom, with eight protons, eight neutrons and eight electrons.*

Tip: The proton and neutron masses given in Figure 5 are for when they're in isolation i.e. when they're not part of a nucleus. Once part of a nucleus, their masses change (see p.442).

The particles in an atom have different charges and masses, shown in Figure 5. Charge is measured in coulombs (C) and mass is measured in kilograms (kg).

Particle	Charge (C)	Mass (kg)
Proton	$+1.60 \times 10^{-19}$	1.673×10^{-27}
Neutron	0	1.675×10^{-27}
Electron	-1.60×10^{-19}	9.11×10^{-31}

Figure 5: *Masses and charges of atomic particles.*

Exam Tip
You'll be given the masses (in kg) of all three particles and the elementary charge, *e*, in the data and formulae booklet in the exam.

1.60×10^{-19} C is the elementary charge, e. The charge on an electron, $-e$, is equal and opposite to the charge on a proton, $+e$.

The nucleus only makes up a tiny proportion of an atom — it's only about one 10 000th of the diameter of the whole atom. The electrons orbit at relatively vast distances from the nucleus, so most of the atom is empty space.

The proton and neutron are roughly 2000 times more massive than the electron, so the nucleus makes up nearly all of the mass of the atom.

Proton and nucleon number

The **proton number** of an atom is the number of protons in its nucleus. It is sometimes called the atomic number, and has the symbol Z. It's the proton number that defines the element — no two different elements will have the same number of protons.

In a neutral atom, the number of electrons equals the number of protons. The element's reactions and chemical behaviour depend on the number of electrons. So the proton number tells you a lot about its chemical properties.

The **nucleon number** is also called the mass number, and has the symbol A. It tells you how many protons and neutrons are in the nucleus.

Each proton or neutron in a nucleus has a mass of approximately 1 unified atomic mass unit (= 1.661×10^{-27} kg, see p.427). The mass of an electron compared with a nucleon is virtually nothing, so the number of nucleons is about the same as the atom's mass (in atomic mass units).

Tip: A neutral atom must have the same number of protons and electrons so that the positive and negative charges cancel each other out.

Tip: Protons and neutrons have slightly different masses when they are in a nucleus compared to when they are isolated (see page 442). The atomic mass unit is the average nucleon mass, calculated from the mass of a carbon-12 nucleus, divided by 12.

Standard notation

Standard notation summarises all the information about an element's nucleus. Figure 6 shows the standard notation for an element X, with nucleon number A and proton number Z:

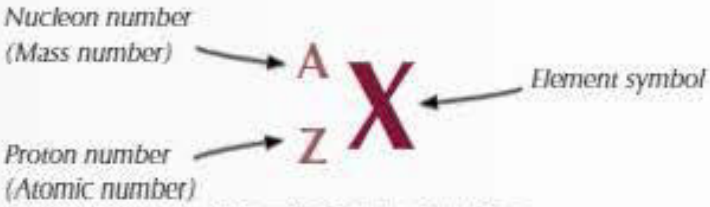
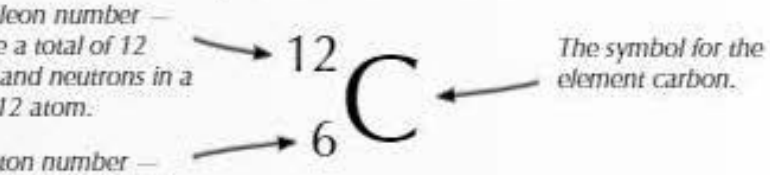


Figure 6: *Standard notation.*

Example

A carbon-12 atom has 6 protons and 6 neutrons.


The nucleon number — there are a total of 12 protons and neutrons in a carbon-12 atom.
12
C
The symbol for the element carbon.
The proton number — there are 6 protons in a carbon atom.
6

Isotopes

Atoms with the same number of protons but different numbers of neutrons are called **isotopes**. Changing the number of neutrons doesn't affect the atom's chemical properties. The number of neutrons affects the stability of the nucleus though. Too few or too many neutrons can make a nucleus unstable. Unstable nuclei may be radioactive and decay to make themselves more stable (see page 427).

Isotopes are often named using their nucleon number, e.g. carbon's isotopes include carbon-12 and carbon-13. Some isotopes of elements have special names e.g. hydrogen's isotopes include deuterium and tritium.

Figure 7: *Tritium can be used to illuminate fire exit signs and watch faces without the need for electricity.*

Example

Hydrogen has three isotopes — protium, deuterium and tritium.

Protium has 1 proton and 0 neutrons.

Deuterium has 1 proton and 1 neutron.

Tritium has 1 proton and 2 neutrons.

Practice Questions — Application

Q1 An atom of oxygen, O, has 8 protons and 8 neutrons.

a) What is the proton number of this atom?

b) What is the nucleon number of this atom?

c) Write this information in standard notation.

Q2 Element X has 21 protons and 24 neutrons.

a) Write this in standard notation.

b) Suggest the standard notation for a different isotope of element X.

Q3 Helium is written in standard notation as: $^{4}_{2}\mathrm{He}$

a) How many protons does an atom of helium have?

An isotope of helium has a nucleon number of 3.

b) How many protons does this isotope have?

Practice Questions — Fact Recall

Q1 Explain the conclusions about the structure of the atom that could be drawn from the results of the alpha-particle scattering experiment.

Q2 Describe the simple nuclear model of the atom.

Q3 What is the proton number of an atom?

Q4 What is the nucleon number of an atom?

Q5 What are isotopes?

2. The Nucleus

The nucleus may be tiny, but it contains most of the mass of the atom. It takes a lot to hold it all together in such a small space...

Learning Objectives:

- Know the relative sizes of the atom and the nucleus.
- Know that the radius of the nucleus can be calculated using the equation $R = r_0 A^{1/3}$, where r_0 is a constant and A is the nucleon number.
- Know how to calculate the mean densities of atoms and nuclei.
- Know what is meant by the strong nuclear force.
- Understand the short-range nature of the strong nuclear force.
- Know that the strong nuclear force is repulsive at distances below about 0.5 fm, and attractive up to a distance of about 3 fm.

Specification Reference 6.4.1

The size of the nucleus and atom

By probing atoms using scattering and diffraction methods, we know that the diameter of an atom is about 0.1 nm (1×10^{-10} m) and the diameter of the smallest nucleus is a few fm (1 fm = 1×10^{-15} m). So nuclei are really, really tiny compared with the size of the whole atom.

To make this easier to visualise, try imagining a large Ferris wheel as the size of an atom. If you then put a grain of rice in the centre, this would be approximately the size of the atom's nucleus.

Molecules are just a number of atoms joined together. As a rough guide, the size of a molecule equals the number of atoms in it multiplied by the size of one atom.

Nuclear radius

As you know from p.412, the particles that make up the nucleus (i.e. protons and neutrons) are called nucleons. The number of nucleons in an atom is called the nucleon (or mass) number, A. Unsurprisingly, as more nucleons are added to the nucleus, it gets bigger.

You can measure the size of a nucleus by firing particles at it. If you plot the radius of the nucleus, R, against the nucleon number, you get a graph like the one shown in Figure 1.

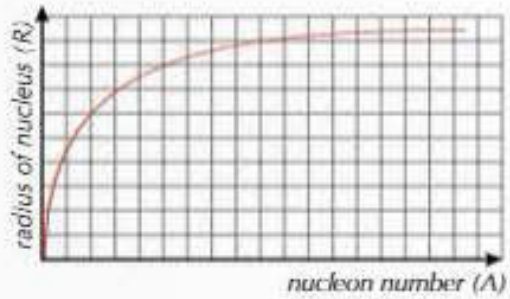


Figure 1: *A graph of nuclear radius, R, against nucleon number, A.*

In fact, nuclear radius increases roughly as the cube root of the nucleon number. You can see this by plotting nuclear radius against the cube root of the nucleon number — as in Figure 2.

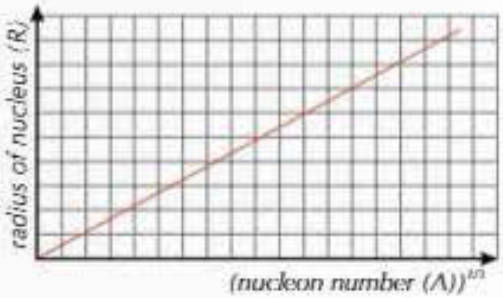


Figure 2: *A graph of nuclear radius, R, against the cube root of nucleon number, A.*

The fact that this graph is a straight line through the origin shows that nuclear radius is directly proportional to the cube root of the nucleon number. This relationship can be written as: $R \propto A^{1/3}$.

By introducing a constant, r_0, we can make this into an equation:

R = nuclear radius → $R = r_0 A^{1/3}$ ← A = nucleon number

Some current experimental results suggest that r_0 is about 1.4 fm, whilst others can give slightly different values.

Tip: You don't need to learn the value of r_0. If you're asked to do calculations like this in the exam, you'll be given all the information you need to answer the question.

Tip: Since A is just a number, with no units, the units of R will depend on what units you use for r_0. If you plug in r_0 in fm, your answer for R will be in fm too.

Example — Maths Skills

An atom of rubidium has a nucleon number of 85. Calculate the radius of its nucleus. $r_0 = 1.4$ fm.

$R = r_0 A^{1/3}$

$= 1.4 \times 10^{-15} \times 85^{1/3}$

$= 6.1555... \times 10^{-15}$ m $= 6.2$ fm (to 2 s.f.)

Density of the nucleus

To calculate the mean density of a nucleus you need to know the mass and volume of the nucleus — and the equation for density:

$$\rho = \frac{m}{V}$$

If you're asked to estimate mean nuclear density, you might have to work out the volume of the nucleus from its radius. Assume the nucleus is a sphere and substitute the radius into the equation:

$$V = \frac{4}{3}\pi r^3$$

Nuclear density is pretty much the same, regardless of the element — roughly 10^{17} kg m^{-3}. Nuclear density is much higher than atomic density. This suggests that:

- Most of an atom's mass is in its nucleus.
- The nucleus is small compared to the atom.
- An atom must contain a lot of empty space.

Tip: The symbol for density, ρ, is the Greek letter 'rho'. Its unit is kgm^{-3}.

Tip: Be careful of units if you're calculating densities — lengths need to be in metres and masses need to be in kilograms.

Tip: You can find the atomic density in the same way as you find the density of the nucleus. Just use the radius of the atom instead of the radius of the nucleus. You can assume the electron mass is negligible for density calculations, so the mass will be the same in both calculations.

Examples — Maths Skills

A carbon atom has a mass of 2.00×10^{-26} kg, an atomic radius of 7.0×10^{-11} m, and a nuclear radius of 3.2×10^{-15} m.

Calculate its mean atomic density.

$V_{atom} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times (7.0 \times 10^{-11})^3 = 1.436... \times 10^{-30}$ m^3

$\rho_{atom} = \frac{m}{V}$ so $\rho = \frac{2.00 \times 10^{-26}}{1.436... \times 10^{-30}} = 1.392... \times 10^4$

$= 1.4 \times 10^4$ kg m^{-3} (to 2 s.f.)

Calculate its mean nuclear density.

$V_{nucleus} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times (3.2 \times 10^{-15})^3 = 1.372... \times 10^{-43}$ m^3

$\rho_{nucleus} = \frac{m}{V}$ so $\rho = \frac{2.00 \times 10^{-26}}{1.372... \times 10^{-43}} = 1.457... \times 10^{17}$

$= 1.5 \times 10^{17}$ kg m^{-3} (to 2 s.f.)

Forces in the nucleus

There are several different forces acting on the nucleons in a nucleus. The electrostatic force causes the positively charged protons in the nucleus to repel each other (p.369). The gravitational force causes all the nucleons in the nucleus to attract each other due to their mass (p.297).

However, the repulsion from the electrostatic force is much, much bigger than the gravitational attraction. If these were the only forces acting in the nucleus, the nucleons would fly apart. So there must be another attractive force that holds the nucleus together — called the **strong nuclear force**.

The strong nuclear force

To hold the nucleus together, the strong nuclear force must be an attractive force that overcomes the electrostatic force. Experiments have shown that the strong nuclear force between nucleons has a short range. It can only hold nucleons together when they are separated by up to a few femtometres — the size of a nucleus. The strength of the strong nuclear force between nucleons quickly falls beyond this distance.

Experiments also show that the strong nuclear force works equally between all nucleons. This means that the size of the force is the same whether for proton-proton, neutron-neutron or proton-neutron interactions.

At very small separations, the strong nuclear force must be repulsive — otherwise there would be nothing to stop it crushing the nucleus to a point.

Tip: The gravitational attraction in the nucleus is so small compared to the other forces, you can just ignore it.

Comparing strong nuclear and electrostatic forces

Figure 3 shows how the strong nuclear force changes with the distance between nucleons. It also shows how the electrostatic force changes so that you can see the relationship between these two forces (although only protons feel the electrostatic force).

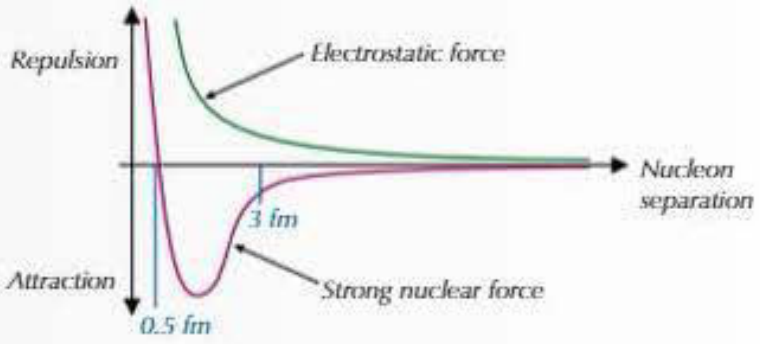


Figure 3: *A graph to show how the strong nuclear and electrostatic forces vary with nucleon separation.*

The graph shows that:

- The strong nuclear force is repulsive for very small separations of nucleons (below about 0.5 fm).
- As nucleon separation increases past about 0.5 fm, the strong nuclear force becomes attractive. It reaches a maximum attractive value and then falls rapidly to zero. After about 3 fm it can no longer hold nucleons together.
- The electrostatic repulsive force extends over a much larger range (indefinitely, actually).

Figure 4: *The Large Hadron Collider at CERN collides lead ions to try to find out more about the strong nuclear force.*

Tip: You can assume $r_0 = 1.4$ fm for Q1.

Practice Questions — Application

Q1 The nucleus of an atom of an element has a radius of 7.0 fm and contains 73 neutrons. Find the number of protons in the nucleus.

Q2 Explain how the strong nuclear force acts between nucleons that are:

a) 0.4 fm apart.

b) 1.5 fm apart.

c) 4.2 fm apart.

Q3 A vanadium atom has a mass of approximately 8.5×10^{-26} kg, and an atomic density of 8.4×10^{4} kg m^{-3}.

a) Calculate the radius of the vanadium atom.

b) The vanadium nucleus has a radius of approximately 5.2×10^{-15} m. Calculate the nuclear density of the vanadium atom.

Practice Questions — Fact Recall

Q1 Give the approximate diameters of an atom and a small nucleus.

Q2 State how the nuclear radius changes with nucleon number.

Q3 State the shape that a nucleus is assumed to be, and give the equation that can be used to calculate its volume.

Q4 a) Sketch a graph to show how the strong nuclear force changes with nucleon separation, marking on any key separation distances.

b) State the range in which the strong nuclear force is repulsive.

c) State the range in which the strong nuclear force is attractive and can hold nucleons together.

3. Particles and Antiparticles

Particles come in two types — matter and antimatter. Antimatter can be produced in pairs with its corresponding matter, but if the two come into contact, they destroy each other.

Hadrons

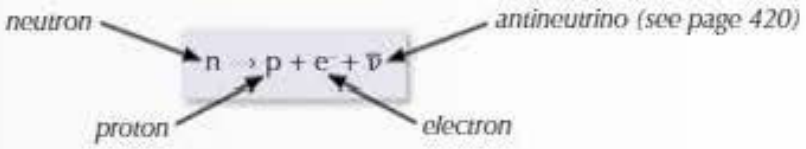
Not all particles can feel the strong nuclear force — the ones that can are called **hadrons**. Hadrons also feel the weak nuclear force. Hadrons aren't **fundamental particles**. They're made up of smaller particles called quarks (see page 424).

Protons and neutrons are hadrons. This is why they can make atomic nuclei — the nucleus of an atom is made up from protons and neutrons held together by the strong nuclear force (p.417). As well as protons and neutrons, there are other hadrons that you don't get in normal matter, like sigmas (Σ) and mesons — luckily you don't need to know a lot about them.

The decay of hadrons

Most hadrons will eventually decay into other particles.
The exception is protons — most physicists think that protons don't decay.

The neutron is an unstable particle that decays into a proton (along with an electron and an antineutrino). The decay of a neutron is really just an example of β^- decay (see p.427), which is caused by the **weak nuclear force**.



Free neutrons (i.e. ones not in a nucleus) have a half-life of about 15 minutes. (The neutron is much more stable when it is part of a nucleus.)

Leptons

Leptons are fundamental particles and they don't feel the strong nuclear force. They interact with other particles via the weak nuclear force and gravity (and the electrostatic force if they're charged).

There are two types of lepton you need to know about — electrons (e^-), which should be familiar, and **neutrinos** (ν).

Neutrinos have zero (or almost zero) mass and zero electric charge — so they don't do much. Neutrinos only take part in weak interactions (see p.425). In fact, a neutrino can pass right through the Earth without anything happening to it.

Name	Symbol	Relative charge
electron	e^-	–1
neutrino	ν	0

Figure 1: *The symbols and relative charges of two leptons you need to know — the electron and the neutrino.*

Learning Objectives:

- Know that all particles classified as hadrons are subject to both the strong nuclear force and the weak nuclear force.
- Know that protons and neutrons are examples of hadrons.
- Know that all particles classified as leptons are subject to the weak nuclear force but not the strong nuclear force.
- Know that electrons and neutrinos are examples of leptons.
- Know what particles and antiparticles are, including the pairs electron-positron, proton-antiproton, neutron-antineutron and neutrino-antineutrino.
- Know that particles and their corresponding antiparticles have the same mass.
- Know that an electron and a positron have opposite charges and that protons and antiprotons have opposite charges.
- Know Einstein's mass-energy equation $\Delta E = \Delta mc^2$.
- Understand the creation and annihilation of particle-antiparticle pairs.

Specification References 6.4.2 and 6.4.4

Tip: Relative charge is the charge of the particle expressed a multiple of the elementary charge, e. A particle with relative charge –1 has a charge of -1.60×10^{-19} C.

Figure 2: *Paul Dirac, the English physicist who predicted the existence of antimatter.*

Tip: All particles are known as matter, and antiparticles are known as antimatter.

Tip: The opposite of a zero charge is just zero. So the antiparticle of any neutral particle will also be neutral.

Tip: Most antiparticles have the same symbol as their corresponding particle, but with a line over the top, i.e. $\bar{x}$. The exceptions to this are charged leptons, like the electron. These tend to have their particles marked with a minus sign, e.g. e^-, to indicate their negative charge, while their antiparticles have a plus sign, e.g. e^+, since they have a positive charge.

Antimatter

Each particle type has a corresponding **antiparticle** with the same mass but with opposite charge. Antiparticles were predicted before they were discovered. When Paul Dirac wrote down an equation obeyed by electrons, he found a kind of mirror image solution. It predicted the existence of a particle like the electron but with opposite electric charge — the **positron**. The positron turned up later in a cosmic ray experiment. Positrons have identical mass to electrons but they carry a positive charge. It is the antimatter version of the electron.

Figure 3 shows the relative charges of the proton, neutron, electron, neutrino and their antiparticles.

Particle/Antiparticle	Symbol	Relative Charge	Rest Mass (kg)
proton	p	+1	1.673×10^{-27}
antiproton	$\bar{p}$	–1	
neutron	n	0	1.675×10^{-27}
antineutron	$\bar{n}$		
electron	e^-	–1	9.11×10^{-31}
positron	e^+	+1	
neutrino	ν	0	0
antineutrino	$\bar{\nu}$		

Figure 3: *Relative charges and rest masses of particles and their corresponding antiparticles.*

The masses in Figure 3 are all rest masses — the mass of the particle when it's not moving. This is because the masses of objects change when they're moving at very high speeds, but you don't need to know about that.

In the exam, you'll be given the masses of protons, neutrons and electrons. Just remember that the mass of an antiparticle is the same as the mass of its corresponding particle. You need to learn the relative charges on each type of particle though (these are all relative to $e = 1.60 \times 10^{-19}$ C). Neutrinos have zero charge and (almost) zero mass.

Einstein's mass-energy equation

You've probably heard about the equivalence of energy and mass. It all comes out of Einstein's special theory of relativity. Energy can turn into mass and mass can turn into energy, as described by the fantastic and rather famous formula, Einstein's mass-energy equation:

ΔE = change in energy in J

$$\Delta E = \Delta m c^2$$

c = speed of light in ms^{-1} $= 3.00 \times 10^8\ ms^{-1}$

Δm = change in mass in kg

You can use this equation to calculate the energy of a particle at rest with a given mass. This is just how much energy would be released if all the mass was converted to energy.

Example — **Maths Skills**

The mass of a proton at rest is 1.673×10^{-27} kg. Calculate the energy that would be released if this proton were completely converted into energy.

$\Delta E = \Delta mc^2$

$= 1.673 \times 10^{-27} \times (3.00 \times 10^8)^2$

$= 1.5057 \times 10^{-10}$

$= 1.51 \times 10^{-10}$ J (to 3 s.f.)

Tip: In this example, Δm (the change in mass) is just the total mass of the proton, as it's all being converted to energy.

Pair production

If you fire two protons at each other at high speed then you'll end up with a lot of energy at the point of impact. This energy might be converted into more particles.

When energy is converted into mass you get equal amounts of matter and antimatter. So if an extra proton is formed then there will always be an antiproton to go with it. It's called **pair production**.

Tip: Protons repel each other, so it takes a lot of energy to make them collide. The energy supplied is released when they collide, so proton-proton collisions release a lot of energy.

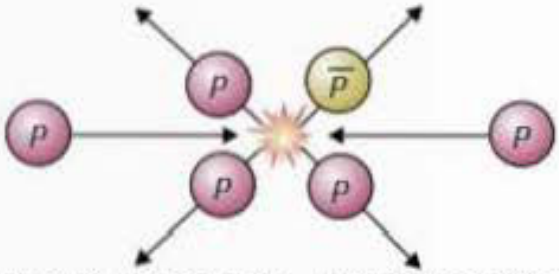


Figure 4: *Pair production — two protons colliding and producing a proton-antiproton pair.*

You can also get pair production from a photon. This only happens if one photon has enough energy to produce that much mass (e.g. a gamma ray photon, γ). It also tends to happen near a nucleus, which helps conserve momentum. You usually get electron-positron pairs produced (rather than any other pair) — because they have a relatively low mass.

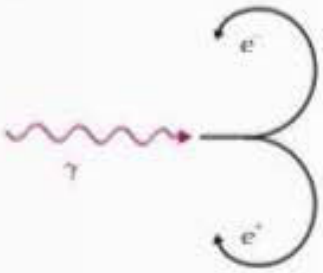


Figure 5: *Pair production — an electron-positron pair produced from a gamma ray photon.*

The minimum amount of energy the photon must have is the combined energy of the two particles at rest (i.e. assuming that the particles have negligible kinetic energy). Remember, you can find the energy equal to a particle at rest using $\Delta E = \Delta mc^2$. So you can calculate the minimum energy, E_γ, of the photon.

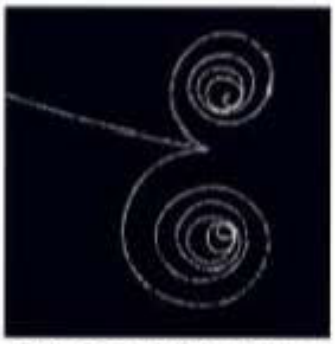


Figure 6: *A pair-production event in a bubble tank. A gamma ray photon has come from the right (but is invisible here). The spiral paths show the motion of an electron-positron pair after their production from the photon. The straighter path is another particle produced as a by-product — but you don't need to know what that is.*

Tip: The tracks are curved as there's usually a magnetic field present in experiments. They curve in opposite directions because of their opposite charges — see p.389.

A particle and its antiparticle have the same rest mass (m), which means that:

E_γ = *minimum energy of photon in J* → $E_\gamma = 2mc^2$ ← **c** = *speed of light in* ms^{-1}

m = *rest mass of one of the pair-produced particles in kg*

You can go further and find the maximum wavelength, λ_{max}, or minimum frequency, f_{min}, of the photon using the equation for the energy of a photon (see page 232):

E_γ = *energy of photon in J* → $E_\gamma = \frac{hc}{\lambda} = hf$ ← **f** = *frequency of photon in Hz*

λ = *wavelength of photon in m* → ← **h** = *Planck's constant in Js*

Just put these two equations for E_γ together and rearrange to find λ_{max} or f_{min}:

$$\lambda_{max} = \frac{h}{2mc} \qquad f_{min} = \frac{2mc^2}{h}$$

Example **Maths Skills**

Calculate the minimum energy a photon must have to produce an electron-positron pair.

The mass of an electron (and so positron) is 9.11×10^{-31} kg.

So, $E_\gamma = 2mc^2$

$= 2 \times 9.11 \times 10^{-31} \times (3 \times 10^8)^2$

$= 1.6398 \times 10^{-13} = 1.64 \times 10^{-13}$ J (to 3 s.f.) (= 1.02 MeV)

Calculate the minimum frequency of this photon.

$E_\gamma = hf$, so $f = \frac{E_\gamma}{h}$

$f = \frac{1.6398 \times 10^{-13}}{6.63 \times 10^{-34}} = 2.4733... \times 10^{20} = 2.47 \times 10^{20}$ Hz (to 3 s.f.)

Tip: Energies in particle physics are often given in MeV. To convert J to MeV, divide the value by 1.6×10^{-19} (to change J into eV) and then divide it by 10^6 (to change the eV to MeV).

Annihilation

When a particle meets its antiparticle the result is **annihilation**.
All the mass of the particle and antiparticle gets converted to energy, in the form of a pair of photons. Antiparticles can usually only exist for a fraction of a second before this happens, so you don't get them in ordinary matter.

Tip: The two photons produced in annihilation are always emitted in opposite directions to conserve momentum.

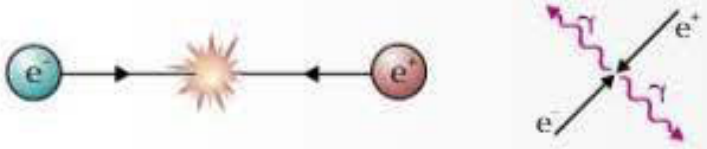
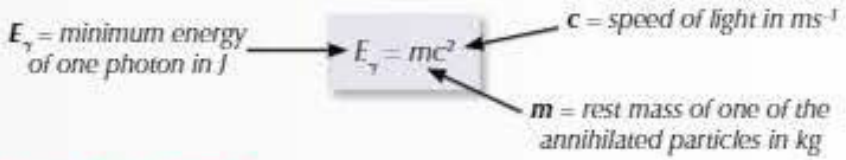


Figure 7: *Electron-positron annihilation.*

Just like with pair production, you can calculate the minimum energy of each photon produced (i.e. assuming that the particles have negligible kinetic energy). The combined energy of the photons will be equal to the combined energy of the particles, so $2E_\gamma = 2mc^2$ and so:


E_γ = minimum energy of one photon in J
$E_\gamma = mc^2$
c = speed of light in ms^{-1}
m = rest mass of one of the annihilated particles in kg

Tip: Just like with pair production, you can use the equation $E_\gamma = hf = \frac{hc}{\lambda}$ to find the minimum frequency, or maximum wavelength, of the photons emitted by annihilation events.

Example — **Maths Skills**

A neutron and antineutron collide and annihilate.
Calculate the minimum energy of one of the photons produced.

The mass of a neutron is 1.675×10^{-27} kg.

So, $E_\gamma = mc^2$
$= 1.675 \times 10^{-27} \times (3.00 \times 10^8)^2$
$= 1.5075 \times 10^{-10}$ J
$= 1.51 \times 10^{-10}$ J (to 3 s.f.) (= 942 MeV)

Practice Questions — Application

Q1 A collision between protons can release enough energy to create a proton-antiproton pair. What is the minimum energy in J required for this pair production to take place?

Q2 A Λ^0 particle is a hadron with a mass of 1.983×10^{-27} kg.

a) Calculate the minimum energy of a photon produced when a Λ^0 particle annihilates with its antiparticle.

b) Calculate the minimum frequency and maximum wavelength of this photon.

Q3 A muon is a type of lepton, with a mass of 1.88×10^{-28} kg. Calculate the minimum frequency a photon must have to pair-produce a muon-antimuon pair.

Exam Tip
Don't worry if you're given a question about a particle you're not familiar with in the exam. You'll be given all the information you need in order to answer the question.

Practice Questions — Fact Recall

Q1 State the type of particle that is affected by the strong nuclear force.

Q2 Name one force through which an electron can interact with other particles.

Q3 Give the relative charge and relative mass of a neutrino, and state whether it is a hadron or lepton.

Q4 a) How does an antiparticle differ from its corresponding particle?

b) How is it the same?

Q5 Name the electron's antiparticle.

Q6 Write down the relative charge of an:

a) antiproton b) antineutron c) antineutrino

Q7 State Einstein's mass-energy equation. Define all symbols used.

Q8 Describe the process of pair production.

Q9 What is produced in the annihilation of a particle and its antiparticle?

Learning Objectives:

- Understand the simple quark model of hadrons in terms of up (u), down (d) and strange (s) quarks and their respective anti-quarks.
- Know the charges of the up (u), down (d), strange (s), anti-up ($\bar{u}$), anti-down ($\bar{d}$) and anti-strange ($\bar{s}$) quarks, as fractions of the elementary charge *e*.
- Know the quark model of the proton (uud) and of the neutron (udd).
- Understand the decay of particles in terms of the quark model.
- Understand beta-minus (β^-) decay in terms of the quark model.

  $d \rightarrow u + {}^{0}_{-1}e + \bar{\nu}$
- Understand beta-plus (β^+) decay in terms of the quark model.

  $u \rightarrow d + {}^{0}_{+1}e + \nu$
- Be able to balance quark transformation equations in terms of charge.

Specification Reference 6.4.2

4. Quarks and Anti-quarks

Quarks are fundamental particles — they're the smallest amount of matter that you can get that interacts with the strong nuclear force.

The simple quark model

Quarks are the building blocks for hadrons like protons and neutrons. In total, there are 6 types of quark. But at A-level, we can limit ourselves to looking at only three types of quark — the up quark (u), the down quark (d), the strange quark (s) and their antiparticles, known as anti-quarks (see Figure 1). The anti-quarks have opposite charges to the quarks. Antiparticles of hadrons (like antiprotons and antineutrons) can be made with anti-quarks.

Name	Symbol	Charge
up	u	+2/3 e
down	d	−1/3 e
strange	s	−1/3 e

Name	Symbol	Charge
anti-up	$\bar{u}$	−2/3 e
anti-down	$\bar{d}$	+1/3 e
anti-strange	$\bar{s}$	+1/3 e

Figure 1: *Properties of up, down and strange quarks (left), and their anti-quarks (right).*

You'll be given the charges of the quarks in the data and formulae booklet in the exam. You won't be given the charges of the anti-quarks, but remember — the charge on an anti-quark is the same size as the charge on its quark, but with the opposite sign.

Strange quarks and down quarks may look pretty similar, but there are other properties of quarks not given here that distinguish them from each other. Luckily, you don't need to know about these other properties.

Quark composition of hadrons

Evidence for quarks came from hitting protons with high energy electrons. The way the electrons scattered showed that there were three concentrations of charge (quarks) inside the proton. The properties of a particle depend on the properties of the quarks that make it up. A proton has a charge of +1 because the quarks that make it up have an overall charge of +1.

Protons are made of two up quarks and one down quark (uud).

- The total charge of a proton is 2/3 + 2/3 + (− 1/3) = +1.

Neutrons are made up of one up quark and two down quarks (udd).

- The total charge of a neutron is 2/3 + (−1/3) + (−1/3) = 0.

Tip: You only need to know the quark composition of protons and neutrons for your exam. If you're given any other hadrons, you'll be given their quark composition.

Figure 2: *The quark composition of nucleons.*

Antiprotons are $\bar{u}\bar{u}\bar{d}$ and antineutrons are $\bar{u}\bar{d}\bar{d}$. Not all hadrons have three quarks though. Protons and neutrons are a type of hadron called baryons, which are made up of three quarks. There are also hadrons made up of a quark and an anti-quark, called mesons. You don't need to know anything more about them, but don't be surprised if you see one pop up in a question.

Quark confinement

It's not possible to get a quark by itself — this is called quark confinement. If you blasted a proton with a lot of energy, a single quark would not be removed. The energy that you supplied would just get changed into a quark-anti-quark pair.

A proton. *Energy supplied to remove u quark.* *When enough energy is supplied, a u and $\bar{u}$ pair is produced and the u quark stays in the proton.*

Figure 3: *Quark confinement — the energy used trying to remove a u quark only creates a u and $\bar{u}$ pair in a pair production.*

Tip: Figure 3 just shows an example of pair production (see page 421). In this example, a meson (quark-anti-quark pair) is created.

Weak nuclear force

Hadrons can decay into other particles via the weak nuclear force (p.419). This is the only thing that can change one type of quark into another. A quark changing into another quark is known as changing a quark's flavour. In any other kind of interaction, the number of quarks of any type must be the same before the interaction as after it. In all interactions, charge must be conserved.

Beta-minus decay

Beta-minus (β^-) decay is a type of weak interaction. In β^- decay a neutron is changed into a proton — in other words udd changes into uud. It means turning a d quark into a u quark.

Beta-minus decay in terms of quarks: $\mathrm{d} \rightarrow \mathrm{u} + {}^{0}_{-1}\mathrm{e} + \bar{\nu}$

Beta-minus decay in terms of charge: $(-\frac{1}{3}) \rightarrow (+\frac{2}{3}) + (-1) + 0$

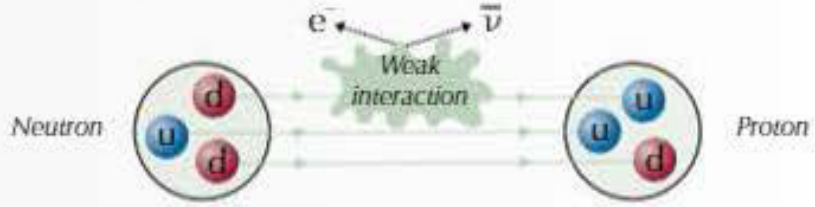


Figure 4: *A quark changing flavour in β^- decay.*

Tip: A β^- particle is just an electron (see p.427).

Tip: You'll notice that charge is conserved in both of β^- and β^+ interactions — the total charge on the left of the arrow equals the total charge on the right.

Beta-plus decay

Some unstable isotopes like carbon-11 decay by beta-plus (β^+) emission. β^+ is another type of weak interaction. β^+ decay just means a positron (a β^+ particle) is emitted. In this case a proton changes to a neutron, so a u quark changes to a d quark.

Beta-plus decay in terms of quarks: $\mathrm{u} \rightarrow \mathrm{d} + {}^{0}_{+1}\mathrm{e} + \nu$

Beta-plus decay in terms of charge: $(+\frac{2}{3}) \rightarrow (-\frac{1}{3}) + (+1) + 0$

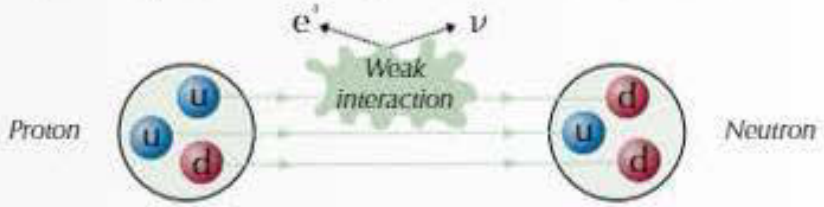


Figure 5: *A quark changing flavour in β^+ decay.*

Tip: Remember, ${}^{0}_{+1}\mathrm{e}$ and β^+ are both symbols for a positron.

Tip: Don't get confused between the equations for beta-plus and beta-minus decay. There should always be one particle and one antiparticle emitted — so if an electron is emitted, an antineutrino is also emitted, and if a positron is emitted, a neutrino is also emitted.

Conservation of charge

In any particle reaction, the total charge after the reaction must equal the total charge before the reaction. You can use this rule to balance quark transformation equations.

Example — **Maths Skills**

Σ^+ and π^+ particles are both hadrons. A Σ^+ particle can decay to produce a π^+ particle and another particle A.

$$\Sigma^+ \rightarrow A + \pi^+$$
$$uus \rightarrow ?dd + u\bar{d}$$

By considering the charge on each quark, identify the missing quark (labelled '?') in this reaction, and hence state the name of particle A.

Write down the relative charge on each quark and then find the total charge on each particle.

u	u	s	→	?	d	d	+	u	$\bar{d}$	
$+\frac{2}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	→	?	$-\frac{1}{3}$	$-\frac{1}{3}$	+	$+\frac{2}{3}$	$+\frac{1}{3}$	
	+1		→		$(? - \frac{2}{3})$		+		+1	

For charge to be conserved, the missing quark must have a relative charge of $+\frac{2}{3}$. This means it must be an up quark.

This means particle A is made up of the quarks udd, so it is a neutron.

Tip: Remember, relative charge is the charge on a particle expressed as a multiple of the elementary charge e, 1.60×10^{-19} C.

Practice Questions — Application

Q1 Calculate the charge of the following quark compositions.

a) $u\bar{u}$ b) uus c) dss d) sss

Q2 A sigma-minus particle Σ^- is a hadron with –1 charge. It contains one strange quark, and two other quarks. Using u and/or d quarks, write down its quark composition.

Q3 A Xi-minus particle, Ξ^-, undergoes the following decay. The quark compositions of the particles are shown, with an unknown quark marked X.

$$\Xi^- \rightarrow \Lambda^0 + \pi^-$$
$$dss \rightarrow Xds + d\bar{u}$$

Using conservation of charge, work out the identity of quark X.

Practice Questions — Fact Recall

Q1 State the names and symbols of three quarks and three anti-quarks in the simple quark model.

Q2 State the charges of three anti-quarks in the simple quark model.

Q3 State the quark composition of a proton and a neutron.

Q4 Explain beta-minus decay in terms of the quark model.

Q5 Copy and complete the quark transformation equation for beta-plus decay: u → d + ☐ + ☐

Q6 Which quantity is always conserved in all particle interactions?

5. Radioactive Decay

Some nuclei are unstable, and decay to a more stable state by emitting nuclear radiation. There are four different types of nuclear radiation, and each has different properties.

Learning Objectives:

- Know what is meant by radioactive decay.
- Understand the spontaneous and random nature of radioactive decay.
- Know that α-particles, β-particles and γ-rays are types of nuclear radiation.
- Understand the nature, penetration and range of α-particles, β-particles and γ-rays.
- Understand techniques and procedures that can be used to investigate the absorption of α-particles, β-particles and γ-rays by appropriate materials (PAG7).

Specification Reference 6.4.3

Unstable nuclei

The nucleus is under the influence of the strong nuclear force holding it together and the electrostatic force pushing the protons apart. It's a very delicate balance, and it's easy for a nucleus to become unstable.

If a nucleus is unstable, it will break down to become more stable. Its instability could be caused by:

- too many neutrons
- too few neutrons
- too many nucleons in total (it's too heavy)
- too much energy in the nucleus

The nucleus decays by releasing energy and/or particles (nuclear radiation), until it reaches a stable form — this is called **radioactive decay**. An individual radioactive decay is spontaneous and random — the decay happens of its own accord (nothing triggers it) and it can't be predicted.

Although you can't predict the decay of an individual nucleus, if you take a very large number of nuclei, their overall behaviour shows a pattern. Any sample of a particular isotope (p.413) has the same rate of decay, i.e. the same proportion of nuclei will decay in a given time (p.433).

Types of nuclear radiation

There are four types of **nuclear radiation** — **alpha**, **beta-minus**, **beta-plus** and **gamma**. They are listed in Figure 1. The masses here have been given in unified atomic mass units (u) — one unified atomic mass unit = 1.661×10^{-27} kg, and is about the same as the mass of a proton or neutron (see page 412).

Radiation	Symbol	Constituent	Relative Charge	Mass (u)
Alpha	α	A helium nucleus — 2 protons & 2 neutrons	+2	4
Beta-minus (Beta)	β^- or β	Electron	–1	(negligible)
Beta-plus	β^+	Positron	+1	(negligible)
Gamma	γ	Short-wavelength, high-frequency electromagnetic wave.	0	0

Figure 1: *Types of nuclear radiation.*

When radiation hits an atom it can knock off electrons, creating an ion — so, radioactive emissions are also known as **ionising radiation**. The different types of radiation have different ionising powers as well as different speeds and penetrating powers — these are summarised in Figure 2.

Tip: Ionisation can be harmful. If it occurs in the cells of a living organism, it can do serious damage to it. This is what makes nuclear radiation so dangerous.

Radiation	Symbol	Ionising Power	Speed	Penetrating power	Affected by magnetic field
Alpha	α	Strong	Slow	Absorbed by paper or a few cm of air	Yes
Beta-minus (Beta)	β^- or β	Weak	Fast	Absorbed by ~3 mm of aluminium	Yes
Beta-plus	β^+	Annihilated by electron — so virtually zero range			
Gamma	γ	Very weak	Speed of light	Absorbed by many cm of lead, or several m of concrete	No

Figure 2: *A summary of the properties of the four types of nuclear radiation.*

Tip: Gamma radiation isn't made up of charged particles, so it won't be deflected by a magnetic field.

Tip: Any material that stops beta radiation will also stop alpha radiation, and any material that stops gamma radiation will also stop alpha and beta radiation.

The stronger the ionising power of radiation, the more energy it loses in a given distance, so the shorter the range of the radiation. As you can see from Figure 2, the more penetrating the type of radiation, the thicker or denser a material needs to be to absorb it — see Figure 3.

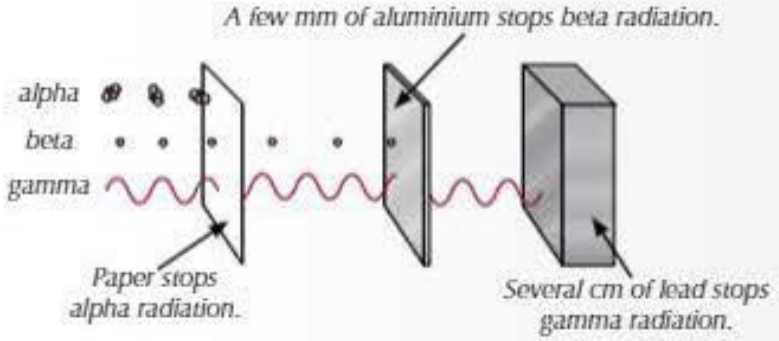


Figure 3: *Examples of materials that stop each type of nuclear radiation.*

Both the thickness and the density of a material affect whether radiation will penetrate it — a very thick piece of aluminium could stop gamma radiation.

Figure 4: *The components inside a smoke alarm, including a source of alpha radiation. Alpha radiation's low penetrating power means it is easily absorbed by smoke particles, and so can be used to detect smoke.*

Background radiation

Background radiation is the low level of radiation that surrounds us at all times. It is made up of radiation from many different sources — these include naturally occurring isotopes in the environment, cosmic radiation from space, and man-made sources from industrial and medical radiation use. You'll need to take background radiation into account when you're investigating radiation (see the next page and page 438).

Figure 5: *Radioactive isotopes in granite rock contribute to background radiation.*

Investigating radiation

PRACTICAL ACTIVITY GROUP 7

You can investigate the penetration of different kinds of radiation by using different radioactive sources. These can be dangerous if you don't use them properly:

- Radioactive sources should be kept in a lead-lined box when they're not being used.
- They should only be picked up using long-handled tongs or forceps.
- Take care not to point them at anyone, and always keep a safe distance from them.

Tip: Lead will absorb all types of ionising radiation if it's thick enough.

Tip: Any experiments involving radiation can be dangerous if done incorrectly. As always, a full risk assessment must be carried out beforehand.

Measuring count rate

To begin, set up the equipment shown in Figure 6. A Geiger-Müller tube produces a 'count' in the form of an electrical pulse each time radiation enters it. The Geiger counter measures the total number of counts over a period of time. The count rate (counts per second) can be calculated by dividing the total number of counts recorded by the time (in seconds) over which this was measured.

Figure 6: *The experimental set-up used to investigate radiation.*

Tip: Wear gloves when performing this experiment, to help protect your hands from radiation.

To measure the count rate of the radioactive source:

1. First of all, without the source present, measure the background counts. Radioactive decay is random, so to get an accurate and precise reading, the background counts need to be measured over a long enough time interval. For most experiments in the lab, around 30 seconds should do. Do this three times, and find the mean number of counts in this time interval.
2. Divide your counts by the number of seconds in the time interval to get the background count rate. You only need to do this once during a given experiment.
3. Place the source in front of the Geiger-Müller tube, as shown in Figure 6, making sure that it is close enough that the counter records a high number of counts (compared to the background counts).
4. Calculate the count rate of the radioactive source, using the same method you used for the background count rate. Again, take three sets of count measurements for three 30 second time intervals, and find the mean before you divide by the 30 seconds to find the count rate.
5. Subtract the background count rate from this value to find the count rate of the source.

Tip: You met background radiation on the previous page.

Tip: Repeating your count measurement and taking an average helps make your results more precise.

Tip: You could instead attach the Geiger-Müller tube to a data-logger and a computer to reduce human error. This may be able to calculate the count-rate for you straight away. Or you could connect the tube to a ratemeter, which measures the count rate directly. In either case, measure the count rate three times, and then average the results.

Tip: If you're comparing penetration through different materials, you should make sure they all have the same thickness. Thickness is a control variable here.

Figure 7: *A Geiger-Müller tube attached to a Geiger counter.*

Investigating penetration properties

To investigate the penetration of radiation through different materials, place a material between the source and the Geiger-Müller tube. Measure the count rate of the source both with and without the material. If the count rate remains about the same regardless of whether the material is present, then the radiation can penetrate the material. If the count rate drops by a large amount, then some of the radiation is being absorbed and blocked by the material. If the count rate drops to zero (after the background count rate is subtracted), the radiation is being completely absorbed. Repeat this for different materials and plot your results on a bar graph (since your results are categoric data).

You can also do this experiment with different sources to compare how different kinds of radiation are blocked by different materials. You'll probably need to change the distance between the source and the Geiger-Müller tube for this, as different kinds of radiation have different penetrating powers in air (see Figure 2 on p.428).

You could also adjust this experiment to investigate how the count rate for a particular source is affected by the thickness of a particular material — e.g. by using sheets of aluminium for beta radiation or different thicknesses of lead for gamma radiation. Or you could investigate how the count rate changes with distance between the source and the Geiger-Müller tube. For these variations, you can plot a scatter graph of your results as thickness and distance are continuous variables. This will allow you to see any trends in your results.

Practice Question — Application

Q1 A radioactive source emits one unknown type of radiation. Using a Geiger-Müller tube placed in front of the source, a high count rate was measured. There was no decrease in the count rate recorded when a few sheets of paper were placed between the source and Geiger-Müller tube. The count rate dropped to almost zero when the sheets of paper were replaced with a thin sheet of aluminium.

a) Identify the type of radiation being emitted by the source.

b) The aluminium is replaced with a thick sheet of lead. The count rate is measured again. Explain why the count rate does not drop all the way to zero.

Practice Questions — Fact Recall

Q1 What is radioactive decay?

Q2 State the type of nuclear radiation that consists of:

a) 2 protons and 2 neutrons

b) a high frequency electromagnetic wave

c) an electron

Q3 List all the types of nuclear radiation which could be blocked by a thick sheet of lead.

6. Nuclear Decay Equations

In any nuclear interaction, charge and nucleon number are always conserved. You need to use this fact to balance nuclear decay equations.

Learning Objectives:

- Be able to balance nuclear transformation equations.
- Understand and be able to write nuclear decay equations for alpha, beta-minus and beta-plus decays.

Specification Reference 6.4.3

Representing nuclear decay

We can write equations to see exactly what's going on with a nucleus when it emits radiation. These equations are called nuclear decay equations (or nuclear transformation equations). To do this, we write the particles in standard notation (see page 413). This lets us clearly see how the numbers of protons and neutrons are changing.

Nuclear decay equations need to be balanced — in every nuclear reaction, including fission and fusion (p.445-448), charge and nucleon number must be conserved.

Tip: If you write out an equation for an interaction and the proton and nucleon numbers don't balance, that interaction cannot happen.

Example

The nuclear decay equation for the decay of americium-241 to neptunium-237 by emission of an alpha particle is:

$$^{241}_{95}\text{Am} \longrightarrow {}^{237}_{93}\text{Np} + {}^{4}_{2}\alpha$$

There are 241 nucleons before the decay (in the americium-241 atom), and 241 nucleons after the decay (237 in the neptunium-237 atom and 4 in the alpha particle), so nucleon number is conserved.

You can see that charge is conserved by looking at the proton number (the blue numbers) — there are 95 protons before the decay and 95 after it.

Tip: Remember, neutrons have no charge, so we just look at proton number for charge conservation.

Mass isn't conserved in nuclear reactions — the mass of an alpha particle is less than the individual masses of two protons and two neutrons. The difference in mass is called the mass defect (p.442), and the energy released when the nucleons bond together to form the alpha particle accounts for the missing mass. Once you have taken the conversion of mass to energy into account, then you can see that 'mass-energy' and momentum are conserved in all nuclear reactions. There's more on this on pages 442-444.

You need to know how to write balanced nuclear decay equations for the main types of radioactive decay.

Alpha emission

Alpha emission only happens in very heavy atoms, like uranium and radium. The nuclei of these atoms are too massive to be stable. When an alpha particle is emitted, the proton number decreases by two, and the nucleon number decreases by four.

Example

Uranium-238 decays to thorium-234 by emitting an alpha particle:

nucleon number decreases by 4

$$^{238}_{92}\text{U} \longrightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\alpha$$

proton number decreases by 2

$238 = 234 + 4$ — *nucleon numbers balance*

$92 = 90 + 2$ — *proton numbers balance, so charge is conserved*

Tip: An alpha particle is the nucleus of a helium atom, so you may see an alpha particle written with the symbol $^{4}_{2}\text{He}$.

Beta-minus emission

Beta-minus decay happens in isotopes that are neutron-rich (i.e. have many more neutrons than protons in their nucleus). During beta-minus decay, one of the neutrons in the nucleus decays into a proton and ejects a beta-minus particle and an antineutrino. The proton number increases by one, and the nucleon number stays the same. Beta-minus particles have a negative charge, so they are written with a negative proton number ($^{0}_{-1}\beta$).

Tip: A β^- particle is an electron, so you may see it written in equations as $^{0}_{-1}e$.

Example

Rhenium-188 decays to osmium-188 by emitting a beta-minus particle:

nucleon number stays the same

$$^{188}_{75}\text{Re} \longrightarrow {}^{188}_{76}\text{Os} + {}^{0}_{-1}\beta + {}^{0}_{0}\bar{\nu}$$

proton number increases by 1

$188 = 188 + 0 + 0$ — *nucleon numbers balance*

$75 = 76 - 1 + 0$ — *proton numbers balance, so charge is conserved*

Beta-plus emission

Beta-plus decay happens in isotopes that are proton-rich (i.e. have a high proton to neutron ratio). A proton in the nucleus changes into a neutron, releasing a beta-plus particle and a neutrino. The proton number decreases by one, and the nucleon number stays the same.

Tip: A β^+ particle is a positron. You may see it written as $^{0}_{+1}e$.

Example

Sodium-22 decays to neon-22 by emitting a beta-plus particle:

nucleon number stays the same

$$^{22}_{11}\text{Na} \longrightarrow {}^{22}_{10}\text{Ne} + {}^{0}_{+1}\beta + {}^{0}_{0}\nu$$

proton number decreases by 1

$22 = 22 + 0 + 0$ — *nucleon numbers balance*

$11 = 10 + 1 + 0$ — *proton numbers balance, so charge is conserved*

Gamma emission

Even after a decay, the nucleus often has excess energy — it's in an excited state. This energy is lost by emitting a gamma ray. This often happens after an alpha or beta decay has occurred. During gamma emission, there is no change to the nuclear constituents — the nucleus just loses excess energy. We don't often write nuclear decay equations for just gamma decay, because the nuclear constituents don't change, but gamma ray photons may appear in equations showing other types of decay. The symbol for a gamma ray photon is $^{0}_{0}\gamma$.

Practice Questions — Application

Q1 Complete the following nuclear decay equation for the beta-minus decay of caesium-137: $^{137}_{55}\text{Cs} \longrightarrow {}^{?}_{?}\text{Ba} + ? + ?$

Q2 Write out the nuclear decay equation for the alpha decay of astatine-211 ($Z = 85$) to an isotope of bismuth. (The chemical symbols of astatine and bismuth are At and Bi respectively.)

Q3 Write out the nuclear decay equation for the beta-plus decay of aluminium-26 ($Z = 13$) to an isotope of magnesium. (The chemical symbols of aluminium and magnesium are Al and Mg respectively.)

Tip: Remember, Z is the proton number of a nucleus — see page 413.

7. Exponential Law of Decay

The number of unstable nuclei that decay each second in a radioactive sample depends on how many unstable nuclei are left in the sample.

Learning Objectives:

- Understand what is meant by the activity of a radioactive source.
- Know what is meant by the decay constant, λ, of an isotope.
- Be able to use the equation $A = \lambda N$.
- Understand graphical methods and spreadsheet modelling of the equation $\frac{\Delta N}{\Delta t} = -\lambda N$ for radioactive decay.
- Be able to use the equation $N = N_0 e^{-\lambda t}$ where N is the number of undecayed nuclei.
- Understand the simulation of radioactive decay using dice.
- Be able to use the equation $A = A_0 e^{-\lambda t}$ where A is the activity.

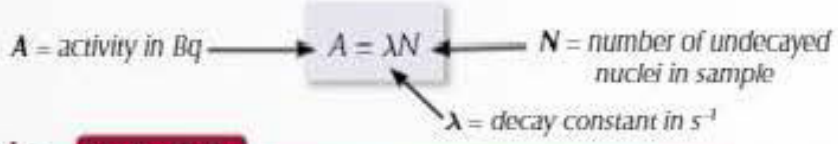
Specification Reference 6.4.3

The rate of radioactive decay

Radioactive decay is completely random — you can't predict which atom's nucleus will decay when. But although you can't predict the decay of an individual nucleus, if you take a very large number of nuclei, their overall behaviour shows a pattern and you can predict how many nuclei will decay in a given amount of time. Any sample of a particular isotope has the same rate of decay — i.e. the same proportion of atomic nuclei will decay in a given time. Each unstable nucleus within the isotope will also have a constant decay probability.

The decay constant and activity

The **activity**, A, of a sample is the number of nuclei that decay each second. It is proportional to the number of undecayed nuclei in the sample, N. For a given isotope, a sample twice as big would give twice the number of decays per second. The **decay constant**, λ, is the constant of proportionality. It is the probability of a specific nucleus decaying per unit time, and is a measure of how quickly an isotope will decay — the bigger the value of λ, the faster the rate of decay. The decay constant has units s^{-1} and the activity is measured in becquerels (1 Bq = 1 nucleus decaying per second).



Example — **Maths Skills**

A sample of a radioactive isotope contains 5.0×10^{-5} mol of undecayed nuclei. Its activity is measured to be 2.4×10^{12} Bq. Calculate the isotope's decay constant.

N = number of mol $\times N_A = 5.0 \times 10^{-5} \times 6.02 \times 10^{23} = 3.01 \times 10^{19}$

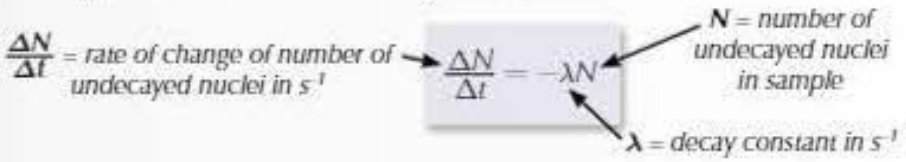
Rearrange $A = \lambda N$ to give $\lambda = \frac{A}{N} = \frac{2.4 \times 10^{12}}{3.01 \times 10^{19}} = 7.973... \times 10^{-8}\ s^{-1}$

$= 8.0 \times 10^{-8}\ s^{-1}$ (to 2 s.f.)

Because the activity, A, is the number of nuclei that decay each second, you can write it as the change in the number of undecayed nuclei, ΔN, during a given time (in seconds) Δt:

$$A = -\frac{\Delta N}{\Delta t}$$

There's a minus sign in this equation because ΔN is always a decrease. Combining these two equations for the activity then gives the rate of change of the number of undecayed nuclei:


ΔN/Δt = rate of change of number of undecayed nuclei in s⁻¹
ΔN/Δt = −λN
N = number of undecayed nuclei in sample
λ = decay constant in s⁻¹

Tip: Don't get the decay constant confused with wavelength — they both use the symbol λ.

Tip: Remember that 1 mol contains 6.02×10^{23} nuclei (from Avogadro's constant, N_A — see p.262).

Tip: The minus sign has just been taken over to the other side by multiplying both sides by –1 here. This is how you'll get this equation in your data and formulae booklet.

Example — **Maths Skills**

A radioactive isotope has a decay constant of $1.2 \times 10^{-4}\ s^{-1}$.
What is the rate of change of N for a sample containing 7.5×10^{20} nuclei?

$$\frac{\Delta N}{\Delta t} = -\lambda N = -(1.2 \times 10^{-4}) \times (7.5 \times 10^{20}) = -9.0 \times 10^{16}\ s^{-1}$$

Tip: So N is currently decreasing by 9.0×10^{16} nuclei per second.

Modelling radioactive decay

Radioactive decay is an iterative process (the number of nuclei that decay in one time period determines the number that are available to decay in the next). You can use the formula on the last page to plot a graph of N against t, by working out how N changes for each small change in t. The easiest way to do this is to use a spreadsheet to model how a sample of an isotope will decay if you know the decay constant, λ, and the number of undecayed nuclei in the initial sample, N_0:

- Set up a spreadsheet with column headings for total time (t), ΔN and N, and a data input cell for each of Δt, λ and N_0.
- Decide on a value of Δt that you want to use — this is the time interval between the values of N that the spreadsheet will calculate. The most sensible time interval will depend on your decay constant.
- You can then enter formulas into the spreadsheet to calculate the number of undecayed nuclei left in the sample after each time interval. You'll need to use $\Delta N = -\lambda \times N \times \Delta t$ (rearranged from the equation on the previous page).

Tip: You could also do this by hand, using the same equations shown in Figure 1. However, this can take a long time, so using a spreadsheet is usually much faster.

Data input cells		
	Δt	E.g. 100 s
	λ	E.g. $1 \times 10^{-4}\ s^{-1}$
	N_0	E.g. 7×10^{25}

t (s)	ΔN	N
$t_0 = 0$		N_0
$t_1 = t_0 + \Delta t$	$(\Delta N)_1 = -\lambda \times N_0 \times \Delta t$	$N_1 = N_0 + (\Delta N)_1$
$t_2 = t_1 + \Delta t$	$(\Delta N)_2 = -\lambda \times N_1 \times \Delta t$	$N_2 = N_1 + (\Delta N)_2$

Figure 1: *An example of the formulas that can be used to create an iterative spreadsheet of the number of undecayed nuclei over time in a radioactive sample.*

Tip: If you write the formulas properly, the spreadsheet can automatically fill them in for as many rows (iterations) as you want.

- Plot a graph of the number of undecayed nuclei against time — see Figure 2.

Tip: You may have to fiddle with your value for Δt to get a graph with a nice shape.

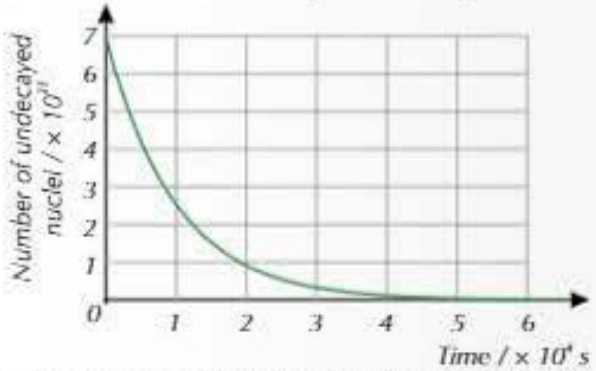


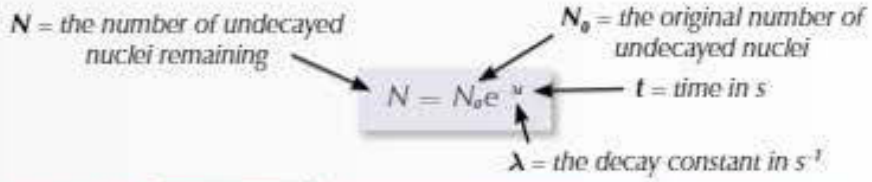
Figure 2: *A graph of N against t for an isotope with a decay constant of $1 \times 10^{-4}\ s^{-1}$ that originally contained 7×10^{25} undecayed nuclei. Plotted from an iterative spreadsheet using a time interval of 100 s.*

Tip: Make sure you iterate enough times, so that you have enough data points to form a graph with a clear curve.

The decay equation

The graph produced by the spreadsheet model (see Figure 2) shows that radioactive decay is a form of **exponential** decay — the number of undecayed nuclei decreases exponentially with time.

It is known that the number of undecayed nuclei remaining, N, depends on the number originally present, N_0, the decay constant, λ, and how much time has passed, t. This is described by the equation:


N = the number of undecayed nuclei remaining
N₀ = the original number of undecayed nuclei
N = N₀e^-λt
t = time in s
λ = the decay constant in s⁻¹

Example — **Maths Skills**

A sample of the radioactive isotope ^{13}N contains 5.00×10^6 nuclei. The decay constant for this isotope is 1.16×10^{-3} s^{-1}.

How many nuclei of ^{13}N will remain after exactly 800 seconds?

$$N = N_0 e^{-\lambda t} = (5.00 \times 10^6) \times e^{-(1.16 \times 10^{-3}) \times 800}$$
$$= 1.976... \times 10^6$$
$$= 1.98 \times 10^6 \text{ nuclei (to 3 s.f.)}$$

Plotting the natural log (ln) of the number of undecayed nuclei (or the activity, see next page) against time gives a straight-line graph (see Figure 3).

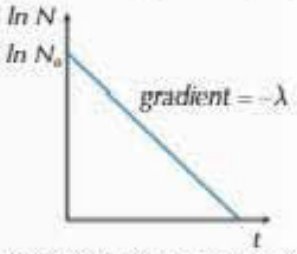


Figure 3: *A graph showing the natural log of the number of undecayed nuclei against time.*

Dice simulation of radioactive decay

Radioactive decay is a random process where there is a constant probability that an undecayed nucleus will decay. The probability is given by the decay constant. The result of rolling a fair 6-sided dice is also random with a constant probability — the probability of rolling any one number is 1/6. These similarities mean that you can simulate radioactive decay using dice, where each dice represents an undecayed nucleus.

You need at least 100 dice for a good simulation of the undecayed nuclei in a small radioactive sample. Roll all the dice and count how many of them landed on a 6 — these dice represent the nuclei that have decayed. Record the total number of dice rolled and the number of dice that have 'decayed' in a table. Remove the 'decayed dice' and roll the remaining dice again.

Repeat this process until all of the dice have 'decayed'. Each roll counts as 1 unit of time passing in the 'lifespan' of the radioactive sample. If you plot a graph of the number of dice rolled each time (i.e. the number of undecayed nuclei left in the sample, N) against time, then you'll see the same exponential relationship as shown in Figure 2 for radioactive decay.

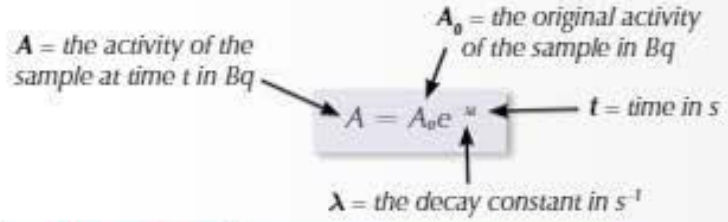
Tip: Remember to always round your final answer to the same no. of significant figures (s.f.) as the least precise data you use in your calculation. Here the 800 seconds is an exact value, and the other pieces of data are to 3 s.f. — so the answer to this calculation should be given to 3 s.f.

Tip: Taking natural log of both sides of $N = N_0 e^{-\lambda t}$ and using the log rules on p.472 gives $\ln N = \ln N_0 - \lambda t$. Comparing this to the straight-line graph equation $y = mx + c$, you can see that a plot of $\ln N$ against t will be a straight line with a y-intercept of $\ln N_0$ and a gradient of $-\lambda$ (see p.484).

Figure 4: *Wooden cubes being used to simulate radioactive decay. Each cube represents an undecayed nucleus that 'decays' when it lands with the black side facing upwards.*

Activity and the decay equation

The number of unstable nuclei decaying per second (the activity) is proportional to the number of nuclei remaining. As a sample decays, its activity goes down — there's an equation for that too:


A_0 = the original activity of the sample in Bq
A = the activity of the sample at time t in Bq
$A = A_0 e^{-\lambda t}$
t = time in s
λ = the decay constant in s^{-1}

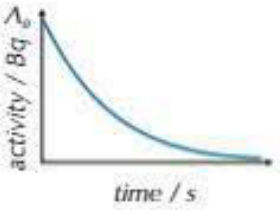


Figure 5: *Graph showing the exponential decay of the activity against time.*

Example — Maths Skills

The isotope radon-220 has a decay constant of $1.25 \times 10^{-2}\ s^{-1}$. How long will it take for the activity of a sample of radon-220 to fall from 85 Bq to 55 Bq?

First rearrange $A = A_0 e^{-\lambda t}$ to make t the subject.
Divide by A_0, take the natural log (ln) of both sides then divide by $-\lambda$.

$$\frac{A}{A_0} = e^{-\lambda t} \Rightarrow \ln\left(\frac{A}{A_0}\right) = -\lambda t \Rightarrow t = \frac{\ln\left(\frac{A}{A_0}\right)}{-\lambda}$$

Now just plug the numbers in and solve for t.

$$t = \frac{\ln\left(\frac{A}{A_0}\right)}{-\lambda} = \frac{\ln\left(\frac{55}{85}\right)}{-1.25 \times 10^{-2}} = 34.825... = 35 \text{ s (to 2 s.f.)}$$

Tip: Remember, in all the equations for radioactivity the time needs to be in seconds.

Practice Questions — Application

Q1 A sample contains 4.5×10^{18} atoms of a radioactive isotope. If it has a decay constant of $1.1 \times 10^{-13}\ s^{-1}$, what is the activity of the sample?

Q2 A sample of a radioactive isotope has an activity of 3.2 kBq. If the isotope has a decay constant of $1.3 \times 10^{-4}\ s^{-1}$, what will the sample's activity be 6.5 hours later?

Q3 An isotope of protactinium has a decay constant of $9.87 \times 10^{-3}\ s^{-1}$. If a sample initially contains 2.5×10^{15} unstable nuclei, how many will there be after 35 minutes?

Practice Questions — Fact Recall

Q1 a) What is the decay constant of a radioactive isotope?
b) What is meant by the activity of a radioactive sample?

Q2 a) Write down an equation for the rate of change in the number of unstable nuclei in a radioactive sample. Define all symbols used.
b) State one method you could use to model the decay of an isotope over time using the formula for rate of change given in part a).

Q3 Sketch a graph to show how the number of undecayed nuclei in a radioactive sample changes over time.

Q4 A student rolls a number of ordinary dice. She records how many dice show a 6, removes those showing 6, and rolls the remaining dice again. She repeats the process until she has removed all of the dice. Explain why this process is used to demonstrate radioactive decay.

8. Half-life and Radioactive Dating

The time it takes the number of unstable nuclei in a sample to halve is constant for a given substance. This quantity is called half-life, and you need to be able to calculate it...

Learning Objectives:

- Know what is meant by the half-life of an isotope.
- Understand techniques and procedures used to determine the half-life of an isotope such as protactinium (PAG7).
- Be able to calculate the half-life of an isotope using the equation $\lambda t_{\frac{1}{2}} = \ln(2)$.
- Know how isotopes can be used in radioactive dating, e.g. carbon-dating.

Specification Reference 6.4.3

Half-life of radioactive isotopes

The **half-life** ($t_{\frac{1}{2}}$) of an isotope is the average time it takes for the number of undecayed nuclei to halve.

Measuring the number of unstable nuclei isn't the easiest job in the world. In practice, half-life isn't measured by counting nuclei, but by measuring the time it takes the activity to halve. The longer the half-life of an isotope, the longer it takes for the radioactivity level to fall.

Calculating half-life from decay curves

You can calculate the half-life of an isotope from a decay curve — see Figure 1.

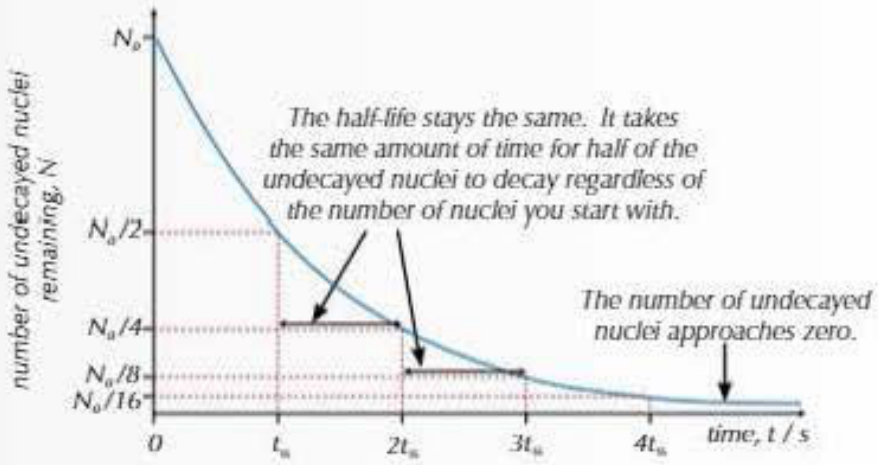

number of undecayed nuclei remaining, N
N_0
$N_0/2$
$N_0/4$
$N_0/8$
$N_0/16$
0
$t_{\frac{1}{2}}$
$2t_{\frac{1}{2}}$
$3t_{\frac{1}{2}}$
$4t_{\frac{1}{2}}$
time, t / s
The half-life stays the same. It takes the same amount of time for half of the undecayed nuclei to decay regardless of the number of nuclei you start with.
The number of undecayed nuclei approaches zero.

Figure 1: *A graph showing that the time taken for the number of undecayed nuclei remaining to halve is always equal to the half-life.*

- Read off the value for the number of unstable nuclei when $t = 0$.
- Go to half the original number of unstable nuclei on the y-axis.
- Draw a horizontal line to the curve, then a vertical line down to the x-axis. Read off the half-life where the line crosses the x-axis.
- It's always a good idea to check your answer — repeat these steps for a quarter of the original value and divide your answer by two. That will also give you the half-life. You can do the same for an eighth of the original value (divide the time by 3), and a sixteenth of the original value. Check that you get the same answer each time.

You can calculate half-life from an activity-time or count rate-time graph in exactly the same way, see Figure 2. N, A and count rate for an isotope will halve in the same amount of time, giving the half-life. (You met count rate on page 429 — it's the number of decays detected by a detector in 1 second.)

Tip: Make sure you check your units carefully when you're calculating half-life.

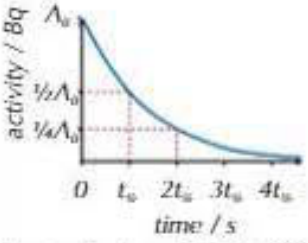
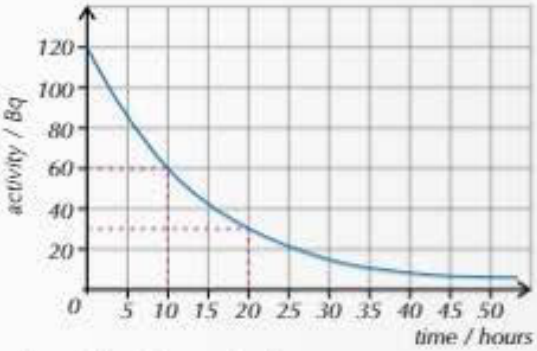


Figure 2: *A graph of activity against time. The activity halves after each half-life $t_{\frac{1}{2}}$.*

Tip: Be careful — the count rate is the number of decays detected per second, but activity is the total number of decays per second. So the count rate of a source will always be lower than the activity.

Example — **Maths Skills**

The graph on the right shows the activity-time graph of a sample of a radioisotope. Calculate the half-life of the isotope.



The starting activity of the sample is 120 Bq. So to find the half-life, read off the time at the point when the activity has dropped to half this value.

120 ÷ 2 = 60. When the activity is 60 Bq, time = 10 hours.

Check this by finding the time taken for the activity to fall to a quarter of its initial value and halving it. When the activity is 30 Bq, time = 20 hours. 20 ÷ 2 = 10 hours. So, the half-life is 10 hours.

Investigating half-life

PRACTICAL ACTIVITY GROUP 7

You can determine the half-life of an isotope for yourself by measuring how the count rate detected from a sample decreases with time. You're most likely to do this using the isotope protactinium-234. Protactinium-234 is formed when uranium decays (via another isotope). You can measure protactinium-234's decay rate using a protactinium generator — a bottle containing a uranium salt and two solvents, which separate out into layers, as shown in Figure 3. The uranium salt is only soluble in the bottom layer. The uranium salt decays into protactinium-234 (as well as some other isotopes, but you don't need to worry about those).

When the generator is shaken, the solvents are mixed. Only the protactinium-234 can form a solution with the top layer solvent, so when the layers in the bottle are allowed to separate, the top layer will only contain the protactinium-234 in solution, while the uranium salt and other decay products are still left in the bottom layer.

Tip: Make sure you do a risk assessment before carrying out this experiment.

Figure 4: *Uranyl nitrate, a uranium salt. It can be dissolved in water and used in protactinium generators.*

Tip: The uranium in the salt is always decaying into protactinium-234, so you can reuse the protactinium generator by shaking the bottle again to dissolve more protactinium-234 in the top layer.

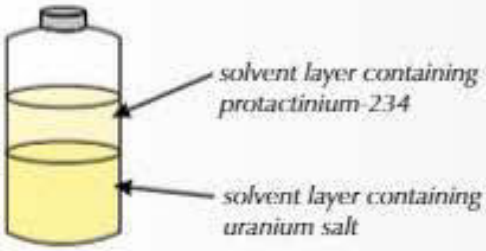


Figure 3: *A protactinium generator.*

So if a Geiger-Müller tube is directed towards the top layer only, the activity of protactinium-234 can be monitored without confusing its counts with those from other decaying isotopes.

1. Start by setting up the equipment shown in Figure 5. Don't shake the protactinium generator yet. Find the background counts by measuring the number of counts detected over a time period (at least 30 s). Repeat this two more times, find the mean of your three results, and divide the mean counts by the number of seconds. This is the background count rate you will need to subtract from your future results.

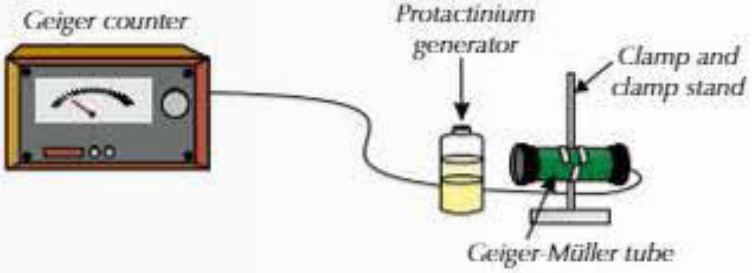


Figure 5: *The experimental set-up for an investigation into the half-life of protactinium-234.*

2. Shake the protactinium generator to mix the solvents together, then place it back in its position shown in Figure 5. Wait for the liquids to separate.
3. As soon as the liquids separate, record the count rate of the protactinium-234 in the top layer (e.g. how many counts you get in 10 seconds, divided by 10). Measure the count rate at sensible intervals (e.g. every 30 seconds) for at least 5 minutes.
4. Subtract your background count rate from each of your results, then plot a graph of count rate against time. It should look like the graph in Figure 6. You can use this graph to find the half-life in exactly the same way as in Figure 1. In this case the half-life is the time taken for the count rate (or activity) to halve.

Tip: Make sure the Geiger-Müller tube is directed at the top layer of the protactinium generator, and that it is close enough that it records a large number of counts.

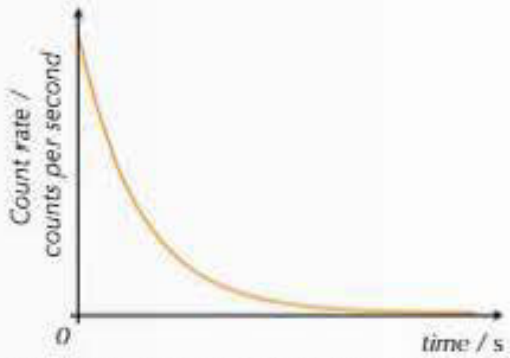


Figure 6: *A graph of count rate against time for a decaying sample of protactinium-234.*

Tip: If you have enough data, calculate the half-life from the time taken for the count rate to drop to a quarter, and an eighth of the original count rate, as shown in Figure 1. Take an average of these values to get your final half-life result.

The half-life equation

You can derive an equation for half-life from the formula $N = N_0 e^{-\lambda t}$ on p.435.

- When $t = t_{½}$, the number of undecayed nuclei has halved, so $N = \frac{1}{2}N_0$.
- Substituting these values into the equation for N:

$$\frac{1}{2}N_0 = N_0 e^{-\lambda t_{½}}$$

- Cancelling N_0 and taking the natural log of both sides:

$$\ln\left(\frac{1}{2}\right) = -\lambda t_{½}$$

- $\frac{1}{2} = 2^{-1}$ so $\ln\left(\frac{1}{2}\right) = \ln(2^{-1}) = -\ln(2)$ using the rules on page 472. So:

Tip: You'll need to use the power law $\frac{1}{a^n} = a^{-n}$ here.

$$\lambda t_{½} = \ln(2)$$

λ = *the decay constant, s^{-1}*

$t_{½}$ = *the half-life, s*

Tip: Here it's fine to give the half-life in seconds. Some isotopes have much longer half-lives though, which are more sensible to give in years.

Example — **Maths Skills**

A radioactive isotope has a decay constant of 1.16×10^{-3} s^{-1}. What is the half-life for this isotope?

Divide both sides of $\lambda t_{\frac{1}{2}} = \ln(2)$ by λ, and substitute in the values.

$$t_{\frac{1}{2}} = \frac{\ln(2)}{\lambda} = \frac{\ln(2)}{1.16 \times 10^{-3}} = 597.54... = 598 \text{ s (to 3 s.f.)}$$

Figure 7: *The carbon-14 in living tissue decays after an organism dies. By measuring how much carbon-14 remains in dead tissue, the age of a sample can be estimated.*

Radioactive dating

HOW SCIENCE WORKS

The radioactive isotope carbon-14 is used in **radioactive dating**. Living plants take in carbon dioxide from the atmosphere as part of photosynthesis, including some molecules containing the radioactive isotope carbon-14. Animals then take this carbon-14 in when they eat the plants. All living things contain the same percentage of carbon-14.

When they die, and so stop taking in any more carbon-14, the activity of carbon-14 in the plant or animal starts to fall, with a half-life of around 5730 years. Archaeological finds of once-living material (like wood or animal bone) can be tested to find the current amount of carbon-14 in them. This can be used to calculate how long the organism has been dead, and so how old the material is.

Example — **Maths Skills**

A sample of 6.5×10^{23} carbon atoms is taken from a spear, and found to have an activity of 0.052 Bq. The ratio of radioactive carbon-14 to stable carbon-12 in living wood is $1 : 1.4 \times 10^{12}$, and the half-life of carbon-14 is 5730 years. How old is the wood the spear is made from?

The half-life of carbon-14 in seconds is

$$t_{1/2} = 5730 \times (365 \times 24 \times 3600) = 1.8070... \times 10^{11} \text{ s}$$

So the decay constant, $\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{1.8070... \times 10^{11}} = 3.8358... \times 10^{-12} \text{ s}^{-1}$

Rearrange and use the equation for activity to find the number of carbon-14 nuclei in the wood:

$$N = \frac{A}{\lambda} = 0.052 \div (3.8358... \times 10^{-12}) = 1.3556... \times 10^{10}$$

Use the ratio given in the question to calculate the expected number of carbon-14 nuclei in a sample of 6.5×10^{23} carbon atoms from living wood:

$$N_0 = (1 \div (1.4 \times 10^{12})) \times 6.5 \times 10^{23} = 4.6428... \times 10^{11}$$

N and N_0 are related by $N = N_0 e^{-\lambda t}$. You can rearrange this by dividing by N_0 and taking the natural log (ln) of both sides to make t the subject and find the age of the wood:

$$N = N_0 e^{-\lambda t} \Rightarrow \frac{N}{N_0} = e^{-\lambda t} \Rightarrow \ln\left(\frac{N}{N_0}\right) = \ln(e^{-\lambda t}) \Rightarrow \ln\left(\frac{N}{N_0}\right) = -\lambda t$$

$$\text{So } t = -\frac{1}{\lambda} \times \ln\left(\frac{N}{N_0}\right) = -\frac{1}{3.8358... \times 10^{-12}} \times \ln\left(\frac{1.3556... \times 10^{10}}{4.6428... \times 10^{11}}\right)$$

$$= 9.2121... \times 10^{11} \text{ s}$$

$$= 29\,000 \text{ years (to 2 s.f.)}$$

Tip: You'll need to use the equations for N back on pages 433 and 435 to answer this question.

Tip: $\ln(e^a) = a$, which is why $\ln(e^{-\lambda t})$ becomes $-\lambda t$ when you're rearranging to find t. For more help on logs, skip on over to page 472.

Practice Questions — Application

Q1 The activity of a radioactive sample fell from 2400 Bq to 75 Bq over a period of 24 hours. How long is the isotope's half-life?

Q2 An isotope has a half-life of 483 seconds. What is its decay constant?

Q3 A sample of carbon atoms from an ancient bone has an activity of 0.45 Bq. The same number of carbon atoms from a piece of living tissue has an activity of 1.2 Bq. Given that the half-life of carbon-14 is 5730 years, how old is the bone?

Tip: You'll need to use the activity equation from page 436 to answer Q3.

Practice Questions — Fact Recall

Q1 What is the half-life of an isotope?

Q2 Describe how you would find an isotope's half-life from a graph showing its activity against time.

Q3 Nobelium-255 is a radioactive alpha-emitter with a half-life of a few minutes. Describe an experiment you could perform to find the half-life of a sample of Nobelium-255.

Q4 Give an example of an isotope used in radioactive dating.

9. Binding Energy

The binding energy is a measure of how strongly a nucleus is held together — the greater the binding energy per nucleon, the more stable the nucleus.

Learning Objectives:

- Know what is meant by the mass defect.
- Know what is meant by the binding energy of a nucleus.
- Be able to calculate the binding energy of nuclei using the equation $\Delta E = \Delta mc^2$ and the masses of nuclei.
- Be able to calculate the binding energy per nucleon for a nucleus.
- Know the shape of the curve for the graph of binding energy per nucleon against nucleon number.

Specification Reference 6.4.4

Mass defect and binding energy

The mass of a nucleus is less than the mass of its constituent nucleons — the difference is called the **mass defect**.

Example — **Maths Skills**

The mass of a nucleus of potassium, $^{40}_{19}K$, is 39.9536 u. The mass of a proton is 1.00728 u and the mass of a neutron is 1.00867 u. Calculate the mass defect of the nucleus in u.

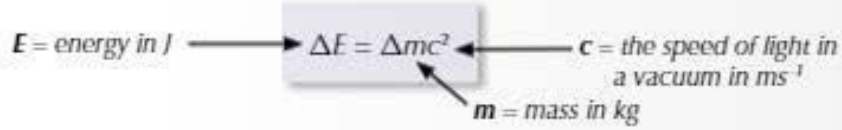
Number of protons = 19, number of neutrons = (40 − 19) = 21

Mass of nucleons = (19 × 1.00728 u) + (21 × 1.00867 u) = 40.32039 u

So mass defect = mass of nucleons − mass of nucleus

= 40.32039 u − 39.9536 u = 0.36679 u

Einstein's equation says that mass and energy are equivalent:



As nucleons join together, the total mass decreases — this 'lost' mass is converted into energy and released. You can calculate this energy using the equation above. The amount of energy released is equivalent to the mass defect.

If you pulled the nucleus completely apart, the energy you'd have to use to do it would be the same as the energy released when the nucleus formed. The energy needed to separate all of the nucleons in a nucleus is called the **binding energy** (measured in MeV), and it is equivalent to the mass defect.

Example — **Maths Skills**

Calculate the binding energy in MeV of the nucleus of a lithium-6 atom, $^{6}_{3}Li$, given that its mass defect is 0.0343 u.

Convert the mass defect into kg:

Mass defect = $0.0343 \times (1.661 \times 10^{-27}) = 5.697... \times 10^{-29}$ kg

Use $\Delta E = \Delta mc^2$ to calculate the binding energy:

$\Delta E = (5.697... \times 10^{-29}) \times (3.00 \times 10^{8})^2 = 5.127... \times 10^{-12}$ J

Convert to MeV:

$\Delta E = ((5.127... \times 10^{-12}) \div (1.60 \times 10^{-19})) \div 10^{6} = 32.046... = 32.0$ MeV (to 3 s.f.)

The binding energy per unit of mass defect can also be calculated. Using the fact that 1 u = 1.661×10^{-27} kg:

$$\Delta E = \Delta mc^2 = 1.661 \times 10^{-27} \times (3 \times 10^{8})^2$$
$$= 1.4949 \times 10^{-10} \text{ J}$$
$$\approx 930 \text{ MeV (to 2 s.f.)}$$

Tip: You've already met this equation on page 420.

Tip: Remember — 1 u = 1.661×10^{-27} kg. You'll be given this in the data and formulae booklet in the exams.

Tip: To convert from J to eV, you divide by the magnitude of the charge on an electron, $e = 1.60 \times 10^{-19}$ C. Then to convert to MeV you just divide by 10^6.

This means that a mass defect of 1 u is equivalent to about 930 MeV of binding energy.

$$1 \text{ u} \approx 930 \text{ MeV}$$

Example — **Maths Skills**

Using the binding energy per unit mass defect, the binding energy in MeV of lithium-6 from the last example on the previous page is:

$\Delta E \approx 0.0343 \times 930 = 31.899 \text{ MeV} = 32 \text{ MeV}$ (to 2 s.f.)

Tip: This is good for checking your answers, or making estimates, but it's not very accurate. If you're asked to find the binding energy, you should always use $\Delta E = \Delta mc^2$.

Binding energy per nucleon

A useful way of comparing the binding energies of different nuclei is to look at the binding energy per nucleon.

$$\text{Binding energy per nucleon} = \frac{\text{Binding energy } (B)}{\text{Nucleon number } (A)}$$

Example — **Maths Skills**

What is the binding energy per nucleon for a $^{6}_{3}$Li nucleus?

You know from the second example on the previous page that binding energy = 32.046... MeV. Nucleon number = $A = 6$

Binding energy per nucleon = $\frac{B}{A}$ = 32.046... ÷ 6 = 5.341...

= 5.34 MeV (to 3 s.f.)

If you plot a graph of binding energy per nucleon against nucleon number, for all elements, the line of best fit is a curve — see Figure 2. A high binding energy per nucleon means that more energy is needed to remove nucleons from the nucleus. In other words the most stable nuclei occur around the maximum point on the graph — which is at nucleon number 56 (i.e. iron, Fe).

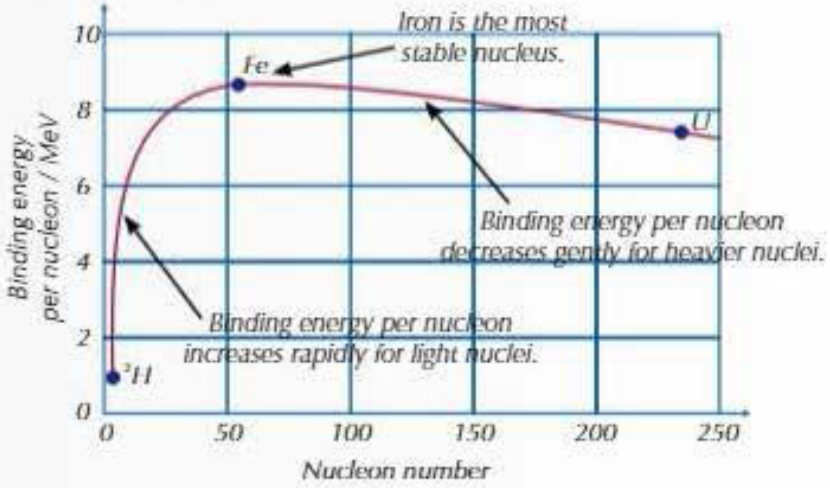


Figure 2: *Graph showing how the binding energy per nucleon varies with nucleon number. The red line shows the line of best fit.*

Figure 1: *Some blocks of pure iron. Iron has the most stable nucleus of all the elements.*

Exam Tip

Make sure you know the shape of this graph, as well as the axes labels — you could be asked to sketch it in an exam.

Tip: You can use this graph to find the energy released in nuclear reactions — see page 447.

Tip: For a reminder on standard notation, see page 413.

Tip: The mass of a proton is 1.00728 u, and the mass of a neutron is 1.00867 u.

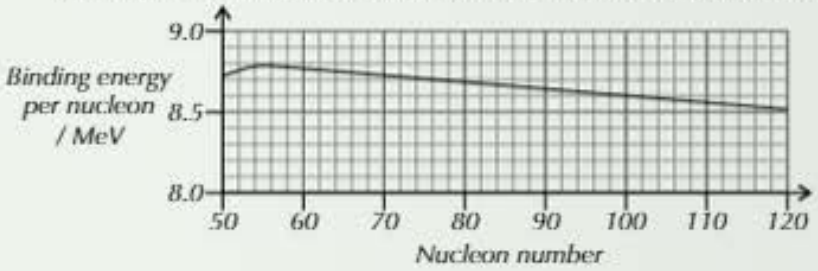
Practice Questions — Application

Q1 The mass defect of a carbon-12 nucleus is 0.0989 u. Calculate the binding energy of the nucleus in MeV.

Q2 Calculate the mass defect of a nucleus of $^{16}_{8}O$ (mass = 15.994915 u) in atomic units, u.

Q3 The binding energy per nucleon of a nucleus of iron-56 is 8.79 MeV. What is its mass defect in u?

Q4 A student finds a graph of binding energy per nucleon against nucleon number in her textbook. A section of the graph is shown below.



a) Using the graph, find the binding energy per nucleon of a molybdenum-100, $^{100}_{42}Mo$, nucleus.

b) Calculate the total binding energy of a molybdenum-100 nucleus.

Practice Questions — Fact Recall

Q1 What is meant by the binding energy of a nucleus?

Q2 Sketch the graph of the binding energy per nucleon against nucleon number. Label the positions of ^{2}H and ^{56}Fe on the curve.

10. Nuclear Fission and Fusion

Radioactive decay isn't the only way that nuclei can change — they can also split into two smaller nuclei, or fuse with other nuclei to form larger ones.

Learning Objectives:

- Understand what is meant by induced nuclear fission.
- Understand what is meant by nuclear fusion.
- Know why fusion reactions require very high temperatures.
- Be able to balance nuclear transformation equations.
- Be able to use the graph of binding energy per nucleon against nucleon number to calculate energy changes in reactions.
- Be able to calculate the energy released (or absorbed) in simple nuclear reactions.

Specification Reference 6.4.4

Nuclear fission

Heavy nuclei (e.g. uranium) are unstable, and some can randomly split into two smaller nuclei — this is called **nuclear fission**. This process is called spontaneous if it just happens by itself, or induced if we encourage it to happen. When nuclear fission occurs, in addition to the two smaller nuclei, a large amount of energy is released, along with a number of free neutrons.

Example

Fission can be induced by making a neutron enter a ^{235}U nucleus, causing it to become very unstable. Only low-energy neutrons can be captured in this way. A low-energy neutron is called a **thermal neutron**.

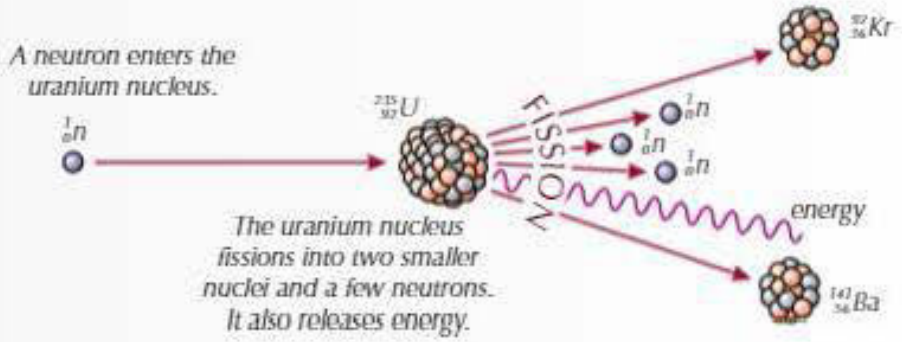


Figure 1: *A possible fission of a uranium-235 nucleus.*

Energy is released during nuclear fission because the new, smaller nuclei have a higher average binding energy per nucleon (see page 443). The larger the nucleus, the more unstable it will be — so large nuclei are more likely to spontaneously fission. This means that spontaneous fission limits the number of nucleons that a nucleus can contain — in other words, it limits the number of possible elements.

Tip: Heavy nuclei don't always fission to form the same daughter nuclei — ^{235}U can split into lots of different pairs of nuclei (with nucleon numbers around 90 and 140).

Nuclear fusion

Two light nuclei can combine to create a larger nucleus. This is called **nuclear fusion**.

Example

In the Sun, hydrogen nuclei fuse in a series of reactions to form helium. One of the reactions is: $^{2}_{1}H + {}^{3}_{1}H \longrightarrow {}^{4}_{2}He + \text{energy}$.

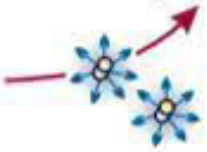
Figure 2: *Two isotopes of hydrogen fuse to form helium.*

Tip: Just like decay equations, equations showing nuclear fission and fusion need to be balanced — see page 431.

Nuclei can only fuse if they are moving very fast, and so have enough energy to overcome the electrostatic (Coulomb) repulsion between them (p.369) and get close enough for the strong interaction to bind them. This means fusion reactions require much higher temperatures than fission, as well as high pressures (or high densities). Under such conditions, generally only found inside stars, matter turns into a state called a plasma.

Tip: The strong interaction can only attract nucleons together when they're less than a few femtometres apart (see p.417) — so the fusing nuclei must have enough energy to get very close to each other.

Low-energy nuclei are deflected by electrostatic repulsion.



High-energy nuclei overcome electrostatic repulsion and are attracted by the strong interaction.

Figure 3: *Nuclei must overcome their mutual electrostatic repulsion to fuse together.*

A lot of energy is released during nuclear fusion because the new, heavier nucleus has a much higher binding energy per nucleon (and so a lower total mass, see p.443). The energy released helps to maintain the high temperatures needed for further fusion reactions.

Nuclear equations

Just like other nuclear transformations (see p.431-432), we can express nuclear fission and fusion reactions as nuclear equations. Just like all equations though, we have to make sure these balance, since charge must be conserved. The total nucleon number and proton number on each side must be equal.

Tip: $2^{1}_{0}n$ just means two free neutrons.

Examples

The nuclear equation for the spontaneous fission of californium-238 into cadmium-115 and tin-121 is:

$$^{238}_{98}Cf \longrightarrow {}^{115}_{48}Cd + {}^{121}_{50}Sn + 2{}^{1}_{0}n$$

The nuclear equation for the fusion of lithium-6 and deuterium into beryllium-7 is:

$$^{6}_{3}Li + {}^{2}_{1}H \longrightarrow {}^{7}_{4}Be + {}^{1}_{0}n$$

Tip: You can often get another small particle produced in nuclear fusion, alongside the main fusion product. This is usually a proton or neutron. If you're fusing larger nuclei (e.g. carbon, oxygen, etc) you may even get a small nucleus, like a hydrogen or helium isotope.

You may be expected to solve problems involving balancing nuclear fission and fusion equations in your exam.

Example **Maths Skills**

The nuclear equation below shows the induced fission of a uranium-235 nucleus.

$$^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{96}_{37}Rb + {}^{A}_{Z}Cs + 3{}^{1}_{0}n$$

Find the nucleon number, A, and proton number, Z, of the Cs nucleus.

To find the nucleon number, subtract the total nucleon number on the right from the total on the left.

$A = (235 + 1) - (96 + (3 \times 1)) = 236 - 99 = 137$

Do the same for proton number.

$Z = (92 + 0) - (37 + 0) = 92 - 37 = 55$

So the Cs nucleus has a nucleon number of 137 and a proton number of 55.

Tip: Cs is the chemical symbol for caesium.

Energy released by fission and fusion

You can tell whether it is energetically favourable for an element to undergo fission or fusion by looking at the graph of binding energy per nucleon against nucleon number. Only elements to the right of ^{56}Fe can release energy through nuclear fission. Similarly only elements to the left of ^{56}Fe can release energy through nuclear fusion. This is because energy is only released when the binding energy per nucleon increases.

Tip: Energetically favourable means that the fusion or fission will give out energy. Reactions that aren't energetically favourable take in energy rather than release it.

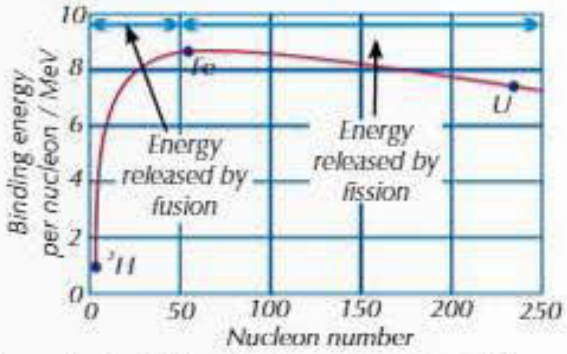


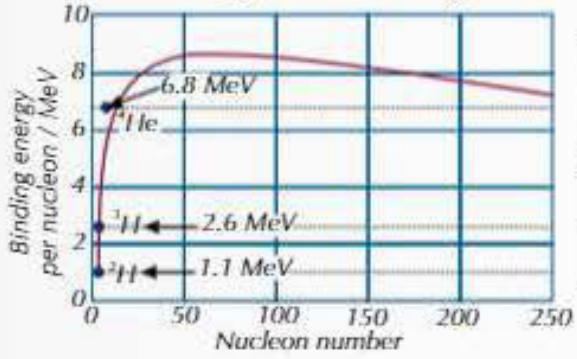
Figure 4: *Graph showing the regions where fusion and fission reactions are energetically favourable.*

Tip: For a reminder on this graph, take a look back at page 443.

The change in binding energy gives the energy released. The binding energy per nucleon graph can be used to estimate the energy released in nuclear reactions.

Example — **Maths Skills**

Calculate the energy released through the fusion of ^2H and ^3H into ^4He.



Binding energy of $^4He = 4 \times 6.8 = 27.2$ MeV

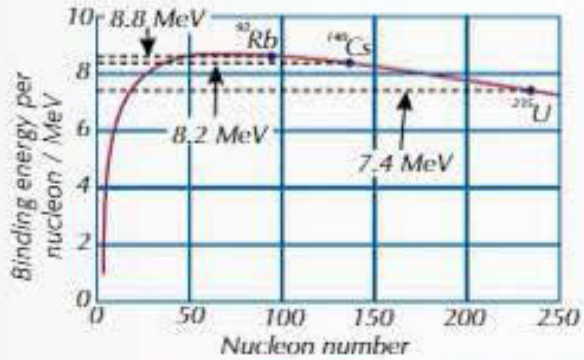
Binding energy of ^2H and $^3H = (2 \times 1.1) + (3 \times 2.6)$
$= 10.0$ MeV

Therefore total energy released $= 27.2 - 10.0$
$= 17.2$
$= 17$ MeV (to 2 s.f.)

Tip: There would be some free neutrons produced in these interactions as well, but you don't need to include them in binding energy calculations. The binding energy of a free neutron is 0 MeV.

Example — **Maths Skills**

Calculate the energy released in the induced nuclear fission of ^{235}U into ^{92}Rb and ^{140}Cs.



Binding energy of ^{235}U
$= 235 \times 7.4 = 1739$ MeV

Binding energy of ^{92}Rb and ^{140}Cs
$= (92 \times 8.8) + (140 \times 8.2)$
$= 1957.6$ MeV

So the total energy released $= 1957.6 - 1739$
$= 218.6$
$= 220$ MeV (to 2 s.f.)

The energy released per reaction is generally lower in nuclear fusion than fission. But the nuclei used in fusion have a lower mass, so one mole of the reactants in a fusion reaction weighs less than one mole of the reactants in a fission reaction. Gram for gram, fusion can release more energy than fission.

In both fission and fusion reactions, the total mass decreases, but mass-energy is conserved. So you can calculate the energy released during a fission or fusion reaction using the equation $\Delta E = \Delta mc^2$, where Δm is the total difference in mass between the reactants and products.

Nucleus / Particle	Mass (u)
^{235}U	234.99333
^{89}Kr	88.89783
^{144}Ba	143.89215
^{139}Te	138.90613
^{94}Zr	93.88431
^{2}H	2.01355
proton	1.00728
neutron	1.00867
positron	0.00055
neutrino	0

Figure 5: *The masses of some nuclei and particles.*

Example — **Maths Skills**

Calculate the energy released (in MeV) by the following fission reaction using the data in Figure 5: $^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{89}_{36}Kr + {}^{144}_{56}Ba + 3{}^{1}_{0}n$

$\Delta m = m_{Kr\text{-}89} + m_{Ba\text{-}144} + 3m_n - m_{U\text{-}235} - m_n$

$= 88.89783 + 143.89215 + (3 \times 1.00867) - 234.99333 - 1.00867$

$= -0.18601$ u

$= (-0.18601 \times 1.661 \times 10^{-27})$ kg $= -3.0896... \times 10^{-28}$ kg

$\Delta E = \Delta mc^2 = -3.0896... \times 10^{-28} \times (3.00 \times 10^8)^2$

$= -2.7806... \times 10^{-11}$ J

$= ((-2.7806... \times 10^{-11}) \div (1.60 \times 10^{-19})) \div 10^6$ MeV

$= -173.79...$ MeV

$= -174$ MeV (to 3 s.f.)

So the fission reaction releases 174 MeV of energy.

In extreme conditions, nuclei can undergo transformations which are not energetically favourable. For example, the large amount of energy released in supernovae allows iron in an exploding star to fuse into bigger nuclei, absorbing some of that energy. Scientists think this is how almost all naturally occurring elements heavier than iron came to exist.

You can calculate the energy absorbed by an energetically unfavourable reaction using the same method as in the example above. In this case the binding energy per nucleon decreases during the transformation, so the total mass increases, and you get a positive values for Δm and ΔE. This indicates that energy was absorbed instead of emitted.

Tip: There's more on supernovae on page 322.

Tip: The peak of the binding energy per nucleon against nucleon number graph is at a nucleon number of 56.

Tip: Remember, if you're using $\Delta E = \Delta mc^2$, you need to account for all the particles that you started with, and all the particles produced.

Practice Questions — Application

Q1 For each of the nuclei below, state whether it is more energetically favourable for them to undergo nuclear fission or nuclear fusion:
a) $^{28}_{14}Si$ b) $^{227}_{89}Ac$ c) $^{56}_{24}Cr$ d) $^{128}_{52}Te$

Q2 A deuterium nucleus, $^{2}_{1}H$, fuses with a tritium nucleus, $^{3}_{1}H$, to form a helium-4 nucleus, $^{4}_{2}He$, and another particle, X. Write a balanced nuclear equation for this reaction and identify particle X.

Q3 Using the data in Figure 5, calculate the energy (in MeV) released during the fusion of two protons: $^{1}_{1}p + {}^{1}_{1}p \longrightarrow {}^{2}_{1}H + {}^{0}_{1}e + {}^{0}_{0}\nu$.

Q4 Using the data in Figure 5, calculate the energy (in MeV) released when a ^{235}U nucleus is hit by a neutron and fissions into ^{94}Zr and ^{139}Te and a number of neutrons.

Practice Questions — Fact Recall

Q1 What is nuclear fission? How can it be induced?

Q2 What is nuclear fusion?

Q3 Explain why light nuclei require high temperatures to fuse together.

11. Fission Reactors

You need to know about the different bits and pieces that make up a fission reactor, as well as the potential environmental problems that come from them.

Learning Objectives:

- Understand what is meant by a chain reaction.
- Know the basic structure of a fission reactor.
- Know and be able to explain the roles of components of a fission reactor, including fuel rods, control rods and the moderator.
- Understand the environmental impact of nuclear waste.

Specification Reference 6.4.4

Structure of fission reactors

We can harness the energy released during nuclear fission reactions in a thermal fission reactor (see Figure 1), but it's important that these reactions are very carefully controlled.

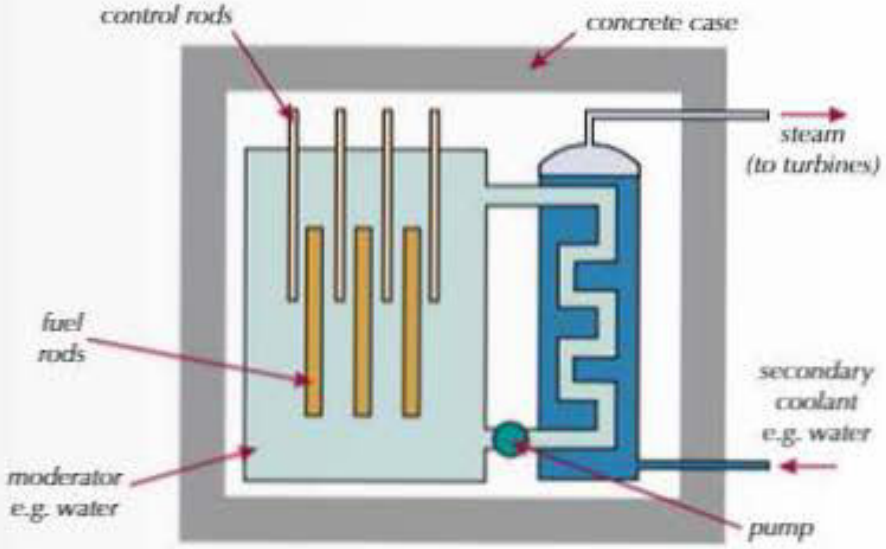


Figure 1: *The key features of a thermal fission reactor.*

Chain reactions

Fission reactors use rods of uranium that are rich in ^{235}U (or sometimes plutonium rods rich in ^{239}Pu) as 'fuel' for fission reactions. (The rods also contain other isotopes, but they don't undergo fission.)

These fission reactions produce neutrons which then induce other nuclei to fission, which produce more neutrons which go on to induce more fission. This is called a **chain reaction**. The neutrons will only cause a chain reaction if they are slowed down, which allows them to be absorbed by the uranium nuclei — these slowed down neutrons are called thermal neutrons (see page 445).

If the chain reaction in a fission reactor is left to continue unchecked, large amounts of energy are released in a very short time. Many new fission reactions will follow each fission, causing a runaway reaction which could lead to an explosion. This is what happens in a nuclear fission bomb.

Moderator

Fuel rods need to be placed in a **moderator** (for example, water) to slow down and/or absorb neutrons. You need to choose a moderator that will slow down some neutrons enough so they can cause further fission, keeping the reaction going at a steady rate.

Tip: In Figure 1, the moderator also acts as a coolant (see page 450).

Tip: Make sure you're familiar with the basic structure and features of a fission reactor.

Tip: The fuel rods are placed into the reactor remotely, which keeps workers as far away from the radiation as possible.

Figure 2: *Fuel rods being lowered into the reactor at a nuclear power station. The core is surrounded by water, which acts as a moderator.*

Control rods

You want the chain reaction to continue on its own at a steady rate, where one fission follows another. The amount of 'fuel' you need to do this is called the **critical mass** — any less than the critical mass (sub-critical mass) and the reaction will just peter out. Fission reactors use a supercritical mass of fuel (i.e. more than the critical mass, so that several new fissions normally follow each fission) and control the rate of fission using **control rods**.

Figure 3: *Cooling towers at a nuclear power plant. The steam used to drive the turbines is passed through the cooling tower to cool it down.*

Control rods control the chain reaction by limiting the number of neutrons in the reactor. They absorb neutrons so that the rate of fission is controlled. Control rods are made up of a material that absorbs neutrons (e.g. boron), and they can be inserted by varying amounts to control the reaction rate. In an emergency, the reactor will be shut down automatically by the release of the control rods into the reactor, which will stop the reaction as quickly as possible.

Coolant

A coolant is a substance which transfers heat in a reactor. The material used should be a liquid or gas at room temperature, so that it can be pumped around the reactor, and be efficient at transferring heat. The moderator can serve as a primary coolant, which transfers the heat produced by fission to another coolant (the secondary coolant), such as water, which is converted to steam as it's heated. This steam then passes through and powers electricity-generating turbines.

Tip: When we say 'nuclear power station', we mean a power station which generates electricity using a fission reactor.

Environmental impact of fission reactors

Deciding whether or not to build a nuclear power station (and if so, where to build it) is a tricky business.

Nuclear fission doesn't produce carbon dioxide, unlike burning fossil fuels, so it doesn't contribute to global warming. It also provides a continuous energy supply, unlike many renewable sources (e.g. wind/solar).

Tip: Fission doesn't produce any greenhouse gases, but parts of the process do, e.g. transporting uranium fuel rods to a power station.

However, some of the waste products of nuclear fission are highly radioactive and difficult to handle and store. When material is removed from the reactor, it is initially very hot, so it is placed in cooling ponds until the temperature falls to a safe level. The radioactive waste is then stored in sealed containers in specialist facilities. These facilities often have to be built deep underground. Digging into the ground to build these facilities can have a significant effect on the local landscape and environment, and damage local wildlife habitats. The waste is kept there until its activity has fallen sufficiently for it to be considered safe. This can take many years, and there's a risk that material could escape from these containers.

Tip: Alpha radiation is particularly dangerous inside the body, as its short penetration range (see p.428) means it will almost certainly be absorbed by cells, which can damage them.

A leak of radioactive material could be harmful to the environment and local human populations both now and in the future, particularly if the material contaminated water supplies. This would allow the isotopes to contaminate plants and animals, and so enter the food chain, exposing organisms (including humans) to radiation. Radiation sources are very dangerous to living cells — if radiation reaches a cell it can ionise atoms inside it. This can kill the cell, immediately harming the organism. Ionisation can also cause a mutation in the cell's DNA, which can result in cancer in animals, or produce a harmful genetic mutation which could be passed on to offspring.

Accidents or natural disasters pose a risk to fission reactors. In 2011 an earthquake and subsequent tsunami in Japan caused a meltdown at the Fukushima nuclear power plant. Over 100 000 people were evacuated from the area, and many tonnes of contaminated water leaked into the sea.

The perceived risk of this kind of disaster leads many people to oppose the construction of nuclear power plants near their homes. Because of all of the necessary safety precautions, building and decommissioning nuclear power plants is very time-consuming and expensive.

Tip: Scientists are trying to develop fusion reactors so that we can generate nuclear electricity without the waste you get from fission reactors, but they haven't yet succeeded in creating one that makes more electricity than it uses.

Practice Questions — Fact Recall

Q1 Describe a fission chain reaction.

Q2 State why neutrons need to be slowed down by a moderator in a fission reactor.

Q3 How are control rods used to control the rate of reaction in a fission reactor? Give an example of a material used to make control rods.

Q4 What properties does the material used for the coolant in a fission reactor need to have? Give an example of a material that could be used as a coolant.

Q5 Give one way that nuclear waste can impact the environment.

Section Summary

Make sure you know...

- How the alpha-particle scattering experiment disproved the Thomson model of the atom, and provided evidence for the atom having a small, charged nucleus.
- That the nuclear model of the atom states that the atom consists of a small, positively-charged nucleus, made up of protons and neutrons, orbited by negatively-charged electrons.
- What the proton number, Z, and nucleon number, A, of a nucleus are.
- How to represent an atom using standard notation, ${}^{A}_{Z}X$, where X is the element's chemical symbol.
- That isotopes are forms of an element with different numbers of neutrons.
- That atoms have a diameter of around 1×10^{-10} m and a nucleus has a typical diameter of a few fm.
- How to use the equation $R = r_0 A^{1/3}$ to calculate the radius of a nucleus, R, where r_0 is constant.
- How to calculate the mean densities of atoms and nuclei by approximating them as spheres.
- That the strong nuclear force is responsible for holding nuclei together, and its nature at different ranges.
- That hadrons, such as protons and neutrons, are particles that feel the strong nuclear force.
- That leptons, such as electrons and neutrinos, are fundamental particles that do not feel the strong nuclear force but are subject to the weak nuclear force.
- That every particle has an equivalent antiparticle which has the same mass but opposite charge.
- That the antiparticles of the electron, proton, neutron and neutrino are the positron, antiproton, antineutron and antineutrino respectively.
- That Einstein's mass-energy equation, $\Delta E = \Delta mc^2$, states that mass and energy are equivalent.
- That pair production is when energy changes into matter in the form of a particle-antiparticle pair.
- That when a particle comes into contact with its corresponding antiparticle, they annihilate and form a pair of photons.
- How to use Einstein's mass-energy equation to make calculations for pair production and annihilation.
- That hadrons are made up of fundamental particles called quarks.
- The basic properties of the up quark, u, the down quark, d, and the strange quark, s, as well as their respective anti-quarks.
- That a proton is made of two up quarks and a down quark, uud, and a neutron is made of one up quark and two down quarks, udd.
- How to describe the decay of particles in terms of the quark model.

cont...

- That in beta-minus decay, a d quark turns into a u quark and emits an electron and an antineutrino.
- That in beta-plus decay, a u quark turns into a d quark and emits a positron and a neutrino.
- That in any particle interaction charge must be conserved, and how to use charge conservation to balance particle interaction equations.
- That unstable nuclei will undergo radioactive decay to become more stable by emitting a particle or energy, and that this process is spontaneous and random.
- The four types of nuclear radiation: alpha particles (helium nuclei), beta-minus particles (electrons), beta-plus particles (positrons) and gamma rays, and their range and penetration properties.
- How to perform techniques and procedures to investigate the absorption of alpha particles, beta particles and gamma rays by different materials.
- How to write and balance nuclear decay equations for alpha, beta-minus and beta-plus decays.
- That the activity of a radioactive source is the number of nuclei in the source that decay each second.
- That the decay constant, λ, of a radioactive isotope is a measure of the rate of decay of an isotope.
- How to use the equation $A = \lambda N$ to calculate the activity of a sample of a radioactive isotope, where N is the number of nuclei of that isotope in the sample.
- How to use the equation $\frac{\Delta N}{\Delta t} = -\lambda N$ to calculate the rate of change of the number of undecayed nuclei of an isotope at a given time, and model radioactive decay using a spreadsheet.
- How to calculate the number of undecayed nuclei of an isotope at time t using the equation $N = N_0 e^{-\lambda t}$, where N_0 is the initial number of undecayed nuclei.
- How to use dice to simulate radioactive decay.
- How to calculate the activity of a source at time t using the equation $A = A_0 e^{-\lambda t}$, where A_0 is the initial activity of the source.
- That the half-life, $t_{\frac{1}{2}}$, of an isotope is the average time taken for the number of undecayed nuclei of the isotope in a sample to halve, and how to find the half-life of an isotope from its decay graph.
- How to perform techniques and procedures to determine the half-life of protactinium.
- How to calculate the half-life of an isotope using the equation $\lambda t_{\frac{1}{2}} = \ln(2)$.
- How radioactive isotopes such as carbon-14 can be used in radioactive dating.
- That the 'mass defect' is the difference between the mass of a nucleus and the total mass of its constituent nucleons.
- That the binding energy of a nucleus is the energy required to separate all the nucleons in a nucleus, and is equivalent to its mass defect converted into energy.
- How to calculate the binding energy of a nucleus from its mass defect using $\Delta E = \Delta mc^2$, and how to calculate the binding energy per nucleon for a given nucleus.
- The shape of the graph of binding energy per nucleon against nucleon number.
- That nuclear fission is when a large nucleus splits into two smaller nuclei with a higher binding energy per nucleon, and that this process releases energy and free neutrons.
- That induced nuclear fission is when fission is caused by the absorption of a low energy neutron.
- That nuclear fusion is when two nuclei combine to form a larger nucleus with higher binding energy per nucleon, and that this process releases energy, but requires very high temperatures and pressures.
- How to write and balance nuclear equations for fission and fusion reactions.
- How to calculate the energy released (or absorbed) in nuclear reactions from the binding energy per nucleon against nucleon number graph and by using $\Delta E = \Delta mc^2$.
- That a chain reaction is where neutrons produced by a fission reaction induce more fission reactions.
- The basic structure of a fission reactor, including details of the moderator, control rods, fuel rods and coolant.
- The environmental impact of nuclear waste.

Exam-style Questions

1 At what range does the strong nuclear force lead to the greatest attraction between nucleons?

A Less than 0.5 fm.

B Between 0.5 fm and 3 fm.

C Between 3 and 6 fm.

D More than 6 fm.

(1 mark)

2 Which of the following interactions is possible?

A $e^+ \rightarrow e^- + \nu$

B $p \rightarrow n + e^- + \nu$

C $n \rightarrow p + e^- + \bar{p}$

D $p \rightarrow n + e^+ + \nu$

(1 mark)

3 A sample contains 8.04×10^{21} atoms of the radioisotope ^{60}Co. It takes 10.4 years for the number of ^{60}Co atoms remaining in the sample to fall to 2.01×10^{21}. What is the decay constant of ^{60}Co?

A $4.23 \times 10^{-9}\ s^{-1}$

B $2.11 \times 10^{-9}\ s^{-1}$

C $2.36 \times 10^{8}\ s^{-1}$

D $1.64 \times 10^{8}\ s^{-1}$

(1 mark)

4 Subatomic particles can be classified into categories such as hadrons and leptons.

(a) Give one similarity and one difference between hadrons and leptons.

(2 marks)

The Δ^{++} particle is a hadron. It has a charge of $+2e$, and a mass of 2.19×10^{-27} kg.

(b) The quark composition of the Δ^{++} particle is uux. Identify quark x.

(1 mark)

(c) All particles have a corresponding antiparticle. State the mass and charge of the antiparticle of the Δ^{++} particle.

(1 mark)

(d) Quarks cannot be found in isolation. When energy is supplied to a hadron to remove a quark, it instead pair-produces a quark-anti-quark pair. Given that the mass of a strange quark is approximately 1.71×10^{-28} kg, calculate the minimum energy, in MeV, required to produce a strange-anti-strange pair.

(3 marks)

5 A plutonium isotope can be represented by $^{240}_{94}Pu$.

(a) State how many neutrons and protons are in the nucleus of an atom of $^{240}_{94}Pu$.

(1 mark)

(b) Name the force that stabilises an atomic nucleus by acting against the repulsive electrostatic force between the protons.

(1 mark)

(c) $^{240}_{94}Pu$ decays by α decay to form an element with symbol U.

Write an equation for this decay using standard notation.

(2 marks)

Another isotope of plutonium, with nucleon number 241, decays by emitting α and β^- particles in stages (a decay chain), eventually forming $^{205}_{81}Tl$.

(d) There are nine α decays in the decay chain from the plutonium isotope to $^{205}_{81}Tl$. Calculate the number of β^- decays in the decay chain.

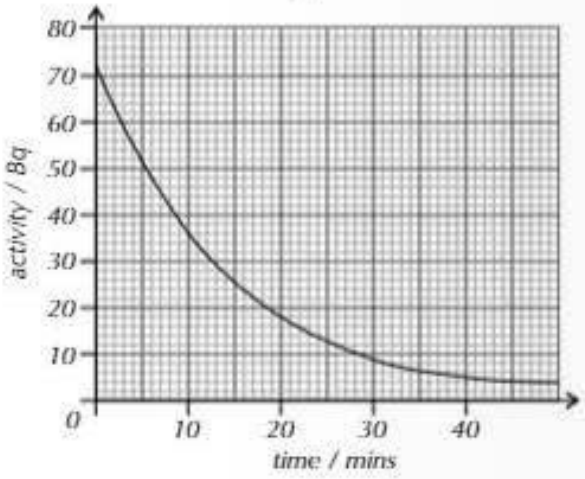
(2 marks)

(e) Describe the process of β^- decay in terms of quark flavour change.

(2 marks)

6 **Fig. 6.1** shows the activity-time graph for a sample of a radioactive isotope.

Fig. 6.1



(a) Find the half-life of the isotope in minutes.

(1 mark)

(b) Calculate the number of undecayed nuclei in the sample after 17 minutes.

(3 marks)

When the isotope decays, the nucleus emits a positron and a neutrino.

(c) The positron goes on to annihilate with an electron. Calculate the minimum frequency of one of the photons produced in this annihilation.

(3 marks)

7 ^{33}P is an isotope of phosphorus.

(a) Calculate the radius of a nucleus of ^{33}P. Use the value $r_0 = 1.4 \times 10^{-15}$ m.

(2 marks)

(b) A nucleus of ^{33}P has a mass of 32.97 u. Calculate the nuclear density of ^{33}P. You may assume the nucleus is spherical.

(3 marks)

^{33}P emits β^- radiation as it decays to an isotope of sulfur.
^{33}P has a half-life of 25.4 days, and a proton number of 15.

(c) Write the nuclear equation for the decay of ^{33}P. (The chemical symbol of sulfur is S.)

(2 marks)

(d) Calculate the decay constant of ^{33}P.

(2 marks)

(e) A sample of ^{33}P contains 1.6×10^{15} undecayed atoms. How long will it take for the number of undecayed atoms to fall to 7.0×10^{13}? Give your answer in days.

(2 marks)

8 This question is about binding energy.

(a) What is meant by the term *binding energy*?

(1 mark)

(b) Copy the axes in **Fig 8.1** and sketch the graph of binding energy per nucleon against nucleon number. Indicate which nucleus is found at the peak of the graph.

Fig. 8.1

(2 marks)

(c) Calculate the binding energy per nucleon of zinc-66, given that it has a mass defect of 0.62065 u. Give your answer in MeV.

(2 mark)

After absorbing a neutron, ^{235}U can fission into ^{94}Sr and ^{140}Xe, along with a number of neutrons:

$$^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{140}_{54}Xe + {}^{94}_{a}Sr + b\,{}^{1}_{0}n$$

(d) Calculate the proton number of ^{94}Sr, a, and the number of neutrons produced, b.

(2 marks)

(e) Explain how the fission of a heavy nucleus releases energy.

(2 marks)

(f) Calculate the energy released by the above reaction in MeV.
(Nuclear masses: ^{235}U = 234.99333 u, ^{94}Sr = 93.89446 u, ^{140}Xe = 139.89194 u, ^{1}n = 1.00867 u.)

(3 marks)

Learning Objectives:

- Know the basic structure of an X-ray tube and that its components include a heater (cathode), an anode, a target metal and a high voltage supply.
- Understand how X-ray photons are produced from an X-ray tube.
- Know what is meant by the attenuation of X-rays and be able to calculate the intensity of an X-ray beam using $I = I_0 e^{-\mu x}$, where μ is the attenuation (absorption) coefficient.
- Know that the X-ray attenuation mechanisms are simple scatter, photoelectric effect, Compton effect and pair production.
- Explain how barium and iodine can be used as contrast media in X-ray imaging.
- Know what computerised axial tomography (CAT) scanning is, and that the components of a CAT scanner include a rotating X-tube producing a thin fan-shaped beam, a ring of detectors, and computer software and display.
- Know advantages of a CAT scan over an X-ray image.

Specification Reference 6.5.1

1. X-ray Imaging

Non-invasive medical imaging techniques let doctors see what's going on (or going wrong) inside the body, so they won't need to resort to surgery to have a look. X-ray imaging is one of these techniques.

X-ray tubes

The X-rays used for diagnostic imaging are produced in X-ray tubes. An **X-ray tube** is a glass tube containing an electrical circuit, with a **cathode** where electrons are emitted, and an **anode** (called the target metal) that the electrons are directed towards. X-ray tubes are surrounded by a lead housing (to keep the X-rays contained), which has a small window in it through which X-rays can pass. To prevent emitted electrons from colliding with gas particles while still in the tube, the X-ray tube is evacuated (i.e. the air is removed).

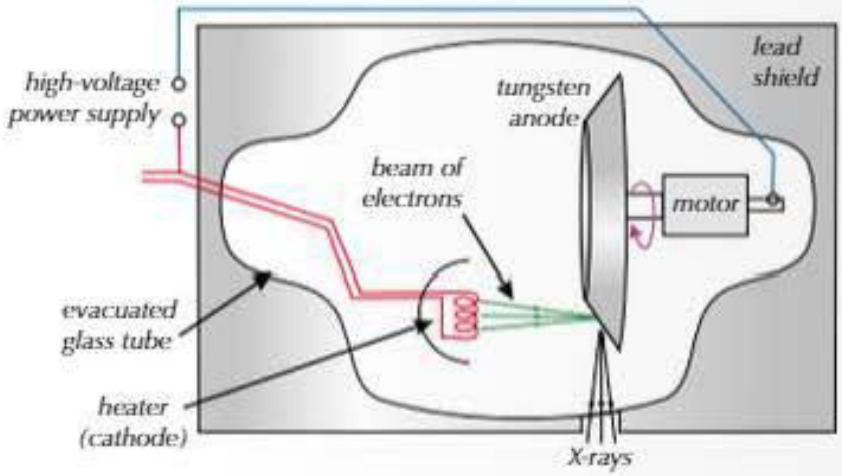


Figure 1: *An X-ray tube.*

At the cathode, electrons are emitted (boiled off) by the hot filament. This filament is heated by passing a current through it — this current is separate from the current that is flowing to the anode. The cathode filament is usually contained in cup-shaped housing, which focuses the beam of electrons onto the target metal.

The target metal (tungsten) acts as the anode of the circuit. The high potential difference across the tube (the 'tube voltage') causes the electrons to accelerate towards it. When the electrons smash into the anode, they decelerate and some of their kinetic energy is converted into electromagnetic energy, in the form of X-ray photons. Whatever energy is lost by the electron is gained by the photon — the result is that the tungsten anode emits a continuous spectrum of X-ray radiation.

The maximum kinetic energy of the electrons (and therefore the maximum energy of the X-ray photons) is equal to the potential difference of the X-ray tube multiplied by the charge of an electron:

$E = eV$

E = maximum kinetic energy of the electrons in J

e = charge of an electron in C

V = potential difference of the X-ray tube in V

So, if a potential difference of 50 kV is used in the tube, the maximum X-ray energy will be 50 keV.

On average, only about 1% of the electrons' kinetic energy is converted into X-rays, while the rest is converted into heat. To avoid overheating, the tungsten anode is rotated at about 3000 rpm (revolutions per minute) to spread heat generated around the whole anode. The anode is also mounted on copper — this conducts the heat away effectively.

Figure 2: *William David Coolidge, American physicist, holding an early version of the X-ray tube he invented.*

Example — **Maths Skills**

Electrons hitting an anode in an X-ray tube produce X-ray photons whose maximum energy corresponds to a wavelength of 5.0×10^{-11} m. The tube voltage is then halved but all other factors are kept the same. Calculate the wavelength of the most energetic X-ray photons produced at this lower voltage.

The maximum energy of the electrons produced in the X-ray tube is $E = e \times V$. Halving the voltage would halve the maximum energy of the electrons and hence halve the maximum energy of the X-ray photons.

Since the energy of a photon (E) is given by $E = \frac{hc}{\lambda}$ (where h and c are constant), halving the energy would mean the wavelength is doubled.

So the new wavelength of the most energetic photons is:

$$2 \times 5.0 \times 10^{-11} = 1.0 \times 10^{-10} \text{ m}$$

X-rays are also produced when beam electrons knock out electrons from the inner shells of the tungsten atoms. Electrons in the atoms' outer shells fall into the vacancies in the inner energy levels, and release energy in the form of X-ray photons — see Figure 3.

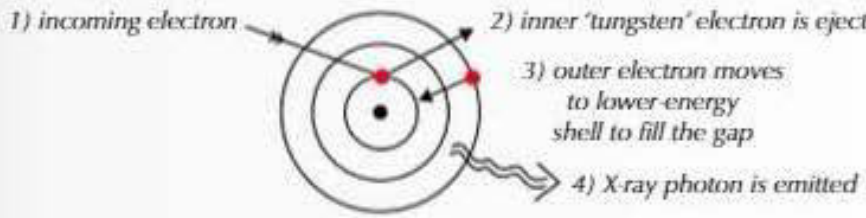

1) incoming electron
2) inner 'tungsten' electron is ejected
3) outer electron moves to lower-energy shell to fill the gap
4) X-ray photon is emitted

Figure 3: *X-rays are emitted when outer electrons move to inner energy shells to fill vacancies in tungsten atoms.*

Varying beam intensity

The intensity of an X-ray beam is the energy per second that passes through a unit area at right angles to the beam. There are two ways to increase the intensity of the X-ray beam:

- Increase the tube voltage. Increasing the potential difference between the cathode and anode gives the electrons more kinetic energy — meaning they have more energy available to be converted into photons on deceleration. Higher energy electrons can also knock out electrons from shells deeper within the tungsten atoms. Both these factors lead to higher energy X-ray photons being produced.
- Increase the current supplied to the filament. This liberates more electrons per second from the cathode, which then produce more X-ray photons per second. Individual photons have the same energy as before.

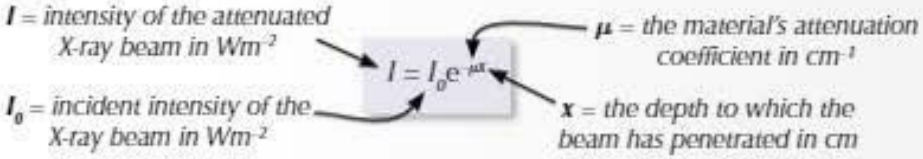
Tip: The X-ray photon is released when the outer electron moves into a lower energy level, not when the inner electron is ejected. This means these photons can only be emitted with very specific energies (which depend on the energies of the shells the electron moves between — see p.329 for more information).

Tip: Because X-rays are harmful, the intensity of medical X-rays needs to be carefully controlled — you need the lowest possible intensity that will give a clear image.

Tip: Current is a measure of the number of electrons passing a point, not of their energy. So increasing the current flowing through the filament increases the number of X-ray photons produced, but doesn't change the maximum photon energy.

X-ray attenuation

When X-rays pass through matter (e.g. a patient's body), they are absorbed and scattered, which leads to a fall in their intensity. The intensity of the X-ray beam decreases (attenuates) exponentially as they travel deeper into the material — the intensity at a depth of x cm depends on the material's **attenuation (absorption) coefficient**:


I = intensity of the attenuated X-ray beam in Wm⁻²
I = I0e^−μx
μ = the material's attenuation coefficient in cm⁻¹
I0 = incident intensity of the X-ray beam in Wm⁻²
x = the depth to which the beam has penetrated in cm

Tip: You might also see the attenuation coefficient given in m^{-1}. In this case you'll need to either convert the attenuation coefficient to cm^{-1} or make sure the penetration depth is in m before doing any calculations.

Example — **Maths Skills**

The attenuation coefficient of a patient's body tissue is 0.2 cm^{-1}. At what depth inside the tissue will an X-ray be attenuated to 40% of the intensity of the incident beam?

You know that the intensity is 40% of the original intensity, so:

$$\frac{I}{I_0} = 0.4 \Rightarrow e^{-\mu x} = 0.4$$

Take the natural logarithm of both sides and rearrange to make x the subject:

$$-\mu x = \ln 0.4 \Rightarrow x = -\frac{\ln 0.4}{\mu} = -\frac{\ln 0.4}{0.2} = 4.58... = 5 \text{ cm (to 1 s.f.)}$$

Tip: 'ln' is the natural logarithm and you can use it to get rid of exponentials because $\ln e^x = x$. You might see 'ln' written as '$\log_e$'. There's more on this on page 472.

Tip: You learnt about the photoelectric effect on pages 237-240.

X-rays are attenuated as they pass through a material in four main ways:

- Simple scattering — at low energies a photon may be deflected by atoms in the material, but without losing energy (i.e. it's elastically scattered).
- The photoelectric effect — a photon with around 30 keV of energy can be absorbed by an electron — the electron is then ejected from its atom. The gap in the electron shell is then filled by another electron, which results in a photon being emitted.
- The Compton effect — a photon with around 0.5-5 MeV of energy loses some of its energy as it interacts with an electron in an atom. The electron is knocked out of the atom and the photon is deflected (i.e. scattered).
- Pair production — a high (> 1.1 MeV) energy photon decays into an electron and a positron (i.e. photon energy is converted to mass) — see page 421.

One factor that affects the amount of energy absorbed by a material is the atomic number of atoms within it. So different tissues (e.g. soft tissue and bone) containing atoms with different atomic numbers will appear different in an X-ray image because they're absorbing different amounts of energy. For example, X-rays are absorbed more by bone than soft tissue, so bones show up more brightly.

If the tissues in the region of interest have similar attenuation coefficients then artificial **contrast media** can be used so these different tissues appear more distinct in an X-ray image. Iodine and barium are both relatively harmless and have high atomic numbers, so they show up clearly in X-ray images. They can also be followed as they move through a patient's body. For example, a barium meal can be swallowed and its path through the digestive system imaged, while iodine is usually injected into blood vessels or tissues so that they can be viewed more clearly.

Figure 4: *An X-ray image of a human skull. The white parts of the image are where the film behind the subject has been exposed to X-rays of lower intensity due to their attenuation as they pass through the subject's body. The more intense an X-ray beam is when it hits the film, the darker that part of the film will be.*

CAT scanning

Computed axial tomography (CT or CAT) scans produce an image of a two-dimensional slice through the body. The patient lies on a table, which slides in and out of a ring containing a rotating X-ray tube (or X-tube) and X-ray detectors.

A thin X-ray beam in the shape of a fan is emitted by the X-ray tube, and the tube is rotated around the body. This beam is picked up by the detectors. Computer software works out how much attenuation has been caused by each part of the body, and converts this information into a high quality image of the patient's tissues. CAT scans produce more detailed images than regular X-rays, especially for soft tissue. The data can also be manipulated to generate a 3D image.

Figure 5: *A patient undergoing an upper-body CAT scan.*

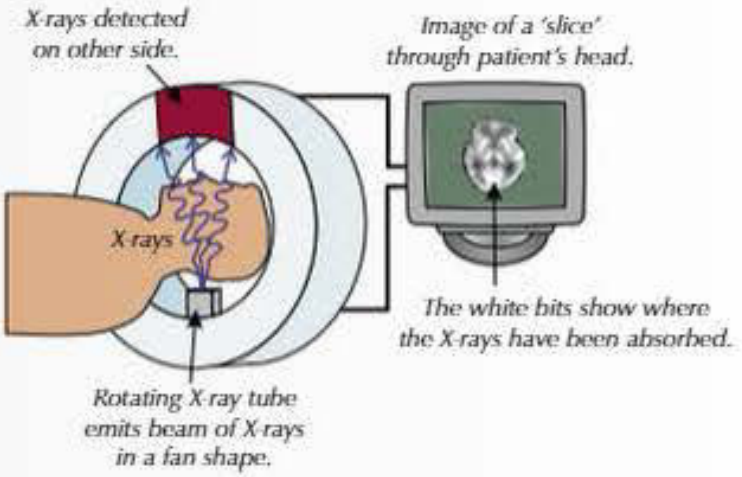

X-rays detected on other side.
Image of a 'slice' through patient's head.
X-rays
The white bits show where the X-rays have been absorbed.
Rotating X-ray tube emits beam of X-rays in a fan shape.

Figure 6: *Diagram showing how a CAT scan works.*

Practice Questions — Application

Q1 In an X-ray tube, electrons are accelerated through a potential difference of 85.0 kV towards an anode. What's the maximum energy an emitted X-ray photon could have in joules?

Q2 X-rays are used to image a bone in a patient's leg. The X-rays have an intensity of 30.0 Wm^{-2} when incident on the bone. The attenuation coefficient of bone (μ) is given by $\mu = 2.5\ cm^{-1}$. Find the intensity of the X-rays at a depth of 0.69 cm beneath the surface of the bone.

Practice Questions — Fact Recall

Q1 a) Draw and label a diagram of an X-ray tube.

b) Explain why the anode in the X-ray tube rotates.

Q2 State the names of four mechanisms by which X-rays can be attenuated.

Q3 Explain how a barium meal can help to improve the contrast between tissues in an X-ray image of the digestive tract.

Q4 Describe how a CAT scanner works.

Q5 Give two advantages of CAT scans over standard X-ray images.

Learning Objectives:

- Know how medical tracers, including technetium-99m and fluorine-18, are used.
- Know how a gamma camera can be used to aid diagnosis.
- Be able to describe the components of a gamma camera and their functions, including the collimator, scintillator, photomultiplier tubes, computer and display.
- Be able to describe how an image is formed in a gamma camera.
- Know that a positron emission tomography (PET) scanner relies on the annihilation of positron-electron pairs, and describe how an image is formed.
- Know how PET scanning can be used to aid diagnosis.

Specification Reference 6.5.2

Tip: Don't worry, you don't need to know what the 'm' at the end of technetium-99m means.

Tip: The beta-plus decay of fluorine-18 in a PET scan produces positrons. These annihilate with electrons in the body, producing high-energy gamma rays — see the next page.

2. Medical Uses of Nuclear Radiation

Medical tracers move through the patient's body to the region of interest and emit radiation. This radiation can be detected with a gamma camera or a PET scanner to map and study the function and structure of tissues or organs.

Medical tracers

Many types of imaging, e.g. X-rays (pages 458–459), only show the structure of organs. **Medical tracers** (or radiotracers) are radioactive substances that are used to show the function, as well as the structure, of tissues or organs.

Medical tracers usually consist of a radioactive isotope (e.g. fluorine-18 or technetium-99m) bound to a substance that is used by the body (e.g. glucose or water). The tracer is injected into or swallowed by the patient and then moves through the body to the region of interest. Where the tracer goes depends on the substance the isotope is bound to — i.e. it goes anywhere that the substance would normally go as it's used by the body. The radiation emitted due to the isotope is detected (e.g. by a gamma camera or PET scanner, see below and page 461) and used to produce an image of inside the patient. This can be useful in different areas — for example:

- Tracers can show areas of damaged tissue in the heart by indicating areas with decreased blood flow. This can reveal coronary artery disease and damaged or dead heart muscle caused by heart attacks.
- They can help identify active cancer tumours. Cancer cells have a much higher metabolism than healthy cells because they're growing fast, so they take up more tracer.
- Tracers can show blood flow and activity in the brain. This helps to research and treat neurological conditions such as Parkinson's, Alzheimer's disease, epilepsy and depression.

Technetium-99m is widely used in medical tracers because the gamma radiation it emits can easily pass out of the body to reach the detector, and it has a half-life of 6 hours (long enough for data to be recorded, but short enough to limit the radiation to which the patient is exposed to an acceptable level). It also decays to a much more stable (less radioactive) isotope.

Fluorine-18 is used in PET scans as it usually undergoes beta-plus decay (which is necessary for a PET scan). It also has a short half-life (110 minutes), meaning the patient's exposure to dangerous radioactivity is kept as low as possible.

Gamma cameras

The gamma rays emitted by some medical tracers in a patient's body are detected using a gamma camera. Once the medical tracer has had time to move to the tissue of interest in the patient, areas where the tracer has been taken up will emit gamma radiation. The patient can then lie on a table and a gamma camera positioned above or below the body can be used to detect the emitted gamma rays.

Gamma cameras consist of five main parts (see Figure 1):

- Lead shield — this surrounds most of the camera and stops radiation from other sources being detected.
- Lead collimator — a piece of lead with thousands of vertical holes in it, which only gamma rays travelling parallel to the holes can pass through.
- Scintillator — a sodium iodide crystal that emits a flash of light (scintillates) whenever a gamma ray hits it.
- Photomultiplier tubes — these detect the flashes of light from the scintillator and turn them into pulses of electricity.
- Electronic circuit — this collects the signals from the photomultiplier tubes and sends them to a computer for processing into an image.

Tip: The scintillator turns the energy of a gamma photon into a large number of visible light photons, which are easier to detect.

Tip: Photomultiplier tubes are also used in PET scanners (see below).

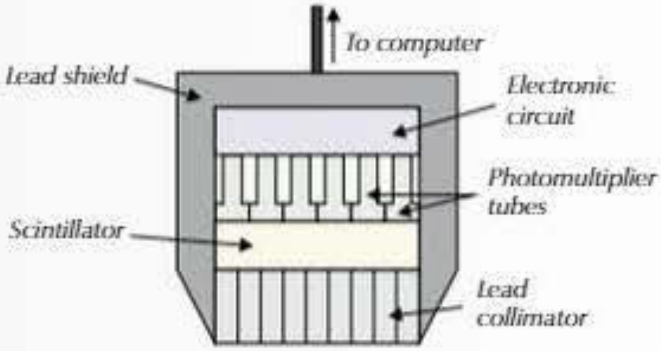


Figure 1: *The components of a gamma camera.*

The computer maps the spatial distribution and the frequency of gamma ray emissions detected by the gamma camera to make a two-dimensional image, effectively forming a snapshot of regions where the medical tracer has been taken up. The gamma camera is often rotated to take multiple images from different angles to provide different perspectives of the area.

Gamma cameras are useful in helping to diagnose patients without the need for surgery. They are cheaper than a PET scanner but are still fairly expensive, and they also require a patient to be exposed to ionising radiation, which is bad for you — see the next page.

PET scans

Positron Emission Tomography (PET) is another medical imaging technique that allows patients to be diagnosed without having to have surgery. PET scans involve injecting a patient with a substance used by the body, (e.g. glucose) that is bound to a positron-emitting radiotracer with a short half-life, (e.g. ^{13}N, ^{15}O or ^{18}F). The patient is left for a time to allow the radiotracer to move through the body to the organs that need to be examined. The positrons emitted by the radioisotope collide with electrons in the organs, and annihilate. This annihilation results in two high-energy gamma rays being emitted in opposite directions (see page 422).

Detectors all around the body detect and record these gamma rays (see Figure 2). The detectors then send the information to a computer, which builds up a map of the radioactivity in the body — this corresponds to the concentration of radiotracers in different parts of the body.

Tip: Because annihilation produces two gamma photons in opposite directions, the PET scanner can locate exactly where the annihilation took place, by measuring how soon one photon arrives at one side of the scanner compared to the other photon arriving at the opposite side.

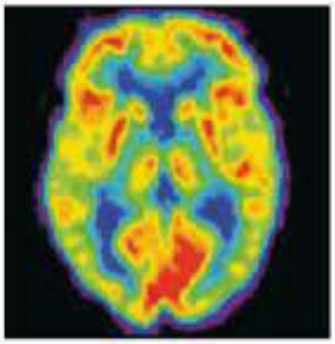


Figure 3: *Image of a healthy brain from a PET scanner.*

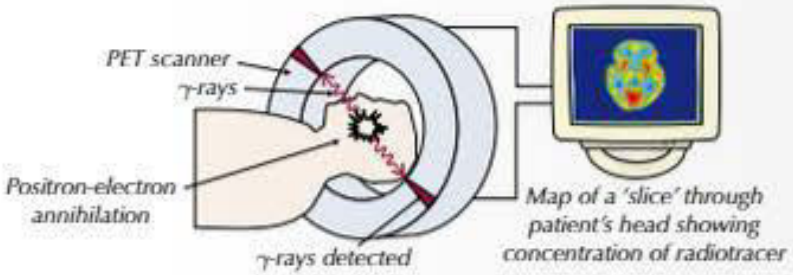


Figure 2: *Diagram showing how a PET scanner works.*

If radioactive glucose is used as a radiotracer, the distribution of radioactivity will correspond to metabolic activity in different parts of the body. This is because more of the radioactive glucose is taken up and used by cells that are doing more work, i.e. cells with a higher metabolism.

By looking at which cells are doing more work, doctors can help diagnose illnesses in patients — for example, cancer cells have a higher level of activity than healthy cells. Another advantage of PET scans is that brain activity can be investigated, which can be difficult with other non-invasive methods.

PET scan radiotracers have a short half-life, so the patient is exposed to radiation for only a short time. However, this short time period means there is only a limited time when a patient can be scanned — unlike with gamma cameras, where the tracer used takes much longer to decay. PET scanners are also incredibly expensive, meaning not many hospitals have one. This means doctors may have to balance the benefits of their patients having PET scans against the costs, inconvenience and time needed to send ill patients to a hospital with a PET scanner.

Tip: Other substances can also be labelled with medical tracers to investigate functions such as blood flow and oxygen use.

The risks of using ionising radiation

X-rays, γ-rays, and α and β particles are all types of ionising radiation. They interact with atoms or molecules to form ions — usually by removing an electron. This can damage cells, and can lead to:

- Cell mutations and cancerous tumours, if the cell's DNA is damaged or altered.
- Cell sterility — i.e. the cell may no longer be able to reproduce.
- Cell death — i.e. the cell could be destroyed completely.

The macroscopic effects of ionising radiation (i.e. the effects on the patient as a whole) can include tumours, skin burns, sterility, radiation sickness, hair loss and death. The result is that radiation is only used in the diagnosis and treatment of disease when absolutely necessary — i.e. when the benefits to the patient outweigh the risks. Even then, radiation doses are limited to the minimum possible.

Tip: See page 427 for more about γ-rays, α particles and β particles.

Practice Questions — Fact Recall

Q1 Describe how medical tracers are used to investigate body tissues and organ function.

Q2 Name a gamma-emitting isotope often used as a medical tracer. Describe the properties that make it suitable for this use.

Q3 Which medical imaging technique is fluorine-18 generally used in?

Q4 Name the main components of a gamma camera and describe what each of them does.

Q5 A tissue's rate of metabolic activity is linked to its rate of uptake of glucose. Describe how a PET scanner produces an image of metabolic activity in different parts of the body.

3. Medical Uses of Ultrasound

You'll have heard of ultrasound from its wide use in prenatal scans. The main reason for its use is that it's the safest and cheapest form of non-invasive imaging we have — but unfortunately it comes with its downsides.

Learning Objectives:

- Know that ultrasound is a longitudinal wave with a frequency greater than 20 kHz.
- Be able to calculate the acoustic impedance of a medium using $Z = \rho c$.
- Know that ultrasound is reflected at boundaries, and how to calculate the fraction of wave intensity that is reflected using $\frac{I_r}{I_0} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$.
- Know what the piezoelectric effect is.
- Know that an ultrasound transducer is a device that emits and receives ultrasound.
- Understand the importance of impedance (acoustic) matching, and how it can be achieved using a special gel in ultrasound scanning.
- Know what is meant by an ultrasound A-scan and B-scan.
- Understand how the Doppler effect can be used in ultrasound to find the speed of blood in a patient.
- Be able to use $\frac{\Delta f}{f} = \frac{2v \cos\theta}{c}$ to determine the speed, v, of blood in a patient.

Specification Reference 6.5.3

What is ultrasound?

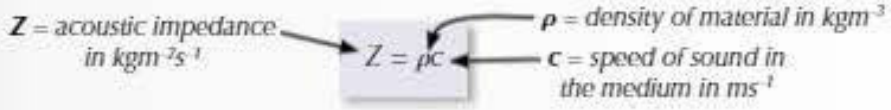
Ultrasound waves are longitudinal waves with higher frequencies than humans can hear (> 20 000 Hz, or 20 kHz). Ultrasound waves travel at the speed of sound.

When an ultrasound wave meets a boundary between two different materials, it is partially reflected and partially transmitted (undergoing refraction if the angle of incidence is not 0°). The reflected waves can be detected by an ultrasound scanner and used to generate an image.

Ultrasound scanning is a relatively safe medical imaging technique, and is particularly useful for viewing soft tissue. The most common frequencies used for medical purposes are usually from 1 to 15 MHz.

Acoustic impedance

The amount of reflection an ultrasound wave experiences at a boundary between different materials depends on the difference in **acoustic impedance**, Z, between them. The acoustic impedance of a material is defined as:



Example — **Maths Skills**

At 20 °C the density of air is approximately 1.2 kgm^{-3} and the speed of sound in air is 340 ms^{-1}. Find the acoustic impedance of air at 20°C.

$Z = \rho c = 1.2 \times 340 = 408 = 410$ kgm^{-2}s^{-1} (to 2 s.f.)

Suppose an ultrasound wave with an intensity of I_0 travels through a material with acoustic impedance Z_1. It hits the boundary between this material and another material with acoustic impedance Z_2. If the two materials have a large difference in acoustic impedance, then most of the energy is reflected, and the intensity of the reflected wave I_r will be high — see Figure 1. If the acoustic impedance of the two materials is the same then there is no reflection.

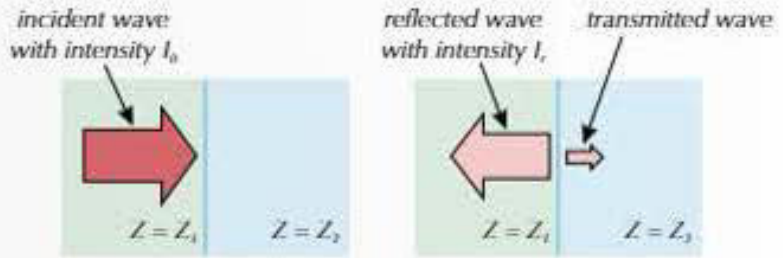


Figure 1: *When a wave travelling through a material of acoustic impedance Z_1 (left) hits a boundary with a material of a very different acoustic impedance Z_2, most of the energy is reflected (right).*

Exam Tip
You don't need to understand where this equation comes from. And it'll be given in your data and formulae booklet, so you don't need to remember it — just make sure you know how to use it.

The equation for the fraction of wave intensity that is reflected is:

$$\frac{I_r}{I_0} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$$

I_r = intensity of reflected wave in Wm^{-2}
I_0 = intensity of incident wave in Wm^{-2}
Z_1 = acoustic impedance of first material in $kgm^{-2}s^{-1}$
Z_2 = acoustic impedance of second material in $kgm^{-2}s^{-1}$

The fraction of wave intensity that is reflected has no units — it's just a ratio. To calculate the fraction of wave intensity that is transmitted, subtract this reflected fraction from 1.

Example — **Maths Skills**

An ultrasound wave meets a boundary between two materials with acoustic impedances of $Z_1 = 1.34 \times 10^6$ $kgm^{-2}s^{-1}$ and $Z_2 = 2.57 \times 10^6$ $kgm^{-2}s^{-1}$. Find the fraction of the wave's intensity that is reflected.

$$\frac{I_r}{I_0} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2} = \frac{(2.57 \times 10^6 - 1.34 \times 10^6)^2}{(2.57 \times 10^6 + 1.34 \times 10^6)^2}$$
$$= 0.09895...$$
$$= 0.0990 \text{ (to 3 s.f.)}$$

The piezoelectric effect

Ultrasound images are produced using the **piezoelectric effect**. This describes the ability of certain materials, such as piezoelectric crystals, to produce a potential difference across them when they are deformed (squashed or stretched).

The particles inside piezoelectric crystals have one end that is more positively-charged, and one end that is more negatively-charged. Normally these particles are randomly oriented, but when stress is applied to the crystal, it shifts the orientation of their electric charges. Some particles move so that their charged ends line up in one direction — this generates a potential difference across the crystal (see Figure 2).

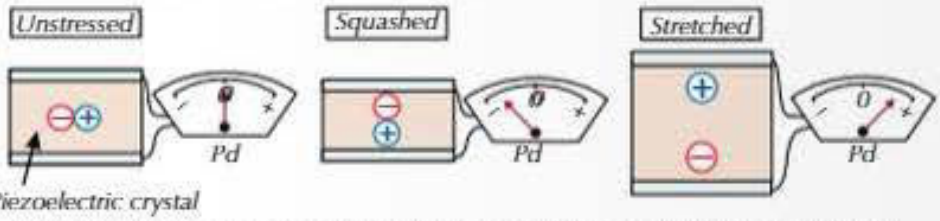


Figure 2: *When a piezoelectric crystal is deformed, one end becomes positively charged and one end becomes negatively charged, resulting in a p.d. across the crystal.*

Figure 3: *An ultrasound probe containing an array of ultrasound transducers.*

Similarly, when you apply a p.d. across a piezoelectric crystal, the crystal deforms to rebalance the charge across it. If the p.d. is alternating, then the crystal vibrates at the same frequency.

Ultrasound waves are generated and detected by a device called an ultrasound transducer. Ultrasound transducers convert ultrasound into electrical signals, and vice versa. They contain lead zirconate titanate (PZT) crystals, which exhibit the piezoelectric effect. Inside the transducer, an alternating potential difference is applied to the PZT crystals, which vibrate to create an ultrasound wave. When the wave is reflected and returns, it causes the crystals to vibrate, which in turn creates an alternating potential difference that is detected in an adjoining circuit.

The ultrasound waves produced by an ultrasound transducer have a wavelength equal to twice the thickness of the PZT crystals inside. Ultrasound with this wavelength will make the crystals resonate (see p.290) and produce a large signal. This means the ultrasound transducer only receives large signals from waves of the same wavelength that it produces, reducing unwanted interference from other sources.

The resolution of the ultrasound transducer increases as the length of each ultrasound pulse decreases, so the vibrations of the PZT crystal are reduced with damping material to produce short pulses.

Tip: The vibrations of the crystal are greatest when the frequency of the potential difference equals the natural frequency of the crystals (see page 290).

Tip: Resolution describes the ability of a device to distinguish two points that are close together.

Coupling media

Soft tissue has a very different acoustic impedance from air, so almost all the ultrasound energy is reflected from the surface of the body if there is air between the transducer and a patient's body. To avoid this, you need a **coupling medium** between the transducer and the body — this displaces the air and has an impedance much closer to that of body tissue. The coupling medium is usually an oil or gel that is smeared onto the skin. The use of coupling media is an example of impedance (acoustic) matching.

Using ultrasound in medical scans

Advantages

- There are no known hazards or side effects (unlike with X-rays, where the patient is exposed to ionising radiation, see page 462).
- Real-time images can be obtained.
- Ultrasound devices are relatively cheap and portable.
- The procedure is relatively comfortable for a patient, and they are allowed to move during the scan.

Disadvantages

- Ultrasound can't penetrate bone, since the difference in acoustic impedance between bone and the surrounding tissue is very large. So ultrasound can't be used to detect fractures or examine the brain.
- The large difference in acoustic impedance between air and body tissues also means most of an ultrasound wave's energy is reflected when it meets air spaces in the body. This means ultrasound can't be used to produce images from behind the lungs.
- Ultrasound images are low resolution (i.e. they're quite 'grainy'), so you can't see fine detail.
- Ultrasound can be used to identify solid masses, but it doesn't provide much information about their composition.

Tip: Remember, most of the energy from ultrasound waves is reflected at boundaries between materials with large differences in acoustic impedance — this includes where bone meets other tissues.

Types of ultrasound scan

There are two types of ultrasound scan you need to know about. A-scans are mostly used for measuring distances, while B-scans are used to form images.

The A-scan

An amplitude scan (A-scan) sends a short pulse of ultrasound into the body as an electron beam inside a cathode ray oscilloscope (CRO) starts to sweep across its screen. The receiver detects reflected ultrasound pulses, which are displayed as peaks on the CRO screen.

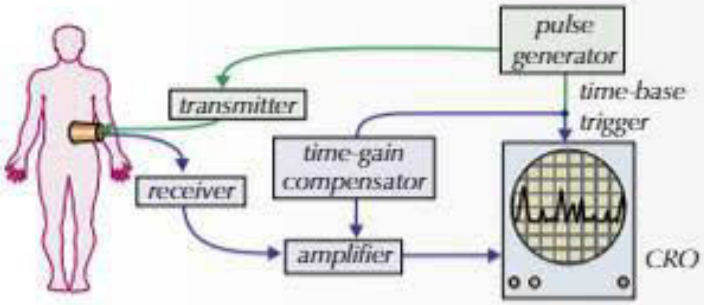


Figure 4: *The set-up of an amplitude scan.*

The horizontal positions of the reflected pulses indicate the time the 'echo' took to return, and are used to work out distances between structures in the body (e.g. the distance between the front and back of an eyeball). Weaker pulses (that have travelled further in the body and arrive later) are amplified more than the stronger, earlier pulses to avoid the loss of valuable data — this process is called time-gain compensation (TGC). A stream of pulses can produce the appearance of steady peaks on the screen, although modern CROs can store a digital image of the peaks after just one pulse.

Tip: Because the images in a B-scan are obtained in real time and the scan doesn't require anaesthesia, the patient can watch the images as they are formed.

The B-scan

In a brightness scan (B-scan), the amplitude of the reflected pulses is displayed as the brightness of a spot on a screen. You can use a linear array of transducers (i.e. lots of transducers arranged in a line) to produce a two-dimensional image, such as in the prenatal scanning of a fetus.

Figure 5: *An ultrasound scan of a fetus at 13 weeks.*

The Doppler effect

Ultrasound waves reflected at an angle off moving cells undergo a change of frequency (or wavelength). This is caused by the **Doppler effect** (p.333). This change of frequency (known as the 'beat frequency') can allow doctors to find the speed at which those cells are moving (for example, blood cells in an artery). You can calculate the change in frequency as a fraction of the initial frequency using this equation:

Δf = change in frequency of the ultrasound (beat frequency) in Hz

f = initial frequency of the ultrasound in Hz

$$\frac{\Delta f}{f} = \frac{2v\cos\theta}{c}$$

v = speed of the moving cell in ms^{-1}

θ = angle between the ultrasound receiver and the line along which the cell is moving in °

c = speed of sound in the medium in ms^{-1}

Tip: The change in frequency will be positive if the blood is travelling towards the receiver, and negative if the blood is travelling away from the receiver.

Or:

$$v = \frac{c\Delta f}{2f\cos\theta}$$

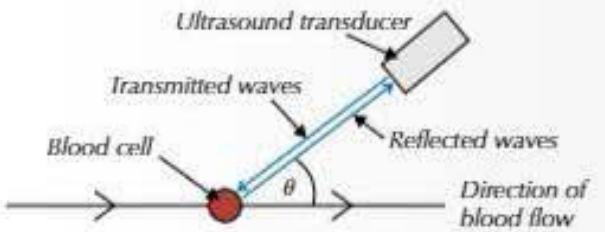
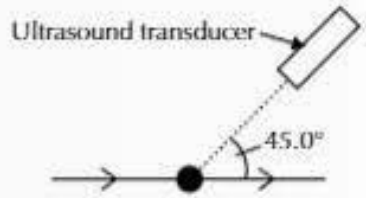


Figure 6: *The ultrasound transducer detects ultrasound waves at an angle θ to the line along which the blood cells are moving.*

Tip: This technique is often used in conjunction with B-scan ultrasound. The B-scan provides an image of the area being studied, making it easier for the technician to target the Doppler beam correctly and adjust the angle of the receiver.

Example — **Maths Skills**

An ultrasound transducer directs longitudinal waves at a frequency of 8.5 MHz at an artery. The waves reflect off blood cells in the artery and are detected at an angle of 45° to the direction of blood flow. The change in frequency of the waves due to the Doppler effect is 3.1 kHz. If the speed of sound in the observed area is 1550 ms^{-1}, calculate the speed at which the blood cells in the artery are moving.



$f = 8.5$ MHz and $\Delta f = 3.1$ kHz, so:

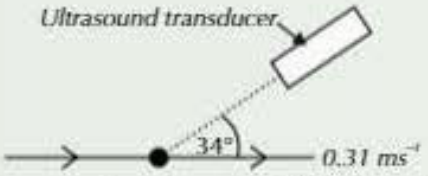
$$v = \frac{c\Delta f}{2f\cos\theta} = \frac{1550 \times 3.1 \times 10^{3}}{2 \times 8.5 \times 10^{6} \times \cos 45^{\circ}} = 0.3997... = 0.40\ \text{ms}^{-1}\ \text{(to 2 s.f.)}$$

Practice Questions — Application

Q1 a) The density of water at 20 °C is 1000 kgm^{-3} (to 2 s.f.) and its acoustic impedance is 1.5×10^{6} $kgm^{-2}s^{-1}$. What is the speed of sound, c, in water at this temperature?

b) A sound wave travels from bone ($Z = 8.0 \times 10^{6}$ $kgm^{-2}s^{-1}$) to water. What will the intensity of the reflected wave be as a percentage of the intensity of the incident wave?

Q2 An ultrasound transducer is used to direct longitudinal waves at a frequency of 12 MHz at an artery. The figure below shows the waves reflecting back to the transducer at an angle of 34° to the direction of blood flow.



The blood cells are moving at a speed of 0.31 ms^{-1} in the artery, and the speed of sound in the area is 1550 ms^{-1}. Calculate the change in frequency of the wave you would expect to see due to the Doppler effect.

Practice Questions — Fact Recall

Q1 What is ultrasound?

Q2 Give the equation for calculating the acoustic impedance of a material. Define all the symbols used and give the units of each quantity.

Q3 What's the piezoelectric effect?

Q4 Explain how an ultrasound wave is generated and detected.

Q5 Why is a coupling medium needed to obtain a clear ultrasound image of body tissue?

Q6 a) Briefly describe what an A-scan is. Give one use of an A-scan.

b) Briefly describe what a B-scan is. Give one use of a B-scan.

Section Summary

Make sure you know...

- The structure of an X-ray tube and the function of its components, including the heater (cathode), anode (target metal) and high voltage supply.
- How X-ray photons can be produced using an X-ray tube.
- That the attenuation (absorption) coefficient, μ, of a material is a measure of how much the intensity of an X-ray beam decreases as it penetrates deeper into the material.
- That the intensity of an attenuated X-ray beam can be calculated using $I = I_0 e^{-\mu x}$.
- That X-ray attenuation mechanisms include simple scatter, the photoelectric effect, the Compton effect and pair production.
- That the high atomic numbers of barium and iodine cause them to show up very differently in X-ray images compared to body tissues, making them useful as contrast media in X-ray imaging.
- How a computerised axial tomography (CAT) scanner works, including the roles of the rotating X-ray tube, the ring of detectors, the computer software and the display.
- That CAT scans produce more detailed images than regular X-rays, especially for soft tissue.
- That the data from CAT scans can be manipulated to generate a 3D image.
- How radioactive substances such as technetium-99m and fluorine-18 can be used as medical tracers to show the structure and function of organs.
- How a gamma camera works, including the roles of the collimator, scintillator, photomultiplier tubes, computer and display, and are able to describe the way an image is formed.
- That a gamma camera can be used to help diagnose a patient without the need for surgery.
- That a positron emission tomography (PET) scanner relies on the annihilation of positron-electron pairs.
- That the positrons needed for a PET scanner are produced by a suitable radiotracer.
- That the detectors in a PET scanner detect and record the gamma rays produced by positron-electron annihilation and send the information to a computer, which builds up a map of the radioactivity in the body.
- That the distribution of radioactivity in a PET scan corresponds to the distribution of the medical tracer in the body, and knowledge of this can aid doctors when diagnosing disease.
- That ultrasound is a longitudinal wave with a frequency greater than 20 kHz.
- How to calculate the acoustic impedance of a medium using $Z = \rho c$.
- That ultrasound is partially reflected at boundaries, and how to calculate the fraction of wave intensity that is reflected using $\frac{I_r}{I_0} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$.
- That the piezoelectric effect is when a potential difference is produced by piezoelectric crystals when they're deformed, or vice versa.
- That an ultrasound transducer is a device that emits and receives ultrasound.
- That soft tissue has a very different acoustic impedance from air, so almost all the ultrasound energy is reflected from the surface of the body if there is air between it and the transducer.
- The importance of impedance (acoustic) matching in ultrasound scanning, and how it can be achieved by using a special gel as a coupling medium.
- That an ultrasound amplitude scan (A-scan) is used to measure distances and diameters and an ultrasound brightness scan (B-scan) can be used to produce a two-dimensional image of a patient's body.
- That ultrasound waves reflected off moving cells at an angle to their line of movement undergo a change of frequency due to the Doppler effect, and how the speed of blood in a patient can be determined using $\frac{\Delta f}{f} = \frac{2v \cos\theta}{c}$.

Exam-style Questions

1 Calculate the fraction of wave intensity that is reflected when ultrasound waves are passing from soft tissue to muscle. The acoustic impedance of soft tissue is 1.58×10^6 kgm^{-2}s^{-1} and the acoustic impedance of muscle is 1.70×10^6 kgm^{-2}s^{-1}.

A 0.929

B 0.0731

C 0.0366

D 0.00134

(1 mark)

2 The intensity of an X-ray beam is reduced by 95% after passing through 4.5 cm of a material. Calculate the attenuation coefficient of the material.

A 0.011 cm^{-1}

B 0.36 cm^{-1}

C 0.57 cm^{-1}

D 0.67 cm^{-1}

(1 mark)

3 A doctor wants to use a non-invasive medical imaging technique to study the uptake of glucose in the lung cells of a patient. Which of the following techniques would be the most suitable?

A CAT scan

B PET scan

C Ultrasound scan

D X-ray imaging

(1 mark)

4 Gamma cameras can be used to help diagnose patients without the need for surgery.

(a) Describe how gamma rays incident on a gamma camera are converted into pulses of electricity.

(3 marks)

(b)* White blood cells are produced by the body to target and fight infections. Technetium-99m is one radioactive isotope that can be used to label white blood cells.

Use this information to discuss how a medical tracer could be used to identify areas of infection in the body. Explain why technetium-99m is suitable for use in medical tracers.

(6 marks)

* The quality of your response will be assessed in this question.

5 An X-ray tube can be used to generate X-rays for medical purposes.

(a) The power supply in an X-ray tube provides a potential difference of 31 kV across the circuit. Calculate the minimum wavelength of the X-rays produced by the X-ray tube.

(3 marks)

(b) As the X-rays pass through body tissues, they are attenuated. Besides simple scattering, suggest one attenuation mechanism that may cause this.

(1 mark)

(c) Iodine is often injected into the blood of a patient before X-rays of their blood vessels are taken. Explain why this is done.

(2 marks)

6 Ultrasound transducers can be used to image soft tissue.

(a) Explain how an A-scan can be used to measure the depth of an eyeball.

(3 marks)

(b) At 15 °C at sea level, the density of air is 1.23 kgm^{-3} and the speed of sound in air is 340 ms^{-1}. Calculate the acoustic impedance of air at this temperature and height.

(1 mark)

(c) Explain why it is difficult to use ultrasound to produce images from behind the lungs.

(2 marks)

An ultrasound operator wants to measure the speed of blood in an artery.
He uses a transducer to direct pulses with a frequency of 8.1 MHz at the artery.
The transducer is held at different angles to the direction of blood flow, and these angles are measured using B-scan ultrasound. The difference in frequency between the incident wave and the reflected wave is recorded for each angle. **Fig 6.1** shows the change in frequency plotted against the cosine of the angle between the receiver and the direction of blood flow. A line of best fit has been drawn.

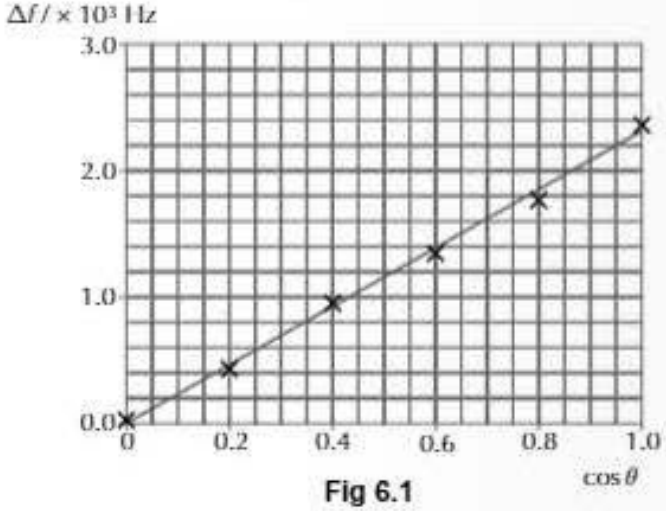


Fig 6.1

(d) Show that the gradient of the line of best fit in **Fig 6.1** is equal to $\frac{2vf}{c}$.

(1 mark)

(e) Assuming the speed of sound is 1550 ms^{-1}, calculate the speed of the blood cells.

(3 marks)

1. Calculations

Sometimes the numbers you use in Physics are just plain awkward — they're either too big, too small or go on forever. The next few pages show how numbers can be written in different ways to make calculations a lot easier.

Standard form

When you are doing a calculation, it's sometimes easier to give your answer in standard form — or you might be given values in standard form. **Standard form** is used for writing very big or very small numbers in a more convenient way. Standard form must always look like this:

This number must always be between 1 and 10. → $A \times 10^n$ ← *'n' is the number of places the decimal point moves.*

Tip: If you don't write a number in standard form, it's known as decimal form — e.g. 0.00012 or 34 500.

Tip: Standard form is a great way to show significant figures (see page 473). For example, the number 230 is ambiguous, it could be to 2 or 3 significant figures. Writing it as 2.30×10^2 makes it clear that it is to 3 significant figures.

Examples — Maths Skills

Here's how to write 3 500 000 in standard form.

- First, move the decimal point until the number you've got is between 1 and 10. In this example, the number is 3.5.
- Then count how many places the decimal point has moved to the left. This number sits to the top right of the 10.

$$3\overset{6}{5}\overset{5}{0}\overset{4}{0}\overset{3}{0}\overset{2}{0}\overset{1}{0} = 3.5 \times 10^6$$

- Et voilà... that's 3 500 000 written in standard form.

Here are some more examples.

- You can write 450 000 as 4.5×10^5.
- The number 0.000056 is 5.6×10^{-5} in standard form — the n is negative because the decimal point has moved to the right instead of the left.
- You can write 0.003456 as 3.456×10^{-3}.

There's a special button on your calculator for using standard form in a calculation — it's the 'Exp' button. So if, for example, you wanted to type in 2×10^7, you'd only need to type in: '2' 'Exp' '7'. Some calculators may have a different button that does the same job, for example it could say 'EE' or '$\times 10^x$' instead of 'Exp' — see Figure 1.

Figure 1: *The 'Exp' or '$\times 10^x$' button is used to input standard form on calculators. Don't confuse it with the power buttons x^2 or $x^\blacksquare$, which are used for raising numbers to a power.*

Orders of magnitude and logarithms

Figure 2: *There are buttons on your calculator for log (log base 10) and ln. If you press shift or 'second function' first, you'll get the $10^{\blacksquare}$ or $e^{\blacksquare}$ functions, the inverse functions of log and ln. There's also a button for the constant e.*

Orders of magnitude can be used to roughly compare the size of two (or more) numbers. If two numbers are roughly the same size, they will have the same order of magnitude. Usually, the order of magnitude of a number is the power that 10 would be raised to if the number were written in standard form. They are mostly used for approximate comparison.

Example — **Maths Skills**

- $12\,340 = 1.234 \times 10^4$, so its order of magnitude is 4.
- $0.000321 = 3.21 \times 10^{-4}$, so its order of magnitude is -4.
- So 12 340 is 8 orders of magnitude (10^8 times) larger than 0.000321.

Scales that use orders of magnitude are known as logarithmic. They allow data that ranges over many orders of magnitude to be displayed and interpreted easily, because instead of going up in equal amounts, each step on the scale is 'so many times bigger' than the last (e.g. 10 times bigger).

A logarithm base 10 ($\log_{10}$ or lg) of a number is defined as the power to which ten must be raised in order to get that number:

$$\log_{10} x = y \quad \text{means} \quad 10^y = x$$

You can have bases other than 10. The other common base in physics is the constant 'e'. e is equal to 2.71828... — it's stored in your calculator (see Figure 2). $\log_e$ is known as the natural logarithm, or 'ln'.

$$\ln x = y \quad \text{means} \quad e^y = x$$

There are a few log rules that work for both log and ln that you need to know:

$$\log AB = \log A + \log B \qquad \log \frac{A}{B} = \log A - \log B \qquad \log x^n = n \log x$$

Tip: These rules will be given in the data and formulae booklet in the exams.

Tip: ln and e are inverse functions, so $\ln e^x = e^{\ln x} = x$.

Tip: Remember, $e^{-ax} = (e^x)^{-a}$.

Example — **Maths Skills**

Show that if $\ln y = \ln(ke^{-ax})$, the graph of $\ln y$ against x is a straight line.

If it is a straight line, it can be expressed in the form $\ln y = mx + c$.

$$\ln y = \ln(ke^{-ax}) = \ln k + \ln e^{-ax} = \ln k - a \ln e^x = \ln k - ax$$
$$= -ax + \ln k$$

So plotting $\ln y$ against x gives a straight line with gradient $-a$ and intercept $\ln k$.

Significant figures

You've already seen that you should always give your answer to the lowest number of significant figures (s.f.) used in the calculation (p.9).

So, in exam questions you can use the number of significant figures given in the question as a guide for how many to give in the answer. It's always good to write down the full unrounded answer, followed by your rounded answer.

You should write down the number of significant figures you've rounded to after your answer too — it shows the examiner you really know what you're talking about.

Tip: You should never round before you get to your final answer. It will introduce rounding errors and you may end up with the wrong answer.

Examples — Maths Skills

In this question the data given to you is a good indication of how many significant figures you should give your answer to:

1 a) Calculate the extension measured by the student in m. (The force constant of the metal is 1.24×10^3 Nm^{-1}.)

The data in the question is given to 3 s.f. so it makes sense to give your answer to 3 s.f. too. But sometimes it isn't as clear as that:

2 b) A force of 48 N is applied to a steel girder with a cross-sectional area of 0.519 m^2. Calculate the tensile stress in the girder.

There are two types of data in this question, force data and cross-sectional area data. The force data is given to 2 s.f. and the cross-sectional area data is given to 3 s.f. You should give your answer to the lowest number of significant figures given — in this case that's to 2 s.f. The answer is 92.48... Nm^{-2} so the answer rounded correctly would be 92 Nm^{-2} (to 2 s.f.).

Exam Tip
In your exam, you might lose a mark on an otherwise correct answer if it's not given to an appropriate number of significant figures. Make sure you always round your answers to the correct number of significant figures.

Unfortunately it's not so simple when you're working with logarithms and raising to powers. There are a few more rules that you need to know:

If you're taking a logarithm, the value of the result is accurate to the same number of decimal places as there are significant figures in the number you're taking a logarithm of.

If you're raising to a power, the value of the result is accurate to the same number of significant figures as there are decimal places in the power.

Examples — Maths Skills

The number of undecayed nuclei in a sample of a radioactive isotope is given by $N = N_0e^{-\lambda t}$, where N_0 is the original number of undecayed nuclei, t is the time elapsed and λ is the decay constant.

Calculate N after exactly 2 minutes in a sample of an isotope with decay constant 3.40×10^{-2} s^{-1} that originally had 4.28×10^{24} undecayed nuclei.

$N = N_0e^{-\lambda t}$

$= 4.28 \times 10^{24} \times e^{-(3.40 \times 10^{-2}) \times (2 \times 60)}$

$= 4.28 \times 10^{24} \times e^{-4.08}$

$= 7.236... \times 10^{22}$

4.08 has 2 decimal places, so the $e^{-\lambda t}$ bit can be given to 2 s.f. and N_0 is given to 3 s.f.
The least number of s.f. is 2, so the answer is $N = 7.2 \times 10^{22}$ (to 2 s.f.)

Calculate the time taken for the number of undecayed nuclei to drop from 4.28×10^{24} to 2.14×10^{23}.

Rearrange the equation for t:

$$t = \frac{\ln(N_0) - \ln(N)}{\lambda} = \frac{\ln(4.28 \times 10^{24}) - \ln(2.14 \times 10^{23})}{3.40 \times 10^{-2}} = \frac{56.7159... - 53.7202...}{3.40 \times 10^{-2}}$$

$= 88.10977...$ s

4.28×10^{24} and 2.14×10^{23} are both given to 3 s.f. So the logarithms are accurate to 3 decimal places, which is equivalent to 5 s.f. for these numbers (i.e. 56.716 and 53.702). λ is given to 3 s.f., which is the least number of s.f., so the answer should be given to 3 s.f. too. So $t = 88.1$ s (to 3 s.f.)

Ratios, fractions and percentages

Ratios, fractions and percentages are all ways of expressing proportions. They're often used to show relationships in Physics, and you'll be expected to convert between them effortlessly. Here's a quick reminder:

Tip: Simplifying a ratio can be done in the same way as simplifying a fraction. It's just that instead of the numbers being on top of each other, they're next to each other.

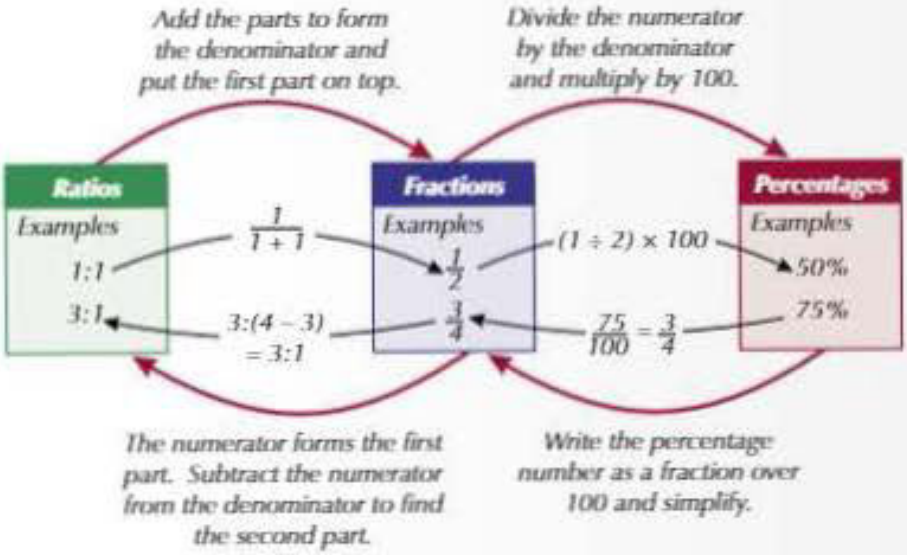

Add the parts to form the denominator and put the first part on top.
Divide the numerator by the denominator and multiply by 100.
Ratios
Examples
1:1
3:1
Fractions
Examples
Percentages
Examples
50%
75%
1/(1 + 1)
3:(4 − 3) = 3:1
(1 ÷ 2) × 100
75/100 = 3/4
The numerator forms the first part. Subtract the numerator from the denominator to find the second part.
Write the percentage number as a fraction over 100 and simplify.

Figure 3: *Converting between ratios, fractions and percentages.*

Example — **Maths Skills**

The length of a desk is measured as 1.62 m with a ruler that has a scale with 5 mm increments. Calculate the percentage uncertainty for the measurement.

When measuring with a ruler you get an uncertainty from both ends, the 0 end and the end you're reading from.

5 mm = 0.005 m, so the uncertainty on each end is $\frac{0.005}{2} = 0.0025$.

So the absolute uncertainty is 0.0025 + 0.0025 = 0.005.

Percentage uncertainty = (0.005 ÷ 1.62) × 100
= 0.3086... %
= 0.31% (to 2 s.f.)

Tip: See page 16 for more on uncertainty when measuring lengths.

Probabilities are likelihoods that something will happen.
Probabilities are often expressed as fractions, percentages or ratios.

Example — **Maths Skills**

Alpha particles are being fired at a thin sheet of gold foil. A scientist calculated that the probability of an alpha particle being deflected by the gold foil (i.e. not passing straight through) is 1.2%.

Write the ratio of the number of particles that are likely to be deflected to the number of particles that are likely to pass straight through.

1.2% is equal to $\frac{1.2}{100} = \frac{12}{1000}$ as a fraction.

So the first part of the ratio is 12 — the number of particles (out of 1000) that are likely to be deflected.

The second part is 1000 − 12 = 988 — the number of particles (out of 1000) that are likely to pass straight through.

The ratio is then written as 12 : 988, which simplifies to 3 : 247.

Tip: The probability that an alpha particle will be deflected is 1.2% — i.e. 1.2 out of 100 are likely to be deflected.

Tip: Remember that the numerator and denominator in fractions should be whole numbers.
Some calculators will do this for you — type $\frac{0.005}{1.62}$ into your calculator and press '='. Hey presto, the fraction it returns is $\frac{1}{324}$.

If not, you can always do it the old fashioned way.

You can use your calculator to simplify the ratio too by entering it as a fraction.

Showing your working

In all calculation questions, whether you are converting units (p.32), switching between fractions and decimals or rearranging equations (see next page), you always need to show your working.

This is important because there are sometimes marks available for intermediate steps in a calculation question — so even if you get the answer wrong, you might get some credit if you've shown your working. You are also more likely to spot any accidental mistakes if you can see the steps you took in your working — particularly useful if you have some spare time at the end of the exam.

2. Algebra

Physics involves a lot of rearranging formulas and substituting values into equations. Easy stuff, but it's also easy to make simple mistakes.

Algebra symbols

Here's a reminder of some of the symbols that you will come across:

Symbol	Meaning
$=$	*equal to*
$<$	*less than*
$\leq$	*less than or equal to*
$<<$	*much less than*
$>$	*greater than*
$\geq$	*greater than or equal to*
$>>$	*much greater than*
$\propto$	*directly proportional to*
$\approx$	*roughly equal to*
Δ	*change in (a quantity)*
Σ	*sum of*

Tip: An example of using $\propto$ can be found on page 193.

Tip: Δ is the Greek capital letter 'delta'. An example of using Δ can be found on page 56.

Figure 1: *It can be easy to make a mistake rearranging equations when you're stressed in an exam. It's a good idea to double check rearrangements, especially if it's a tricky one where you've had to combine and rearrange equations.*

Tip: Taking a value outside a bracket is known as factorising. You often need to factorise to solve quadratic equations like this one. Just write the value to the left of the bracket and write each term, divided by the number you've taken outside, inside the brackets. You should have done this at GCSE.

Rearranging equations

Being able to rearrange equations is a must in Physics — you'll often need to make a different quantity the subject in an equation. Just remember the golden rule — whatever you do to one side of the equation, you must do to the other side of the equation.

Example — **Maths Skills**

The equation for the volume of a sphere is $V = \frac{4}{3}\pi r^3$. Rearrange the equation to make r the subject.

$V = \frac{4}{3}\pi r^3$

$\frac{3V}{4\pi} = r^3$ ↙ Multiply by 3 and divide by 4π

$\sqrt[3]{\frac{3V}{4\pi}} = r$ ↙ Take the cube root

Example — **Maths Skills**

One of the equations of motion is $s = ut + \frac{1}{2}at^2$. When $t = 0$, $s = 0$. Find an expression for t at another point where $s = 0$.

If $s = 0$, the equation can be written as: $ut + \frac{1}{2}at^2 = 0$

Take t outside the bracket: $t\left(u + \frac{1}{2}at\right) = 0$

Two things multiplied together to give 0 means one of them must be 0,

so $t = 0$ or $\left(u + \frac{1}{2}at\right) = 0$

We're interested in the non-zero solution, which can be rearranged to give:

$t = -\frac{2u}{a}$

Substituting into equations

Once you've rearranged your equation, you'll probably need to substitute values into the equation to find your answer. Pretty easy stuff — make sure you avoid the common mistakes by putting values in the right units and putting numbers in standard form before you substitute.

Example — **Maths Skills**

A proton is accelerated to a speed of 3.00 kms^{-1}.

The proton will diffract if it interacts with an object of about the same size as its de Broglie wavelength, λ.

Use the equations $p = mv$ and $\lambda = \frac{h}{p}$ to work out its de Broglie wavelength.

h = the Planck constant = 6.63×10^{-34} Js

$m = 1.673 \times 10^{-27}$ kg

h and m are in the right units but v is not, so you need to convert it to ms^{-1}.

$\boldsymbol{v} = 3.00$ kms^{-1} $= 3.00 \times 10^{3}$ ms^{-1}

So $\boldsymbol{p} = m\boldsymbol{v} = 1.673 \times 10^{-27} \times 3.00 \times 10^{3} = 5.019 \times 10^{-24}$ kg ms^{-1}

Substitute this into de Broglie's equation:

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{5.019 \times 10^{-24}}$$

$= 1.320... \times 10^{-10}$ m $= 1.32 \times 10^{-10}$ m (to 3 s.f.)

Exam Tip
The values of h and m are given in your data and formulae booklet.

Tip: Be careful when using numbers in standard form in your calculator. Double-check you've typed it right and make sure your answer seems a sensible order of magnitude (page 34).

Example — **Maths Skills**

A train pulls out of a station and is initially travelling at 2.0 cms^{-1}. The train accelerates with a constant acceleration over a distance of 74 km. At this distance, the train reaches a final velocity of 150 kmh^{-1}. Calculate the acceleration of the train.

All given values are in different units, so you need to convert them.

$\boldsymbol{u} = 2.0$ cms^{-1} $= 2.0 \times (1 \times 10^{-2})$ ms^{-1} $= 0.020$ ms^{-1}

$\boldsymbol{s} = 74$ km $= 74 \times 1 \times 10^{3}$ m $= 74\,000$ m

$\boldsymbol{v} = 150$ kmh^{-1} $= 150 \times 1 \times 10^{3}$ mh^{-1} $= 150\,000$ mh^{-1}

$= 150\,000 \div 3600$ ms^{-1} $= 41.66...$ ms^{-1}

Rearrange the equation of motion $\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{as}$ to make $\boldsymbol{a}$ the subject:

$$\boldsymbol{a} = \frac{\boldsymbol{v}^2 - \boldsymbol{u}^2}{2\boldsymbol{s}}$$

$$= \frac{41.66...^2 - 0.020^2}{2 \times 74\,000}$$

$= 0.0117...$ ms^{-2}

$= 0.012$ ms^{-2} (to 2 s.f.)

Tip: Converting all the values into the correct units before putting them into the equation helps you avoid making silly mistakes.

Tip: This equation is on page 49.

3. Graphs

You can get a lot of information from a graph — you'll need to know what the area under a graph and the gradient represent, and be able to sketch and recognise simple graphs, given an equation.

Area under a graph

Many quantities in A-level physics can be found from the area between a curve or line and the horizontal axis of a graph. Here are a few examples:

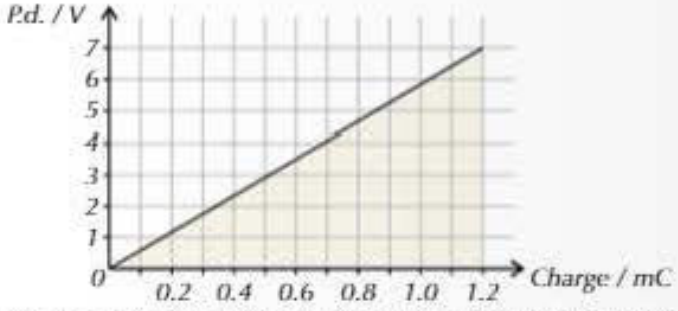
- The work done to move an object from one point to another in a radial gravitational field is the area under a graph of gravitational force against radial distance (see p.305).
- Energy stored by a capacitor is the area under a graph of V against Q (p.348-349).

To find an area under a graph, you'll either need to work it out exactly or estimate the area — it depends on the graph's shape. You'll need to estimate the area if the graph is not made up of straight lines.

Tip: You'll need to remember all the examples you've seen in this book.

Example — **Maths Skills**

A capacitor is charged and the p.d.-charge graph is shown below. Use the graph to find the energy stored by the capacitor when the p.d. across it reaches 12 V.



Energy stored (work done) = area under graph. The area is a triangle, so the energy stored $= \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times (1.2 \times 10^{-3}) \times 7$

$= 4.2 \times 10^{-3}$ J

Tip: This is a straight-line graph, which you've seen before on page 13. Remember, they have the form $y = mx + c$, where m is the gradient and c is the y-intercept (the value of y when $x = 0$).

Tip: The charge is given in mC, but you need it in C to be able to calculate the energy. So the change in charge is 0.7×10^{-3} C.

Example — **Maths Skills**

The graph shows how the gravitational force on an object changes with distance from a point mass. Find the work done to move an object from 1×10^5 m to 4×10^5 m away from the mass.

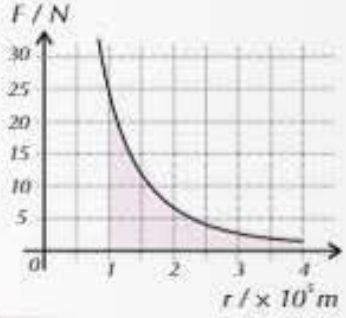
For a curved graph, you can estimate the area by counting the number of squares under the graph, which is approximately 8.

The area of one square

$= 5$ N $\times 0.5 \times 10^5$ m $= 2.5 \times 10^5$ J.

So the work done $= 2.5 \times 10^5 \times 8$

$= 2 \times 10^6$ J



Tip: There's more on gravitational forces on pages 297-307.

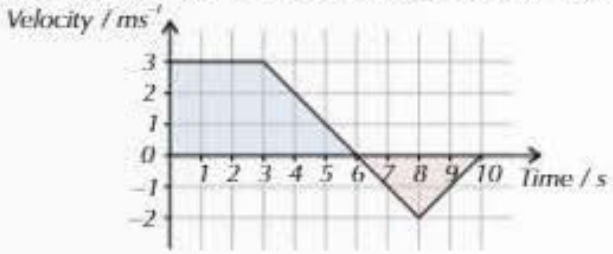
Tip: Draw a dot or a cross inside every square you count, to help you keep track of them.

Negative areas

You can end up with negative areas when the quantity on the vertical axis is a vector. For example, on a graph of velocity against time, the area between the curve and the horizontal axis is displacement. The velocity can be negative, so the area enclosed by the negative part of the curve and the horizontal axis is the 'negative area'. So displacement is negative (i.e. in the negative direction).

Example — **Maths Skills**

The velocity-time graph for a model train is shown below. The displacement of the train can be worked out from the area under the graph. The blue area indicates the 'positive area' and the red area indicates the 'negative area'.



Blue area = 13.5 m, red area = −4 m

So the total displacement (i.e. the displacement of the train from its starting position) = 13.5 − 4 = 9.5 m.

Tip: To find the area between the line and the horizontal axis, you could use the formula for a trapezium and a triangle, or split it up into a square and two triangles. The formulas for calculating the areas of common shapes are over on page 486.

Rates of change

A graph is a plot of how one variable changes with another. The rate of change of the variable on the vertical axis with respect to the variable on the horizontal axis at any point is given by the gradient of the graph.

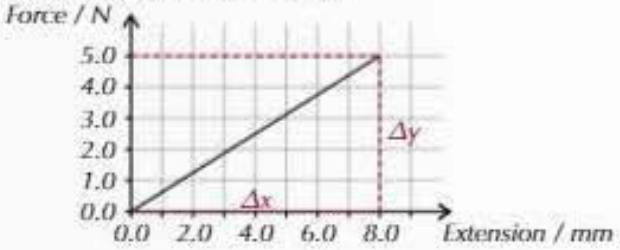
$$\text{Rate of change of } y \text{ with } x = \frac{\Delta y}{\Delta x} = \text{gradient of a } y\text{-}x \text{ graph.}$$

Often, the gradient represents a useful rate of change that you want to work out. For example, the gradient of a velocity-time graph is rate of change of velocity, which is acceleration.

For a linear graph, the gradient can just be found by calculating the change in y over the change in x between any two points.

Example — **Maths Skills**

The gradient of a force-extension graph for a spring obeying Hooke's law is equal to the spring constant of the spring.



The gradient of this graph is equal to: $\frac{\Delta y}{\Delta x} = \frac{5.0}{8.0 \times 10^{-3}} = 625$

So the spring constant of this spring = 625 Nm^{-1}.

Tip: Don't forget to check the units of the axes before you calculate the gradient. In this case, the extension is given in mm, so the change in x is 8×10^{-3}, not just 8.

Gradients of curved graphs

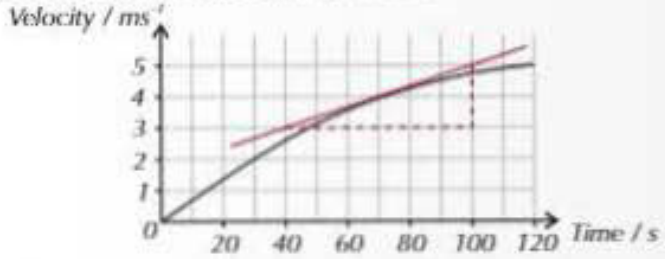
For a curved graph, the gradient, and so the rate of change, is always changing. So if you use the method on the previous page to calculate the gradient of a curved graph, you get the average gradient, and so the average rate of change.

To find the instantaneous rate of change (at a point), draw a **tangent** to the curve at the point where you want to know the gradient and find the gradient of the tangent.

Example — **Maths Skills**

The velocity-time graph of a cyclist is shown below.

a) Find the acceleration of the cyclist at 70 s.

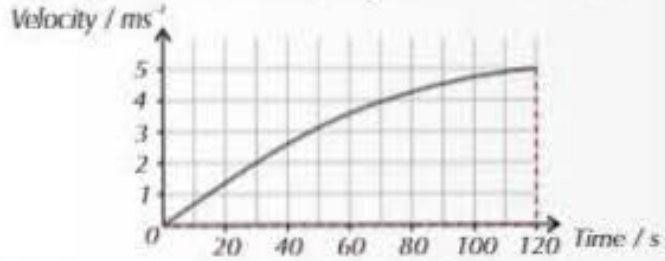


A tangent to the curve at 70 s is drawn on the graph with a gradient of:

$$\frac{\Delta y}{\Delta x} = \frac{5-3}{100-40} = \frac{2}{60} = 0.03333... = 0.03 \text{ (to 1 s.f.)}$$

So the acceleration at 70 s = 0.03 ms^{-2}

b) Find the average acceleration of the cyclist over the 120 s.



Over the 120 s, the average rate of change is:

$$\frac{\Delta y}{\Delta x} = \frac{5-0}{120-0} = \frac{5}{120} = 0.0416... = 0.04 \text{ (to 1 s.f.)}$$

So the average acceleration over 120 s = 0.04 ms^{-2}

Tip: When drawing a tangent, it helps to make it long — it will be easier to draw, and the tangent line will be more likely to intersect some grid lines, making the gradient easier to calculate.
You should aim to make the hypotenuse of the triangle you use to calculate the gradient at least half as long as the line on the graph.

Figure 1: *Make sure you use a really sharp pencil and a ruler whenever you're drawing graphs and tangents.*

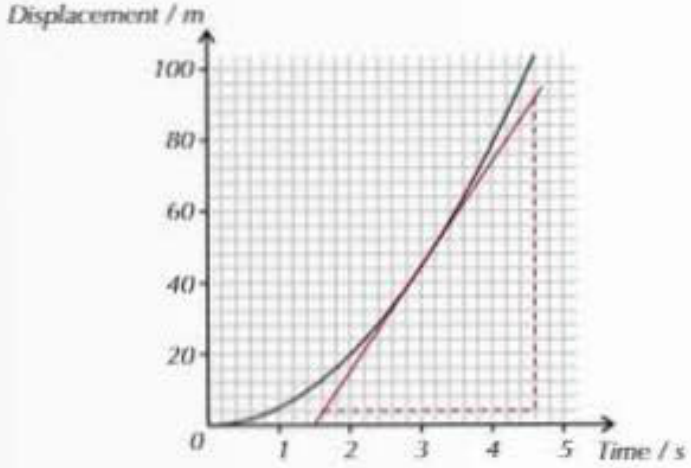
Rate of change of a gradient

The gradient is already a 'rate of change of something', so the rate of change of a gradient is the 'rate of change of the rate of change'. Sometimes these represent useful quantities too.

A common example of this is on a displacement-time graph. The rate of change of its gradient is equal to the acceleration.

Example — **Maths Skills**

The graph shows a displacement-time graph for an object falling from a height from rest. Its acceleration is due to gravity. You can use this graph to find the value for acceleration due to gravity, g — you just need to find the rate of change of the gradient.



The gradient (velocity) at $t = 0$ is 0 because the object was dropped from rest.

At 3 s, the gradient is found by drawing a tangent:

$$\text{Gradient at 3 s} = \frac{92 - 4}{4.6 - 1.6} = \frac{88}{3} = 29.333...$$

Rate of change of gradient between $t = 0$ s and $t = 3$ s:

$$\frac{\text{change in gradient}}{\text{change in } t} = \frac{29.333... - 0}{3 - 0} = \frac{29.333...}{3} = 9.777...$$
$$= 10 \text{ (to 1 s.f.)}$$

So the value of $g \approx 10$ ms^{-2} (to 1 s.f.).

Tip: The gradient of a displacement-time graph at a particular point is equal to the instantaneous velocity at that point. Finding the change in the gradient over time is the same as finding the change in the velocity over time — which is acceleration (see page 55).

Tip: The actual value of g is 9.81 ms^{-2}. It is found to be 9.777... ms^{-2} here because reading off a graph is not as accurate as calculating the value of g directly.

Modelling rates of change

You can use equations involving rates of change to model quantities with an iterative spreadsheet. If you know the rate of change of something, you can plot a graph for how it changes over time.

For example, acceleration $a = \frac{\Delta v}{\Delta t}$, so $\Delta v = a\Delta t$, where v is velocity and t is time. Consider the situation in which an object is fired vertically upwards with a known initial velocity, v_{init}. The acceleration is $-g$ (it's slowing down), so $\Delta v = -g\Delta t$. Since we know v_{init}, we can pick a small value of Δt and see how v changes over time. You can also see how displacement (s) varies.

- Set up a spreadsheet with column headings for t, v and s, as well as a single data input cell for each of Δt, v_{init} and Δv. Δv is calculated using the equation above, $\Delta v = -g\Delta t$.
- Decide on a Δt that you want to use, e.g. 0.1 s — this is the time interval between the values of v (and s) that the spreadsheet will calculate.
- You can then enter formulas into the spreadsheet to calculate the new velocity after each time interval. The displacement for each time interval is just the average velocity during the time interval, multiplied by the time interval Δt.

Tip: Make sure you're familiar with your spreadsheet program before you start trying to model anything. You'll need to know how to reference and do calculations involving cells.

Tip: Data input cells are the cells in which you write the variables that aren't changing, e.g. initial velocity, time interval, etc. Make sure the references to them in your formulas are fixed when you autofill the rows (iterations) later.

Tip: If you write the formulas properly, the spreadsheet can automatically fill them in for as many rows (iterations) as you want.

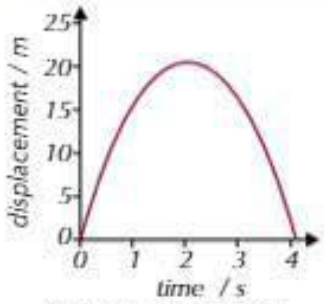


Figure 2: *A graph of **s** against t, for an object fired vertically with an initial velocity of 20 ms^{-1}.*

Data input cells	Δt	0.1 s
	$\boldsymbol{v}_{ini}$	E.g. 20 ms^{-1}
	$\Delta \boldsymbol{v}$	$-9.81 \times \Delta t$

***t* in s**	***v* in ms^{-1}**	***s* in m**
$t_0 = 0$	$\boldsymbol{v}_0 = \boldsymbol{v}_{ini}$	$\boldsymbol{s}_0 = 0$
$t_1 = t_0 + \Delta t$	$\boldsymbol{v}_1 = \boldsymbol{v}_0 + \Delta \boldsymbol{v}$	$\boldsymbol{s}_1 = \boldsymbol{s}_0 + \left(\frac{\boldsymbol{v}_1 + \boldsymbol{v}_0}{2} \times \Delta t\right)$
$t_2 = t_1 + \Delta t$	$\boldsymbol{v}_2 = \boldsymbol{v}_1 + \Delta \boldsymbol{v}$	$\boldsymbol{s}_2 = \boldsymbol{s}_1 + \left(\frac{\boldsymbol{v}_2 + \boldsymbol{v}_1}{2} \times \Delta t\right)$

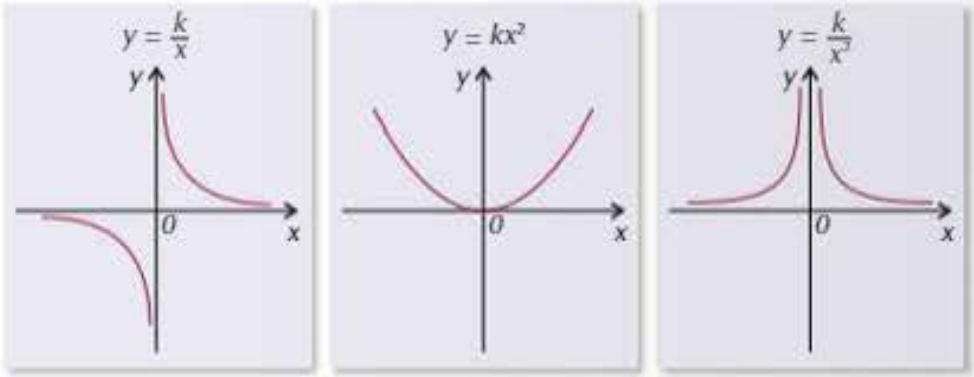
Figure 3: *An example of the formulas that can be used to create an iterative spreadsheet of velocity over time for an object with constant acceleration due to gravity.*

- You can then plot a graph of either **v** against *t* or **s** against *t* — see Figure 2.

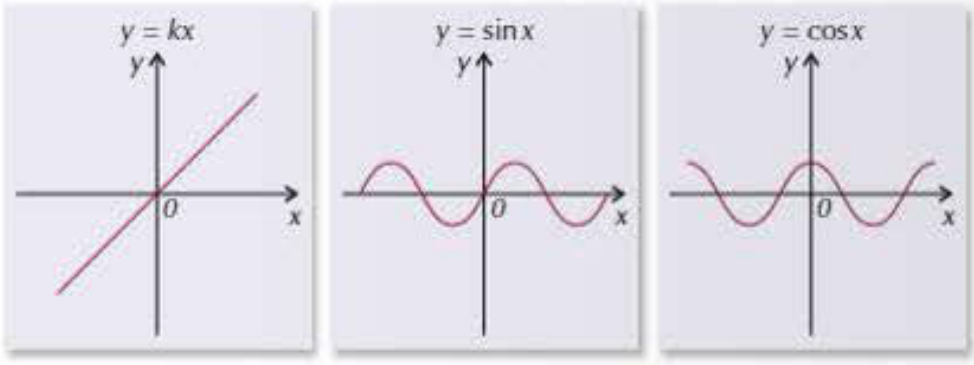
Sketching graphs

There are some graph shapes that crop up in Physics all the time. The following graphs are examples of the types of graphs you need to know how to recognise and sketch. *k* is constant in all cases.

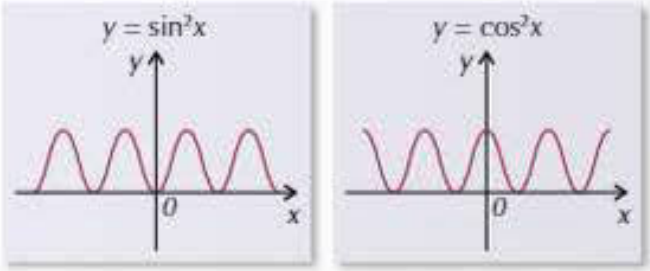
Tip: $R = \frac{\rho l}{A}$ on p.156 is an example of an equation of the form $y = \frac{k}{x}$, where $y = R$, $x = A$ and $k = \rho l$.



Tip: The *x*-*t*, *v*-*t* and *a*-*t* graphs for simple harmonic motion on page 281 are sin and cos graphs, e.g.: $x = A\cos(\omega t)$. The velocity and acceleration graphs can both be calculated from the displacement graph.



Tip: The graph for kinetic energy of an object in simple harmonic motion against time on p.283 has the same shape as the $y = \cos^2 x$ graph.



4. Exponential and Log Graphs

There are quite a few exponential relationships in A-level Physics, and you need to know what they'll look like as a graph. You also need to know how you can use logarithms to plot a seemingly complicated relationship as a nice straight line. Read on...

Exponential graphs

A fair few of the relationships you need to know about are exponential — where the rate of change of a quantity is proportional to the amount of the quantity left. Here are a few that crop up in the A-level course (if they don't ring a bell, go have a quick read about them)...

- Charge on a capacitor (p.360) — the rate of decay of charge on a discharging capacitor is proportional to the amount of charge left on the capacitor:

$$Q = Q_0 e^{\frac{-t}{CR}}$$

There are also exponential relationships for I and V, and for charging capacitors (see pages 360-361).

- Radioactive decay (p.435) — the rate of decay of a radioactive sample is proportional to the number of undecayed nuclei in the sample:

$$N = N_0 e^{-\lambda t}$$

The activity of a radioactive sample behaves in the same way.

- X-ray attenuation (p.458) — the reduction of intensity of X-rays over distance is proportional to their intensity:

$$I = I_0 e^{-\mu x}$$

Because the rate of change is proportional to the amount, exponential growth gets faster and faster as the amount gets bigger. Exponential decay is just the opposite of exponential growth — it gets slower and slower as the amount gets smaller. Because of this, exponential graphs have a characteristic shape that you need to be able to recognise and sketch:

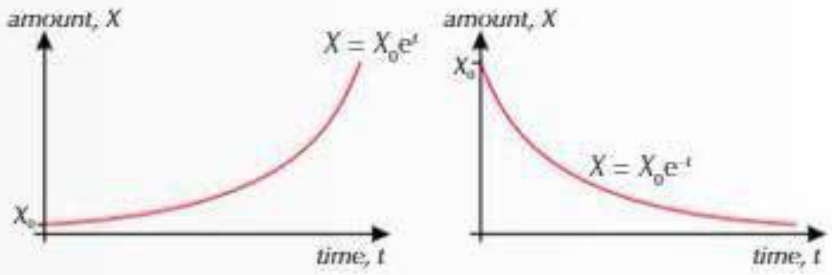


Figure 2: *(Left) The characteristic graph of exponential growth. The rate of growth gets faster and faster. (Right) The characteristic graph of exponential decay. The rate of decay gets slower and slower, never reaching zero.*

Tip: Don't forget, e is a constant equal to 2.718... — it'll be stored in your calculator. You can raise e to a power using the button $e^{\blacksquare}$ or e^{x} (see page 472).

Tip: These equations are all examples of exponential decay — see Figure 2.

Figure 1: *The growth of bacteria is a classic example of something that can be modelled with exponential growth. The more bacteria there are, the faster the rate of growth becomes.*

Tip: You should recognise the decay graph from the sections on capacitors (p.357) and nuclear decay (p.437).

Tip: Remember the log rules from p.472 — you'll need them here.

Log-linear graphs

You can plot an exponential relationship as a straight line using the natural log, ln. For the relationship $y = ke^{-ax}$, if you take the natural log of both sides of the equation you get:

$$\ln y = \ln (ke^{-ax}) = \ln k + \ln (e^{-ax}) \quad \text{so} \quad \ln y = \ln k - ax$$

Then all you need to do is plot (ln y) against x. You get a straight-line graph with (ln k) as the vertical intercept, and $-a$ as the gradient.

Tip: Graphs with the natural logarithm of a function on one of the axes are called log-linear graphs. There are examples of how they can be used on pages 363 and 435.

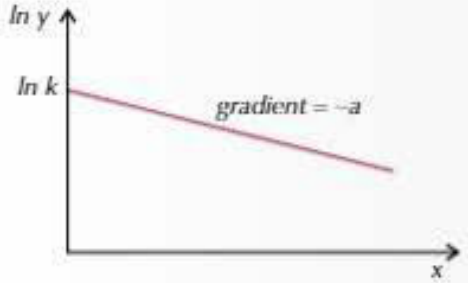


Figure 3: *The function $y = ke^{-ax}$ plotted on a log-linear graph of ln y against x.*

Log-log graphs

You can use logs to plot a straight-line graph for other relationships too. Say the relationship between two variables x and y is:

$$y = kx^n$$

Tip: This relationship is called a power law relationship.

Take the log (base 10) of both sides to get:

$$\log y = \log k + n \log x$$

So, if you plot log y against log x, log k will be the y-intercept and n will be the gradient of the graph.

Example — **Maths Skills**

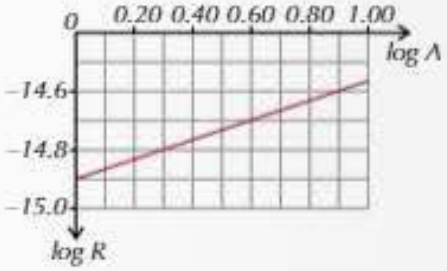
A physicist carries out an experiment to determine the nuclear radius, R (in m), of various elements with various nucleon numbers, A. She plots a line of best fit for her results on a graph of log R against log A. Part of the graph is shown. Using the equation $R = r_0 A^{1/3}$, find the value of the constant r_0 from the graph.

Tip: Look back at pages 415-416 for more on nuclear radius.

First take logs of both sides:

$$\log R = \log (r_0 A^{1/3}) = \log r_0 + \log A^{1/3}$$
$$= \log r_0 + \frac{1}{3} \log A$$

Comparing this to the equation of a straight line (in the form $y = mx + c$), you can see that the gradient of the graph is $\frac{1}{3}$ and the vertical intercept is $\log r_0$.



Tip: If $a = \log_{10} b$, then $b = 10^a$.

So, reading from the graph, the vertical intercept is about –14.9.

$\log r_0 = -14.9$, so $r_0 = 10^{-14.9} = 1.258... \times 10^{-15} = 1.3$ fm (to 2 s.f.)

5. Geometry and Trigonometry

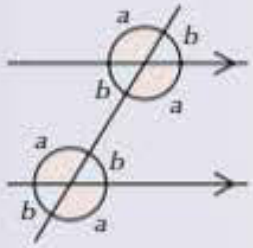
You'll often find that you need to deal with different 2D and 3D shapes in Physics. Sometimes you'll need to resolve forces, which could mean using all sorts of angle rules, as well as Pythagoras and trigonometry. Here's a recap on the basics.

Geometry basics

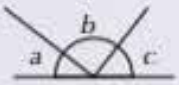
Angle rules

These angle rules should be familiar — make sure you're happy with them.

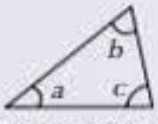
When two parallel lines are intercepted by the same line:



Vertically opposite angles are equal, and $a + b = 180°$



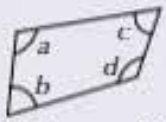
Angles on a straight line add up to 180°.



Angles in a triangle add up to 180°.

$a + b + c = 180°$

Angles around a point add up to 360°.



Angles in a quadrilateral add up to 360°.

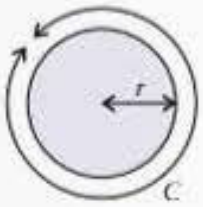
$a + b + c + d = 360°$

Angles can be measured in degrees or radians — make sure you know how to convert between them:

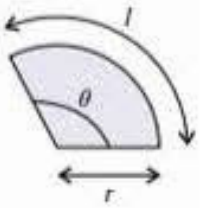
- To convert from degrees to radians, multiply by $\frac{\pi}{180°}$.
- To convert from radians to degrees, multiply by $\frac{180°}{\pi}$.

Circumference and arc length

You may need to calculate the distance around the edge of a circle (or part of it).



Circumference, $C = 2\pi r$



Arc length, $l = r\theta$, θ in radians

Tip: Remember, the arrows on the lines in the diagram mean that they're parallel.

Tip: You'll need to use radians when working with circular motion and simple harmonic motion.

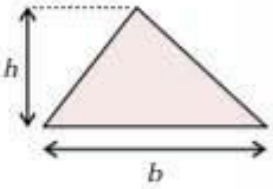
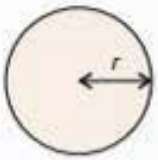
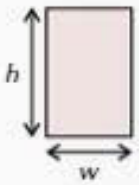
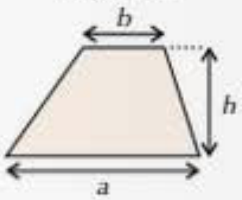
Exam Tip
Don't forget to put your calculator into either degrees or radians mode depending on what you're working in. It's a common mistake that could cost you marks.

Exam Tip
You'll be given these in the data and formulae booklet, just remember that θ is in radians.

Areas of shapes

Make sure you remember how to calculate the areas of these shapes:

Exam Tip
You'll be given the formulae for the area of a circle and the area of a trapezium in the data and formulae booklet.

Triangle	Circle	Rectangle	Trapezium
			
$A = \frac{1}{2} \times b \times h$	$A = \pi \times r^2$	$A = h \times w$	$A = \frac{1}{2} \times (a + b) \times h$

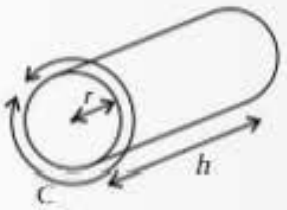
Surface areas

If you need to work out the surface area of a 3D shape, you just need to add up the areas of all the 2D faces of the shape. The exception to this is a sphere, where the surface area is given by $A = 4\pi r^2$ — this will be given to you in the data and formulae booklet.

Exam Tip
The curved surface area of a cylinder is given in the data and formulae booklet. This is the surface area of a hollow cylinder, i.e. with no ends. If you need to calculate a closed cylinder, you'll need to add the areas of the two circular ends ($2 \times \pi r^2$).

Example — **Maths Skills**

A piece of wire has the shape of a cylinder. It has radius 2.0 mm and length 0.020 m. Calculate the surface area of the wire.



The surface of a cylinder is made of two circles (the ends of the wire) and a rectangle with a width equal to the circumference of one of those circles (which is rolled to make the length of the wire).

Surface area = (2 × area of the circle) + (area of the rectangle)

$= (2 \times \pi r^2) + (2\pi r \times h)$

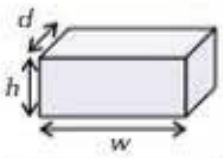
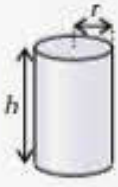
$= (2 \times \pi \times (2.0 \times 10^{-3})^2) + (2 \times \pi \times (2.0 \times 10^{-3}) \times 0.020)$

$= 2.7646... \times 10^{-4} = 2.8 \times 10^{-4}\ m^2$ (to 2 s.f.)

Volumes of shapes

Make sure you remember how to calculate the volumes of a cuboid, a sphere and a cylinder:

Exam Tip
You'll be given the volume of a sphere and a cylinder in the data and formulae booklet — hoorah.

		
$V = w \times h \times d$	$V = \frac{4}{3}\pi r^3$	$V = \pi r^2 h$

Example — **Maths Skills**

Estimate the density of a nucleus.

Protons and neutrons have nearly the same mass, $m_{nucleon} \approx 1.7 \times 10^{-27}$ kg.

A nucleus with a nucleon number A has mass $A \times m_{nucleon}$.

Assuming nuclei are spherical, the volume of a nucleus is given by $V = \frac{4}{3}\pi R^3$. The radius of a nucleus, R, is given by $R = r_0 A^{1/3}$.

Density = mass ÷ volume, so:

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{A \times m_{nucleon}}{\frac{4}{3}\pi R^3} = \frac{A \times m_{nucleon}}{\frac{4}{3}\pi (r_0 A^{1/3})^3} = \frac{A \times m_{nucleon}}{\frac{4}{3}\pi r_0^3 A} = \frac{3m_{nucleon}}{4\pi r_0^3}$$

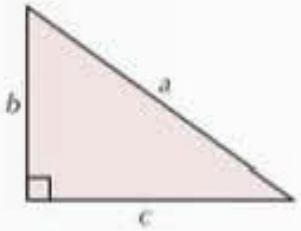
$$= \frac{3 \times 1.7 \times 10^{-27}}{4\pi \times (1.4 \times 10^{-15})^3} = 1.47... \times 10^{17} = 1.5 \times 10^{17} \text{ kg m}^{-3} \text{ (to 2 s.f.)}$$

Tip: r_0 is a constant, roughly equal to 1.4 fm — see page 416.

Trigonometry basics

Right-angled triangles

You can use Pythagoras' theorem for all right-angled triangles — the square of the hypotenuse is equal to the sum of the squares of the two smaller sides.



$$a^2 = b^2 + c^2$$

To work out a length or angle within a right-angled triangle, remember SOH CAH TOA (covered on page 42). There are plenty of examples of this being used to resolve vectors over on page 43. Just remember:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Figure 1: *You can find the buttons for sine (sin) cosine (cos) and tangent (tan) on your calculator as well as their inverse functions ($\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$).*

Small-angle approximations

For really small angles in radians, you can make the following assumptions for the values of sin, cos and tan:

$$\sin\theta \approx \theta \qquad \tan\theta \approx \theta \qquad \cos\theta \approx 1$$

This is really useful in Physics as the angles are often small (see pages 217, 221, 317 and 328).

Exam Tip

You'll be given these small-angle approximations in the data and formulae booklet.

Sine and cosine rules

If a triangle is not right angled, you need to use the sine and cosine rules to work out angles and side lengths.

Tip: These rules work on any triangle.

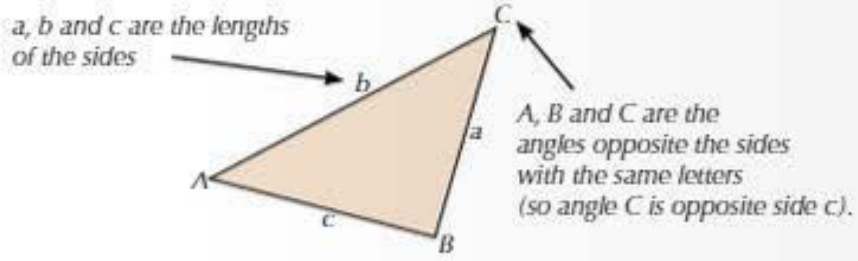
The sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The cosine rule:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

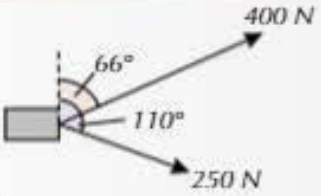
Tip: You can use any two bits of the sine rule to make a normal equation with just one = sign. The sine rule also works if you flip all the fractions upside down: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.


a, b and c are the lengths of the sides
C
b
a
A
c
B
A, B and C are the angles opposite the sides with the same letters (so angle C is opposite side c).

Which rule you can use depends on which sides and angles you already know.

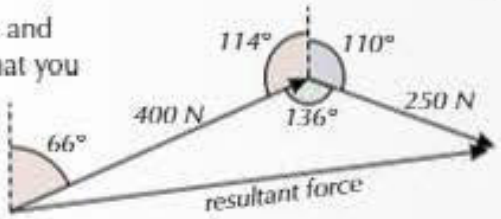
Example — Maths Skills

A cart is pulled by two horses, both pulling at different angles and with different forces, as shown. Calculate the magnitude and direction of the resultant force.



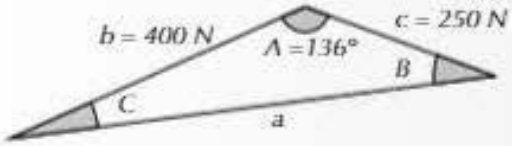
Tip: You'll often be required to represent 3D problems in 2D (e.g. with a force diagram) and solve them using these trigonometry rules.

Draw a vector triangle and work out any angles that you can with angle rules.



Tip: When working out angles, remember that the angles on a line must add up to 180° and the angles around a point must add up to 360°.

You can use the cosine rule, so label the angle that you know A.



The cosine rule gives: $a = \sqrt{b^2 + c^2 - 2bc\cos A}$

$= \sqrt{400^2 + 250^2 - 2 \times 400 \times 250 \times \cos 136°}$

$= 605.28... = 610\text{ N (to 2 s.f.)}$

Tip: You could have also re-labelled the triangle so that C was A instead, and used the cosine rule again.

You can now use the sine rule in the form $\frac{\sin C}{c} = \frac{\sin A}{a}$ to find C.

$C = \sin^{-1}\left(\frac{\sin A}{a} \times c\right) = \sin^{-1}\left(\frac{\sin 136°}{605.28...} \times 250\right)$

$= 16.673... = 17°\text{ (to 2 s.f.)}$

Tip: The angle from north is 66° + 17° = 83°.

So the resultant force is 610 N (to 2 s.f.) at a bearing of 083° (to 2 s.f.).

Exam Structure and Technique

Passing exams isn't all about revision — it really helps if you know how the exam is structured and have got your exam technique nailed so that you pick up every mark you can.

Figure 1: *The Room of Doom awaits you. But don't panic — prepare properly and there's no reason you can't ace the exam.*

Course structure

OCR A A-Level Physics is split into six modules, all of which are covered in this book:

Module 1 — Development of practical skills in physics

Module 2 — Foundations of physics

Module 3 — Forces and motion

Module 4 — Electrons, waves and photons

Module 5 — Newtonian world and astrophysics

Module 6 — Particles and medical physics

Exam structure

For OCR A A-Level Physics you're gonna have to sit through three exams at the end of your course:

- Paper 1 — Modelling Physics — 37% of the A Level
- Paper 2 — Exploring Physics — 37% of the A Level
- Paper 3 — Unified Physics — 26% of the A Level

Papers 1 and 2 are both 2 hours and 15 minutes long. Paper 3 is 1 hour and 30 minutes long. The exams will cover material from both years of your A-Level course, so you need to learn everything from Year 1 as well as Year 2. Papers 1 and 2 are worth 100 marks each, and Paper 3 is worth 70 marks.

- Paper 1 covers material from Modules 1, 2, 3 and 5. It's split up into two sections. Section A has 15 marks' worth of multiple choice questions. Section B has 85 marks' worth of short and extended answer questions.
- Paper 2 covers material from Modules 1, 2, 4 and 6. It's split up into two sections. Section A has 15 marks' worth of multiple choice questions. Section B has 85 marks' worth of short and extended answer questions.
- Paper 3 covers content from all six modules. It has 70 marks' worth of short and extended answer questions.

Exam Tip
Make sure you have a good read through this exam structure. It might not seem important now but you don't want to get any nasty surprises at the start of an exam.

Exam Tip
It can be easy to lose track of time in long exams. Make sure to ration your time between questions (p.491) and check the clock as you go to make sure you're not surprised at the end of the exam.

Command words

It sounds obvious, but it's really important you read each question carefully, and give an answer that fits. Look for command words in the question — they'll give you an idea of the kind of answer you should write.

Exam Tip
Make sure you read the whole question carefully. Sometimes they specify the details (or the level of detail) they want you to include in your answer. For example, they may ask something like "Describe how the apparatus can be used to accurately and safely determine the *I-V* characteristic of a filament lamp." Make sure you talk about accuracy and safety to get all of the marks.

Some command words, like calculate, draw and complete are pretty self-explanatory. But command words for written questions can be a bit trickier. Common command words for these questions are:

- State — give a definition, example or fact.
- Identify — pick out information from data provided in the question, or say what something is.
- Describe — don't waste time explaining why a process happens — that's not what the question is after. It just wants to know what happens.
- Explain — give reasons for why something happens, not just a description.
- Suggest/Predict — use your scientific knowledge to work out what the answer might be.
- Compare — make sure you relate the things you're comparing to each other. It's no good just listing details about each one, you need to say how these things are similar or different.
- Discuss — you'll need to include more detail. Depending on the question you could need to cover what happens, what the effects are, and perhaps include a brief explanation of why it happens.
- Justify — show or prove that something is correct.
- Evaluate — Give the arguments both for and against an issue, or the advantages and disadvantages of something. You also need to give an overall judgement.

Quality of extended responses

Exam Tip
Questions where you're marked on the quality of your extended response are marked with an asterisk (*) by the question.

For some extended answer questions, you'll be marked on the 'quality of your extended response'. These questions are designed to test how well you can put together a well structured and logical line of reasoning. They'll often require you to give a long answer in full written English, e.g. to explain, analyse or discuss something. To get top marks, you need to make sure that:

Exam Tip
Don't worry too much about quality of extended response questions. Really, you should be looking to give full and coherent explanations for every question.

- you answer the question and all the information you give is relevant to the question you've been asked,
- you back up your points with clear evidence using the data given to you in the question,
- you organise your answer clearly, coherently and in a sensible order,
- you use specialist scientific vocabulary where it's appropriate.

These questions could also involve other tasks, like a calculation or having to draw an experimental set-up, like a circuit. Make sure any drawings are clear and use correct symbols where appropriate. When doing calculations, make sure your working is laid out logically and it's clear how you've reached your answer. That includes making sure any estimates and assumptions you've made in your working are clearly stated, e.g. assuming air resistance is negligible.

Exam Tip
Whilst these questions don't test your handwriting, spelling or grammar, if the examiner can't read what you've written or understand the point you're trying to make, it will lose you marks.

There's usually a lot to think about with this type of question, and it can be easy to write down a lot of great and relevant physics but forget to answer all parts of the question. It's always a good idea to double check you've done everything a question has asked you to do before moving on.

Time management

This is one of the most important exam skills to have. How long you spend on each question is really important in an exam — it could make all the difference to your grade.

Everyone has their own method of getting through the exam. Some people find it easier to go through the paper question by question and some people like to do the questions they find easiest first. The most important thing is to find out the way that suits you best before the exam — and that means doing all the practice exams you can before the big day.

Check out the exam timings given by OCR that can be found on page 489 and on the front of your exam paper. These timings give you just over 1 minute per mark.

However, some questions will require lots of work for only a few marks and other questions will be much quicker. So don't spend ages struggling with questions that are only worth a couple of marks — move on. You can come back to them later when you've bagged loads of other marks elsewhere.

It's worth keeping in mind that the multiple choice questions in Paper 1 and Paper 2 are all only worth 1 mark, even though some of them could be quite tricky and time-consuming. Don't make the mistake of spending too much time on these. If you're struggling with some of them, move on to the written answer questions where there are more marks available and then go back to the harder multiple choice questions later.

If you find that you're running out of time, go through the remaining questions and jot down what you can — for some of them you may get a mark for recalling the correct equation, or stating a definition. And if you've left any multiple choice questions, with only minutes to go, just make your best guess — you've got a 1 in 4 chance of getting an extra mark.

Exam Tip
Make sure you read the rest of the information given on the front of the exam paper before you start. It'll help make sure you're well prepared.

Exam Tip
Don't forget to go back and do any questions that you left the first time round — you don't want to miss out on marks because you forgot to do a question.

Exam Tip
When you're doing practice papers, set yourself a time limit so you get used to the exam timings.

Strange questions

You may get some weird questions that seem to have nothing to do with anything you've learnt. DON'T PANIC. Every question will be something you can answer using physics you know, it just may be in a new context.

Check the question for any key words that you recognise. For example, if a question talks about acceleration, think about the rules and equations you know, and whether any of them apply to the situation in the question. Sometimes you might have to pull together ideas from different parts of physics — read the question and try to think about what physics is being used. That way you can list any equations or facts you know to do with that topic and try to use them to answer the question.

Exam data and formulae booklet

When you sit your exams, you'll be given a data and formulae booklet as an insert within the exam paper. On it you'll find a lot of equations from the course, but not all of them. Make sure you know which equations you'll be given, and which you need to learn off by heart for the exam. There's also some useful data in the booklet to help you with your exam, including...

- the Planck constant, h
- the gravitational constant, G
- the length of a light year in metres, m

Constants that you'll be given in the exam are listed on page 572.

Exam Tip
If you're stuck on a question, have a quick look through the data and formulae booklet — you may find an equation that helps.

Answers

Module 1

Development of Practical Skills

1. Planning an Experiment

Page 8 — Application Question

Q1 a) Independent variable — angle of the ramp
Dependent variable — the final velocity of trolley

b) Any one from: e.g. the distance the trolley travels / the initial velocity of the trolley.

Page 8 — Fact Recall Questions

Q1 To make sure that it is only the independent variable that is affecting the dependent variable, otherwise the results won't be valid.

Q2 An experiment that is carried out using the same equipment before the real experiment. It is used to roughly work out the effect that changing the independent variable will have on the dependent variable, and to help choose appropriate units, range and increments of measurement.

Q3 Any three from: whether the experiment tests what it was meant to test / if the method is clear enough for someone else to follow / whether all control variables are going to be kept constant / whether the apparatus and techniques are appropriate for the investigation / if the method gives precise results / that the experiment is going to be carried out safely.

2. Recording and Processing Data

Page 10 — Application Questions

Q1 a) $4.53 \times 3.142 = 14.23326 =$ **14.2 (to 3 s.f.)**
The number with the lowest number of significant figures in this calculation is 4.53, which is to 3 significant figures. So the answer must also be to 3 significant figures.

b) $0.315 \div 0.025 = 12.6 =$ **13 (to 2 s.f.)**
The number to the lowest number of significant figures in this calculation is 0.025, which is to 2 significant figures.

Q2 Extension at weight of 2.0 N:
Mean = $(2.9 + 3.2 + 3.1) \div 3 = 3.066... =$ **3.1 cm (to 2 s.f.)**
Extension at weight of 4.0 N:
Mean = $(5.8 + 5.9) \div 2 = 5.85 =$ **5.9 cm (to 2 s.f.)**
Extension at weight of 6.0 N:
Mean = $(8.4 + 8.4 + 8.5) \div 3 = 8.433... =$ **8.4 cm (to 2 s.f.)**
The result for trial 2 of the 4.0 N weight is an anomaly and so should be left out of the calculation for the mean. All the answers should be given to 2 s.f. as the data is only to 2 s.f.

Page 10 — Fact Recall Questions

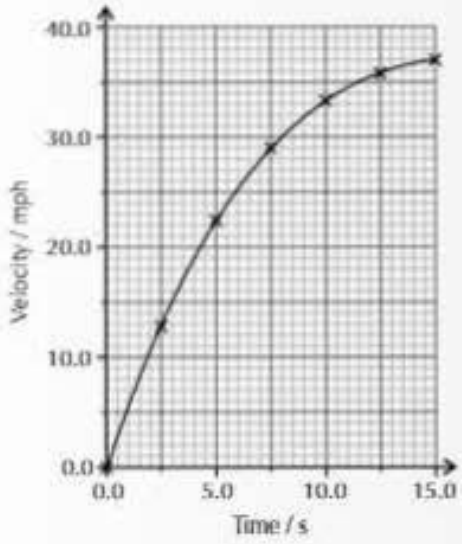
Q1 Data with a numerical value.

Q2 A result that doesn't fit in with the rest of the results and is likely to be wrong.

3. Presenting Results

Page 12 — Application Question

Q1



Page 12 — Fact Recall Questions

Q1 You can present categoric data in a pie chart / bar chart.

Q2 The independent variable.

4. Analysing Results

Page 15 — Application Question

Q1 a) Negative correlation.

b) Rearrange the equation to match $y = mx + c$.
$V = -rI + \varepsilon$, so, y-intercept, $c = \varepsilon$
so, $\varepsilon =$ **16 V**

c) $V = -rI + \varepsilon$, so gradient, $m = -r$
gradient $= \frac{\Delta y}{\Delta x} = \frac{-16}{20} = -0.8\ \Omega$
$r = -m = -(-0.8) =$ **0.8 Ω**

5. Evaluating and Drawing Conclusions

Page 20 — Application Questions

Q1 a) **±0.1 g**

b) (0.1 ÷ 50) × 100 = **0.2%**

Q2 a) **±0.05 A**

The current is given to 2 significant figures, so the assumed uncertainty is half of 0.1 A.

b) (3.4 + 3.6 + 4.0 + 3.9 + 3.6) ÷ 5 = **3.7 V**

Q3 a) The results are not valid as the temperature changed during the experiment.

b) Any two from: e.g. take more repeats for each length / take readings over a greater range of wire lengths / keep the temperature the same throughout the experiment / use more sensitive apparatus to measure the diameter of the wire.

Page 20 — Fact Recall Questions

Q1 The uncertainty given as a percentage of the measurement.

Q2 Any two from: e.g. by taking more repeats / by using more sensitive equipment for recording results / by using more appropriate equipment for recording results.

Q3 Precision depends on the amount of spread of your data from the mean of your results. Accuracy is how close your results are to the true value.

Exam-style Questions — Page 22

1 D ***(1 mark)***

2 a) Absolute uncertainty = ± 0.5 °C

As a percentage of 90:

(0.5 ÷ 90) × 100 = 0.555... = **0.6% (to 1 s.f.)**

(2 marks for the correct answer, otherwise 1 mark for giving the absolute uncertainty)

b) Mean of the current:

2.8 A is an anomaly, and so shouldn't be included in the calculation.

(1.9 + 1.8 + 2.0 + 2.0 + 2.1) ÷ 5 = 1.96 = 2.0 A (to 2 s.f.)

Range of values = 2.1 − 1.8 = 0.3 A

So the uncertainty = 0.15 A

So the mean current = **2.0 ± 0.15 A**

(3 marks for the correct answer, otherwise 1 mark for using the correct equation to calculate the mean and 1 mark for using the range of values to calculate the uncertainty)

c) How to grade your answer (pick the description that best matches your answer):

0 marks: There is no relevant information.

1-2 marks: At least one point from evaluation and one point from improvements have been included in the answer. Answer is basic and has lack of structure. Information is missing and there are few backed up arguments.

3-4 marks: Point 1 or 2 and point 3, 4 or 5 from evaluations is included in the answer. At least two points from improvements have been included in the answer. Answer has some structure, and information is presented with a clear line of reasoning, with arguments that are partially backed up.

5-6 marks: Point 1 or 2, point 3 or 4 and point 5 from evaluations is included in the answer. Points 1 or 2 and at least one other point from improvements have been included in the answer. The answer is well structured. There is a clear and logical line of reasoning, with information and arguments that are fully backed up.

Here are some points your answer may include:

Evaluation

1. She took a large number of repeat readings for the current at each temperature, allowing her to calculate a more precise mean.
2. The set of current values for each water temperature are all quite close to the mean, which shows that the results are quite precise/repeatable.
3. One limitation is that the potential difference of the power supply may not be the same as the potential difference across the thermistor, as there will be some resistance in the ammeter and the wires. This means their calculations of the resistance will be inaccurate/ there will be a systematic error in the results.
4. The experiment does not answer the original question of how the resistance of the thermistor varies with temperature as the potential difference across the thermistor was not monitored or controlled.
5. The intervals between the temperatures are quite large, and the student has not investigated a large enough range of temperatures. So it would not be possible to determine the type of mathematical relationship between resistance and temperature (e.g. linear or non-linear).

Improvements

1. To monitor and control the potential difference across the thermistor, a voltmeter could be connected in parallel to the thermistor, and a variable resistor could be connected in series with the thermistor. This would give a more accurate calculation of resistance.
2. The student should take more readings for temperatures between 50 °C and 90 °C, e.g. at 5 °C intervals.
3. The student could use an ammeter with a higher resolution to reduce the uncertainty in each measurement.
4. A thermometer with a higher resolution could be used, or a digital thermometer to reduce any human error in the readings.

Module 2

Foundations of Physics

1. Quantities and Units

Page 33 — Application Questions

Q1 $0.3 \times (1 \times 10^{12}) =$ **300 000 000 000 Hz** ($= 3 \times 10^{11}$ Hz)

Q2 Mass, m, is measured in kilograms, kg
Velocity, v = distance ÷ time
Distance is measured in metres, m, time is measured in seconds, s, so velocity is measured in m ÷ s, or ms^{-1}.
$p = mv$, so the units of momentum = kg × ms^{-1} = $kgms^{-1}$

Page 33 — Fact Recall Questions

Q1 kelvin / K

Q2 Any two from: e.g. J (joule) / Ω (ohm) / V (volt) / N (newton) / ms^{-1} / $kgms^{-1}$

Q3 a) 1×10^{-12}
b) 0.01 or 1×10^{-2}
c) 1×10^{6}

2. Making Estimates

Page 35 — Application Questions

Q1 a) A room is about the height of one-and-a-half people.
A person is around 1.8 m in height.
One room = 1.5 × 1.8 = 2.7 m.
A two-storey house is as tall as 2 rooms, plus the height of the roof and any material between rooms.
So a two-storey house is about as tall as 3 rooms.
3 × 2.7 = 8.1 m.
So a two-storey house is about 8 m tall (to 1 s.f.).

b) The mass of a person is about 70 kg. Mass of a child is about half the mass of an adult, so 70 ÷ 2 = 35 kg.
Gravitational potential energy = 35 × 9.81 × 3
= 1030.05 J
So the gravitational potential energy of the child is about 1000 J (to 1 s.f.).

Q2 The actual value of the change in velocity (over a given time) was larger, so the actual acceleration of the ball would be larger. This means that the actual force on the ball would be larger than the one calculated.

3. Errors and Uncertainties

Page 38 — Application Questions

Q1 $v = l\lambda = 125 \times 2.72 = 340$ ms^{-1}
The uncertainty is found by adding together the percentage uncertainties.
1% + 1.5% = 2.5%
So $v =$ **340 ± 2.5% ms^{-1}**

Q2 volume = $l^3 = 4.0^3 = 64$ cm^3
Now find the percentage uncertainty in l:
(0.05 ÷ 4.0) × 100 = 1.25%
Multiply this by the power to find the uncertainty in l^3:
1.25% × 3 = 3.75%
So volume = **64 ± 3.75% cm^3**

Page 38 — Fact Recall Questions

Q1 a) E.g. taking repeated measurements and averaging the results / using more sensitive apparatus.
b) E.g. calibrating equipment.

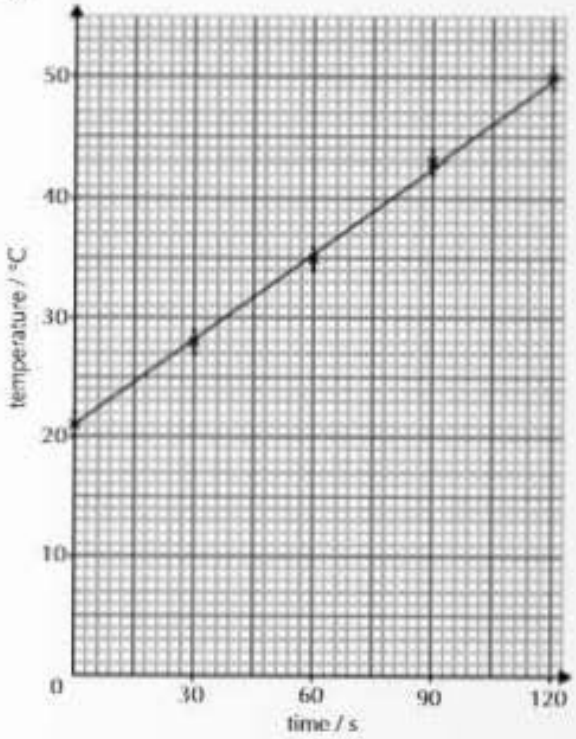
Q2 Add together their absolute uncertainties.

Q3 $$\text{percentage difference} = \frac{\text{experimental value} - \text{accepted value}}{\text{accepted value}} \times 100$$
Percentage difference shows the accuracy of an experimental result.

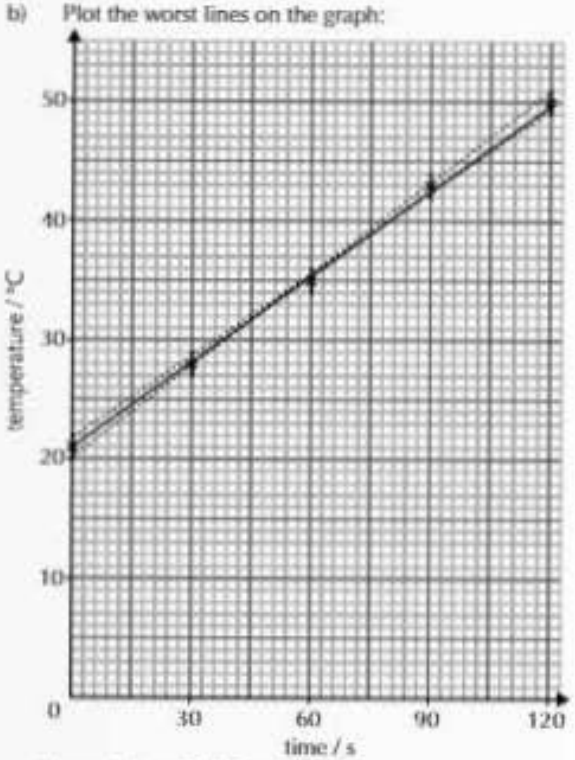
4. Graphical Representations of Uncertainties

Page 40 — Application Question

Q1 a)



b) Plot the worst lines on the graph:



The gradient of the line of best fit
= (49.5 − 21) ÷ 120 = 0.2375
The gradient of the steepest worst line
= (51 − 20) ÷ 120 = 0.2583...
The gradient of the shallowest worst line
= (49 − 22) ÷ 120 = 0.225
The difference between the gradient of the steepest worst line and the line of best fit is 0.2583... − 0.2375 = 0.0208...
The difference between the line of best fit and the gradient of the shallowest worst line is 0.2375 − 0.225 = 0.0125
The result for the steepest worst line is larger, so take this as the uncertainty in the final result.
So the gradient = **0.24 ± 0.02 °C s^{-1} (to 2 s.f.)**

The results in the table are to 2 s.f., so the gradient should be given to 2 s.f. too. You should then quote the uncertainty rounded to the same number of decimal places as the rounded value of the gradient.

5. Scalars and Vectors

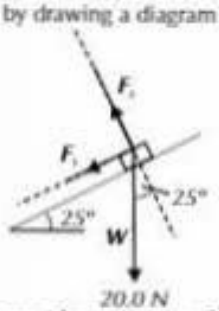
Page 45 — Application Questions

Q1 $v = \sqrt{2.0^2 + 0.75^2} = 2.136... = 2.1\ \text{ms}^{-1}$ (to 2 s.f.)
$\theta = \tan^{-1}\frac{0.75}{2.0} = 20.556... = 21°$ (to 2 s.f.)
So the resultant velocity is **2.1 ms^{-1} at 21° down from the horizontal**.
Once you've found the magnitude of the vector you could use sin or cos to find the angle θ too, but it's safer to use tan as you know the values of the opposite and adjacent sides are correct.

Q2 Horizontal component $F_x = F\cos\theta = 12\cos 56° = 6.710...$
= 6.7 N (to 2 s.f.)
Vertical component $F_y = F\sin\theta = 12\sin 56° = 9.948...$
= 9.9 N (to 2 s.f.)

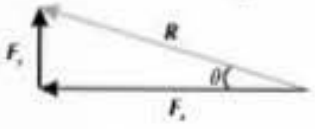
Q3 Start by drawing a scale diagram with an appropriate scale (e.g. 5.0 cm = 1.0 N). Then measure the side and angle:
The resultant force is 5.0 N on a bearing of 055°.

Q4 Start by drawing a diagram to show the forces on the brick:



Then use trigonometry to find the size of F_1 and F_2:
a) $F_1 = \sin 25° \times 20.0 = 8.452...$ **= 8.5 N (to 2 s.f.)**
b) $F_2 = \cos 25° \times 20.0 = 18.126...$ **= 18 N (to 2 s.f.)**

Q5 Let force **A** be the 22 N force, and force **B** be the 18 N force.
Resolving force **A**:
$A_x = 22\cos 40° = 16.85...$
$A_y = 22\sin 40° = 14.14...$
Resolving force **B**:
$B_x = 18\cos 50° = 11.57...$
$B_y = 18\sin 50° = 13.78...$
Then calculate the resultant horizontal component:
$F_x = A_x - B_x = 16.85... - 11.57... = 5.28...$
Calculate the resultant vertical component:
$F_y = A_y - B_y = 14.41... - 13.78... = 0.35...$
Find the magnitude of the resultant force:
$R = \sqrt{F_x^2 + F_y^2} = \sqrt{5.28...^2 + 0.35...^2} = 5.29...$
Now draw a diagram to help you find the angle of the resultant force on the toy:



$\tan\theta$ = opposite ÷ adjacent, so $\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$
$\theta = \tan^{-1}\left(\frac{0.35}{5.28...}\right) = 3.817...°$
So the resultant force is **5.3 N (to 2 s.f.) to the left at an angle of 3.8° (to 2 s.f.) above the horizontal**.

Page 45 — Fact Recall Questions

Q1 A scalar quantity has only a magnitude (size), while a vector quantity has size and direction.

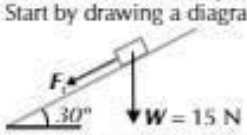
Q2 The resultant (vector).

Exam-style Questions — Page 47

1 D ***(1 mark)***

The error bar for the measurement of 0.4 V shows an absolute uncertainty of ± 0.05 V. (0.05 ÷ 0.4) × 100 = 12.5%.

2 a) Start by drawing a diagram to show the forces:



Component down the slope, $F_1 = W\sin\theta$
$= 15\sin 30 = 7.5$ N

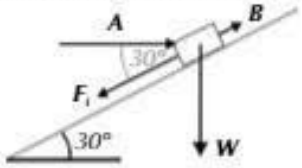
The percentage uncertainty in the weight is:
(0.5 ÷ 15) × 100 = 3.33...%

This is equal to the percentage uncertainty in the component of the box's weight that acts down the slope.

So the component = **7.5 ± 3.3% N (to 2 s.f.)**

(3 marks for correct answer and uncertainty, otherwise 1 mark for correct answer without uncertainty, and 1 mark for correct method of calculation)

b) Start by drawing a diagram to show the forces:



For the box to have a resultant force of 3.0 N up the slope, the component of ***A*** up the slope minus F_1 must equal ***B***.

The component of ***A*** up the slope = $23\cos 30°$

The angle between A and the slope is 30° because for two parallel lines that are being intersected, the alternate angles are equal.

$F_1 = W\sin\theta = W\sin 30°$,
$B = 3.0$ N
so $23\cos 30° - W\sin 30° = 3.0$
$W = (23\cos 30° - 3.0) \div \sin 30°$
$= 33.83...$ N = **34 N (to 2 s.f.)**

(3 marks for correct answer, otherwise 1 mark for correct resolution of force A up the slope and 1 mark for correct calculation of W)

3 a) It is a systematic error ***(1 mark)***. Random errors cannot be predicted and cause results to change in unpredictable ways. Systematic errors usually shift all of the measurements by the same amount ***(1 mark)***.

b) 3 cm = 0.03 m

force constant = force ÷ extension = 60 ÷ 0.03
$= 2000$ Nm^{-1}

Now find the percentage uncertainties in both force and extension:

% uncertainty in force = (5 ÷ 60) × 100 = 8.33... %
% uncertainty in extension = (0.05 ÷ 3) × 100 = 1.66...%

Add the percentage uncertainties to find the uncertainty in the force constant:

8.33...% + 1.66...% = 10 %

So force constant = **2000 ± 10% Nm^{-1}**

(4 marks for correct answer, otherwise 1 mark for correct force constant, 1 mark for correct percentage uncertainties of force and extension and 1 mark for correct method used to combine uncertainties)

Module 3

Section 1 — Motion

1. Constant Acceleration

Page 49 — Application Questions

Q1 $u = 0$ ms^{-1} $v = 10$ ms^{-1} $t = 20$ s $a = ?$ $s = ?$
so use $s = \frac{1}{2}(u + v)t \Rightarrow s = \frac{1}{2}(0 + 10) \times 20 =$ **100 m**

Q2 $u = 25$ ms^{-1} $v = 0$ ms^{-1} $t = 18$ s $a = ?$ $s = ?$
$v = u + at \Rightarrow a = \frac{v - u}{t} = \frac{0 - 25}{18} = -1.3888...$
So the deceleration is **1.4 ms^{-2} (to 2 s.f.)**.

Q3 $s = 103$ m $u = 0$ ms^{-1} $t = 9.2$ s $v = ?$ $a = ?$

$s = ut + \frac{1}{2}at^2 \Rightarrow a = \frac{s - ut}{\frac{1}{2}t^2} = \frac{103 - (0 \times 9.2)}{\frac{1}{2} \times 9.2^2}$
$= 2.4338... =$ **2.4 ms^{-2} (to 2 s.f.)**

Page 49 — Fact Recall Questions

Q1 a) The velocity of an object is its rate of change of displacement.

b) The acceleration of an object is its rate of change of velocity.

Q2 The equations are:
$v = u + at$
$s = \frac{1}{2}(u + v)t$
$s = ut + \frac{1}{2}at^2$
$v^2 = u^2 + 2as$

2. Acceleration Due To Gravity

Page 52 — Application Questions

Q1 $s = \frac{1}{2}gt^2 = \frac{1}{2} \times -9.81 \times 6.19^2 = -187.940...$

So height = **188 m (to 3 s.f.)**

Height is a scalar quantity so you don't need to worry about the minus signs here.

Q2 $v^2 = u^2 + 2as \Rightarrow v = \sqrt{0^2 + (2 \times -9.81 \times -6.83)}$
$= (-)11.576...$ ms^{-1}

So it's travelling at **11.6 ms^{-1} (to 3 s.f.)**.

Q3 E.g. use a small, heavy ball to negate the effect of air resistance and use a mechanism to automatically release and time the ball bearing's fall.

Page 52 — Fact Recall Questions

Q1 The only force present in free fall motion is weight.

Q2 a) E.g. Measure the length of time it takes for a metal ball bearing to fall a known distance using a switch to release the ball bearing from an electromagnet and start a timer, and a trap door to catch the ball bearing and stop the timer. Measure the height h from the bottom of the ball bearing to the trapdoor. Flick the switch to simultaneously start the timer and disconnect the electromagnet, releasing the ball bearing. The ball bearing will fall, knocking the trapdoor down and breaking the circuit — which stops the timer. Record the time t shown on the timer. Repeat this experiment three times and average the time taken to fall from this height. Repeat this experiment but drop the ball bearing from several different heights.

b) For each height, rearrange $h = \frac{1}{2}gt^2$ to get $g = 2h \div t^2$ then substitute in the height and average time that you have measured.

3. Projectile Motion

Page 54 — Application Questions

Q1 First find how long it's in the air by considering only the vertical velocity:

$s = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2s}{g}} = \sqrt{\frac{2 \times -1.61}{-9.81}} = 0.5729...$ s

Then find out how far it travels in this time:

$s = ut = 502 \times 0.5729... = 287.60...$ = **288 m (to 3 s.f.)**

Remember there's no acceleration in the horizontal direction.

Q2 a) Consider only the vertical velocity:

$u_v = \sin 60.0° \times 25 = 21.65...$ ms^{-1}

$v_v^2 = u_v^2 + 2gs$

$\Rightarrow v_v = \sqrt{21.65...^2 + 2 \times (-9.81) \times (0)} = -21.65...$ ms^{-1}

The rock will be moving towards the ground, so it will have a negative final velocity.

$v_v = u_v + at \quad \Rightarrow t = \frac{v_v - u_v}{a} = \frac{-21.65... - 21.65...}{-9.81}$

$= 4.413...$ = **4.4 s (to 2 s.f)**

b) Consider the horizontal velocity:

$u_h = \cos 60.0° \times 25 = 12.5$ ms^{-1}

So $s = u_h t = 12.5 \times 4.413... = 55.174...$ = **55 m (to 2 s.f.)**

Q3 The vertical velocity, $u_v = \sin 31.5° \times 12.1 = 6.322...$ ms^{-1}

At the highest point, the ball's vertical velocity will be 0 ms^{-1}.

$v_v = 0$ ms^{-1}, $a = -9.81$ ms^{-2}.

$v_v^2 = u_v^2 + 2as$, so $s = \frac{v_v^2 - u_v^2}{2a} = \frac{0^2 - 6.322...^2}{2 \times -9.81} = 2.0372...$ m

Total height above ground = $2.0372... + 4.20 = 6.2372...$

= **6.24 m (to 3 s.f.)**

Page 54 — Fact Recall Questions

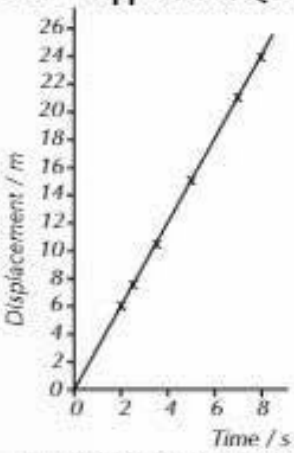
Q1 Free fall motion with an initial velocity is called projectile motion.

Q2 Resolve the initial velocity into horizontal and vertical components, then use the vertical component to work out how long it's in the air and/or how high it goes. Then use the horizontal component to work out how far it goes in the horizontal direction while it's in the air.

4. Displacement-Time Graphs

Page 58 — Application Questions

Q1



Q2 a) The cyclist is moving forwards at a constant velocity.

b) The cyclist is moving forwards but decelerating.

c) The cyclist isn't moving.

d) The cyclist is moving back towards the starting point at a constant velocity.

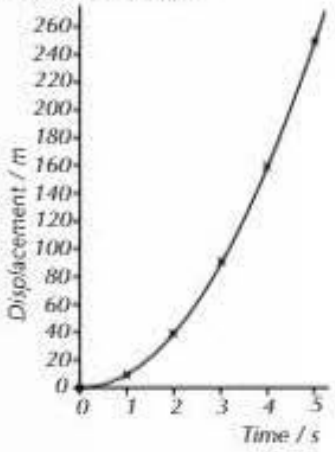
Q3 First find the rocket's acceleration:

$v = u + at \Rightarrow a = \frac{v - u}{t} = \frac{100.0 - 0}{5.0} = 20$ ms^{-2}

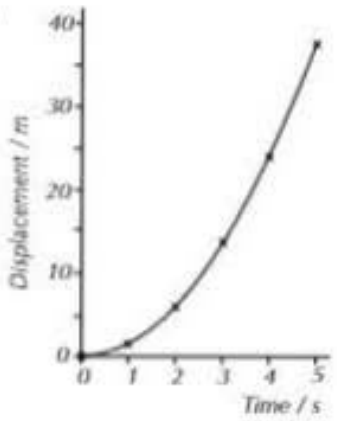
Then work out the rocket's displacement at intervals with $s = ut + \frac{1}{2}at^2$:

t (s)	s (m)
0	0
1	10
2	40
3	90
4	160
5	250

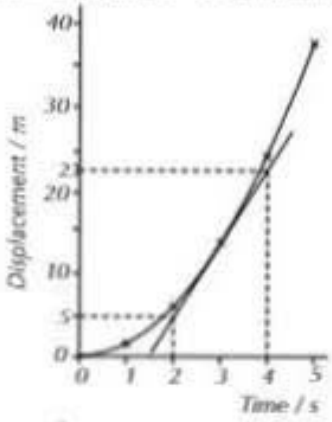
Then plot the graph:



Q4 a)



b) Draw a tangent at $t = 3.0$ and measure its gradient:



$$v = \frac{\Delta y}{\Delta x} = \frac{23.0 - 5.0}{4.0 - 2.0} = \frac{18.0}{2.0} = \mathbf{9.0\ ms^{-1}}$$

Page 59 — Fact Recall Questions

Q1 A curved line on a displacement-time graph shows acceleration (or deceleration).

Q2 A straight line on a displacement-time graph shows constant velocity.

Q3 Acceleration.

Q4 Instantaneous velocity.

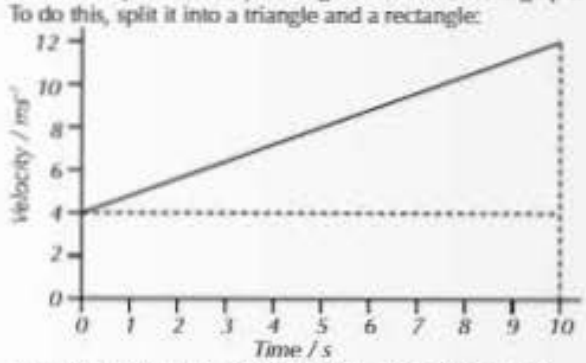
Q5 Instantaneous velocity is an object's velocity at a particular moment in time, average velocity is the overall displacement divided by time taken.

Q6 Divide the overall change in displacement by time taken.

5. Velocity-Time Graphs

Page 63 — Application Question

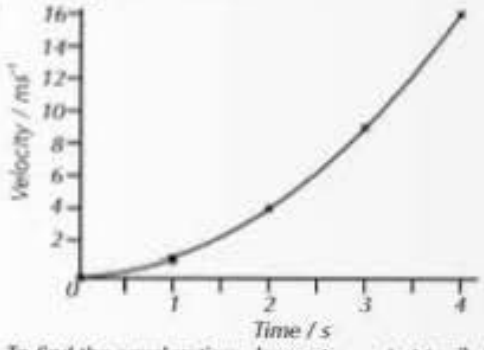
Q1 Find the displacement by finding the area under the graph. To do this, split it into a triangle and a rectangle:



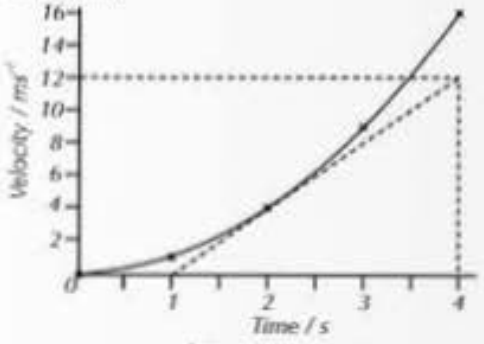
Area of triangle = ½ × base × height = ½ × 10 × 8 = 40
Area of rectangle = base × height = 10 × 4 = 40
So displacement = 40 + 40 = **80 m**
You could also treat it as a trapezium and work out the area directly using the formula for the area of a trapezium: area = ½(a + b) × h

Page 65 — Application Questions

Q1 a)

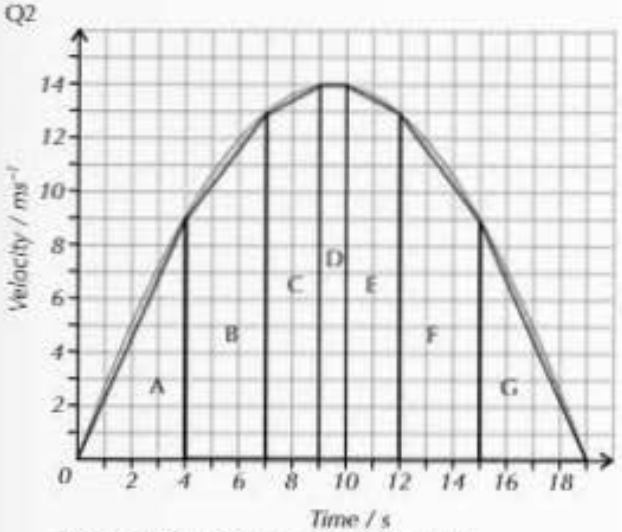


b) To find the acceleration, draw a tangent at $t = 2$ and find its gradient:



$$\text{Acceleration} = \frac{\Delta y}{\Delta x} = \frac{12 - 0}{4 - 1} = \frac{12}{3} = \mathbf{4\ ms^{-2}}$$

Q2



Area of first triangle (A) = ½(4 × 9) = 18 m
Area of first trapezium (B) = ½(9 + 13) × 3
= 33 m
Area of second trapezium (C) = ½(13 + 14) × 2
= 27 m
Area of rectangle (D) = 14 × 1 = 14 m
Area of third trapezium (E) = ½(14 + 13) × 2
= 27 m
Area of fourth trapezium (F) = ½(13 + 9) × 3
= 33 m
Area of second triangle (G) = ½((19 – 15) × 9) = 18 m
Total area = 18 + 33 + 27 + 14 + 27 + 33 + 18
= **170 m (accept 168 – 172 m)**

You'd still get the marks if you got the correct answer using a different method, for example if you counted the squares.

Page 65 — Fact Recall Questions

Q1 The gradient of a velocity-time graph tells you the acceleration.

Q2 Uniform acceleration on a velocity-time graph is shown by a straight line.

Q3 The area under a velocity-time graph tells you the displacement.

Q4 Non-uniform acceleration is shown on a velocity-time graph by a curved line.

6. Investigating Motion

Page 69 — Application Question

Q1 The ticker timer makes dots at a rate of 2 dots per second, so time between dots = 1 ÷ 2 = 0.5 s
Distance between dots = 7 cm = 0.07 m
$v = \frac{s}{t} = \frac{0.07}{0.5} =$ **0.14 ms^{-1}**

Page 69 — Fact Recall Questions

Q1 E.g. light gates have a lower uncertainty than stopwatches because they do not depend on human reaction times.

Q2 E.g. Place a smooth ramp 1 m away from a wall. Place a metre rule along the 1 m flat stretch between the wall and the ramp. Set up a video camera to film the set-up from the side. Measure the length of a trolley, hold it in place on the ramp and begin filming. Let go of the trolley and let it roll down the ramp and hit the wall. Once the trolley has come to a stop, stop the recording. Use video analysis software to go through the video frame-by-frame, and count how many frames it takes for the trolley to completely pass a reference point on the meter rule before the collision. Use the frame rate of the video to find the time taken for the trolley to pass the reference point. Use this time and the length of the trolley to calculate the speed of the trolley. Repeat this video analysis for after the collision.

7. Stopping Distances

Page 71 — Application Questions

Q1 Stopping distance = 15 + 38 = **53 m**

Q2 Dahlia travels 14 m during her reaction time.
$s = vt$, so:
$t = s \div v$
$= 14 \div 25$
$=$ **0.56 s**

Page 71 — Fact Recall Questions

Q1 The stopping distance is the distance covered by the vehicle in the time between the driver first spotting a hazard and the vehicle coming to a complete stop. It's the sum of the thinking distance and the braking distance.
The thinking distance is the distance the vehicle travels during the driver's reaction time.
The braking distance is the distance the vehicle travels after the brakes are applied until it comes to a complete stop.

Q2 Any three from: e.g. mass of the vehicle, ice (or water, oil, leaves etc.) on the road, quality of the tyres, quality of the vehicle's brakes, weather conditions.

Q3 a) Braking
b) Thinking
c) Thinking
d) Braking

Exam-style Questions — Page 73-75

1 C ***(1 mark)***

The equation of motion you would use is $s = \frac{1}{2}gt^2$ as you know that the ball's initial velocity (u) is zero. So you need a value for s (distance from the starting point to the light gate) and t (time taken for the ball bearing to travel this distance).

2 D ***(1 mark)***

Use thinking distance = speed x reaction time and stopping distance = thinking distance + braking distance.

3 B ***(1 mark)***

Use $v^2 = 2gs$, where $g = 9.81$ ms^{-2}, as initial velocity is zero.

4 D ***(1 mark)***

The horizontal component of the velocity is $2.50\cos 60°$.
The horizontal distance travelled is $3.00 \times 2.50\cos 60°$.

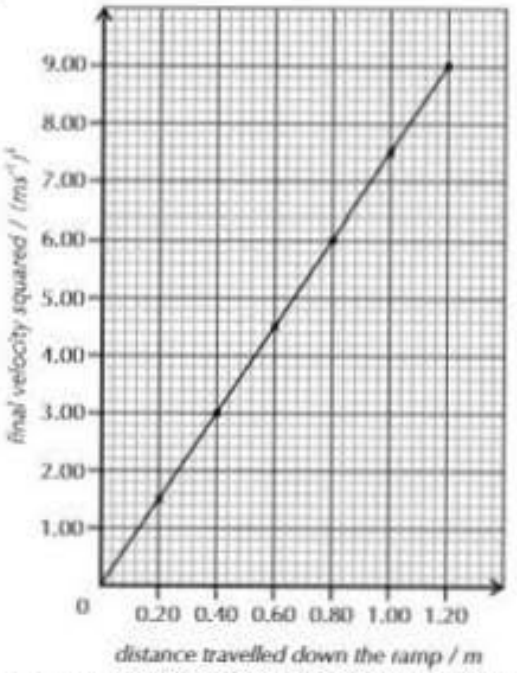
5 a) E.g. the student accelerates for the first 50 s. She then walks at a constant speed for 100 s, before decelerating to a stop in the next 75 s.

(1 mark for correct description of motion and 1 mark for stating the correct times)

b) total displacement = 250 m
total time taken = 225 s
average velocity = total displacement ÷ total time taken
$= 250 \div 225$
$= 1.111...$ ms^{-1} = **1.1 ms^{-1} (to 2 s.f.)**

(2 marks for correct answer, otherwise 1 mark for correct working if answer incorrect)

6 a)



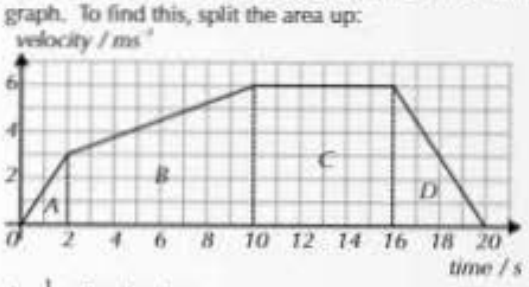
(1 mark for all points correctly plotted, 1 mark for line of best fit drawn correctly)

b) $v^2 = u^2 + 2as$ and $u = 0$, so $v^2 = 2as$
Equation of a straight line through the origin is $y = mx$.
Comparing $v^2 = 2as$ and $y = mx$, you can see that
$y = v^2$, $x = s$ and $m = 2a$
so $a = 0.5 \times m$
change in $y = 9.00 - 0 = 9.00$ (ms^{-1})^2
change in $x = 1.20 - 0 = 1.20$ m
so $a = 0.5 \times (9.00 \div 1.20)$
$= 3.75$ **ms^{-2} (to 3 s.f.)**

(3 marks for correct answer, otherwise 1 mark for stating that gradient = 2a and 1 mark for correct working if answer incorrect)

Since the graph is a straight line, the acceleration is constant.

7 a) The cyclist's displacement is given by the area under the graph. To find this, split the area up:



A: $\frac{1}{2} \times 2 \times 3 = 3$ m
B: $8 \times \frac{6+3}{2} = 36$ m
C: $6 \times 6 = 36$ m
D: $\frac{1}{2} \times 4 \times 6 = 12$ m

So displacement = 3 + 36 + 36 + 12 = **87 m**

(2 marks for correct answer, otherwise 1 mark for correct working if answer incorrect)

b) change in $y = 0 - 6 = -6$ ms^{-1}
change in $x = 20 - 16 = 4$ s
gradient = change in y ÷ change in x
$= -6 \div 4 = -1.5$ ms^{-2}
so acceleration = **-1.5 ms^{-2}**

(2 marks for correct answer, otherwise 1 mark for correct working if answer incorrect)

Remember, deceleration is just a negative acceleration.

c) $v^2 = u^2 + 2as$
$\Rightarrow v = \sqrt{(0)^2 + (2 \times 3.40 \times 22.5)}$
$= 12.369...$ ms^{-1}
$=$ **12.4 ms^{-1} (to 3 s.f.)**

(2 marks for correct answer, otherwise 1 mark for correct working if answer incorrect)

d) $v^2 = u^2 + 2as$
$\Rightarrow v = -\sqrt{(\sin 29.0° \times 6.50)^2 + (2 \times -9.81 \times -1.31)}$
$= -5.969...$ ms^{-1} = **-5.97 ms^{-1} (to 3 s.f.)**

(2 marks for correct answer, otherwise 1 mark for correct working if answer incorrect)

Use the negative solution for v as it's travelling downwards when it hits the ground.

e) $v = u + at \Rightarrow t = \frac{v-u}{a} = \frac{-5.969... - (\sin 29.0° \times 6.50)}{-9.81}$
$= 0.9297...$ = **0.930 s (to 3 s.f.)**

(2 marks for correct answer, otherwise 1 mark for correct working if answer incorrect)

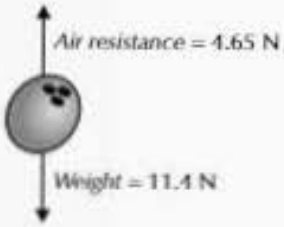
Section 2 — Forces in Action

1. Mass, Weight and Force Basics

Page 78 — Application Questions

Q1 $W = mg$, so:
$m = W \div g$
$= 7.75 \div 0.620$
$= \mathbf{12.5\ kg}$

Q2



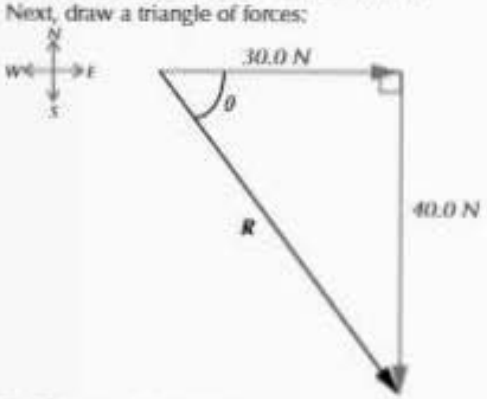
Page 78 — Fact Recall Questions

Q1 $W = mg$ / weight = mass × gravitational field strength
Q2 The centre of mass of an object is the single point that you can consider its whole weight to act through.
Q3 At the centre of the object.
Q4 upthrust
Q5 free-body diagram

2. Net Forces

Page 81 — Application Questions

Q1 $F = ma \Rightarrow F = 24.1 \times 3.5 = 84.35\ N = \mathbf{84\ N\ (to\ 2\ s.f.)}$
Q2 $F = ma \Rightarrow m = \frac{F}{a} = \frac{18}{29} = 0.6206...\ kg = \mathbf{0.62\ kg\ (to\ 2\ s.f.)}$
Q3 a) First, find the net force in the north-south direction.
Let north be positive and south be negative.
North-south force = 20.0 − 60.0 = −40.0 N
Next, draw a triangle of forces:



By Pythagoras' theorem:
$R = \sqrt{30.0^2 + 40.0^2} = 50.0\ N$
Find the direction of R as a clockwise angle from east.
$\theta = \tan^{-1}\left(\frac{40.0}{30.0}\right) = 53.13...° = 53.1°$ (to 3 s.f.)
So the net force is **50.0 N at 53.1° clockwise from east.**
b) $F = ma$
$\Rightarrow a = \frac{50.0}{89.0} = 0.56179...\ ms^{-2} = \mathbf{0.562\ ms^{-2}\ (to\ 3\ s.f.)}$

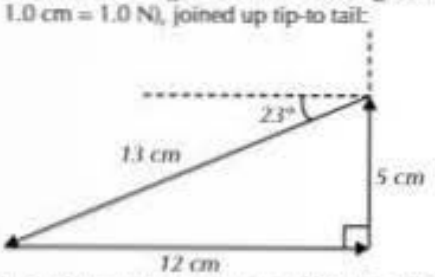
Page 81 — Fact Recall Questions

Q1 The net force is the sum of all the forces acting on the object, accounting for their relative directions.
Q2 F = net force (N), m = mass (kg) and a = acceleration (ms^{-2}).

3. Equilibrium

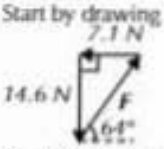
Page 84 — Application Questions

Q1 Draw a scale diagram of the forces (e.g. using a scale of 1.0 cm = 1.0 N), joined up tip-to tail:



They form a closed triangle, so the object's in equilibrium.
This diagram has been shrunk to fit in the column — but you should draw it full-size.

Q2 Either:
Start by drawing the forces in a closed triangle:



$F = \sqrt{7.1^2 + 14.6^2} = 16.234...\ N = \mathbf{16\ N\ (to\ 2\ s.f.)}$
Or:
Use the horizontal components:
$(\cos 64° \times F) - 7.1 = 0 \Rightarrow F = \frac{7.1}{\cos 64°} = 16.196...\ N$
$= \mathbf{16\ N\ (to\ 2\ s.f.)}$
You could have also used vertical components, in which case you'd use F sin 64° and 14.6 N instead.

Q3 Since the box is in equilibrium, the force F must have the same magnitude as the component of the 16 N force parallel to force F.
So:
$F = \sin 39° \times 16 = 10.069...\ N = \mathbf{10\ N\ (to\ 2\ s.f.)}$
You could also have used a triangle of forces to answer this question.

Page 84 — Fact Recall Questions

Q1 0 N
Q2 Splitting a force into two perpendicular components (usually horizontal and vertical).

4. Moments and Torques

Page 89 — Application Questions

Q1 $M = Fx = 73.1 \times 0.25 = 18.275$ Nm = **18 Nm (to 2 s.f.)**

Q2 Anticlockwise moment applied by child on left:
$M = Fx = 450.0 \times 1.50 = 675$ Nm
So clockwise moment applied by child on right = 675 Nm
$M = Fx \Rightarrow x = \frac{M}{F} = \frac{675}{500.0} =$ **1.35 m**

Q3 a) weight $W = mg = 24\,000 \times 9.81 = 235\,440$ N
moment $= Fx = 235\,440 \times 3.5 = 824\,040$ kN m
= 820 kN m anticlockwise (to 2 s.f.)
Don't forget to convert the mass in kg to weight in N before you calculate the moment.
b) clockwise moment about support =
$(55\,000 \times 7.5) + (X \times 6.5) = 412\,500 + 6.5X$
anticlockwise moment about support = 824 040
(you worked this out in part a)).
clockwise moments = anticlockwise moments, so
$412\,500 + 6.5X = 824\,040$
$X = \frac{824\,040 - 412\,500}{6.5} = 63\,313.8...$ kN
= 63 kN (to 2 s.f.)

Q4 $T = Fd \Rightarrow F = \frac{T}{d} = \frac{50.0}{0.35} = 142.85...$ N = **143 N (to 2 s.f.)**

Page 89 — Fact Recall Questions

Q1 A moment is the turning effect of a force around a turning point. It's the force × the perpendicular distance from the pivot to the line of action of the force.

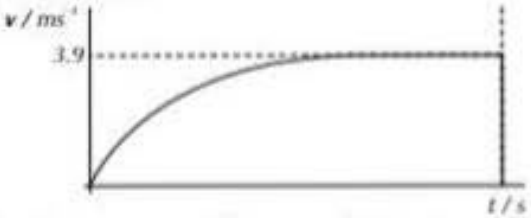
Q2 A pair of forces of equal size which act parallel to each other but in opposite directions is called a couple.

5. Drag and Terminal Velocity

Page 93 — Application Questions

Q1 a) The ball starts with a large (positive) acceleration as it falls through the air. The acceleration is decreasing slightly (due to air resistance). After it hits the water it experiences a sudden deceleration, which decreases until the ball has reached terminal velocity (3.9 ms^{-1}). It then falls at a constant velocity until it hits the bottom of the cylinder.

b)



Q2 Air resistance on a car will increase with the car's speed until the frictional force balances the driving force. The larger the air resistance on the car, the lower the speed it will reach before the driving and frictional forces become balanced.

Page 93 — Fact Recall Questions

Q1 Friction is a force that opposes motion. Drag is the name given to friction caused by a fluid.

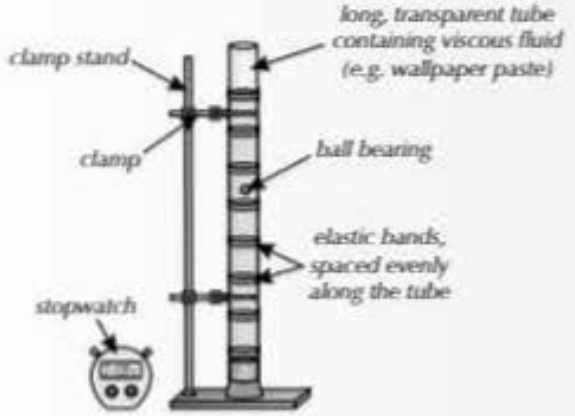
Q2 Friction acts in the opposite direction to the motion.

Q3 When an object reaches terminal velocity, the frictional forces are equal in size to the driving force(s) and in the opposite direction.

Q4 The correct graph is a).

Q5 Skydivers reduce their terminal velocity by using a parachute to increase the drag they experience.

Q6



6. Density, Pressure and Upthrust

Page 96 — Application Questions

Q1 $\rho = \frac{m}{V} = \frac{360}{0.45} =$ **800 kg m^{-3}**

Q2 $p = \frac{F}{A}$
so, $A = \frac{F}{p} = \frac{26\,500}{159\,000} = 0.1666...$ m^2 = **0.167 m^2 (to 3 s.f.)**

Q3 $\rho = \frac{m}{V}$, so $m = \rho \times V$
$m = 2700 \times 9.1 \times 10^{-4} = 2.457$ kg = **2.5 kg (to 2 s.f.)**

Q4 $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.040)^3 = 2.68... \times 10^{-4}$ m^3
$\rho = \frac{m}{V} = \frac{0.15}{2.68... \times 10^{-4}} = 559.52...$ kg m^{-3}
= 560 kg m^{-3} (to 2 s.f.)

Q5 upthrust $= (h_2 - h_1)\rho g A$
Convert cm to m and g cm^{-3} to kg m^{-3}:
$h_2 = 0.760$ m, $h_1 = 0.650$ m, $r = 0.0500$ m,
$\rho = 1.00 \times 1000 = 1000$ kg m^{-3}
$A = \pi r^2 = \pi \times 0.0500^2$
upthrust $= (0.760 - 0.650) \times 1000 \times 9.81 \times (\pi \times 0.0500^2)$
$= 8.47523...$ N
= 8.48 N (to 3 s.f.)

Page 96 — Fact Recall Questions

Q1 $\rho = \frac{m}{V}$, where ρ = density, m = mass and V = volume.

Q2 The weight of the amount of water displaced by the object.

Q3 For the object to float in the fluid, the density of the object must be lower than the density of the fluid.

Exam-style Questions — Pages 98-100

1 C ***(1 mark)***

Calculate the mass of the object by dividing the weight on Earth by 9.81 ms $^{-2}$. Then divide the weight on Venus by the mass you calculated to find the gravitational field strength.

2 A ***(1 mark)***

3 B ***(1 mark)***

4 C ***(1 mark)***

Since the diagrams are to scale, you can just measure the distance between the forces for each couple, and use your measurements and the forces to calculate the torques.

5 a) E.g. Fill a glass tube that has been marked at regular intervals (e.g. with rubber bands) with liquid X. Drop a ball bearing into the tube ***(1 mark)***. Use a stopwatch to record the time at which it reaches each mark on the tube. Calculate the time taken by the ball to move between consecutive markings ***(1 mark)***. Use this and the distance between markings to calculate the velocity of the ball between each pair of markings. When the velocity becomes constant, the ball has reached its terminal velocity ***(1 mark)***.

b) mass of liquid = mass of full beaker − mass of empty beaker

$= 862 - 250 = 612$ g

$\rho = \frac{m}{V} = \frac{612}{450} = 1.36$ g cm $^{-3}$ **= 1.4 g cm $^{-3}$ (to 2 s.f.)**

(2 mark for correct answer, 1 mark for correct method if answer incorrect)

c) Convert density into kg m $^{-3}$ and height into m:

$\rho = 1360$ kg m $^{-3}$, $h = 0.040$ m

$p = h\rho g$

$= 0.040 \times 1360 \times 9.81$

$= 533.664$ Pa = **530 Pa (to 2 s.f.)**

(2 mark for correct answer, 1 mark for correct method if answer incorrect)

d) Apple floats, so upthrust is equal to the apple's weight.

mass of apple = 185 ÷ 1000 = 0.185 kg

upthrust = $W = mg$

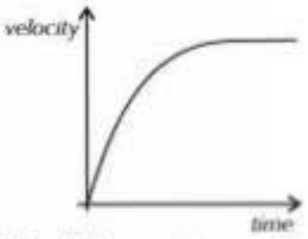
$= 0.185 \times 9.81$

$= 1.8148...$ N = **1.81 N (to 3 s.f.)**

(2 marks for correct answer, 1 mark for correct method if answer incorrect)

6 a) The constant pulling forces on the dinghy cause it to accelerate. As the dinghy's velocity increases, the resistive force on the dinghy increases ***(1 mark)***. This causes the resultant force on the dinghy to decrease, so its acceleration decreases ***(1 mark)***. Eventually, the resistive force will equal the forwards components of the pulling forces, and the resultant force is 0. At this point, the dinghy stops accelerating, the velocity becomes constant, and the dinghy has reached terminal velocity ***(1 mark)***.

b)



(1 mark for correct axes and graph shape)

c) At terminal velocity, the resultant force on the dingy is 0. So F must be equal to the sum of the forwards components of the 80.0 N forces.

forwards component of one 80.0 N force $= \cos 15.0° \times 80.0$

So $F = (\cos 15.0° \times 80.0) + (\cos 15.0° \times 80.0)$

$= 154.548...$ N = **155 N (to 3 s.f.)**

(2 marks for correct answer, 1 mark for correct method if answer incorrect)

d) $F = ma$

$a = F \div m = 154.548... \div 83.2$

$= 1.85754...$ ms $^{-2}$ = **1.86 ms $^{-2}$ (to 3 s.f.)**

(2 marks for correct answer, 1 mark for correct method if answer incorrect)

7 a) The moment of a force about a point/pivot is equal to the force × the perpendicular distance between the line of action of the force and the point/pivot ***(1 mark)***.

b) The front wheels will exert a greater force on the truck ***(1 mark)***. The distance between the front wheels and the centre of mass is shorter than between the rear wheels and the centre of mass, so the force exerted by the front wheels must be larger in order for moments to be balanced ***(1 mark)***.

c) If the front wheels feel no force, the clockwise moments must balance the anticlockwise moments about the pivot (rear wheels).

anticlockwise moments = clockwise moments

$\Rightarrow W \times 0.65 = 21\,000 \times 2.4$

$\Rightarrow W = \frac{21000 \times 2.4}{0.65} = 77538.4...$ N

= 78 000 N (to 2 s.f.)

(2 marks for correct answer, 1 mark for anticlockwise moments = clockwise moments if answer incorrect)

8 a) The centre of gravity is the single point through which all of an object's weight can be considered to act ***(1 mark)***.

b) Suspend the parrot from the edge by a string, and draw a straight line directly downwards from the string across the parrot, using a plumb bob to help ***(1 mark)***. Then move the string to a different position on the edge of the parrot, and repeat the previous step ***(1 mark)***. The parrot's centre of gravity is at the point where the two lines drawn intersect ***(1 mark)***.

c) Because cranes are tall they have a naturally high centre of mass. The base must therefore be heavy to lower the centre of mass ***(1 mark)*** and wide ***(1 mark)*** so that the centre of mass stays within the base area of the crane ***(1 mark)***.

Section 3 — Work, Energy and Power

1. Work and Power

Page 104 — Application Questions

Q1 $W = \boldsymbol{Fx} = 203 \times 2.81 = 570.43 = \mathbf{570\ J\ (to\ 3\ s.f.)}$

Q2 $P = \boldsymbol{Fv} \Rightarrow \boldsymbol{F} = \frac{P}{\boldsymbol{v}} = \frac{60\,100}{34.7} = 1731.98... = \mathbf{1730\ N\ (to\ 3\ s.f.)}$

Q3 $W = \boldsymbol{Fx}\cos\theta = 371 \times 1.39 \times \cos 13.1° = 502.26...$
$= \mathbf{502\ J\ (to\ 3\ s.f.)}$

Q4 Look at only the horizontal component of the force:
$P = \boldsymbol{Fv}\cos\theta = 83.1 \times 2.99 \times \cos 15.2° = 239.77...$
$= \mathbf{240\ W\ (to\ 3\ s.f.)}$

Q5 a) $P = \frac{W}{t} = \frac{980\,000}{22} = 44\,545.4... = \mathbf{45\ kW\ (to\ 2\ s.f.)}$

b) $W = \boldsymbol{Fx}\cos\theta \Rightarrow \boldsymbol{x} = W \div (\boldsymbol{F}\cos\theta)$
$= 980\,000 \div (9500\cos 15°)$
$= 106.79...$
$= \mathbf{110\ m\ (to\ 2\ s.f.)}$

Page 104 — Fact Recall Questions

Q1 Energy is transferred when work is done.

Q2 One joule is the work done when a force of 1 newton moves an object through a distance of 1 metre.

Q3 Power is the rate of doing work (or transferring energy).

Q4 Power = force × velocity (or $P = \boldsymbol{Fv}$)

2. Kinetic Energy and Gravitational Potential Energy

Page 106 — Application Questions

Q1 $E_p = mgh = 32.4 \times 9.81 \times 0.285 = 90.585...$
$= \mathbf{90.6\ J\ (to\ 3\ s.f.)}$

Q2 $E_p = mgh$ where h is change in height.
Change in height = 890 − 760 = 130 m
So $E_p = 770 \times 9.81 \times 130 = 981\,981$
$= \mathbf{980\,000\ J}$ or $\mathbf{980\ kJ\ (to\ 2\ s.f.)}$

Q3 $E_k = \frac{1}{2}mv^2 \Rightarrow m = \frac{2E_k}{v^2} = \frac{2 \times 150\,000}{16^2} = 1171.875$
$= \mathbf{1200\ kg\ (to\ 2\ s.f.)}$

Page 106 — Fact Recall Questions

Q1 An object has kinetic energy when it is moving.

Q2 An object gains gravitational energy when it's lifted up in a gravitational field.

3. Conservation of Energy

Page 110 — Application Questions

Q1 $\text{efficiency} = \frac{\text{useful output energy}}{\text{total input energy}} \times 100$

$\Rightarrow \text{useful output energy} = \frac{\text{efficiency}}{100} \times \text{total input energy}$
$= \frac{20.0}{100} \times 2900 = \mathbf{580\ J}$

Q2 $\frac{1}{2}mv^2 = mgh \Rightarrow h = \frac{v^2}{2g}$
$h = \frac{7.8^2}{2 \times 9.81} = 3.1009... = \mathbf{3.1\ m\ (to\ 2\ s.f.)}$

Q3 a) $E_p = mgh = 0.65 \times 9.81 \times 103 = 656.7...$
$= \mathbf{660\ J\ (to\ 2\ s.f.)}$

b) $\frac{1}{2}mv^2 = 0.95 \times E_p \Rightarrow v = \sqrt{\frac{2 \times 0.95 \times 656.7...}{0.65}}$
$= 43.81... = \mathbf{44\ ms^{-1}\ (to\ 2\ s.f.)}$

c) $mgh = 0.800 \times \frac{1}{2}mv^2 \Rightarrow h = \frac{0.400v^2}{g}$
$= \frac{0.400 \times 43.81...^2}{9.81}$
$= 78.28$
$= \mathbf{78\ m\ (to\ 2\ s.f.)}$

Even though the falcon has just converted gravitational potential energy into kinetic energy and back again, it's lost some height because some of the energy has been transferred to heat energy, sound, etc.

Page 110 — Fact Recall Questions

Q1 Energy cannot be created or destroyed. Energy can be transferred from one form to another but the total amount of energy in a closed system will not change.

Q2 How much of the input energy is transferred usefully by a system.

Exam-style Questions — Page 112

1 C ***(1 mark)***

2 a) Gravity ***(1 mark)***

b) $W = \boldsymbol{Fx}\cos\theta = 660 \times 95 \times \cos 11°$
$= 61\,548.02... = \mathbf{62\ kJ\ (to\ 2\ s.f.)}$
(2 marks — 1 mark for correct answer, 1 mark for correct substitutions)

3 a) $E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 74.8 \times 6.5^2 = 1580.15$
$= \mathbf{1600\ J\ (to\ 2\ s.f.)}$
(2 marks for correct answer, 1 mark for correct working if answer incorrect)

b) The brakes do work against friction to convert all the E_k to other types of energy.
So energy transferred = $W = E_k = 1580.15$ J
$P = \frac{W}{t} = \frac{1580.15}{3.0} = 526.71...$
$= \mathbf{530\ W\ (to\ 2\ s.f.)}$
(2 marks for correct answer, 1 mark for correct working if answer incorrect)

c) $mgh = \frac{1}{2}mv^2 \Rightarrow gh = \frac{1}{2}v^2$
$\Rightarrow v = \sqrt{2 \times 9.81 \times 22.5}$
$= 21.010... = \mathbf{21.0\ ms^{-1}\ (to\ 3\ s.f.)}$
(2 marks for correct answer, 1 mark for correct working if answer incorrect)

Section 4 — Materials

1. Hooke's Law

Page 117 — Application Questions

Q1 1.60 cm = 0.0160 m

$F = kx = 1250 \times 0.0160 = \mathbf{20.0\ N}$

Q2 0.80 mm = 0.00080 m (= 8.0×10^{-4} m)

Rearrange $F = kx$ to get $k = \frac{F}{x}$

$k = \frac{20}{8.0 \times 10^{-4}} = \mathbf{25\,000\ Nm^{-1}}$

Q3 a) extension = new length − original length

$= 22.0 - 20.0 = 2.0\ \text{cm} = \mathbf{0.020\ m}$

b) Rearrange $F = kx$ to get $k = \frac{F}{x}$

$k = \frac{55.0}{0.020} = \mathbf{2750\ Nm^{-1}}$

Q4 First add the top, k_t, and bottom, k_b, spring constants for both sets of springs in series:

$\frac{1}{k_t} = \frac{1}{k_t} + \frac{1}{k_b} = \frac{1}{80} + \frac{1}{60}$

$\Rightarrow k_t = 1 \div \left(\frac{1}{80} + \frac{1}{60}\right) = 34.285...\ \text{Nm}^{-1}$

Add the combined spring constants for the left, k_l, and right, k_r, sides in parallel to get the overall combined spring constant:

$k_{combined} = k_l + k_r = 2 \times k_t = 2 \times 34.285... = 68.571...\ \text{Nm}^{-1}$

$= \mathbf{70\ Nm^{-1}}$ **(to 1 s.f.)**

Page 117 — Fact Recall Questions

Q1 $F = kx$. Where F is the force applied, k is the force constant and x is the extension.

Q2 The point beyond which force is no longer proportional to extension.

Q3 You would need to measure the original length before adding the weight, and the extended length once the weight has been added. Calculate the extension by subtracting the original length of the spring from its extended length.

2. Elastic and Plastic Deformation

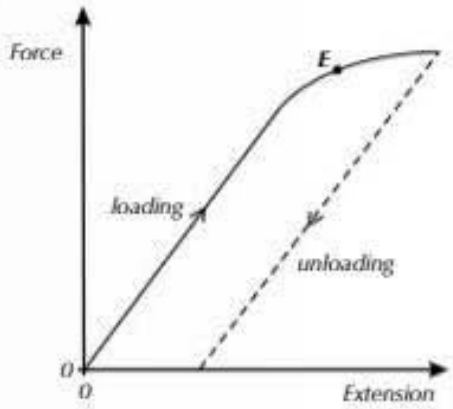
Page 121 — Application Questions

Q1 $E = \frac{1}{2}Fx = \frac{1}{2} \times 30 \times 1.2 \times 10^{-3} = \mathbf{0.18\ J}$

Q2 Work done = area under force-time graph

$= \frac{1}{2} \times 26 \times 10^{-3} \times 12 = 0.156\ \text{J} = \mathbf{0.16\ J}$ **(to 2 s.f.)**

Q3



Q4 The band is elastic, because it returns to its original length (i.e. its extension is zero) after all the load has been removed.

Page 121 — Fact Recall Questions

Q1 The force (or load) beyond which a material will be permanently stretched.

Q2 A material that is deforming elastically returns to its original shape/length once the forces acting on it are removed.

Q3 A material that is deforming plastically is permanently stretched once the forces acting on it are removed.

Q4 The area between the two lines is the work done to permanently deform the material.

3. Stress and Strain

Page 123 — Application Questions

Q1 $\sigma = \frac{F}{A} = \frac{50.0}{3.1 \times 10^{-4}} = 1.6129... \times 10^{7}$

$= \mathbf{1.6 \times 10^{7}\ Pa\ (or\ Nm^{-2})}$ **(to 2 s.f.)**

Q2 extension = new length − original length

$= 12.3 - 12.0 = 0.3\ \text{cm}$

$\varepsilon = \frac{x}{l} = \frac{0.3}{12.0} = \mathbf{0.025}$

Remember, there are no units for strain, as it is a ratio. Just make sure when working it out that the extension and original length are in the same units.

Q3 $\sigma = \frac{F}{A}$

so $F = \sigma \times A = (3.8 \times 10^{8}) \times (1.2 \times 10^{-7})$

$= 45.6 = \mathbf{46\ N}$ **(to 2 s.f.)**

Page 123 — Fact Recall Questions

Q1 a) The force applied to a material divided by its cross-sectional area.

b) The change in length divided by the original length of a material.

c) The stress that a material experiences when it breaks.

d) The maximum stress that a material can withstand before breaking.

4. The Young Modulus

Page 126 — Application Questions

Q1 a) $1.1\text{ mm} = 1.1 \times 10^{-3}\text{ m}$

$$A = \pi\left(\frac{\text{diameter}}{2}\right)^2$$

$$= \pi\left(\frac{1.1 \times 10^{-3}}{2}\right)^2 = 9.503... \times 10^{-7}$$

$$= \mathbf{9.5 \times 10^{-7}\ m^2\ (to\ 2\ s.f.)}$$

b) $x = 0.20\text{ mm} = 2.0 \times 10^{-4}\text{ m}$

$F = 23\text{ N}$, $l = 1.0\text{ m}$, $A = 9.503... \times 10^{-7}\text{ m}^2$

$$E = \frac{Fl}{Ax} = \frac{23 \times 1.0}{(9.503... \times 10^{-7}) \times (2.0 \times 10^{-4})}$$

$$= 1.2101... \times 10^{11} = \mathbf{1.2 \times 10^{11}\ Pa\ (or\ Nm^{-2})\ (to\ 2\ s.f.)}$$

Q2 a) $\sigma = \frac{F}{A} = \frac{100}{8.0 \times 10^{-7}} = 1.25 \times 10^{8}$

$$= \mathbf{1.3 \times 10^{8}\ Pa\ (to\ 2\ s.f.)}$$

b) $F = \frac{\sigma}{\varepsilon}$ so $\varepsilon = \frac{1.25 \times 10^{8}}{3.5 \times 10^{9}} = 0.3571... = \mathbf{0.36\ (to\ 2\ s.f.)}$

Q3 energy per unit volume $= \frac{1}{2} \times 4\,200\,000 \times \frac{0.11}{254}$

$$= 909.44... = \mathbf{910\ Jm^{-3}\ (to\ 2\ s.f.)}$$

Page 126 — Fact Recall Questions

Q1 Nm^{-2} or Pa.

Q2 Original length, extension, weights/load, diameter of wire (to find cross-sectional area).

Q3 E.g. wear safety goggles while carrying out the experiment / stand up so can move out of the way of falling weights.

Q4 The Young modulus of a material.

5. Stress-Strain Graphs

Page 131 — Application Questions

Q1 a) The elastic limit

b) The material is obeying Hooke's law.

c) Point C

d) The Young modulus is the gradient of the graph up to point A, so:

Young modulus $= (150 \times 10^{6}) \div 0.002 = \mathbf{7.5 \times 10^{10}\ Pa}$

Watch out for units — here the graph gives stress in MPa. You need to convert this into a value in Pa before you can calculate the Young modulus for the material.

e) Point A is the limit of proportionality. The energy stored per unit volume is given by the area under the graph up to Point A, which is $0.5 \times 0.002 \times 150 = \mathbf{0.15\ MPa}$.

Q2 Graph B shows a brittle material, as it is just a straight line. Graph A shows a material reaching its elastic limit and undergoing plastic deformation. Brittle materials don't deform plastically.

Page 131 — Fact Recall Questions

Q1 The stress at which a large amount of plastic deformation takes place with a constant or reduced load.

Q2 Any one of: e.g. polythene, rubber.

Q3 Yes. The stress-strain graph for a brittle material is a straight line through the origin, which shows it obeys Hooke's law.

Exam-style Questions — Pages 133-134

1 D ***(1 mark)***

$E = \sigma \div \varepsilon = FL \div Ax$

Read F from graph to be 10 N.

$F = 10\text{ N}$, $L = 0.80\text{ m}$, $x = 0.080\text{ m}$, $A = 5.0 \times 10^{-6}\text{ m}^2$

$E = (10 \times 0.80) \div (5 \times 10^{-6} \times 0.080)$

$= 2.0 \times 10^{7}\text{ Pa}$

2 A ***(1 mark)***

$F = kx \Rightarrow k = F \div x = 1.1 \div (5 \times 10^{-2})$

$= 22.0\text{ Nm}^{-1}$

$E = \frac{1}{2}kx^2 = \frac{1}{2} \times 22.0... \times 0.018^2$

$= 0.0356... = 0.036\text{ J (to 2 s.f.)}$

3 a) How to grade your answer (pick the description that best matches your answer):

0 marks: There is no relevant information.

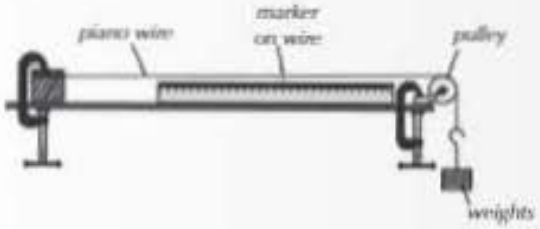
1-2 marks: A diagram has been drawn, and method points 5, 7, 8 and 9 have been covered. Answer is basic and has lack of structure. Information is missing and there are few backed up arguments.

3-4 marks: A diagram has been drawn and most of the method points have been covered. Answer has some structure, and information is presented with a clear line of reasoning, with arguments that are partially backed up.

5-6 marks: A diagram has been drawn, all of the method points have been covered, and at least one point from the accuracy points have been covered. The answer is well structured. There is a clear and logical line of reasoning, with information and arguments that are fully backed up.

Here are some points your answer may include:

Apparatus



Method

1. Add just enough weight to the end of the wire to straighten it.
2. Measure the starting position of the marker on the wire.
3. Use a micrometer or vernier callipers to measure the diameter of wire.
4. Use $A = \pi r^2$ to calculate its cross-sectional area.
5. Add weights to the end of the wire in steps, recording the weight and the marker reading each time.
6. Calculate the extension of the wire after each weight is added by subtracting the original marker position from the current marker reading.
7. Calculate the stress on the wire at each step using $\sigma = \frac{F}{A}$ and the strain using $\varepsilon = \frac{x}{l}$.
8. Plot a stress-strain graph for the wire.
9. The Young modulus is equal to the gradient of the linear graph.

Reducing Error

1. Use a mass meter/digital scales to measure the weight of each weight used.
2. Measure the wire's diameter in at least three different places and take an average.
3. Use a long piece of wire to give a large extension and reduce uncertainty.
4. Use a thin marker on the wire to reduce random errors.
5. Keep the marker and ruler at eye level when measuring the extension.
6. Use a travelling microscope to view the position of the marker when measuring extension.
7. As wire is unloaded, re-measure the extension for each weight, and average your results (as long as you haven't gone past the elastic limit).

b) Young modulus = gradient = $\frac{\Delta\text{stress}}{\Delta\text{strain}}$

$= \frac{5.0 \times 10^{8}}{3.0 \times 10^{-3}}$

$= 1.666... \times 10^{11}$ Pa

$= \mathbf{1.7 \times 10^{11}}$ **Pa (to 2 s.f.)**

(2 marks for the correct answer, otherwise 1 mark for the correct working.)

You could also give your answer in MPa, GPa, or Nm $^{-2}$.

c) Elastic potential energy per unit volume
= area under graph = ½ × $1.8 \times 10^{-3} \times 3.0 \times 10^{8}$
= 270 000 Jm^{-3}
Volume of wire = $1.0 \times 10^{-2} \times 1.6 = 1.6 \times 10^{-2}$ m^3
So elastic potential energy stored in wire
= $270\,000 \times 1.6 \times 10^{-2} = 0.0432$ J = **0.043 J (to 2 s.f.)**

(3 marks for the correct answer, otherwise 1 mark for correctly calculating the area under the graph, and 1 mark for correctly calculating the volume of the wire.)

4 a) $F = \frac{\sigma}{\varepsilon} = \frac{Fl}{A\Delta x} \Rightarrow A = \frac{Fl}{E\Delta x}$

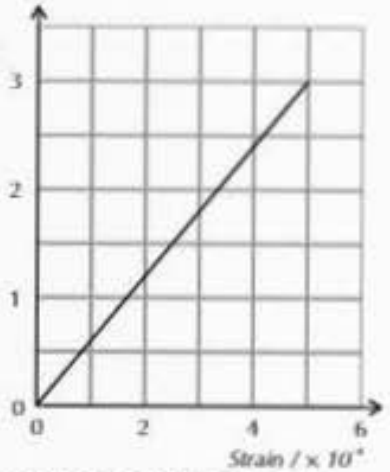
$F = 2.0$ kN $= 2000$ N, $l = 2.0$ m, $E = 2.10 \times 10^{11}$ Pa, $\Delta x = 0.20$ mm $= 2.0 \times 10^{-4}$ m

$A = \frac{2000 \times 2.0}{(2.10 \times 10^{11}) \times (2.0 \times 10^{-4})} = 9.523... \times 10^{-5}$

$= \mathbf{9.5 \times 10^{-5}}$ **m^2 (to 2 s.f.)**

(2 marks for correct answer, otherwise 1 mark for correct working)

b) E.g.



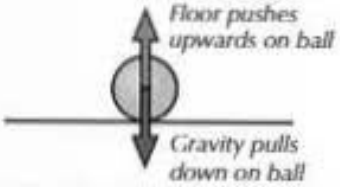
(1 mark for graph showing straight line passing through origin, 1 mark for graph ending at breaking stress, 1 mark for gradient equal to Young modulus)

Section 5 — Newton's Laws of Motion and Momentum

1. Newton's Laws of Motion

Page 137 — Application Questions

Q1



Q2 The rocket is burning through its fuel stores, so is losing mass in the form of fuel. $F = ma$ cannot be used since it requires the mass of the object to remain constant.

Q3 When a bird flaps its wings it pushes down on the air. The air then pushes back up on the bird's wings with the same force, which causes it to lift.

Q4 Rearranging $F = ma$:
$m = \frac{F}{a} = \frac{780}{0.41} = 1902.439... =$ **1900 kg (to 2 s.f.)**

Q5 First find the force they push each other with:
$F = m \times a = 60.0 \times 2.3 = 138$ N
Then find the acceleration of the other skater:
$F = m \times a \Rightarrow a = \frac{F}{m} = \frac{138}{55.0} = 2.5090... =$ **2.5 ms^{-2} (to 2 s.f.)**

Page 137 — Fact Recall Question

Q1 Newton's 1st law: The velocity of an object won't change unless a resultant force acts on it. This means a body will stay still or move in a straight line at a constant speed unless there's a resultant force acting on it.
Newton's 2nd law: Net force = change in momentum ÷ change in time. This means a larger force acting on an object over a given time causes a larger change in the momentum of the object.
Newton's 3rd law: If object A exerts a force on object B, then object B exerts an equal but opposite force on object A. This means every action has an equal and opposite reaction.

2. Momentum

Page 141 — Application Questions

Q1 Momentum before = Momentum after
$0 = (125 \times v) + (1.0 \times 10) = 125v + 10$
$v = -10 \div 125 = -0.08$ ms^{-1}
So the boat moves at **0.08 ms^{-1}** in the opposite direction to the rock.

Q2 Momentum before = Momentum after
$(0.165 \times 2.25) + (0.165 \times -4.75) = (0.165 \times -4.25) + (0.165 \times v)$
$-0.4125 = -0.70125 + 0.165v$
So **$v = 1.75$ ms^{-1}**
Kinetic energy before:
$= (0.5 \times 0.165 \times 2.25^2) + (0.5 \times 0.165 \times (-4.75)^2)$
$= 2.2790...$ J
Kinetic energy after:
$= (0.5 \times 0.165 \times (-4.25)^2) + (0.5 \times 0.165 \times (1.75)^2)$
$= 1.7428...$ J
Kinetic energy is not conserved, so this is an inelastic collision.

Q3 Mass (per second) = density × volume (per second)
$= 1000 \times (8.4 \times 10^{-3})$
$= 8.4$ kg
To find the velocity of the water, consider the length of the cylinder of water shot out of the hose per second. This gives you the distance travelled per second.

$$\text{Velocity} = \frac{\text{volume of water (per second)}}{\text{cross-sectional area of hose}}$$
$$= \frac{8.4 \times 10^{-3}}{5.6 \times 10^{-4}}$$
$$= 15 \text{ ms}^{-1}$$

Linear momentum = mass × velocity
$= 8.4 \times 15$
$= 126 =$ **130 kgms^{-1} (to 2 s.f.)**

Page 141 — Fact Recall Questions

Q1 Linear momentum = mass × velocity / $\boldsymbol{p} = m\boldsymbol{v}$

Q2 Linear momentum is always conserved — i.e. the total linear momentum is the same before and after a collision (provided no external forces act).

Q3 A perfectly elastic collision is one where linear momentum and kinetic energy are conserved. An inelastic collision is one where linear momentum is conserved, but kinetic energy is not.
Some kinetic energy is lost to the surroundings and/or converted to other forms (such as heat and sound).

3. Impulse and Vehicle Safety

Page 145 — Application Questions

Q1 Impulse = area under force–time graph
$= \frac{1}{2} \times 10 \times 10^{-3} \times 300 =$ **1.5 Ns**

Q2 Rigid materials don't deform very easily, so a large proportion of the energy involved in the crash is transferred elsewhere instead of being absorbed.

Q3 a) The seat belt increases the time that the dummy takes to come to a stop by stretching. Since force is equal to change in momentum over time taken, this reduces the overall force. It also slows the dummy down so that it hits the airbag with less force.

b) Air bags inflate very rapidly, with a lot of force. Without a seat belt, the passenger can keep moving forwards quickly as the car slows down. They could hit the air bag as it is inflating with a force big enough to cause injury.

Q4 Velocity in ms^{-1} = (125 × 1000) ÷ 3600 = 34.7222... ms^{-1}
There are 1000 m in a km and 3600 seconds in an hour.

$$F = \frac{\Delta p}{\Delta t} = \frac{\Delta (mv)}{\Delta t}$$
$$\Delta t = \frac{mv - mu}{F}$$
$$= \frac{(18\,000 \times 0) - (18\,000 \times 34.7222...)}{-62\,000}$$
$$= 10.080... =$$ **10 s (to 2 s.f.)**

Page 145 — Fact Recall Questions

Q1 The product of average force and time / the change in momentum of a body.

Q2 Air bags rapidly inflate on collision, providing a cushion which increases the time it takes for the passenger to slow down, and so reduces the force on the passenger. It also prevents them from hitting hard surfaces inside the car.

Q3 Seat belts can cause bruising during a crash, and can be dangerous for small children if booster seats aren't used.

Q4 E.g. crumple zones at the front and back of cars crumple up on impact. This causes the car to take longer to stop, increasing the impact time and decreasing the force on the passengers.

Exam-style Questions — Pages 147-148

1 C ***(1 mark)***

When the ball is fired, the cannon moves backwards and the ball is shot forwards, conserving momentum. Before the ball is fired, the kinetic energy is 0 J as neither object is moving, so point 1 cannot be correct. The cannon has a much larger mass than the ball, so for a given force it has a smaller acceleration than the ball (using F = ma) so point 3 cannot be correct.

2 C ***(1 mark)***

For a scenario to show Newton's Third Law, it has to show a pair of identical forces that act on two separate objects.

3 B ***(1 mark)***

Take the direction that the first ball, of mass 0.25 kg, is travelling in to be the positive direction.

Momentum of first ball = $mv = 0.25 \times 1.2 = 0.3$ kg ms^{-1}.

Mass of second ball = $0.25 \div 2 = 0.125$ kg.

Momentum of second ball = $0.125 \times -0.3 = -0.0375$ kg ms^{-1}.

So total momentum before collision

$= 0.3 - 0.0375 = 0.2625$ kg ms^{-1}.

After the collision, the balls move together so the mass is $0.25 + 0.125 = 0.375$ kg.

$p = mv$ so $v = p \div m = 0.2625 \div 0.375 = 0.7$ ms^{-1}.

4 a) The rate of change of momentum of an object is equal to the net force which acts on the object.

$F = \frac{\Delta p}{\Delta t}$ ***(1 mark)***

b) $F = \frac{\Delta p}{\Delta t} = \frac{\Delta (mv)}{\Delta t}$

So, $F\Delta t = \Delta (mv)$

$F\Delta t$ is constant for both cases, so:

$\Delta (mv)_1 = \Delta (mv)_2$

$\Rightarrow m_1 \Delta v_1 = m_2 \Delta v_2$

$m_1 = 1000$ kg, $\Delta v_1 = 31 - 26.5 = 4.5$ ms^{-1},

$m_2 = 1000 - 50 = 950$ kg

So, $\Delta v_2 = \frac{m_1 \Delta v_1}{m_2} = \frac{1000 \times 4.5}{950} = 4.7368...$ ms^{-1}

So, final velocity = 26.5 + 4.7368...

= 31.2368...

= **31.2 ms^{-1} (to 3 s.f.)**

(2 marks for correct answer, otherwise 1 mark for correct working)

5 a) E.g. this is an inelastic collision ***(1 mark)*** because kinetic energy is not conserved — some is converted to sound energy ***(1 mark)***.

b) Impulse is equal to the change in momentum.

Momentum before = (325 × 2.40) = 780 kg ms^{-1}

Momentum after = 0

So magnitude of impulse = **780 kg ms^{-1}**

(2 marks for correct answer, otherwise 1 mark for correct working.)

c) If the dodgem's initial speed was doubled, its initial momentum would be doubled, and therefore the impulse would be doubled:

Impulse = 780 × 2 = **1560 kg ms^{-1}** ***(1 mark)***

For parts b) and c), you could also give your answer in units of Ns.

d) Say right is positive direction.

Horizontal momentum before collision = horizontal momentum after collision.

$m \times (5.1 \sin 42° + 0) = m \times (-1.5 \sin 25° + v_B \sin 59°)$

Cancelling m:

$5.1 \sin 42° = -1.5 \sin 25° + v_B \sin 59°$

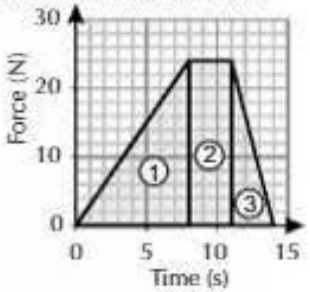
$\Rightarrow v_B = \frac{5.1 \sin 42° + 1.5 \sin 25°}{\sin 59°} = 4.720...$

= **4.7 ms^{-1} (to 2 s.f.)**

(2 marks for correct answer, otherwise 1 mark for correct working.)

You could also work this out using vertical momentum conservation.

6 a) Divide into three sections:



So the total area under the graph

= area 1 + area 2 + area 3

= (½ × 8 × 24) + (3 × 24) + (½ × 3 × 24) = **204 Ns**

(2 marks for correct answer, otherwise 1 mark for calculating the area under the graph)

b) $\Delta p = \Delta (mv) = F\Delta t$

Rearranging:

$m = \frac{F\Delta t}{\Delta v} = \frac{204}{2.2} = 92.727... =$ **93 kg (to 2 s.f.)**

(2 marks for correct answer, otherwise 1 mark for correct working)

c) E.g. There is no resultant force acting on it, as friction is assumed to be negligible, so Newton's First Law states that the velocity of the trolley will not change ***(1 mark)***.

Module 4

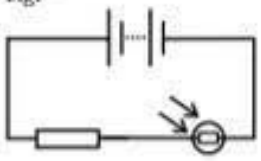
Section 1 — Electricity

1. Circuit Diagrams

Page 149 — Application Questions

Q1

Q2 E.g.



2. Current

Page 152 — Application Questions

Q1 $4e = 4 \times 1.60 \times 10^{-19} =$ **6.4×10^{-19} C**

Q2 $I = \Delta Q \div \Delta t = 91 \div 32 = 2.84375 =$ **2.8 A (to 2 s.f.)**

Q3 $v = I \div Ane$
$= 12 \div (0.50 \times 10^{-6} \times 8.5 \times 10^{28} \times 1.60 \times 10^{-19})$
$= 0.00176... =$ **0.0018 ms^{-1} (to 2 s.f.)**

Page 152 — Fact Recall Questions

Q1 Current is the rate of flow of charge.

Q2 From positive to negative.

Q3 An electrically charged particle that is free to move in an object.

Q4 1.60×10^{-19} C

Q5 Zero

Q6 Any one from: e.g. increase the cross-sectional area of the wire, increase the number density of charge carriers/electrons in the wire, increase the mean drift velocity of the charge carriers/electrons (by increasing the p.d. across the wire).

3. Potential Difference

Page 154 — Application Questions

Q1 $V = W \div Q = 114 \div 56.0 = 2.035... =$ **2.04 V (to 3 s.f.)**

Q2 $VQ = \frac{1}{2}mv^2$ so $V = mv^2 \div 2Q$
$V = (6.64 \times 10^{-27} \times (2.20 \times 10^4)^2) \div (2 \times 2 \times 1.60 \times 10^{-19})$
$= 5.0215$ V $=$ **5.02 V (to 3 s.f.)**

Page 154 — Fact Recall Questions

Q1 The potential difference between two points is the work done to move a unit charge between those two points.

Q2 In parallel across the component.

Q3 $eV = \frac{1}{2}mv^2$ where e = elementary charge, V = potential difference, m = mass of the electron and v = final velocity of the electron.

4. Resistance and Resistivity

Page 158 — Application Questions

Q1 $V = I \times R = 2.10 \times 8.62 = 18.102 =$ **18.1 V (to 3 s.f.)**

Q2 $R = V \div I = 13.4 \div 1.21 = 11.074... =$ **11.1 Ω (to 3 s.f.)**

Q3 $R = \rho L \div A = (1.72 \times 10^{-8} \times 0.40) \div (2.8 \times 10^{-5})$
$= 2.457... \times 10^{-4}\ \Omega =$ **2.5×10^{-4} Ω (to 2 s.f.)**

Q4 Gradient $= \Delta y \div \Delta x = (0.10 \times 10^{-3}) \div 0.30 = 0.00033...$
Cross-sectional area $= \pi r^2 = \pi \times 0.004^2 = 0.0000502...$
Resistivity = gradient × cross-sectional area
$= 0.00033... \times 0.0000502...$
$= 1.675... \times 10^{-8} =$ **1.7×10^{-8} Ωm (to 2 s.f.)**

Page 158 — Fact Recall Questions

Q1 The resistance of something is a measure of how difficult it is to get a current to flow through it.
It is defined as $R = V \div I$.

Q2 Area $= \pi r^2$, assuming the wire is cylindrical, so that the cross-section of the wire is circular.

Q3 Use a test wire of the metal to be investigated, with a known cross-sectional area. Clamp the test wire to a ruler so that one end is aligned with the zero-mark. Connect this wire in a circuit with a power supply, an ammeter and a voltmeter. Use a flying lead to connect the test wire, so the length of test wire connected in the circuit can be varied. Connect the flying lead to the wire and record the length of the wire in the circuit. Measure the current through and potential difference across the wire and use these to calculate the resistance of that particular length of wire. Repeat this process at least once more for the same length of wire and use the values to calculate an average resistance for this length of wire. Repeat this process for different lengths of wire and plot a graph of average resistance against length. Use this graph to calculate the resistivity of the wire material by finding the gradient and multiplying it by the cross-sectional area of the wire.

Q4 Resistivity is dependent on temperature, so the temperature must be kept constant so that it doesn't affect the results of the experiment (it can cause errors). As current flows through a wire, it causes the temperature of the wire to increase. The temperature could be kept constant by ensuring only a small current is used. A switch can also be used to ensure that current is only flowing through the wire in short bursts.

5. Types of Conductor

Page 161 — Fact Recall Questions

Q1 Nothing — the resistance of an ohmic conductor is constant (if the physical conditions it's under are constant).

Q2 Temperature.

Q3

6. *I-V* Characteristics

Page 164 — Application Question

Q1 a) a diode / LED

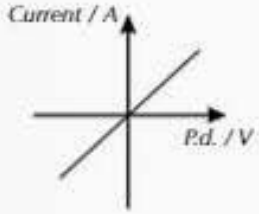
b) The diode requires a voltage of about 0.6 V in the forward bias before it will conduct, so there is no current flow until about 0.6 V on the diagram. After 0.6 V, the current is allowed to flow, and increases with an increase in potential difference.

c) In reverse bias, the resistance of the diode is very high.

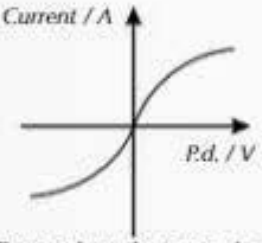
Page 164 — Fact Recall Questions

Q1 A voltmeter is assumed to have infinite resistance and an ammeter is assumed to have zero resistance.

Q2 a)



b)



Q3 A filament lamp is not an ohmic conductor. Its *I-V* characteristic is not a straight line — so current is not proportional to potential difference.

7. Power and Electrical Energy

Page 167 — Application Questions

Q1 **3400 W** (3.4 kW)

Power is defined as the energy transfer per second. It's measured in watts, where 1 watt is equal to 1 joule per second.

Q2 a) $P = \frac{W}{t} = \frac{12\,500}{2.00} = \mathbf{6250\ W}$

b) $I = \frac{P}{V} = \frac{6250}{8.00} = 781.25 = \mathbf{781\ A\ (to\ 3\ s.f.)}$

Q3 $P = \frac{V^2}{R}$, so rearranging: $R = \frac{V^2}{P}$

$R = \frac{230^2}{5200} = 10.173... = \mathbf{10\ \Omega\ (to\ 2\ s.f.)}$

Q4 $W = VIt = 230 \times 1.2 \times 45 = 12\,420 = \mathbf{12\,000\ J\ (to\ 2\ s.f.)}$

Q5 $P = 1250$ W

$P = I^2R$, so rearranging: $I = \sqrt{\frac{P}{R}}$

$I = \sqrt{\frac{1250}{54.2}} = 4.8023... = \mathbf{4.80\ A\ (to\ 3\ s.f.)}$

Page 167 — Fact Recall Questions

Q1 Power is the rate of transfer of energy (or doing work). It's measured in watts (W), where 1 watt is equivalent to 1 joule per second.

Q2 E.g. $P = VI$, $P = \frac{V^2}{R}$, $P = I^2R$

Q3 $W = VIt$

8. Domestic Electricity

Page 169 — Application Questions

Q1 1 kWh = 3.6×10^6 J

So, energy transferred in kWh,

$W = 72\,000 \div (3.6 \times 10^6) = \mathbf{0.02\ kWh}$

Q2 Cost = number of units × price per unit

So, number of units = cost ÷ price per unit

£28.24 = 2824p

number of units = 2824 ÷ 12.2

$= 231.47... = \mathbf{231\ units\ (to\ 3\ s.f.)}$

9. E.m.f. and Internal Resistance

Page 174 — Application Questions

Q1 $\varepsilon = V + v$, so $v = \varepsilon - V = 2.50 - 2.24 = \mathbf{0.26\ V}$

Q2 $\varepsilon = V + Ir = 4.68 + (0.63 \times 0.89) = 5.2407$

$= \mathbf{5.2\ V\ (to\ 2\ s.f.)}$

Q3 $\varepsilon = I(R + r)$, so $\varepsilon \div I = R + r$ and so $R = (\varepsilon \div I) - r$

$R = (15.0 \div 26.1) - 8.28 \times 10^{-3} = 0.566432...$

$= \mathbf{0.566\ \Omega\ (to\ 3\ s.f.)}$

Q4 $P = I^2R = (1.2)^2 \times 0.50 = 0.72$ W

So **0.72 J** is dissipated each second.

Q5 a) $\varepsilon = \mathbf{5.0\ V}$

The e.m.f. is the intercept on the vertical axis.

b) internal resistance = –gradient of the graph

$= -(\Delta y \div \Delta x)$

e.g. internal resistance = $-(-5.0 \div 0.6)$

$= 8.333...$

$= \mathbf{8.3\ \Omega\ (to\ 2\ s.f.)}$

c) The straight line would still have the same *y*-intercept at 5.0 V. The gradient = $-r$, so the gradient would be half as steep as that for the original power supply, so it would intercept the *x*-axis at 1.2 A.

Page 174 — Fact Recall Questions

Q1 The load resistance is the total resistance of all the components in the external part of the circuit.

The load resistance doesn't include the internal resistance of the power source.

Q2 volts (V)

Q3 The energy wasted per coulomb overcoming an internal resistance.

Q4 The gradient is $-r$ (where r is the internal resistance) and the vertical intercept is ε (the electromotive force).

10. Conservation of Energy and Charge in Circuits

Page 178 — Application Questions

Q1 a) $R = R_1 + R_2 + R_3$
$40\ \Omega = R_1 + 20\ \Omega + 10\ \Omega$
$R_1 = 40 - 20 - 10 = \mathbf{10\ \Omega}$

b) 0.4 A
The current flowing through a series circuit is the same at all points of the circuit.

Q2 a) First calculate the total resistance of the 3 Ω and 6 Ω resistors in parallel, using $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$
$\frac{1}{R} = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$, so $R = 2\ \Omega$
Resistors in series add up and the total resistance is 10 Ω, so the total resistance of the 12 Ω and R_1 resistors in parallel is equal to: $10\ \Omega - 2\ \Omega - 5\ \Omega = 3\ \Omega$
The 5 Ω here is the resistance of the resistor in between the two sets of parallel resistors.
So for the R_1 resistor and the 12 Ω resistor in parallel, the total resistance is 3 Ω. So:
$\frac{1}{3} = \frac{1}{R_1} + \frac{1}{12}$
$\frac{1}{R_1} = \frac{1}{3} - \frac{1}{12} = \frac{1}{4}$, so $R_1 = \mathbf{4\ \Omega}$

b) First find the total current through the circuit, using the total resistance and the source p.d.
$I = V \div R$
$I = 12 \div 10 = 1.2$ A
This current flows through the 5 Ω resistor, so find the p.d. across this resistor using:
$V = I \times R = 1.2 \times 5 = \mathbf{6\ V}$

c) The p.d. across the resistors in parallel (3 Ω and 6 Ω) is:
$V = I \times R = 1.2 \times 2 = 2.4$ V
So the current through the 6 Ω is:
$I = V \div R = 2.4 \div 6 = \mathbf{0.4\ A}$
Remember, current splits depending on the resistance — it doesn't just split equally between resistors (unless the resistors have equal resistance).

Page 178 — Fact Recall Questions

Q1 The total current entering a junction = the total current leaving it.

Q2 The total e.m.f. around a series circuit = the sum of the p.d.s across each component.

Q3 They are in series.

Q4 parallel

11. The Potential Divider

Page 181 — Application Questions

Q1 a) $V_{out} = \frac{R_2}{R_1 + R_2} V_{in} = \frac{3.0}{12 + 3.0} \times 16 = \mathbf{3.2\ V}$

b) $V_{out} = \frac{R_2}{R_1 + R_2} V_{in}$, so $5.0 = \frac{R_2}{12 + R_2} \times 16$
$5.0(12 + R_2) = 16R_2 \Rightarrow 60 + 5.0R_2 = 16R_2 \Rightarrow 60 = 11R_2$
$R_2 = \frac{60}{11} = 5.4545... = \mathbf{5.5\ \Omega}$ **(to 2 s.f.)**

Q2 $V_{out} = \frac{R_2}{R_1 + R_2} V_{in}$ so $\frac{V_{out}}{V_{in}} = \frac{R_2}{R_1 + R_2} = \frac{4}{16} = \frac{1}{4}$
$4R_2 = R_1 + R_2$ so $3R_2 = R_1$
So, for example, you could have $R_1 = 9\ \Omega$ and $R_2 = 3\ \Omega$.
Here, you can have any values that $3R_2 = R_1$ is true for.

Q3 a) $V_{out} = \frac{R_2}{R_1 + R_2} V_{in}$ so $V_{out} = \frac{1.5}{3.0 + 1.5} \times 1.5 = \mathbf{0.5\ V}$

b) $V_{out} = \frac{R_2}{R_1 + R_2} V_{in}$ so $0.30 = \frac{R_2}{3.0 + R_2} \times 1.5$
$0.30(3.0 + R_2) = 1.5R_2 \Rightarrow 0.90 + 0.30R_2 = 1.5R_2$
$0.90 = 1.2R_2$
$R_2 = \mathbf{0.75\ \Omega}$

Page 181 — Fact Recall Questions

Q1 A potential divider is a circuit containing a voltage source and a couple of resistors in series. The voltage across one of the resistors is used as an output voltage. If the resistors aren't fixed, the circuit will be capable of producing a variable output voltage.

Q2 $V_{out} = \frac{R_2}{R_1 + R_2} V_{in}$, where V_{out} = the output potential difference, R_2 = the resistance of the resistor connected across the voltage output, R_1 = the resistance of the other resistor in the potential divider and V_{in} = the potential difference of the power supply.

Q3 You can make a light sensor using a potential divider by using an LDR as one of the resistors.

Q4 A potential divider containing a variable resistor instead of two resistors in series. E.g. they are used in a volume control on a stereo.

Exam-style Questions — Pages 183-185

1 B ***(1 mark)***

2 C ***(1 mark)***

3 D ***(1 mark)***

4 C ***(1 mark)***

5 a) $P = VI = 3.00 \times 0.724 = 2.172 =$ **2.17 W (to 3 s.f.)**
(2 marks for correct answer, 1 mark for correct working if answer incorrect)

b) 5 minutes = 300 seconds
$\Delta Q = I \times \Delta t = 0.724 \times 300 = 217.2 =$ **217 C (to 3 s.f.)**
(2 marks for correct answer, 1 mark for correct working if answer incorrect)

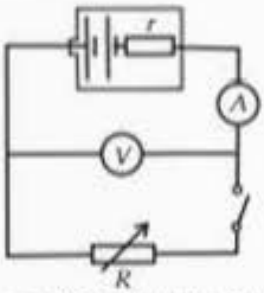
c) $V = W \div Q = 56.5 \div 217.2 = 0.26012...$
$=$ **0.260 V (to 3 s.f.)**
(2 marks for correct answer, 1 mark for correct working if answer incorrect)

d) p.d. across the filament lamp $= I \times R$
$= 0.724 \times 2.00$
$= 1.448$ V
$V = V_1 + V_2 + V_{bulb}$
$3.00 = 0.26012... + V_2 + 1.448$
$V_2 = 1.2918... =$ **1.29 V (to 3 s.f.)**
(2 marks for correct answer, 1 mark for correct working if answer incorrect)

e) No, the filament lamp is not an ohmic conductor as I is not directly proportional to V ***(1 mark)***.

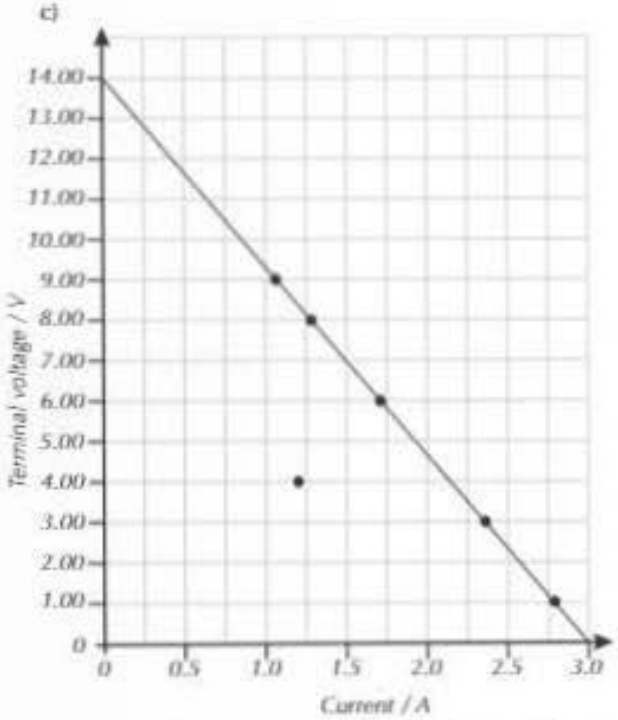
6 a) As the electrons move through the battery, they collide with atoms inside the battery and transfer some of their energy ***(1 mark)***. (The small amount of resistance that causes this energy loss inside the battery is called internal resistance.)

b) E.g.



(1 mark for a sensible circuit that could be used in this experiment. It should contain a battery, a variable resistor, a voltmeter and an ammeter.)
E.g. Set the variable resistor (the load resistance) to its highest resistance. Close the switch and record the current (I) through and potential difference (V) across the circuit ***(1 mark)***. Open the switch and close it again to get another set of current and potential difference readings for this load resistance. Repeat this to get a third set of readings. Calculate the mean current and potential difference for this resistance from your results ***(1 mark)***. Decrease the resistance of the variable resistor by a small amount and repeat the above process for this resistance. Continue until you have a set of mean currents and potential differences for 10 different load resistances ***(1 mark)***. Plot a V-I graph of the mean current and potential difference values. This will give you a straight-line graph whose gradient is $-r$, where r is the internal resistance of the battery ***(1 mark)***.

c)



(1 mark for all points correctly plotted, 1 mark for line of best fit drawn)

d) e.m.f. = y-intercept = **14 V** ***(1 mark)***

e) internal resistance = −gradient $= -\Delta y \div \Delta x$
(E.g. $= 14.00 \div 3.0) = 4.666... =$ **4.67 Ω (to 3 s.f.)**
(2 marks for correct answer, 1 mark for correct working if answer incorrect)

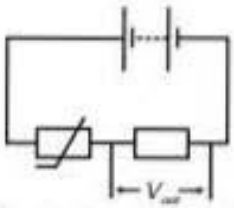
7 a) For the parallel resistors:
$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$
$\frac{1}{R} = \frac{1}{10.0} + \frac{1}{10.0}$
$\frac{1}{R} = \frac{1}{5.00}$
$R = 5.00\ \Omega$
For the whole circuit:
$R = 10.0 + 5.00 =$ **15.0 Ω**
(2 marks for the correct answer, 1 mark for correct working if answer incorrect)

b) Resistance of component A, $R_A = 10.0\ \Omega$
Potential difference splits according to the ratio of resistance of the component to the total resistance, so:
$V_A = \frac{R_A}{R} V$
$= \frac{10.0}{15.0} \times 12$
$=$ **8.0 V**
(2 marks for the correct answer, 1 mark for correct working if answer incorrect)

c) $R = 3.21 \times 10^{-3} + 5.00 = 5.00321\ \Omega$
$I = V \div R$
$I = 12 \div 5.00321 = 2.3984... =$ **2.4 A (to 2 s.f.)**
(2 marks for the correct answer, 1 mark for correct working if answer incorrect)

d) $\rho = (R \times A) \div l = (3.21 \times 10^{-3} \times 3.05 \times 10^{-4}) \div 0.0300$
$= 3.2635 \times 10^{-7}$
$= \mathbf{3.26 \times 10^{-7}\ \Omega m}$ **(to 3 s.f.)**
(2 marks for the correct answer, 1 mark for correct working if answer incorrect)

8 a) E.g.



(1 mark for drawing the cells, thermistor and resistor in series, 1 mark for correctly drawing the circuit such that the output voltage is across the resistor)

b) $V_{out} = \frac{R_2}{R_1 + R_2} V_{in}$

$= \frac{6.0}{9.2 + 6.0} \times 22 = 8.6842... = \mathbf{8.7\ V}$ **(to 2 s.f.)**
(1 mark)

c) $W = \frac{V^2}{R} t = \frac{V_{in}^2}{R_2} t$

$t = 12 \times 60 = 720\ s$

$W = \frac{(8.6842...)^2}{6.0} \times 720$

$= 9049.861...\ J$

$= \mathbf{9000\ J}$ **(to 2 s.f.)**
(2 marks for the correct answer, 1 mark for correct working if answer incorrect)

d) $V_{out} = \frac{R_2}{R_1 + R_2} V_{in}$

Rearrange for R_1:

$V_{out}(R_1 + R_2) = R_2 V_{in}$

$R_1 + R_2 = \frac{R_2 V_{in}}{V_{out}}$

$R_1 = \frac{R_2 V_{in}}{V_{out}} - R_2$

$= \frac{6.0 \times 22}{13} - 6.0$

$= 4.1538...$

$= \mathbf{4.2\ \Omega}$ **(to 2 s.f.)**
(2 marks for the correct answer, 1 mark for correct working if answer incorrect)

Section 2 — Waves

1. Progressive Waves

Page 189 — Application Questions

Q1 The wave is travelling to the left / from right to left.
The point at X is moving upwards, so the higher point of the wave must be moving towards X. So the wave must be moving to the left.

Q2 a) −0.2 m
b) 0.2 m
For *displacement* the *direction* matters (it can be negative), but for *amplitude* you only need a magnitude.
c) 1.6 m

Page 189 — Fact Recall Questions

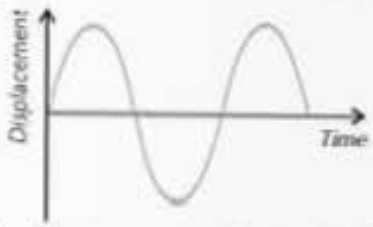
Q1 By causing particles or fields in the region to oscillate.

Q2 *A* shows the amplitude of the wave.
B shows a crest of the wave.
C shows the wavelength of the wave.

Q3 E.g. the amount by which one wave lags behind another / the difference between a point on one wave and the next identical point on the second wave.

Q4 In longitudinal waves, the direction of oscillations are along the same line as the direction of energy propagation. In transverse waves, the direction of oscillations are perpendicular to the direction of energy propagation.

Q5 E.g.



The *displacement–time graph* for a longitudinal wave looks just like the one for a transverse wave.

2. Frequency, Speed and Intensity

Page 193 — Application Questions

Q1 a) Rearrange $I = \frac{P}{A}$ for power:
$P = IA$
$= 5 \times 0.2$
$= \mathbf{1\ W}$

b) intensity $\propto$ (amplitude)^2

So, $\frac{\text{intensity}}{(\text{amplitude})^2} = \text{constant}$

$\frac{\text{final intensity}}{(\text{final amplitude})^2} = \frac{\text{initial intensity}}{(\text{initial amplitude})^2}$

$\text{final intensity} = \text{initial intensity} \times \left(\frac{\text{final amplitude}}{\text{initial amplitude}}\right)^2$

Amplitude is doubled, so
final amplitude ÷ initial amplitude = 2
So, final intensity = $5 \times (2)^2$
$= \mathbf{20\ Wm^{-2}}$

Q2 a) The length of 1.5 oscillations is 0.15 m, so the length of one oscillation is $\frac{0.15}{1.5} = 0.10$ m.
So wavelength, $\lambda = \mathbf{0.10\ m}$.

b) $v = f\lambda$, and $v = \frac{d}{t}$

Rearrange $v = \frac{d}{t}$ for t, and substitute in $v = f\lambda$:

$t = \frac{d}{f\lambda} = \frac{1.0}{30.0 \times 0.10} = 0.33...s$

So the time taken to travel 1.0 m is **0.33 s (to 2 s.f.)**

Page 193 — Fact Recall Questions

Q1 frequency = $\frac{1}{\text{period}}$ / $f = \frac{1}{T}$

Q2 The vertical axis shows voltage.
The horizontal axis shows time.

Q3 Calculate the period of the wave from the oscilloscope display, by counting the number of divisions for one full cycle and multiplying this by the value of one division. Use this value of the period to calculate the frequency using $f = \frac{1}{T}$.

Q4 wave speed = frequency × wavelength / $v = f\lambda$

Q5 The intensity of a progressive wave is the rate of flow of energy per unit area at right angles to the direction of travel of the wave.

Q6 intensity $\propto$ (amplitude)^2

3. Electromagnetic Waves

Page 195 — Application Question

Q1 Speed of EM waves in a vacuum, $v = 3.00 \times 10^8$ ms^{-1}
rearrange $v = f\lambda$ for f:
$f = v \div \lambda$
$= 3.00 \times 10^8 \div 2.40 \times 10^{-5}$
$= \mathbf{1250\ Hz}$

Page 195 — Fact Recall Questions

Q1 Transverse waves.

Q2 Gamma rays

Q3 a) 10^{-3} — 10^{-1} m
b) 10^{-8} — 4×10^{-7} m
c) 10^{11} — 10^{-8} m

Q4 Infrared light, visible light and ultraviolet light.

4. Polarisation

Page 198 — Application Question

Q1 a) This wave must be transverse because it can be polarised.

b) C represents the transmission axis. It must be the same as the first polarising filter, because all the waves that pass through the first one also gets through the second one.

c) The transmission axis would now be at right angles to the waves that passed through the first filter, so no waves would get through the second filter.

5. Reflection and Refraction

Page 201 — Application Questions

Q1 The angle of incidence is measured from the normal, so:
angle of incidence = 90 − 30 = 60°
angle of incidence = angle of reflection, so:
angle of reflection = **60°**

Q2 Side B is deeper. The wave has bent away from the normal and has a longer wavelength in side B, so must be travelling faster here. Water waves travel faster in deeper water so B must be deeper.

Page 201 — Fact Recall Questions

Q1 angle of incidence = angle of reflection

Q2 a) The wave bends towards the normal.

b) The wavelength of the wave decreases, but its frequency stays the same.

6. Refractive Index and Total Internal Reflection

Page 206 — Application Questions

Q1 a) $n = \frac{c}{v} = \frac{3.00 \times 10^8}{1.94 \times 10^8} = 1.546... = \mathbf{1.55\ (to\ 3\ s.f.)}$

b) The light will bend towards the normal — $n_{air} = 1$, so air is less optically dense than the material, so light slows down as it crosses the boundary.

c) $\sin C = \frac{1}{n} \Rightarrow C = \sin^{-1}\left(\frac{1}{n}\right) = \sin^{-1}\left(\frac{1}{1.546...}\right) = 40.290...$
$= \mathbf{40.3°\ (to\ 3\ s.f.)}$

Q2 So that the ray won't refract as it enters the block.

Q3 a) When light travelling from the cage meets the water-air boundary, it's refracted (away from the normal). This makes it look like the light is coming from a different point to where the cage actually is.

b) You know: $n_1 = 1.38$, $n_2 = 1.49$, $\theta_1 = 37.2°$, $\theta_2 = ?$
So rearrange the law of refraction to find θ_2:
$n_1 \sin\theta_1 = n_2 \sin\theta_2 \Rightarrow \theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right)$
$= \sin^{-1}\left(\frac{1.38 \times \sin 37.2°}{1.49}\right)$
$= 34.053... = \mathbf{34.1°\ (to\ 3\ s.f.)}$

Page 206 — Fact Recall Questions

Q1 It's a measure of the optical density of the material, given by the ratio of the speed of light in a vacuum to the speed of light in the material.

Q2 It will bend towards the normal.

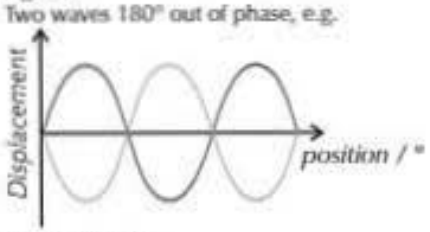
Q3 $n_2 < n_1$ and the angle of incidence must be greater than the critical angle.

7. Superposition and Interference

Page 210 — Application Questions

Q1 E.g. 90°

Q2 Two waves 180° out of phase, e.g.



Q3 a) i) Point G
ii) No points.
iii) Point E
b) Two points (C and E).

Page 210 — Fact Recall Questions

Q1 When two or more waves meet, the resultant displacement equals the vector sum of the individual displacements.

Q2 When two waves pass through each other and their displacements combine to make a displacement with greater magnitude.

Q3 The phase difference of two points on a wave is the difference in their positions in the wave's cycle.

Q4 Phase difference can be measured in degrees, radians or fractions of a cycle.

Q5 When their phase difference is an odd multiple of 180° (π radians or half a cycle).

Q6 Two wave sources are coherent if the waves have the same wavelength and frequency and a fixed phase difference between them.

Q7 Two waves are in phase if they have a phase difference of 0° (or a multiple of 360°).
You could have said 2π radians or 1 full cycle instead of 360° here.

8. Diffraction

Page 212 — Application Question

Q1 No, the gap is much smaller than the waves, so the waves are mostly reflected back.

Page 212 — Fact Recall Questions

Q1 All waves.

Q2 Diffraction effects are strongest when the gap size is similar in size to the wavelength. The size of the doorway and the wavelength of sound waves are roughly equal, so the sound waves diffract through the doorway and can be heard around the corner. Light has a much smaller wavelength than the doorway, so diffraction effects aren't noticeable and the person cannot be seen around the corner.

Q3 Monochromatic light is light made up of only one wavelength (and so frequency).

Q4 A bright central fringe with alternating dark and bright fringes on either side of it.

9. Two-Source Interference

Page 215 — Application Questions

Q1 a) E.g.

b) E.g.

Q2 p.d. = $x\lambda$, so $x = 0.42 \div 0.12 = 3.5$
As x is in the form $n + \frac{1}{2}$ (where $n = 3$), there will be destructive interference.

Page 215 — Fact Recall Questions

Q1 They must be coherent.

Q2 The path difference is the amount by which the path travelled by one wave is longer than the path travelled by the other wave.

Q3 You see constructive interference when the path difference equals $n\lambda$ (n is an integer).

Q4 E.g. Have one amplifier attached to two loudspeakers.

Q5 E.g. Attach two loudspeakers to an amplifier and create sound waves at a set frequency. Walk along a straight line parallel to the line of the speakers. Mark down points of maximum loudness and quietness, which show the position of constructive and destructive interference.

Q6 The probe would detect alternating areas of maximum and minimum signal strength.

10. Young's Double-Slit Experiment

Page 219 — Application Questions

Q1 a) Laser light is coherent/monochromatic, so the pattern is clearer.

b) Light fringes are produced where there is constructive interference and dark fringes are produced where there is destructive interference.

c) $x = \frac{\lambda D}{d} = \frac{(4.5 \times 10^{-7}) \times 12.0}{0.00030} = \mathbf{0.018\ m}$

d) There are 10 maxima with a gap of 0.018 m between each one, so the total width is $9 \times 0.018 = 0.162$ m. So $y = \mathbf{0.16\ (to\ 2\ s.f.)}$.

Even though there are 10 maxima, there are only 9 fringe spacings (the gaps between them).

Q2 Rearrange the double-slit formula: $D = \frac{dx}{\lambda}$

So $D = \frac{(1.29 \times 10^{-2}) \times (0.11 \times 10^{-3})}{615 \times 10^{-9}}$

$= 2.307... = \mathbf{2.3\ m\ (to\ 2\ s.f.)}$

Page 219 — Fact Recall Questions

Q1 By shining light through a double-slit system.

Q2 E.g. Mount a card with two thin slits, with spacing a, a distance D from an observation screen. Shine the laser beam through the slits to create an interference pattern on the screen, then find the fringe spacing, x, by measuring across several fringes with a ruler then dividing by the number of fringe widths between them. Use the double-slit formula $\lambda = \frac{dx}{D}$ to calculate the wavelength of the light.

Q3 Young's double-slit experiment showed that light could diffract and interfere. Both of these qualities are wave properties, which suggested that light was a wave.

11. Diffraction Gratings

Page 222 — Application Question

Q1 a) First work out the slit spacing. It has 4.5×10^5 slits per metre, so the slit spacing $d = \frac{1}{4.5 \times 10^5}$

$= 2.222... \times 10^{-6}$ m

Use the diffraction grating equation, rearranged for θ, and using $n = 3$:

$\theta = \sin^{-1}\left(\frac{n\lambda}{d}\right) = \sin^{-1}\left(\frac{3 \times (5.9 \times 10^{-7})}{2.222... \times 10^{-6}}\right)$

$= 52.797... = \mathbf{53°\ (to\ 2\ s.f.)}$

b) No, because $\frac{n\lambda}{d} = \frac{4 \times (5.9 \times 10^{-7})}{2.222... \times 10^{-6}} = 1.1$ (to 2 s.f.).

The sin function is only defined between −1 and 1, so $\sin^{-1}1.1$ is impossible.

c) The pattern will be more spread out, because the angle is related to n, d and λ. If n and d remain constant, θ will become larger and so each order of maximum will be further from the zero order.

Page 222 — Fact Recall Questions

Q1 E.g. Measure the distance D between the diffraction grating and an observation screen. Shine a laser through the diffraction grating onto the screen, and using a ruler measure the spacing to the nth order maximum (choose a value of n). Calculate the angle to the normal made by the maximum using $\theta = \frac{x}{D}$.

Rearrange the diffraction grating equation to $\lambda = \frac{d \sin\theta}{n}$ and use it to calculate the wavelength.

Q2 White light is made up of a range of different wavelengths. These diffract by different amounts when they pass through the diffraction grating, forming a spectrum.

12. Stationary Waves

Page 227 — Application Questions

Q1 This wave is at the third harmonic so 1.5 wavelengths fit on the string, so $1.5\lambda = 6$, so $\lambda = \mathbf{4\ m}$.

Q2 a) The wave is vibrating at the fundamental frequency, so the length of the string is half a wavelength. So $\lambda = 5$ m.

$v = f\lambda = 100 \times 5 = \mathbf{500\ ms^{-1}}$

b) $v = f\lambda$ so $\lambda = \frac{v}{f} = \frac{500}{200} = 2.5$ m

So one wavelength (or two half wavelengths) would fit on the string at this frequency. This resonant frequency is called the second harmonic.

You could also just look at the frequency — it's twice the frequency of the first harmonic, so $2 \times$ ½ wavelengths fit on the string.

Q3 The length of the pipe is equal to $\frac{\lambda}{4}$.

Q4 At fundamental frequency, length of open pipe is half a wavelength.

So length of open pipe $= \frac{1}{2} \times 3.2 = 1.6$ m

Length of closed pipe $= 1.6 - 0.2 = 1.4$ m

At fundamental frequency, length of closed pipe is one quarter of the wavelength.

So wavelength $= 4 \times$ length of closed pipe

$= 4 \times 1.4 = \mathbf{5.6\ m}$

Page 227 — Fact Recall Questions

Q1 The superposition of two progressive waves with the same wavelength and frequency, moving in opposite directions.

Q2 Unlike progressive waves, no energy is transmitted by a stationary wave.

Q3 A resonant frequency of a string is a frequency at which a stationary wave is formed because an exact number of waves are produced in the time it takes for a wave to get to the end of the string and back again.

Q4 $\frac{\lambda}{2}$

Q5 The lowest possible resonant frequency.

Q6

Q7 E.g. Use a metal plate to reflect microwaves and a probe to observe nodes and antinodes.

Exam-style Questions — Pages 229-231

1 D ***(1 mark)***

Constructive interference occurs when the path difference is a whole number of wavelengths.

2 C ***(1 mark)***

3 A ***(1 mark)***

1.7×10^{-16} ms $= 1.7 \times 10^{-19}$ s

$f = 1 \div t = 1 \div (1.7 \times 10^{-19}) = 5.88... \times 10^{18}$ Hz

All electromagnetic waves travel at the same speed in a vacuum: $v = 3.00 \times 10^{8}$ ms^{-1}.

Rearrange $v = f\lambda$ to $\lambda = v \div f = 3 \times 10^{8} \div 5.88... \times 10^{18}$

$= 5.1 \times 10^{-11}$ m

4 D ***(1 mark)***

Intensity $\propto$ amplitude^2 = constant, so:

initial intensity (I_i) $\propto$ (initial amplitude (x_i))^2 = final intensity (I_f) $\propto$ (final amplitude (x_f))^2

Rearranging: $x_i = x_f\sqrt{\frac{I_i}{I_f}} = 0.78 \times \sqrt{0.11 \div 0.42}$

$= 0.399...$ m $= 40$ cm (to 2 s.f.)

5 a) The third harmonic ***(1 mark)***

b) Tube length $= 3 \times \frac{\lambda}{2}$

Rearranging:

$\lambda = \frac{2}{3} \times$ tube length $= \frac{2}{3} \times 1.25$

$= 0.8333...$ **= 0.833 m (to 3 s.f.)** ***(1 mark)***

c) How to grade your answer (pick the description that best matches your answer):

0 marks: There is no relevant information.

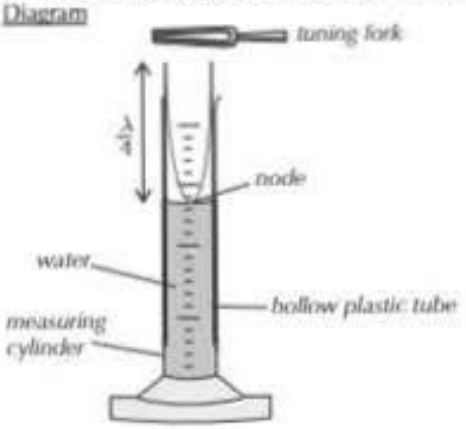
1-2 marks: A diagram has been drawn, and method points 2, 6, and 8 have been covered. Answer is basic and has lack of structure. Information is missing and there are few backed up arguments.

3-4 marks: A labelled diagram has been drawn and method points 1, 2, 4, 6, and 8 have been covered. At least one accuracy point is covered. Answer has some structure, and information is presented with a clear line of reasoning, with arguments that are partially backed up.

5-6 marks: A labelled diagram has been drawn, all of the method points have been covered, and at least one point from the accuracy points has been covered. The answer is well structured. There is a clear and logical line of reasoning, with information and arguments that are fully backed up.

Here are some points your answer may include:

Diagram



Method

1. Suspend the tube in water in a measuring cylinder to create a closed-end pipe.
2. Note down the frequency of the sound produced by the tuning fork.
3. Gently tap tuning fork to generate sound waves.
4. Hold tuning fork just above tube to direct sound waves into the tube.
5. Move the tube up and down in the water until you find the shortest distance between the top of the tube and the water level at which the sound from the fork is loudest.
6. Measure the distance between the top of the tube and the water level.
7. Multiply this distance by four to get the wavelength of the wave.
8. Plug f and λ into $v = f\lambda$ to calculate v.

Improving accuracy

1. Repeat the experiment with different tuning forks and take an average for the value of v.
2. Repeat for each tuning fork to find average λ.

6 a) Each division is 0.500 μs and one cycle is 6 divisions wide, so one cycle takes $6 \times 0.500 = 3.00$ μs

3.00 μs $= 3.00 \times 10^{-6}$ s

$f = \frac{1}{T} = \frac{1}{3.00 \times 10^{-6}} = 3.33... \times 10^{5}$ Hz

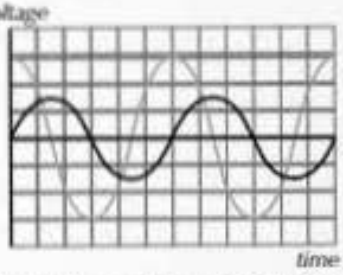
Rearrange $v = f\lambda$ to $\lambda = \frac{v}{f}$

So $\lambda = \frac{3.00 \times 10^{8}}{3.33... \times 10^{5}} =$ **900 m**

(3 marks for correct answer, otherwise 1 mark for calculating the frequency and 1 mark for correct working.)

b) Radio ***(1 mark)***

c) *Voltage*



(1 mark for second wave of equal wavelength and frequency and half the amplitude, and 1 mark for second wave drawn 90° out of phase ahead of 1st wave.)

d) $I = \frac{P}{A} \Rightarrow P = IA$ for a given A

and $I \propto$ (amplitude)^2, so $P \propto$ (amplitude)^2

$\frac{P}{(\text{amplitude})^2} =$ constant

So $\frac{P_1}{(\text{amplitude of 1st wave})^2} = \frac{P_2}{(\text{amplitude of 2nd wave})^2}$

$P_2 = P_1\left(\frac{\text{amplitude of 2nd wave}}{\text{amplitude of 1st wave}}\right)^2$

$= 15.2 \times \left(\frac{1}{2}\right)^2 =$ **3.8 W**

(2 marks for correct answer, otherwise 1 mark for correct working.)

7 a) There are 9 maxima, so 8 fringes.
So fringe spacing $x = \frac{0.24}{8} = 0.030$ m.
To find λ use the double-slit formula:
$\lambda = \frac{ax}{D} = \frac{0.030 \times 0.15 \times 10^{-3}}{7.5}$
$= \mathbf{6.0 \times 10^{-7}}$ **m (= 600 nm)**
(3 marks for correct answer, otherwise 1 mark for finding fringe spacing and 1 mark for attempting to use double-slit formula.)

b) Lasers are coherent ***(1 mark)*** and monochromatic, so the diffraction patterns are clearer ***(1 mark)***.

c) Distance between slits $= \frac{1}{2.55 \times 10^5}$
$= 3.921... \times 10^{-6}$ m
$d \sin\theta = n\lambda \Rightarrow \theta = \sin^{-1}\left(\frac{n\lambda}{d}\right)$
$= \sin^{-1}\left(\frac{1 \times (6 \times 10^{-7})}{3.921... \times 10^{-6}}\right)$
$= 8.800... = \mathbf{8.8°}$ **(to 2 s.f.)**
(4 marks for correct answer. Otherwise 1 mark for finding the space between slits, 1 mark for rearranging formula to find θ, and 1 mark for correct calculations.)

d) The maxima would become much clearer / better defined. ***(1 mark)***

8 a) $n = \frac{c}{v} = \frac{3.00 \times 10^8}{2.03 \times 10^8} = 1.477... = \mathbf{1.48}$ **(to 3 s.f.)**
(2 marks for correct answer, otherwise 1 mark for correct working if answer incorrect.)

b) Use the law of refraction at the air-core boundary:
$n_1 \sin\theta_1 = n_2 \sin\theta_2$ with $n_1 = 1$, $\theta_1 = 34°$, $\theta_2 = 21°$.
Rearrange: $n_2 = \frac{n_1 \sin\theta_1}{\sin\theta_2} = \frac{1 \times \sin 34°}{\sin 21°} = 1.56...$
$= \mathbf{1.6}$ **(to 2 s.f.)**
(3 marks for correct answer, otherwise 1 mark for substituting into the correct formula and 1 mark for correct rearrangement if answer incorrect.)

c) $n = \frac{c}{v}$ and $v = f\lambda$.
When a wave travels through a boundary, its wavelength changes and its frequency remains constant. So n becomes:
$n = \frac{c}{f\lambda}$ so $\lambda = \frac{c}{fn} = \frac{3.00 \times 10^8}{5.00 \times 10^{14} \times 1.56...}$
$= 3.845... \times 10^{-7} = \mathbf{380}$ **nm (to 2 s.f.)**
(3 marks for correct answer, otherwise 1 mark for knowing f is constant and 1 mark for substituting into correct formula.)

d) $\sin C = \frac{n_2}{n_1} = \frac{1.47}{1.56...} = 0.9470...$
so $C = \sin^{-1}(0.9470...) = 71.2... = \mathbf{71°}$ **(to 2 s.f.)**
(2 marks for correct answer, 1 mark for correct working if answer incorrect.)

e) The light ray enters the cladding at an angle of $90° - 21° = 69°$ (the angle has to add to the angle between the cladding and the normal), which is less than the critical angle, so total internal reflection cannot happen and it enters the cladding instead.
(1 mark for $\theta < C$,* 1 mark for mention of total internal reflection.*)

f) Ray B has been totally internally reflected because it hits the core-cladding boundary at an angle greater than the critical angle. ***(1 mark for total internal reflection, 1 mark for critical angle.)***

Section 3 — Quantum Physics

1. The Photon Model

Page 233 — Application Questions

Q1 $2.20 \times 1.60 \times 10^{-19} = \mathbf{3.52 \times 10^{-19}}$ **J**

Q2 $E = hf = (6.63 \times 10^{-34}) \times (6.0 \times 10^{53})$
$= 3.978 \times 10^{20}$
$= \mathbf{4.0 \times 10^{20}}$ **J (to 2 s.f.)**

Q3 $E = \frac{hc}{\lambda}$ so $\lambda = \frac{hc}{E}$
$= ((6.63 \times 10^{-34}) \times (3.00 \times 10^8)) \div (8.2 \times 10^{-20})$
$= 2.425... \times 10^{-6}$
$= \mathbf{2.4 \times 10^{-6}}$ **m (to 2 s.f.)**

Page 233 — Fact Recall Questions

Q1 A photon is a quantum of energy of EM radiation.

Q2 $E = \frac{hc}{\lambda}$

Q3 energy

2. The Planck Constant

Page 236 — Application Questions

Q1 $eV_0 = \frac{hc}{\lambda}$
so $V_0 = \frac{hc}{e\lambda} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^8}{1.60 \times 10^{-19} \times 600 \times 10^{-9}}$
$= 2.071... = \mathbf{2.1}$ **V (to 2 s.f.)**

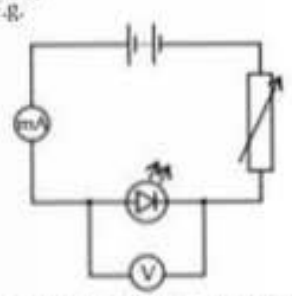
Q2 Gradient of graph $= (2.0 - 1.3) \div |(2.1 \times 10^6) - (1.58 \times 10^6)|$
$= 0.7 \div (0.52 \times 10^6) = 1.34... \times 10^{-6}$ Vm
$h = \frac{e \times \text{gradient}}{c} = \frac{1.60 \times 10^{-19} \times 1.34... \times 10^{-6}}{3.00 \times 10^8}$
$= 7.179... \times 10^{-34} = \mathbf{7.2 \times 10^{-34}}$ **Js (to 2 s.f.)**

Page 236 — Fact Recall Questions

Q1 The minimum voltage needed for a current to flow through an LED / The voltage needed to give electrons in the circuit the same energy as a photon emitted by the LED.

Q2 $eV_0 = \frac{hc}{\lambda}$

Q3 a) E.g.



b) The currents through the circuit will be very small.

3. The Photoelectric Effect

Page 240 — Application Questions

Q1 The work function is the minimum amount of energy required for an electron to be emitted from the zinc's surface. The energy of each photon is greater than this value, and so they will be able to transfer enough energy to release electrons from the zinc sheet.

Q2 a) The maximum kinetic energy of the electrons increases as the frequency of the incident radiation increases. Increasing the frequency of the radiation increases the energy of each photon ($E = hf$), and so more energy can be transferred to the free electrons in the metal. The maximum kinetic energy of the photoelectrons emitted is the energy transferred by a photon minus the work function energy. The work function energy remains the same, so as the energy of the incident photons increases, the maximum kinetic energy of the emitted electrons also increases.

b) $E = hf$, $h = 6.63 \times 10^{-34}$ Js, $f = 1.20 \times 10^{15}$ Hz,
so $E = (6.63 \times 10^{-34}) \times (1.20 \times 10^{15})$
$= 7.956 \times 10^{-19} =$ **7.96×10^{-19} J (to 3 s.f.)**

c) $hf = \phi + KE_{max}$
$KE_{max} = 7.26 \times 10^{-19}$ J, $hf = 7.956 \times 10^{-19}$ J,
so the work function of the metal (ϕ) is
$\phi = hf - KE_{max} = (7.956 \times 10^{-19}) - (7.26 \times 10^{-19})$
$=$ **6.96×10^{-19} J**

Page 240 — Fact Recall Questions

Q1 The photoelectric effect is when electrons are emitted from the surface of a metal when radiation of a high enough frequency (usually ultraviolet light) is shone on it.

Q2 Initially, the gold leaf in the electroscope is repelled from the negatively charged metal. The UV radiation releases photoelectrons from the zinc plate, which removes negative charge from the zinc plate. The gold leaf falls as the charge on the plate is reduced, reducing the repulsion between the leaf and the metal it's attached to.

Q3 More electrons will be emitted, but their maximum kinetic energy will remain the same.

Q4 Before an electron can be emitted by a metal, it needs enough energy to break the bonds holding it there. This means there is a minimum amount of energy an electron needs before it can escape the metal surface. The threshold frequency is the minimum frequency of radiation that has photons with this energy.

Q5 The maximum amount of energy that can be transferred to an electron is the energy of one photon. There is a minimum amount of energy needed to free the electron from the metal (the work function energy). Therefore the maximum amount of kinetic energy an electron can have is the energy of one photon minus the work function energy.

4. Wave-Particle Duality

Page 244 — Application Questions

Q1 $\lambda = \frac{h}{p}$, where p (momentum) = mass × velocity, so the de Broglie wavelength is inversely proportional to the velocity of a particle. Therefore increasing the velocity will decrease the de Broglie wavelength of a particle.

Q2 a) $\lambda = \frac{h}{p}$, so $p = h \div \lambda$

$h = 6.63 \times 10^{-34}$ Js, $\lambda = 0.162$ nm,
$\lambda = 0.162 \times 10^{-9}$ m $= 1.62 \times 10^{-10}$ m,
$p = (6.63 \times 10^{-34}) \div (1.62 \times 10^{-10})$
$= 4.09259... \times 10^{-24}$ kg ms^{-1}
$=$ **4.09×10^{-24} kg ms^{-1} (to 3 s.f.)**

Make sure you always include units with your answer — it could get you some precious extra marks in the exam.

b) $m_e = 9.11 \times 10^{-31}$ kg,
$\boldsymbol{v} = \boldsymbol{p} \div m_e$
$= (4.09259... \times 10^{-24}) \div (9.11 \times 10^{-31})$
$= 4.4924... \times 10^{6}$ ms^{-1}
$E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times (9.11 \times 10^{-31}) \times (4.4924... \times 10^{6})^2$
$=$ **9.19×10^{-18} J (to 3 s.f.)**

Q3 a) $\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{6.646 \times 10^{-27} \times 60.0}$
$= 1.662... \times 10^{-9} =$ **1.66×10^{-9} m (to 3 s.f.)**

b) $\lambda = \frac{h}{p}$, so $v = h \div \lambda m$
$v = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 1.662... \times 10^{-9}}$
$= 437716.7...$ ms$^{-1} =$ **4.38×10^{5} ms^{-1} (to 3 s.f.)**

Page 244 — Fact Recall Questions

Q1 Diffraction shows electromagnetic waves have wave properties, and the photoelectric effect shows electromagnetic waves have particle properties.

Q2 All particles have both particle and wave properties. Waves can also show particle properties.

Q3 Electron diffraction

Exam-style Questions — Pages 246-247

1 C ***(1 mark)***

$2\ GeV = 2 \times 10^9 \times 1.60 \times 10^{-19} = 3.2 \times 10^{-10}$ J.

$E = hf$, so $f = E \div h = (3.2 \times 10^{-10}) \div (6.63 \times 10^{-34})$

$= 4.826... \times 10^{23}$ Hz

2 B ***(1 mark)***

$4.33\ eV = 4.33 \times 1.60 \times 10^{-19} = 6.928 \times 10^{-19}$ J.

$\phi = hf_0 = hc \div \lambda$

so $\lambda = hc \div \phi$

$= (6.63 \times 10^{-34} \times 3.00 \times 10^8) \div (6.928 \times 10^{-19})$

$= 2.870... \times 10^{-7}$ m

3 D ***(1 mark)***

$c = f\lambda$ so $\lambda = c \div f = 5.172... \times 10^{-7}$ m.

$V_0 = \frac{hc}{e\lambda} = (6.63 \times 10^{-34} \times 3.00 \times 10^8)$

$\div (1.60 \times 10^{-19} \times 5.172... \times 10^{-7}) = 2.40...$ V

4 D ***(1 mark)***

The spacing of the grating is 2×10^{-6} m. Diffraction only occurs when the de Broglie wavelength is roughly equal to this spacing.

The momentum of the neutrons is $p = mv$

$= 1.675 \times 10^{-27} \times 3200 = 5.36 \times 10^{-24}$ kg ms^{-1}.

So the de Broglie wavelength is

$(6.63 \times 10^{-34}) \div (5.36 \times 10^{-24}) = 1.23... \times 10^{-10}$.

This is a lot smaller than the spacing of the grating, so diffraction does not occur.

5 a) $E = hc \div \lambda = (6.63 \times 10^{-34} \times 3.00 \times 10^8) \div (5.00 \times 10^{-7})$

$= 3.978 \times 10^{-19}$ J

$(3.978 \times 10^{-19}) \div (1.60 \times 10^{-19}) = 2.48625$

$=$ **2.49 eV (to 3 s.f.)**

(2 marks for correct answer, otherwise 1 mark for correct calculation of energy in joules)

b) eV_0 = energy of one photon, E,

so $V_0 = \frac{E}{e} = \frac{3.978 \times 10^{-19}}{1.60 \times 10^{-19}}$

$= 2.486...$ V ***(1 mark)***

So the LED will not light up, as the potential difference across it is smaller than its threshold voltage ***(1 mark)***.

6 a) All particles have both particle and wave properties. Waves can also show particle properties ***(1 mark)***.

b) i) $m_e = 9.11 \times 10^{-31}$ kg, $E_k = 1.02 \times 10^{-18}$ J

Rearrange $E_k = \frac{1}{2}mv^2$ to give $v = \sqrt{\frac{2E_k}{m}}$

$v = \sqrt{\frac{2 \times 1.02 \times 10^{-18}}{9.11 \times 10^{-31}}} = 149.64...$ ms^{-1}

$=$ **150 ms^{-1} (to 3 s.f.)**

(2 marks for correct answer, otherwise 1 mark for correct working)

ii) $\lambda = \frac{h}{p} = (6.63 \times 10^{-34}) \div (9.11 \times 10^{-31} \times 149.64...)$

$= 4.863... \times 10^{-6} =$ **4.86×10^{-6} m (to 3 s.f.)**

(2 marks for correct answer, otherwise 1 mark for correct working)

iii) The kinetic energy of each electron $= \frac{1}{2}mv^2 = eV$, so if the potential difference is halved, the speed of each electron decreases by a factor of $\sqrt{2}$. This means that the de Broglie wavelength will increase by a factor of $\sqrt{2}$ ***(1 mark)***.

7 a) The work function of a metal is the minimum energy an electron needs to escape the surface of the metal ***(1 mark)***.

b) The energy of photons depends on the frequency of the light ***(1 mark)***. Below a certain frequency (the threshold frequency), the photons don't have enough energy to release an electron from the metal's surface ***(1 mark)***.

c) A zinc plate is attached to the top of an electroscope ***(1 mark)***. The zinc plate is negatively charged, which causes the gold leaf in the electroscope to rise up as it is repelled from the metal ***(1 mark)***. The zinc plate is then irradiated with UV radiation ***(1 mark)***. Electrons are lost from the zinc plate due to the photoelectric effect ***(1 mark)***. The zinc plate and electroscope therefore lose their negative charge and so the gold leaf falls back down as it is no longer repelled ***(1 mark)***.

d) $\phi = hf_0$

$f_0 = 1.03 \times 10^{15}$ Hz, $h = 6.63 \times 10^{-34}$ Js

So $\phi = hf_0 = (6.63 \times 10^{-34}) \times (1.03 \times 10^{15})$

$= 6.8289 \times 10^{-19}$ J

$hf = \phi + KE_{max}$

$hf = 3.0 \times 10^{-18}$ J, $\phi = 6.8289 \times 10^{-19}$ J,

substituting into $hf = \phi + KE_{max}$ gives:

$3.0 \times 10^{-18} = 6.8289 \times 10^{-19} + KE_{max}$

$KE_{max} = (3.0 \times 10^{-18}) - (6.8289 \times 10^{-19})$

$= 2.317... \times 10^{-18} =$ **2.3×10^{-18} J (to 2 s.f.)**

(3 marks for correct answer, otherwise 1 mark for correct calculation of work function and 1 mark for correct working to calculate KE_{max})

e) i) If the intensity of the incident radiation is increased, this increases the number of photons hitting an area in a given time, but not the energy of each photon (as this depends on the frequency of the radiation) ***(1 mark)***. As the electrons can only absorb one photon at a time, their kinetic energy and therefore their speed is unaffected by the increase in intensity ***(1 mark)***.

ii) The rate at which photoelectrons are emitted increases, as there are more photons hitting an area in a given time, so more electrons can be liberated in a given time ***(1 mark)***.

Module 5

Section 1 — Thermal Physics

1. Phases of Matter and Temperature

Page 251 — Application Questions

Q1 $T \approx \theta + 273 \Rightarrow \theta \approx T - 273 = 345 - 273 = \mathbf{72\,°C}$

Q2 a) Net flow of thermal energy is from the filling to the pastry.

b) If the pastry and the filling are both in thermal equilibrium with the room, then the pastry and the filling must be in thermal equilibrium with each other. Since they are in thermal equilibrium with each other, they must be the same temperature.

Page 251 — Fact Recall Questions

Q1 a) Particles in solids vibrate about fixed positions in a lattice, and are close together.

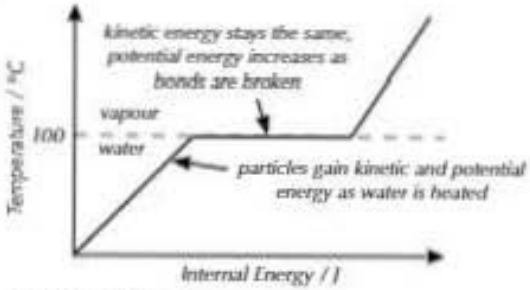
b) Particles in liquids are free to move past one another and are constantly moving. They're fairly close together, in an irregular arrangement.

c) Particles in gases are far apart and free to move around with constant random motion. They are not in any particular order.

Q2 Brownian motion. It is caused by randomly moving water particles hitting the pollen particles unevenly.

Q3 The average kinetic energy stays the same because the energy supplied is altering the bonds, and therefore potential energy, of the particles instead of increasing their speeds.

Q4


kinetic energy stays the same, potential energy increases as bonds are broken
vapour
water
100
Temperature / °C
particles gain kinetic and potential energy as water is heated
Internal Energy / J

Q5 a) Absolute zero.

b) 0 K, –273 °C.

Q6 The internal energy of a body is the sum of the random distribution of kinetic and potential energies associated with the molecules of the body.

Q7 When the two objects are at the same temperature with no net flow of energy between them.

2. Thermal Properties of Materials

Page 256 — Application Questions

Q1 $E = mc\Delta\theta$, so $E = 0.45 \times 244 \times 3.0 = 329.4$
$= \mathbf{330\ J\ (to\ 2\ s.f.)}$

Q2 Convert masses to kg: $270\ g \div 1000 = 0.27\ kg$
$910\ g \div 1000 = 0.91\ kg$

Expand and rearrange $m_b c_b (T_i - T_f) = m_b c_a (T_f - T_i)$ for T_i:

$\Rightarrow m_b c_b T_i - m_b c_b T_f = m_b c_a T_f - m_b c_a T_i$

$\Rightarrow m_b c_b T_i + m_b c_a T_i = m_b c_a T_f + m_b c_b T_f$

$\Rightarrow T_i (m_b c_b + m_b c_a) = m_b c_a T_f + m_b c_b T_f$

$$\Rightarrow T_i = \frac{m_b c_a T_f + m_b c_b T_f}{m_b c_b + m_b c_a}$$

$$= \frac{(0.27 \times 890 \times 82) + (0.91 \times 3600 \times 34)}{(0.91 \times 3600) + (0.27 \times 890)}$$

$= 37.28... = \mathbf{37\ °C\ (to\ 2\ s.f.)}$

You don't need to convert to kelvin here since changes in temperature are the same for both.

Q3 Convert mass to kg: $100.0\ g \div 1000 = 0.1000\ kg$
Energy lost from 25.0 °C to 0 °C
$= mc_{water}\Delta\theta = 0.1000 \times 4180 \times 25.0 = 10\,450\ J$
Energy lost in freezing $= mL_f = 0.1000 \times 334\,000$
$= 33\,400\ J$
Energy lost from 0 °C to –5.00 °C
$= mc_{ice}\Delta\theta = 0.1000 \times 2110 \times 5.0 = 1055\ J$
Total energy lost $= 10\,450 + 33\,400 + 1055$
$= 44\,905 = \mathbf{44\,900\ J\ (to\ 2\ s.f.)}$

Page 256 — Fact Recall Questions

Q1 The amount of energy needed to raise the temperature of 1 kg of the substance by 1 K (or 1 °C).

Q2 E.g. Place an electric heater and digital thermometer in a block of the material and cover it with insulation. Turn the heater on and heat the material so that its temperature increases by about 10 K. Use a stopwatch to measure how long the heater was on for. Use an ammeter and voltmeter attached to the electric heater to measure the current and voltage supplied, and then calculate the energy supplied (E) using $E = W = VIt$. Then use $E = mc\Delta\theta$ to calculate c.

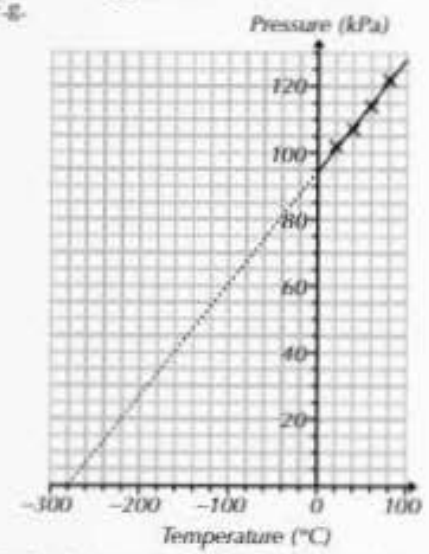
Q3 The quantity of thermal energy required to melt or freeze 1 kg of a substance.

Q4 E.g. Put equal masses of ice in two identical funnels above beakers. Put a heating coil in one of the funnels and turn it on for three minutes. Use an ammeter and voltmeter attached to the heating coil to measure the current and voltage supplied, and use these to calculate the energy transferred by the heating coil to the ice (E) using $E = W = VIt$. After switching off the heating coil, measure the mass of water collected in the beakers. Subtract the mass of water collected from the beaker without the heating coil from the mass of water from the beaker with the heating coil to get the mass of ice, m, that melted solely due to the presence of the heater. Use $E = mL$ to calculate L.

3. The Gas Laws

Page 261 — Application Questions

Q1 E.g.



So absolute zero is approximately **−280°C**.

Depending on your line of best fit, accept any answer between −300°C and −260°C.

Q2 a) The volume has halved. As p and V are inversely proportional, the pressure will have doubled, so $p = 2 \times 1.4 \times 10^5 =$ **2.8×10^5 Pa**.

b) Start temperature in K = 27 + 273 = 300 K.
End temperature in K = −173 + 273 = 100 K.
The temperature is divided by 3. As p and T are directly proportional, the pressure will also be divided by 3, so $p = \frac{2.8 \times 10^5}{3} = 9.333... \times 10^4 =$ **9.3×10^4 Pa (to 2 s.f.)**

Q3 a) 92 mm = 92×10^{-3} m
Volume of air bubble = $\pi r^2 l$
$= \pi \times (3.0 \times 10^{-3})^2 \times (92 \times 10^{-3})$
$= 2.6012... \times 10^{-6}$
$=$ **2.6×10^{-6} m³ (to 2 s.f.)**

b) $pV =$ constant so $p_1V_1 = p_2V_2$
Rearrange to:
$V_2 = \frac{p_1V_1}{p_2} = \frac{350\,000 \times (2.6012... \times 10^{-6})}{420\,000}$
$= 2.167... \times 10^{-6} =$ **2.2×10^{-6} m³ (to 2 s.f.)**

c) So that the temperature has time to stabilise.

d) A straight line graph passing through the origin.

e) Boyle's law.

Page 261 — Fact Recall Questions

Q1 It must have a fixed mass.

Q2 The pressure law states that at constant volume, the pressure p of an ideal gas is directly proportional to its absolute temperature T, or $\frac{p}{T} =$ constant.

4. The Ideal Gas Equation

Page 263 — Application Questions

Q1 $pV = nRT \Rightarrow V = \frac{nRT}{p} = \frac{2.3 \times 8.31 \times (25 + 273)}{2.4 \times 10^5}$
$= 0.23731... =$ **0.24 m³ (to 2 s.f.)**

Q2 $pV = NkT \Rightarrow p = \frac{NkT}{V}$
$= \frac{(8.21 \times 10^{24}) \times (1.38 \times 10^{-23}) \times 500}{4.05}$
$= 1.3987... \times 10^4$
$=$ **1.40×10^4 Pa (to 3 s.f.)**

Q3 $pV = NkT \Rightarrow T = \frac{pV}{Nk}$
$= \frac{(1.29 \times 10^5) \times 0.539}{(1.44 \times 10^{24}) \times (1.38 \times 10^{-23})}$
$= 349.894... =$ **350 K (to 3 s.f.)**

Q4 $pV = nRT \Rightarrow T = \frac{pV}{nR}$
$= \frac{(2.3 \times 10^5) \times 0.39}{20 \times 8.31} = 539.71...$
$=$ **540 K (to 2 s.f.)**

Page 263 — Fact Recall Questions

Q1 The number of particles in one mole of a material. Its value is 6.02×10^{23} mol^{-1}.

Q2 $pV = nRT$
p = pressure (in Pa)
V = volume (in m³)
n = number of moles of gas
R = molar gas constant (= 8.31 Jmol^{-1}K^{-1})
T = temperature (in K)

Q3 $k = \frac{R}{N_A}$
R = ideal gas constant (= 8.31 JK^{-1}mol^{-1})
N_A = Avogadro's constant (= 6.02×10^{23} mol^{-1})

Q4 $pV = NkT$
p = pressure (in Pa)
V = volume (in m³)
N = number of molecules of gas
k = Boltzmann constant (= 1.38×10^{-23} JK^{-1})
T = temperature (in K)

5. The Pressure of an Ideal Gas

Page 266 — Application Questions

Q1 $pV = \frac{1}{3}Nm\overline{c^2} \Rightarrow p = \frac{1}{3} \times \frac{Nm\overline{c^2}}{V}$ so:
$p = \frac{1}{3} \times \frac{(5 \times 6.02 \times 10^{23}) \times (5.31 \times 10^{-26}) \times (8.11 \times 10^6)}{1.44}$
$= 3.0005... \times 10^5 =$ **3.00×10^5 Pa (to 3 s.f.)**

Q2 $pV = \frac{1}{3}Nm\overline{c^2} \Rightarrow \overline{c^2} = \frac{3pV}{Nm}$
905 cm³ = 905×10^{-6} m³
$\overline{c^2} = \frac{3 \times (7.40 \times 10^5) \times (905 \times 10^{-6})}{(0.310 \times 6.02 \times 10^{23}) \times (5.10 \times 10^{-26})}$
$= 2.110... \times 10^5$ m²s^{-2}
$c_{rms} = \sqrt{\overline{c^2}} = \sqrt{2.110... \times 10^5} = 459.44... =$ **459 ms^{-1} (to 3 s.f.)**

Page 266 — Fact Recall Questions

Q1 Any four from: gas contains large number of particles / the particles move rapidly and randomly / the volume of the particles is negligible compared to the volume of the gas / collisions between particles themselves or between particles and the walls of the container are perfectly elastic / the duration of each collision is negligible when compared to the time between collisions / there are no forces between particles except for the moment when they are in a collision.

Q2 Each particle exerts a force on the wall of the container when it collides with it. The combined force from all the particles is spread all over the surface of each wall. This produces a steady, even force on all the walls of the box, which is pressure.

Q3 c_{rms} is the root mean square speed (of the particles in a gas). I.e. the square root of the mean of the squared speeds of the particles.

6. Internal Energy of an Ideal Gas

Page 269 — Application Questions

Q1 Curve B.

Q2 $E = \frac{3}{2}kT = \frac{3}{2} \times (1.38 \times 10^{-23}) \times 112 = 2.3184 \times 10^{-21}$
$= \mathbf{2.32 \times 10^{-21}\ J}$ **(to 3 s.f.)**

Page 269 — Fact Recall Questions

Q1 $pV = NkT$ and $pV = \frac{1}{3}Nmc^2$.

Q2 That the potential energy is 0 J because there are no forces between the particles.

Exam-style Questions — Pages 271-272

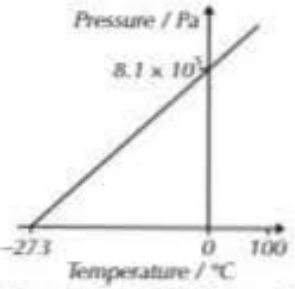
1 B ***(1 mark)***
pV = constant so $p_1V_1 = p_2V_2$. $V = \pi r^2 \times l$ so $p_1(\pi r^2 l_1) = p_2(\pi r^2 l_2)$.
Dividing by πr^2: $p_1l_1 = p_2l_2 \Rightarrow p_2 = (p_1l_1) \div l_2$
$p_2 = (120 \times 30) \div 50 = 72$ kPa

2 D ***(1 mark)***

3 a) Increasing the temperature increases the kinetic energy of the gas molecules ***(1 mark)***, so the r.m.s. speed of the gas molecules increases ***(1 mark)***. So from $pV = \frac{1}{3}Nmc^2$ it can be seen that the pressure must also increase ***(1 mark)***.

You could also have answered this by talking about the change in momentum — at higher temperatures the particles collide with the walls of the container more often and on average there's a larger change in momentum during a collision. This means a greater force is exerted on the walls of the container in the same amount of time, and so the pressure is increased.

b)



(1 mark for straight-line graph and 1 mark for crossing through (–273, 0) and (0, 8.1 × 10⁵).)

c) $pV = NkT \Rightarrow N = \frac{pV}{kT} = \frac{(8.1 \times 10^5) \times 0.51}{(1.38 \times 10^{-23}) \times 273}$
$= 1.096... \times 10^{26} = \mathbf{1.1 \times 10^{26}}$ **(to 2 s.f.)**
(2 marks for correct answer, otherwise 1 mark for correct working if answer incorrect.)

d) Total mass of gas = $(1.096... \times 10^{26}) \times (2.7 \times 10^{-26})$
$= 2.96...$ kg
$E = mc\Delta\theta = 2.96... \times (2.2 \times 10^3) \times 150$
$= 9.7699... \times 10^5 = \mathbf{9.8 \times 10^5\ J}$ **(to 2 s.f.)**
(2 marks for correct answer, otherwise 1 mark for finding the mass of gas if answer is incorrect.)

4 a) $E = mc\Delta\theta = (92 \times 10^{-3}) \times 2110 \times 25 = 4.853 \times 10^3$ J
$= \mathbf{4.9 \times 10^3\ J}$ **(to 2 s.f.)** ***(1 mark)***

b) $E = ml = (92 \times 10^{-3}) \times (3.3 \times 10^5)$
$= 3.036 \times 10^4$ J $= \mathbf{3.0 \times 10^4\ J}$ **(to 2 s.f.)** ***(1 mark)***

c) Total energy needed = $(4.853 \times 10^3) + (3.036 \times 10^4)$
$= 3.5213 \times 10^4$ J
$\text{Time taken} = \frac{\text{total energy needed}}{\text{rate of energy supplied}} = \frac{3.5213 \times 10^4}{50.0}$
$= 704.26 = \mathbf{700\ s}$ **(to 2 s.f.)**
(2 marks for correct answer, otherwise 1 mark for correct working if answer is incorrect.)

d) It would slow down the melting of the ice ***(1 mark)***. Heat is transferred from hotter substances to colder substances — as the ice is colder than 25 °C, insulating the beaker will stop heat transfer into the ice from the surrounding air ***(1 mark)***.

5 a) How to grade your answer (pick the description that best matches your answer):
0 marks: There is no relevant information.
1-2 marks: At least one *K* point for each of solid, liquid and gas have been covered. *B* points 1 and 3 have been covered. Answer is basic and has lack of structure, with information missing.
3-4 marks: At least two *K* points for each of solid, liquid and gas have been covered. All *B* points have been covered. Answer has some structure, and information is generally relevant.
5-6 marks: All *K* and *B* points have been covered. The answer is well structured, and information is relevant and presented clearly.
Here are some points your answer may include:
Kinetic Model of Matter (*K*)
1. Particles in solids are close together.
2. Particles in solids are arranged in a regular lattice structure.
3. Particles in solids vibrate around fixed positions.
4. Particles in liquids are fairly close together.
5. Particles in liquids have an irregular arrangement.
6. Particles in liquids are free to move past one another.
7. Particles in gases are far apart.
8. Particles in gases are free to move with constant random motion.
9. Particles in gases are not in any particular order.

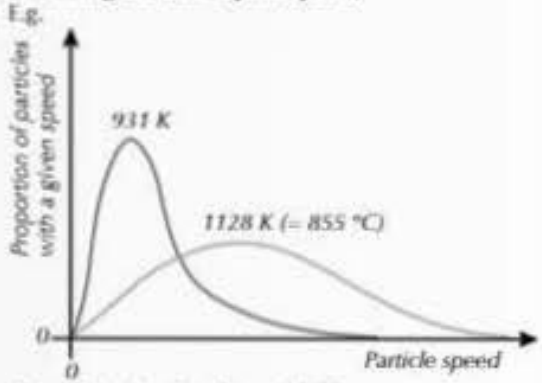
Brownian Motion (*B*)
1. Brownian motion describes the random, zigzag motion of particles suspended in a fluid.
2. It is caused by randomly moving gas or liquid particles hitting the observed particles unevenly.
3. Brownian motion can be observed by putting smoke in an illuminated glass jar and viewing the motion of the smoke particles in the air with a microscope.

b) i) $N = n \times N_A = 54.0 \times (6.02 \times 10^{23}) = 3.2508 \times 10^{25}$
$= \mathbf{3.25 \times 10^{25}}$ **(to 3 s.f.)** ***(1 mark)***

ii) $pV = nRT \Rightarrow T = \frac{pV}{nR}$
$= \frac{1.00 \times 10^5 \times 4.18}{54.0 \times 8.31}$
$= 931.49...\text{ K} = \mathbf{931 K}$ **(to 3 s.f.)**
(2 marks for correct answer, otherwise 1 mark for correct working.)

iii) Convert temperature to K:
$T = \theta + 273 = 855 + 273 = 1128\text{ K}$
$\frac{1}{2}m\overline{c^2} = \frac{3}{2}kT \Rightarrow \overline{c^2} = \frac{3kT}{m} = \frac{3 \times 1.38 \times 10^{-23} \times 1128}{3.40 \times 10^{-26}}$
$= 1.373... \times 10^6\text{ m}^2\text{s}^{-2}$
$c_{rms} = \sqrt{\overline{c^2}} = \sqrt{1.373... \times 10^6} = 1171.96...$
$= \mathbf{1170\text{ ms}^{-1}}$ **(to 3 s.f.)**
(3 marks for correct answer, otherwise 1 mark for converting temperature to kelvin and 1 mark for calculating the mean square speed.)

iv) E.g.



(1 mark for drawing Maxwell-Boltzmann curves, 1 mark for making high temperature curve lower and more spread out than low temperature curve.)

Section 2 — Circular Motion and Oscillations

1. Circular Motion

Page 275 — Application Questions

Q1 Period in seconds $= 28 \times 24 \times 3600 = 2.419... \times 10^6$ s
Angular velocity $= \omega = \frac{2\pi}{T} = \frac{2\pi}{2.419... \times 10^6}$
$= 2.5972... \times 10^{-6}\text{ rads}^{-1}$
$= \mathbf{2.6 \times 10^{-6}\text{ rads}^{-1}}$ **(to 2 s.f.)**
$v = r\omega = (384\,000 \times 1000) \times 2.5972... \times 10^{-6}$
$= 997.331... = \mathbf{1000\text{ ms}^{-1}}$ **(to 2 s.f.)**

Q2 Frequency in rev s^{-1} $= f = \frac{\text{no. of revolutions}}{\text{time taken}} = \frac{4}{3600}$
$\omega = 2\pi f = 2 \times \pi \times \frac{4}{3600} = \frac{\pi}{450}\text{ rads}^{-1}$
$\omega = \frac{\theta}{t} \Rightarrow \theta = \omega t = \frac{\pi}{450} \times 60 = \frac{2}{15}\pi = 0.41887...$
$= \mathbf{0.419\text{ rad}}$ **(to 3 s.f.)**
$v = \omega r = \frac{\pi}{450} \times \frac{125}{2} = 0.43633...\text{ ms}^{-1}$
$= \mathbf{0.436\text{ ms}^{-1}}$ **(to 3 s.f.)**

Q3 Frequency in rev s^{-1} $= f = 460 \div 60$.
2.0 cm from centre:
Angular speed $= \omega = 2\pi f = \frac{46}{3}\pi = 48.171...$
$= \mathbf{48\text{ s}^{-1}}$ **(to 2 s.f.)**
Linear speed $= v = \omega r = \frac{46}{3}\pi \times 0.020$
$= 0.9634...\text{ ms}^{-1} = \mathbf{0.96\text{ ms}^{-1}}$ **(to 2 s.f.)**
4.0 cm from centre: Angular speed $= \mathbf{48\text{ s}^{-1}}$ **(to 2 s.f.)**
Angular speed is the same at any point on a solid rotating object.
Linear speed $= 0.040 \times \frac{46}{3}\pi = 1.926... = \mathbf{1.9\text{ ms}^{-1}}$ **(to 2 s.f.)**

Q4 Kinetic energy $= \frac{1}{2}mv^2$
$v = \omega r$ and $\omega = 2\pi f$
$\Rightarrow v = 2\pi fr$
$f = \frac{1}{T}$ and r is the length of the string l, so $v = \frac{2\pi l}{T}$
So kinetic energy $= \frac{1}{2}m(\frac{2\pi l}{T})^2$
$= \frac{2m\pi^2 l^2}{T^2}$

Page 275 — Fact Recall Questions

Q1 Angle in radians $= \frac{\pi}{180} \times$ angle in degrees

Q2 The angle that an object rotates through per second.

Q3 $v = \omega r$, where v is the linear speed, ω is the angular velocity, and r is the radius of the circle of rotation.

Q4 The period is the time taken for a complete revolution. The frequency is the number of complete revolutions per second.

Q5 $\omega = 2\pi f$, where ω is the angular velocity and f is the frequency.

2. Centripetal Force and Acceleration

Page 279 — Application Questions

Q1 Frequency in $rev\,s^{-1} = f = 15 \div 60.0 = 0.25\ rev\,s^{-1}$
Angular velocity $\omega = 2\pi f = 2 \times \pi \times 0.25 = 0.5\pi\ rad\,s^{-1}$
$F = m\omega^2 r = 60.0 \times (0.5\pi)^2 \times \frac{8.5}{2} = 629.1...$
$= \mathbf{630\ N\ (to\ 2\ s.f.)}$
This might seem like a lot, but it's about the same as the force experienced by the rider due to gravity.

Q2 **C**
$a = \omega^2 r$ so rearranging gives: $\omega = \sqrt{\frac{a}{r}}$
$\omega = \frac{2\pi}{T}$ so rearranging gives: $T = \frac{2\pi}{\omega}$
So $T = \frac{2\pi}{\sqrt{\frac{a}{r}}} \Rightarrow T = 2\pi\sqrt{\frac{r}{a}}$
So time to complete 3 orbits $= 3 \times T = 6\pi\sqrt{\frac{r}{a}}$

Q3 Convert the washer's mass to kg:
$m_w = 310 \div 1000 = 0.31$ kg
So the weight of the washer is
$W = m_w g = 0.31 \times 9.81 = 3.0411$ N
This exerts a centripetal force F on the bung, i.e. $F = m_b\omega^2 r$.
Convert the bung's mass to kg: $m_b = 22 \div 1000 = 0.022$ kg
So $3.0411 = 0.022 \times \omega^2 \times 0.60$
This gives $\omega = \sqrt{\frac{3.0411}{0.022 \times 0.60}} = 15.178...\ rad\ s^{-1}$
Rearrange the formula $\omega = \frac{2\pi}{T}$ to give $T = \frac{2\pi}{\omega}$
So $T = \frac{2\pi}{15.178...} = 0.4139... = \mathbf{0.41\ s\ (to\ 2\ s.f.)}$

Q4 a) $a = \frac{v^2}{r} = 31.1^2 \div 56.8$
$= 17.0283... = \mathbf{17.0\ ms^{-2}\ (to\ 3\ s.f.)}$
b) The formula for centripetal force is $F = \frac{mv^2}{r}$, so if the linear speed is halved (i.e. v becomes $\frac{v}{2}$), then the centripetal force is quartered. So the new centripetal force is $\frac{F}{4}$.

Q5 If the biker's speed is the minimum possible speed for him to not fall, the centripetal acceleration towards the centre (i.e. down) at the top of the cylinder will be $9.81\ ms^{-2}$, due to gravity. The motorcycle will fall if its circular motion has a centripetal acceleration smaller than this.
So $a = 9.81\ ms^{-2} = \frac{v^2}{r}$
Rearranging this: $v = \sqrt{ar} = \sqrt{9.81 \times 5.0}$
$= 7.003...\ ms^{-1} = \mathbf{7.0\ ms^{-1}\ (to\ 2\ s.f.)}$
This is only about 16 mph — not very fast at all.

Page 279 — Fact Recall Questions

Q1 If an object is moving in a circle, centripetal acceleration is the acceleration of the object directed towards the centre of the circle. Centripetal force is the force towards the centre of the circle responsible for the centripetal acceleration.

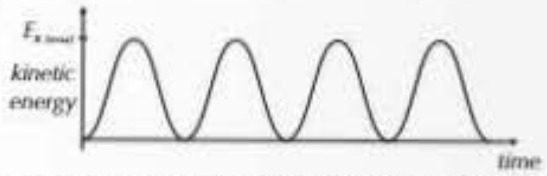
Q2 $a = \omega^2 r$
a = centripetal acceleration in ms^{-2}
ω = angular velocity (in $rad\,s^{-1}$)
r = radius of circular motion in m

Q3 $F = \frac{mv^2}{r}$
F = centripetal force in N
m = mass of object in kg
v = magnitude of linear velocity in ms^{-1}
r = radius of circular motion in m

3. Simple Harmonic Motion

Page 283 — Application Questions

Q1 a) A and C
Maximum velocity occurs at the midpoint/equilibrium position.
b) B and E
Maximum acceleration occurs when displacement is maximum.

Q2


The kinetic energy starts at 0 as the girl starts from rest. It then varies sinusoidally (like a sine wave) between 0 and $E_{k\,(max)}$ as potential energy is converted to kinetic energy and back. The kinetic energy is at a maximum when the swing is at its lowest point. The swing passes through this point 4 times during two complete oscillations.

Q3 a) Maximum displacement = $A = 0.60$ m
Maximum speed = $\omega A = 2\pi fA = 0.90\ ms^{-1}$
Rearrange $v_{max} = 2\pi fA$ to give $f = \frac{v_{max}}{2\pi A}$
So $f = 0.90 \div (2 \times \pi \times 0.60) = 0.2387...$
$= \mathbf{0.24\ Hz\ (to\ 2\ s.f.)}$
b) $f = \frac{1}{T}$ so rearranging gives $T = \frac{1}{f}$
So $T = 1 \div 0.2387... = 4.188... = \mathbf{4.2\ s\ (to\ 2\ s.f.)}$
c) Maximum acceleration = $\omega^2 A$
$= (2\pi f)^2 A = (2 \times \pi \times 0.2387...)^2 \times 0.60 = \mathbf{1.35\ ms^{-2}}$

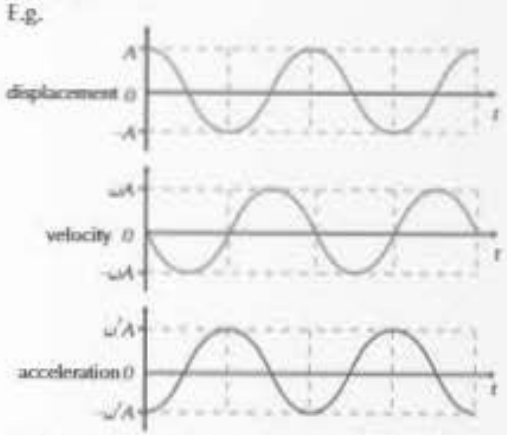
Page 283 — Fact Recall Questions

Q1 $a = -\omega^2 x$

Q2 The frequency of oscillation is the number of complete cycles per second. The period of oscillation is the time taken for a complete cycle.

Q3 The velocity is $\frac{\pi}{2}$ radians ahead of the displacement.

Q4 a) Equilibrium
b) Maximum displacement
c) Maximum displacement
d) Equilibrium

Q5 E.g.


Q6 At its maximum displacement, the object's kinetic energy, E_k, is zero (so it has zero velocity). All of its energy is potential energy, E_p. As the object moves towards the equilibrium position, the restoring force does work on the object and transfers some E_p to E_k. At the equilibrium position, the object's E_p is said to be zero and its E_k is maximum — so its velocity is maximum. As the object moves away from the equilibrium, all that E_k is transferred back to E_p again.

4. Calculations with SHM

Page 286 — Application Questions

Q1 a) $x = A$, so: $a = -\omega^2 A = -1.5^2 \times 1.6 = -\mathbf{3.6\ ms^{-2}}$

The question asks for the acceleration, not the magnitude of acceleration, which is why you can't use $a_{max} = \omega^2 A$. The displacement given in the question is a positive value, so the acceleration is negative.

b) Angular frequency $= \omega = 2\pi f$, rearranging gives $f = \frac{\omega}{2\pi}$

$= 1.5 \div (2 \times \pi) = 0.2387...$ Hz

Time to complete 1 oscillation $= T = \frac{1}{f} = \frac{1}{0.2387...}$

$= 4.188...$ s

Time to complete 15 oscillations

$= 15 \times 4.188... = 62.83...$ s $= \mathbf{63\ s\ (to\ 2\ s.f.)}$

Q2 a) Frequency $= f = \frac{1}{T} = \frac{1}{0.75} = 1.333...\ s^{-1}$

Maximum speed is given by: $v_{max} = \omega A = 2\pi fA$

Substituting this in and rearranging for amplitude gives:

$A = \frac{v_{max}}{2\pi f} = \frac{0.85}{2 \times \pi \times 1.3333...} = 0.101...$ m

$= \mathbf{0.10\ m\ (to\ 2\ s.f.)}$

b) Velocity at $x = 0.080$ m:

$v = \pm \omega\sqrt{A^2 - x^2} = \pm 2\pi f\sqrt{A^2 - x^2}$

$= \pm 2 \times \pi \times 1.333... \times \sqrt{0.101...^2 - 0.080^2}$

$= \pm 0.5228...\ ms^{-1} = \mathbf{\pm 0.52\ ms^{-1}\ (to\ 2\ s.f.)}$

The answer has a ± sign at the front because you don't know the direction of the velocity.

Q3 Period $= T =$ time to complete exactly 5 oscillations $\div 5$

$= \frac{15.5}{5} = 3.1$ s

Frequency $= f = \frac{1}{T} = \frac{1}{3.1} = 0.3225...\ s^{-1}$

Amplitude $= A = 0.45$ m

Displacement at time t: $x = A\cos(\omega t) = A\cos(2\pi ft)$

At time $t = 10.0$, $x = 0.45 \times \cos(2\pi \times 0.3225... \times 10.0)$

$= 0.0681...$ m $= \mathbf{0.068\ m\ (to\ 2\ s.f.)}$

Don't forget to put your calculator into radians here.

Q4 Pendulum passes through equilibrium twice every period, so if it's set to 120 ticks per minute:

Period $= T = 120 \div 2 \div 60 = 1$ s

Frequency $= f = \frac{1}{T} = \frac{1}{1} = 1$ Hz

Amplitude $A = 6.2$ cm $= 0.062$ m

Magnitude of max acceleration $= a_{max} = \omega^2 A = (2\pi f)^2 A$

$a_{max} = (2\pi \times 1)^2 \times 0.062 = 2.447...\ ms^{-2} = \mathbf{2.4\ ms^{-2}\ (to\ 2\ s.f.)}$

Page 286 — Fact Recall Questions

Q1 Displacement $= x = A\cos(\omega t)$, where x is the object's displacement, ω its angular frequency, and A its maximum displacement.

Q2 Acceleration $= a = -\omega^2 x$ Velocity $= v = \pm\omega\sqrt{A^2 - x^2}$

Here, a is the acceleration of the object, x is its displacement, v its velocity, ω its angular frequency, and A its maximum displacement.

Q3 Max speed $= v_{max} = \omega A$, where v_{max} is the object's maximum velocity, ω its angular frequency, and A its maximum displacement.

Q4 Max acceleration $= a_{max} = \omega^2 A$, where a_{max} is the object's maximum acceleration, ω its angular frequency, and A its maximum displacement.

5. Investigating SHM

Page 289 — Application Question

Q1 a) It takes 0.50 s for each complete oscillation, $T = \mathbf{0.500\ s}$.

b) $f = \frac{1}{T}$, so $f = 1 \div 0.500 = \mathbf{2.00\ Hz}$

c) $\omega = 2\pi f = 2\pi \times 2 = 4\pi = 12.566... = \mathbf{12.6\ rad\ s^{-1}\ (to\ 3\ s.f.)}$

Page 289 — Fact Recall Questions

Q1 E.g. You could use a mass attached to the end of a pendulum, which is made to oscillate in front of a piece of card with a reference mark on it to show the midpoint of the oscillations. Attached at the same point that the pendulum is suspended from, there should be a protractor to measure the angle that the pendulum makes with a vertical reference line as it is released.

Q2 a) This means the time period you calculate for a single oscillation will be more accurate, since any human error introduced by starting or stopping the stopwatch too late (or too early) will be 'shared' over several oscillations.

b) Using a fiducial marker helps you start and stop your stopwatch at the same point in the oscillations, meaning the measurement of the time period for one complete oscillation will be more accurate.

6. Free and Forced Oscillations

Page 293 — Fact Recall Questions

Q1 A free vibration involves no transfer of energy between the oscillating object and its surroundings. The object will continue to oscillate at its natural frequency and with the same amplitude forever. A forced vibration occurs if there's a periodic external driving force acting on the object.

Q2 Resonance occurs when the driving frequency approaches the natural frequency of an object and the object begins to oscillate with a rapidly increasing amplitude.

Q3 E.g. any three from: a radio's electric circuit resonating when it's tuned to the same frequency as a radio station / a glass resonating when driven by a sound wave at its natural frequency / a column of air in an organ pipe resonating when driven by the motion of air at its base / a swing in a playground resonating when it's pushed by someone at its natural frequency.

Q4 A damping force is a force that acts on an oscillator and causes it to lose energy to its surroundings, reducing the amplitude of its oscillations.

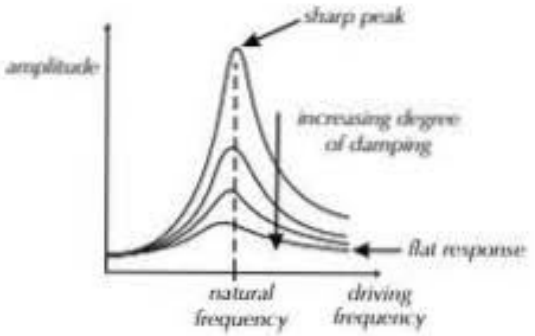
Q5 Light damping — damping such that an oscillating system takes a long time to stop, and the amplitude of the system reduces by only a small amount each period.

Heavy damping — damping such that the system takes less time to stop oscillating than a lightly damped system, and the amplitude gets much smaller each period.

Critical damping — damping such that the amplitude of an oscillating system is reduced (and so the system returns to equilibrium) in the shortest possible time.

Overdamping — extremely heavy damping such that an oscillating system takes longer to return to equilibrium than a critically damped system.

Q6



Exam-style Questions – pages 295-296

1 D ***(1 mark)***

The equation for linear speed is $v = \omega r$, where $r = 3.0$ m.

Frequency = 6 revolutions per minute, so $f = 6 \div 60 = 0.1$ Hz.

$\omega = 2\pi f = 2 \times \pi \times 0.1 = 0.628...$ rads $^{-1}$.

This means $v = 0.628... \times 3.0 = 1.884... = 1.9$ ms $^{-1}$ (to 2 s.f.).

2 C ***(1 mark)***

A critically damped system takes the shortest possible time to return to equilibrium. An overdamped system takes longer to return to equilibrium.

3 C ***(1 mark)***

The correct graph needs to show 1 complete cycle (i.e. 2 'peaks') in 3 seconds and zero potential energy at 0, 1.5 s and 3 s.

4 a) The maximum displacement of the block (A) is 0.20 m and the time period of the oscillations (T) is 1.147 s.

Velocity is given by:

$$v = \pm\omega\sqrt{A^2 - x^2}$$

$$= \pm\frac{2\pi}{T}\sqrt{A^2 - x^2}$$

$$= \pm\frac{2 \times \pi}{1.147} \times \sqrt{0.20^2 - 0.15^2}$$

$$= \pm 0.7246... \text{ ms}^{-1} = \mathbf{\pm 0.72\ ms^{-1}}\ \textbf{(to 2 s.f.)}$$

Don't forget to convert 20 cm and 15 cm into metres.

(2 marks for the correct answer, otherwise 1 mark for correct working if answer incorrect.)

b) Maximum kinetic energy is: $E_{K(max)} = \frac{1}{2}mv_{max}^2$

Maximum velocity is:

$v_{max} = \omega A = \frac{2\pi}{T}A = \frac{2 \times \pi}{1.147} \times 0.20 = 1.095...$ ms $^{-1}$

Substitute this into the equation for maximum kinetic energy:

$$E_{K(max)} = \frac{1}{2} \times 0.60 \times (1.095...)^2 = 0.3600...$$

$$= \mathbf{0.36\ J}\ \textbf{(to 2 s.f.)}$$

(3 marks for correct answer, otherwise 1 mark for calculating maximum velocity, and 1 mark for substituting this into the formula for maximum kinetic energy.)

c) If t is the number of seconds shown on the stopwatch, then at $t = 0$, the block is at its equilibrium position (i.e. when $t = 0$, the displacement (x) = 0). This means x is given by the formula $x = A \sin(\omega t)$. So at $t = 3.0$:

$x = A \sin(\omega t)$

$$= 0.20 \times \sin(\frac{2\pi}{T} \times 3.0)$$

$$= 0.20 \times \sin(\frac{2 \times \pi}{1.147} \times 3.0)$$

$$= -0.1327... \text{ m}$$

So the block is **0.13 m** (to 2 s.f.) from its equilibrium position.

(2 marks for correct answer, otherwise 1 mark for correct working if answer incorrect.)

5 a) The centripetal force acting on the motorcyclist is given by:

$$F = \frac{mv^2}{r} = \frac{210 \times v^2}{5.0}$$

To avoid sliding off the track, the motorcyclist needs to travel with a minimum speed of v_{min}, where:

$$1500 = \frac{210 \times v_{min}^2}{5.0}$$

So $v_{min}^2 = \frac{1500 \times 5.0}{210} = 35.714...$

This means $v_{min} = \sqrt{35.714...} = 5.976...$

$$= \mathbf{6.0\ ms^{-1}}\ \textbf{(to 2 s.f.)}$$

(2 marks for the correct answer, otherwise 1 mark for correct working if answer incorrect.)

b) $v = \omega r$, so $\omega = \frac{v_{min}}{r} = \frac{5.976...}{5.0} = 1.195...$

$$= \mathbf{1.2\ rad\ s^{-1}}\ \textbf{(to 2 s.f.)}$$

(2 marks for the correct answer, otherwise 1 mark for correct working if answer incorrect.)

c) $\omega = \frac{2\pi}{T}$ So $T = \frac{2\pi}{\omega} = \frac{2\pi}{1.195...} = 5.256...$

$$= \mathbf{5.3\ s}\ \textbf{(to 2 s.f.)}$$

(2 marks for the correct answer, otherwise 1 mark for correct working if answer incorrect.)

6 a) The amplitude of the pendulum's oscillation will increase rapidly around its natural frequency ***(1 mark)***. This is called resonance ***(1 mark)***.

b) E.g. friction with the water acts to dampen the oscillation ***(1 mark)***. The resonance will occur at a frequency slightly less than the pendulum's natural frequency ***(1 mark)***. The pendulum's response will be flatter (i.e. the resonance peak will be less sharp) ***(1 mark)***.

Section 3 — Gravitational Fields

1. Gravitational Fields

Page 299 — Application Questions

Q1 $F = \frac{GMm}{r^2}$

$= \frac{(6.67 \times 10^{-11}) \times (2.15 \times 10^{30}) \times (2.91 \times 10^{29})}{(1 \times 10^{11})^2}$

$= 4.17308... \times 10^{28}$

$= \mathbf{4.2 \times 10^{28}\ N}$

We can ignore the minus sign in Newton's law of gravitation here because we're not interested in the direction of the force.

Q2 $\frac{2.5}{0.5} = 5$, so the force will be $5^2 = 25$ times larger:

$25 \times 25 = \mathbf{625\ N}$

Q3 a) 6 370 000 m + 10 000 m = 6 380 000 m = **6380 km**

b) The upwards force must balance the downwards force due to gravity:

$F = -\frac{GMm}{r^2}$

$= -\frac{(6.67 \times 10^{-11}) \times (2500) \times (5.97 \times 10^{24})}{(6380 \times 10^3)^2}$

$= -24\,456.7...\ N$

So upwards force = **24 500 N (to 3 s.f.)**

Page 299 — Fact Recall Questions

Q1 A gravitational field is a force field generated by any object with mass which causes any other object with mass to experience an attractive force.

Q2 a)

b)

Q3 $F = -\frac{GMm}{r^2}$,

where F is the force on mass m due to mass M,
G is the gravitational constant, $6.67 \times 10^{-11}\ Nm^2kg^{-2}$,
and r is the distance between M and m.

2. Gravitational Field Strength

Page 302 — Application Questions

Q1 $g = \frac{F}{m} = \frac{581}{105} = 5.5333... = \mathbf{5.53\ Nkg^{-1}}$ **(to 3 s.f.)**

Q2 $g = \frac{F}{m}$, so $F = gm$. The magnitude of the value of g is lower on the Moon, which means an object with the same mass will experience less force due to gravity (and so be easier to lift).

Q3 $g = -\frac{GM}{r^2} = -\frac{6.67 \times 10^{-11} \times 4.87 \times 10^{24}}{(6050 \times 10^3)^2}$

$= -8.874... = \mathbf{-8.87\ Nkg^{-1}}$ **(to 3 s.f.)**

Q4 a) $F = -\frac{GMm}{r^2}$

You just want to find the radius, so you're not really interested in direction, and you can just use $F = \frac{GMm}{r^2}$

Rearrange for r:

$r = \sqrt{\frac{GMm}{F}} = \sqrt{\frac{6.67 \times 10^{-11} \times 7.34 \times 10^{22} \times 65}{105}}$

$= 1.7408... \times 10^6 = \mathbf{1.7 \times 10^6\ m}$ **(to 2 s.f.)**

If you'd used the minus sign in the equation, you should have used $F = -105$ too, so the minus signs cancel out.

b) $g = -\frac{GM}{r^2} = -\frac{6.67 \times 10^{-11} \times 7.34 \times 10^{22}}{((1.7408... \times 10^6) + (640 \times 10^3))^2}$

$= -0.8636... = \mathbf{-0.86\ Nkg^{-1}}$ **(to 2 s.f.)**

Page 302 — Fact Recall Questions

Q1 g is the gravitational field strength (or the force per unit mass due to gravity), measured in Nkg^{-1}.

Q2 M is the mass of the object creating the gravitational field.

Q3 The correct graph is c).

3. Gravitational Potential and Energy

Page 307 — Application Questions

Q1 a) No change (G is a constant).

b) V_g will double (twice as negative) as it's related to $\frac{1}{r}$ ($V_g = -\frac{GM}{r}$).

c) No change (mass is the same everywhere).

d) g will be four times bigger as it's related to $\frac{1}{r^2}$ ($g = -\frac{GM}{r^2}$).

Q2 $\Delta W = m\Delta V_g = 1.72 \times 531$

$= 913.32$

$= \mathbf{913\ J}$ **(to 3 s.f.)**

Q3 Number of squares underneath the graph between 2×10^6 m and 5×10^6 m is approximately 2 squares. Each square on the graph is worth $20 \times (1 \times 10^6) = 2 \times 10^7$ Nm. Therefore work done = $2 \times 2 \times 10^7 = \mathbf{4 \times 10^7\ J}$.

Q4 a) $E = -\frac{GMm}{r} = -\frac{6.67 \times 10^{-11} \times 7.34 \times 10^{22} \times 95.0}{-1.74 \times 10^6}$

$= 2.6729... \times 10^8 = \mathbf{2.67 \times 10^8\ J}$ **(to 3 s.f.)**

b) Using the equation for escape velocity:

$v = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2 \times (6.67 \times 10^{-11}) \times (7.34 \times 10^{22})}{(1.74 \times 10^6)}}$

$= 2.3722... \times 10^3$

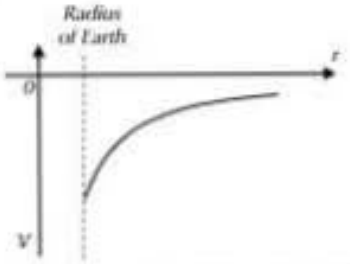
$= \mathbf{2.37 \times 10^3\ ms^{-1}}$ **(to 3 s.f.)** **(= 2.37 kms^{-1})**

Page 307 — Fact Recall Questions

Q1 The gravitational potential at a point is the work done in moving a unit mass from infinity to that point.
It's measured in Jkg^{-1}.

Q2 The negative energy sign means that the gravitational potential is negative and increases towards zero as you move further away from the source of the gravitational field. At infinity, the gravitational potential is zero. You can think of this negative potential as being caused by you having to do work against the gravitational field to move an object out of it.

Q3



Q4 Escape velocity is the velocity needed so an object has just enough kinetic energy to escape a gravitational field.

4. Motion of Masses in Gravitational Fields

Page 311 — Application Questions

Q1 $\frac{mv^2}{r} = \frac{GMm}{r^2} \Rightarrow v^2 = \frac{GMmr}{r^2 m}$

Cancelling m's and r's: $v^2 = \frac{GM}{r}$

So $v = \sqrt{\frac{GM}{r}}$

Q2 $T^2 = \left(\frac{4\pi^2}{GM}\right)r^3 \Rightarrow M = \left(\frac{4\pi^2}{G}\right)\frac{r^3}{T^2}$

Convert T to s:

42 hours = 42 × 3600 = 151 200 s

$M = \left(\frac{4\pi^2}{6.67 \times 10^{-11}}\right)\frac{(3.95 \times 10^8)^3}{(151200)^2}$

$= 1.5955... \times 10^{27}$ kg = **1.6×10^{27} kg (to 2 s.f.)**

Q3 $T^2 \propto r^3$

So $\frac{T^2}{r^3} = \frac{T_{new}^2}{\left(\frac{r}{2}\right)^3}$

$\frac{T^2}{r^3} = \frac{8T_{new}^2}{r^3}$

$T^2 = 8T_{new}^2$

$T_{new}^2 = \frac{T^2}{8}$

$T_{new} = \frac{T}{\sqrt{8}} = \frac{T}{2\sqrt{2}}$

Page 311 — Fact Recall Questions

Q1 A satellite is a small mass that orbits a larger mass.

Q2 The orbital speed of a satellite is inversely proportional to the square root of the radius of its orbit ($v \propto \frac{1}{\sqrt{r}}$), so as the radius increases the speed decreases.

Q3 The orbital period of a satellite is proportional to the square root of the radius cubed ($T \propto \sqrt{r^3}$), so as the radius increases the orbital period increases.

Q4 A geostationary satellite is a satellite that orbits directly over the equator and is always above the same point on Earth. Their orbit takes exactly 1 day. Geostationary satellites are always above the same point of the Earth, so receivers don't need to be repositioned to keep up with them.

Q5 Kepler's first law: Each planet moves in an ellipse around the Sun, with the Sun at one focus.
Kepler's second law: A line joining the Sun to a planet will sweep out equal areas in equal times.
Kepler's third law: The period of the orbit and the mean distance between the Sun and the planet are related by $T^2 \propto r^3$.

Exam-style Questions — pages 313-314

1 B ***(1 mark)***
Gravitational field strength decreases as you move away from the mass by the inverse square law.

2 A ***(1 mark)***

3 a) Time taken for one orbit = T and distance for circular orbit = $2\pi r$, so speed = $\frac{\text{distance}}{\text{time}}$ becomes:

$v = \frac{2\pi r}{T} \Rightarrow T = \frac{2\pi r}{v}$ ***(1 mark)***

Substitute in expression for v:

$T = \frac{2\pi r}{v} = \frac{2\pi r}{\left(\sqrt{\frac{GM}{r}}\right)} = \frac{2\pi r\sqrt{r}}{\sqrt{GM}}$

So $T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$ ***(1 mark)***

b) Kepler's third law states that $T^2 \propto r^3$, so:

$\frac{T_i^2}{r_i^3} = \frac{T_f^2}{r_f^3} \Rightarrow r_f^3 = T_f^2 \frac{r_i^3}{T_i^2}$

So $r_f = \sqrt[3]{T_f^2 \frac{r_i^3}{T_i^2}} = \sqrt[3]{3.6^2 \times \frac{420\,000^3}{1.8^2}}$

$= 666\,708.44... =$ **670 000 km (to 2 s.f.)**

(2 marks for correct answer, otherwise 1 mark for correct working.)

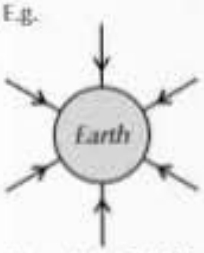
c) $g = \frac{F}{m} \Rightarrow F = mg$

$F = 65 \times 1.8 = 117 =$ **120 N (to 2 s.f.)** ***(1 mark)***

d) A line joining Jupiter to a Carpo will sweep out equal areas in equal times. ***(1 mark)***

4 a) A force field is a region in which a body experiences a non-contact force ***(1 mark)***.

b) E.g.



(1 mark for field lines pointing to the centre of Earth.)

c) No — the value of **g** close to the surface of the Earth is uniform, so the height difference in a classroom will have a negligible effect as the distance to the centre of the Earth is about the same for both students.
(1 mark for correct answer and correct reasoning.)

d) $g = -\frac{GM}{r^2}$

$\Rightarrow r = \sqrt{-\frac{GM}{g}} = \sqrt{-\frac{(6.67 \times 10^{-11}) \times (5.97 \times 10^{24})}{(-6.31)}}$

$= 7.94392... \times 10^6$ m $= 7943.92...$ km

So altitude $= 7943.92... - 6370 = 1573.92...$

$=$ **1570 km (to 3 s.f.)**

(2 marks for correct answer, otherwise 1 mark for correct working.)

e) First find the distance r:

$V_e = -\frac{GM}{r}$

$\Rightarrow r = -\frac{GM}{V_e} = -\frac{(6.67 \times 10^{-11}) \times (5.97 \times 10^{24})}{-20.6 \times 10^6}$

$= 1.933... \times 10^7$ m

Then find g:

$g = -\frac{GM}{r^2} = -\frac{(6.67 \times 10^{-11}) \times (5.97 \times 10^{24})}{(1.933... \times 10^7)^2}$

$= -1.0656... =$ **-1.07 Nkg^{-1} (to 3 s.f.)**

(3 marks for correct answer, otherwise 1 mark for rearranging to make r the subject and 1 mark for correct r.)

f) $v = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2 \times (6.67 \times 10^{-11}) \times (5.97 \times 10^{24})}{1.933... \times 10^7}}$

$= 6.4187... \times 10^3 =$ **6.42×10^3 ms^{-1} (to 3 s.f.)**

(2 marks for correct answer, otherwise 1 mark for use of correct formula)

The value of r in this equation is the distance from the centre of the mass M (in this case Earth), so use the distance you found in e).

g) A geostationary orbit ***(1 mark)***.

5 a) You can use the graph to find the magnitude of the force, F, on the asteroid at a given distance r from the centre of the planet. So you can use Newton's law of gravitation $F = \frac{GMm}{r^2}$.

You don't need the minus sign since you're using the magnitude of the force, F.

Pick a set of values from the graph
e.g. $F = 10 \times 10^5$ N and $r = 1 \times 10^7$ m

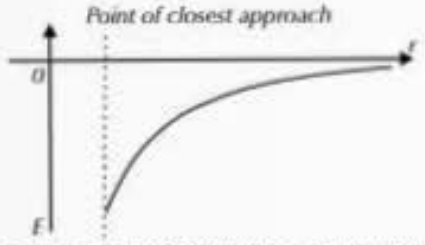
So $m = \frac{Fr^2}{GM} = \frac{10 \times 10^5 \times (1 \times 10^7)^2}{6.67 \times 10^{-11} \times 2.14 \times 10^{21}}$

$= 7.005... \times 10^5 = 7 \times 10^5$ kg (to 1 s.f.)

(2 marks for correct answer, otherwise 1 mark for correct working.)

b) The gravitational potential energy of the asteroid is the gravitational potential of the planet's gravitational field at a distance, r, from the planet multiplied by the asteroid's mass ***(1 mark)***. The gravitational potential is the work done in moving a unit mass from infinity to that point. Work is done moving a mass away from the planet since there is a force of attraction between them, so to move a mass towards a planet, the work done is negative. So the gravitational potential (and so the gravitational potential energy) is negative ***(1 mark)***.

c)



(1 mark for a graph below the r axis showing a 1/r relationship with E tending to zero as r tends to infinity)

Section 4 — Astrophysics and Cosmology

1. The Solar System

Page 316 — Fact Recall Questions

Q1 A cluster of stars and planets held together by gravity.

Q2 A star and all the objects which orbit it (and the objects which orbit those objects).

Q3 The Oort cloud.

Q4 The unit of distance that is equal to the mean distance between the Earth and the Sun.

2. Astronomical Distances

Page 319 — Application Questions

Q1 20 pc $= 20 \times 3.1 \times 10^{16}$ m $= 62 \times 10^{16}$ m

1 ly $= 9.5 \times 10^{15}$ m

so, 20 pc $= (62 \times 10^{16}) \div (9.5 \times 10^{15}) = 65.263...$ ly

$=$ **70 ly (to 1 s.f.)**

Q2 a) First convert 0.37 arcseconds into degrees and then radians:

$0.37 \times \frac{1}{3600} = 1.0277... \times 10^{-4}$°

$1.0277... \times 10^{-4}° \times \frac{\pi}{180} = 1.79... \times 10^{-6}$ rad

Then $d = \frac{L}{\theta} = \frac{1.50 \times 10^{11}}{1.79... \times 10^{-6}}$

$= 8.36... \times 10^{16}$ m $=$ **8.4×10^{16} m (to 2 s.f.)**

b) The distance to Sirius is $8.36... \times 10^{16}$ m.

1 light year $= 9.5 \times 10^{15}$ m,

so Sirius is $(8.36... \times 10^{16}) \div (9.5 \times 10^{15}) = 8.80...$

$= 8.8$ light years away (to 2 s.f.).

So light from Sirius will take **8 years and 10 months** to reach Earth.

Page 319 — Fact Recall Questions

Q1 Half the angle by which a nearby star appears to move in relation to the background stars in 6 months as the Earth moves between opposite points of its orbit.

Q2 A parsec is a unit of distance equal to 3.1×10^{16} m. A star is exactly one parsec (pc) away from Earth if the angle of parallax, $\theta = 1$ arcsecond $= \left(\frac{1}{3600}\right)°$.

Q3 The distance that electromagnetic waves travel in a vacuum in one year.

3. Stellar Evolution

Page 323 — Fact Recall Questions

Q1 gravity

Q2 the main sequence

Q3 The pressure produced from the hydrogen fusion in the core of the star balances the gravitational force compressing the star, so it is stable.

Q4 When the core of a star runs out of hydrogen, hydrogen fusion, and the pressure caused by it, stops and the core starts to contract and heat up. The heat from the core of the star heats up the surrounding hydrogen layer until it is hot enough for the hydrogen to fuse into helium.

Q5 Core hydrogen burning stops in the star and the core of the star begins to contract, causing the outer layers to expand and cool, and the star becomes a red giant.

Q6 Once fusion stops in the core of a low-mass star, the pressure created by fusion is lost, so the core begins to contract under its own weight and heat up. Once the core gets to about the size of the Earth, electron degeneracy pressure stops it contracting any further. The outer layers of the star are ejected, and the hot dense core that is left over is a white dwarf.

Q7 The remnants of the outer layers of a red giant that are ejected as the star becomes a white dwarf.

Q8 The electron degeneracy pressure cannot withstand the gravitational forces at this mass, so when the fusion reactions have stopped, the core will continue to contract beyond the point at which a white dwarf would form.

Q9 The core contracts and the outer layers fall onto the core and rebound, causing a huge shockwave that propels the outer layers into space.

Q10 A neutron star. They are made mostly of neutrons.

Q11 A black hole is an object whose escape velocity is greater than the speed of light. They are formed when a star of core mass more than 3 solar masses contracts and collapses into an infinitely dense point.

4. Stellar Radiation and Luminosity

Page 327 — Application Questions

Q1 a) A is bigger, because A has the greater luminosity. Luminosity is proportional to surface area and the fourth power of temperature, and since A and B are both at the same temperature, A must be bigger.

b) Main sequence

c) White dwarf

d) Star B

Q2 a) Rigel's peak wavelength will be shorter than the Sun's, because Rigel has a higher temperature than the Sun.

b) $L = 4\pi r^2 \sigma T^4$

$$\text{so } r = \sqrt{\frac{L}{4\pi\sigma T^4}} = \sqrt{\frac{4.5 \times 10^{31}}{4\pi \times 5.67 \times 10^{-8} \times 12\,000^4}}$$

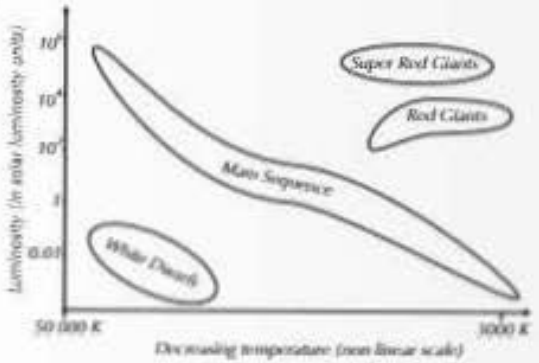
$$= 5.518... \times 10^{10}\ \text{m} = \mathbf{5.5 \times 10^{10}\ m\ (to\ 2\ s.f.)}$$

Page 327 — Fact Recall Questions

Q1 A white star

Q2 $L = 4\pi r^2 \sigma T^4$, where L is the luminosity of the star, r is the radius of the star, σ is Stefan's constant and T is the surface temperature of the star.

Q3



5. Stellar Spectra

Page 332 — Application Questions

Q1 B

Q2 $\Delta E = -3.4 - (-13.6) = 10.2\ \text{eV} = 10.2 \times 1.6 \times 10^{-19}$

$$= 1.632 \times 10^{-18}\ \text{J}$$

$$\Delta E = hf, \text{ so } f = \frac{\Delta E}{h} = \frac{1.632 \times 10^{-18}}{6.63 \times 10^{-34}} = 2.461... \times 10^{15}$$

$$= \mathbf{2.5 \times 10^{15}\ Hz\ (to\ 2\ s.f.)}$$

Q3 $\Delta E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^8}{3.60 \times 10^{-7}} = 5.525 \times 10^{-19}\ \text{J}$

Convert to eV, $\Delta E = 5.525 \times 10^{-19} \div (1.60 \times 10^{-19})$

$$= 3.453...\ \text{eV}$$

So original energy level $= -2.00 - 3.453... = -5.453...$

$$= \mathbf{-5.45\ eV\ (to\ 3\ s.f.)}$$

Q4 $d \sin\theta = n\lambda$

$$\text{so, } d = \frac{n\lambda}{\sin\theta} = \frac{3 \times 4.7 \times 10^{-7}}{\sin(0.020)} = 0.000070504...\ \text{m}$$

$$= \mathbf{7.1 \times 10^{-5}\ m\ (to\ 2\ s.f.)}$$

Page 332 — Fact Recall Questions

Q1 The zero order maximum is a line of maximum brightness at the centre of a diffraction pattern. It's in line with the incident beam.

Q2 White light is made up of a range of different wavelengths. These diffract by different amounts when they pass through the diffraction grating, forming a spectrum.

Q3 A photon. The photon's energy is equal to the difference between the energy levels (the change in energy of the electron).

Q4 Each atom in the star absorbs particular wavelengths of radiation that correspond with the differences between its electron energy levels. So there will only be absorption lines in the spectra at the particular wavelengths corresponding to the elements found in the star.

6. The Big Bang Theory

Page 337 — Application Questions

Q1 1.4×10^7 pc = 14 Mpc

$v = H_0 d = 70 \times 14 = 980$ kms^{-1} = **9.8×10^5 ms^{-1} (to 2 s.f.)**

Q2 Convert v to ms^{-1}

$v = 463 \times 1000 = 463\,000$ ms^{-1}

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

$$\text{so } \Delta\lambda = \frac{v\lambda}{c} = \frac{463000 \times 0.211}{3 \times 10^8} = 0.00032564...\text{m}$$

$$= \mathbf{3.26 \times 10^{-4}\ m\ (to\ 3\ s.f.)}$$

Page 337 — Fact Recall Questions

Q1 On a large scale the universe is homogeneous and isotropic and the laws of physics are the same everywhere.

Q2 The sound waves travelling in the same direction as the police car are 'bunched up' in front of the police car. The bunching up causes the frequency and therefore the pitch of the sound waves to be higher.

Q3 Red shift is where waves emitted from a source that is moving away from the observer are detected by the observer with a longer wavelength and lower frequency than they were emitted at. Blue shift is where waves emitted from a source that is moving towards the observer are detected by the observer with a shorter wavelength and higher frequency than they were emitted at.

Q4 $\boldsymbol{v} = H_0 d$, where $\boldsymbol{v}$ is the recessional velocity of an object in kms^{-1}, H_0 is the Hubble constant in kms^{-1}Mpc^{-1} and d is the distance of the object from Earth in Mpc.
You could also have given the units as ms^{-1} *for* v, s^{-1} *for* H_0 *and* m *for* d.

Q5 The theory that the universe started off very hot and very dense and has been expanding ever since.

Q6 Cosmic microwave background radiation is electromagnetic radiation in the microwave region that is found everywhere in the universe, and is largely homogeneous and isotropic. It is a continuous spectrum of radiation that equates to a temperature of 2.7 K.

7. The Evolution of the Universe

Page 340 — Application Question

Q1 a) 13.7 billion years = $13.7 \times 10^9 \times 365 \times 24 \times 60 \times 60$

$= 4.3204... \times 10^{17}$ s

$t = H_0^{-1}$, so $H_0 = t^{-1} = \dfrac{1}{4.3204... \times 10^{17}}$

$= 2.314... \times 10^{-18}$ s^{-1}

$= (2.314... \times 10^{-18} \times 3.1 \times 10^{22})$ ms^{-1}Mpc^{-1}

$= 71752.08...$ ms^{-1}Mpc^{-1}

$= (71752.08... \div 1000)$ kms^{-1}Mpc^{-1}

$=$ **71.8 kms^{-1}Mpc^{-1} (to 3 s.f.)**

b) That H_0 has been constant since the universe began.

Page 341 — Fact Recall Questions

Q1 If you assume the expansion of the universe has been constant, then the age of the universe is H_0^{-1}, where H_0 is Hubble's constant.

Q2 We can only measure the size of the observable universe, because only the light from within that distance has had enough time to reach us.

Q3 a) The temperature is about 10^{12} K. E.g. quarks join together to form particles like protons and neutrons.

b) The temperature is about 3000 K. E.g. electrons combine with hydrogen and helium nuclei to form atoms.

c) The temperature is about 2.7 K. E.g. stars, galaxies and galactic clusters form, due to gravitational attraction from density fluctuations in the universe.

Q4 Dark matter could explain why galactic clusters have a greater mass than is calculated from their luminosity, and why stars at the edge of a galaxy move faster than they should.

Q5 MACHOs are Massive Compact Halo Objects, e.g. black holes and brown dwarfs.

Q6 WIMPs are Weakly Interacting Massive Particles.

Q7 The expansion of the universe is thought to be accelerating, not decelerating as astronomers expected — dark energy might explain this.

Exam-style Questions — pages 343-345

1 C ***(1 mark)***

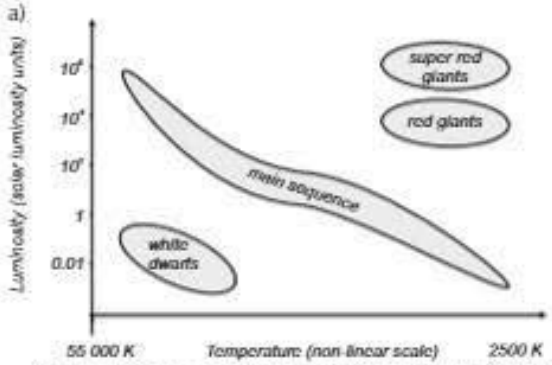
What the star becomes depends on its core's mass compared to the Chandrasekhar limit, which is 1.4 solar masses. The ratio of the core mass to the Sun's mass is 1.09..., so the mass is below the Chandrasekhar limit, and the star becomes a white dwarf.

2 D ***(1 mark)***

3 B ***(1 mark)***

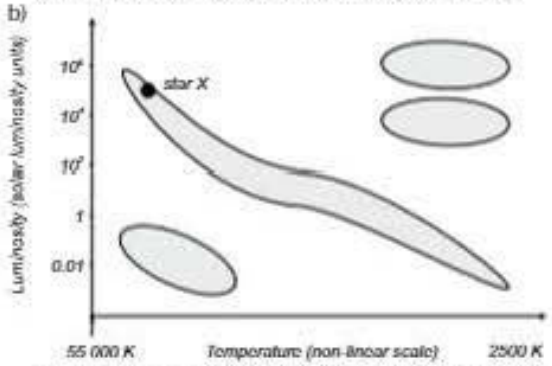
Calculate the energy of the photon using $E = hf$, and convert it to eV by dividing it by 1.6×10^{-19}. Then just add this number to −13.6 eV.

4 a)



(1 mark for correct shape of main sequence, 1 mark for drawing the red giants and super red giants correctly, and 1 mark for drawing white dwarfs correctly.)

b)



(1 mark for correctly plotting Star X on the HR diagram at a roughly correct position for its luminosity and temperature.)

c) When fusion in the core of the star stops, the core will begin to contract ***(1 mark)*** and the outer layers will be ejected in a supernova, leaving the core behind ***(1 mark)***. The gravitational forces will be so large that the core will not stop at a white dwarf or a neutron star, but will collapse to an infinitely dense point (a black hole) ***(1 mark)***.

5 a) Wien's displacement law states that:

$\lambda_{max} \propto \frac{1}{T}$, so $\lambda_{max} T = \text{constant}$

$\lambda_{maxMu} T_{Mu} = \lambda_{maxSun} T_{Sun}$

$T_{Mu} = \frac{\lambda_{maxSun} T_{Sun}}{\lambda_{maxMu}} = \frac{500.0 \times 10^{-9} \times 5800}{828.6 \times 10^{-9}} = 3499.87...\ \text{K}$

$= \mathbf{3500\ K\ (to\ 2\ s.f.)}$

(2 marks for correct answer, otherwise 1 mark for correct method)

b) $L = 4\pi r^2 \sigma T^4$, so

$r = \sqrt{\frac{L}{4\pi\sigma T^4}}$

$= \sqrt{\frac{1.083 \times 10^{12}}{4\pi \times 5.67 \times 10^{-8} \times 3499.87...^4}}$

$= 1.0064... \times 10^{12}\ \text{m}$

$= \mathbf{1.0 \times 10^{12}\ m\ (to\ 2\ s.f.)}$

(2 marks for correct answer, otherwise 1 mark for correct method)

c) For a parallax angle in arcseconds, $p = \frac{1}{d}$

So, $d = \frac{1}{p}$

$d = \frac{1}{5.5 \times 10^{-3}} = 1818.18...\ \text{pc}$

$1\ \text{pc} = 3.1 \times 10^{16}\ \text{m}$

$1\ \text{ly} = 9.5 \times 10^{15}\ \text{m}$

Convert d into light years:

$d = (1818.18... \times 3.1 \times 10^{16}) \div 9.5 \times 10^{15}$

$= 5933.014...\ \text{ly}$

$= \mathbf{5900\ ly\ (to\ 2\ s.f.)}$

(3 marks for correct answer, otherwise 1 mark for correct method for calculation of distance in parsecs, and 1 mark for correct calculation to convert parsecs to light years)

d) The outer layers of the star collapse towards the centre and rebound off the core ***(1 mark)***. This triggers powerful shockwaves which cause a very bright explosion — this is a supernova ***(1 mark)***.

6 a) The universe started off very hot and very dense (perhaps as an infinitely hot, infinitely dense singularity) and expanded to form the present universe ***(1 mark)***.

b) Hubble's law says that the further away a galaxy is, the faster it is moving away from us. It suggests that the universe is expanding so at one point it must have been denser and hotter ***(1 mark)***.

c) $H_0 = 65\ \text{km s}^{-1}\ \text{Mpc}^{-1} = \frac{65 \times 10^3}{3.1 \times 10^{22}} = 2.09... \times 10^{-18}\ \text{s}^{-1}$

$t = \frac{1}{H_0} = \frac{1}{2.09... \times 10^{-18}} = 4.769... \times 10^{17}\ \text{s}$

$= \mathbf{15\ billion\ years\ (to\ 2\ s.f.)}$

(3 marks for correct answer, otherwise 1 mark for correctly converting H_0 to s^{-1} and 1 mark for substituting into correct formula)

d) E.g. Cosmic microwave background radiation: The Big Bang theory predicts that lots of gamma radiation was produced in the early universe ***(1 mark)***. The cosmic microwave background radiation is consistent with this prediction, as the predicted radiation would be Doppler shifted by the expansion of the universe from the gamma range of the spectrum to the microwave range ***(1 mark)***.

7 a) $d \sin\theta = n\lambda$, so:

$\lambda = d \sin\theta \div n = 5.4 \times 10^{-6} \times \sin(0.18) \div 2$

$= 4.833... \times 10^{-7}\ \text{m}$

$= \mathbf{4.8 \times 10^{-7}\ m\ (to\ 2\ s.f.)}$

(2 marks for correct answer, otherwise 1 mark for correct method)

b) $\Delta E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{4.833... \times 10^{-7}} = 4.114... \times 10^{-19}\ \text{J}$

$= 4.114... \times 10^{-19} \div 1.60 \times 10^{-19}$

$= 2.57173...\ \text{eV} = \mathbf{2.6\ eV\ (to\ 2\ s.f.)}$

(3 marks for correct answer, otherwise 1 mark for correct method and 1 mark for correct conversion from J to eV)

c) Electrons in hydrogen atoms absorb and emit photons by moving between energy levels ***(1 mark)***. Since the electron can only move between these discrete energy levels, they absorb the same energies (and so frequencies/wavelengths) of photon that they emit, and these energies would be different for different atoms ***(1 mark)***. The dark lines are caused as the light emitted from inside the Sun passes through hydrogen in the Sun, which absorbs these photons, and prevents them from reaching the observer ***(1 mark)***

d) $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$, so:

$v = \frac{\Delta\lambda\, c}{\lambda} = \frac{0.019 \times 10^{-9} \times 3 \times 10^8}{4.833... \times 10^{-7}}$

$= 11791.96...\ \text{ms}^{-1}$

$= \mathbf{1.2 \times 10^4\ ms^{-1}\ (to\ 2\ s.f.)}$

(2 marks for correct answer, otherwise 1 mark for correct method)

Module 6

Section 1 — Capacitors

1. Capacitors

Page 350 — Application Questions

Q1 $C = \frac{Q}{V}$ rearranged gives $Q = CV = 0.10 \times 230 =$ **23 C**

Q2 $W = \frac{1}{2}V^2C = \frac{1}{2} \times 230^2 \times 40 \times 10^{-3} = 1058$ J

= 1060 J (to 3 s.f.)

Q3 E.g. it would have to be very large to provide enough power, which might hinder the portability of the media player / it could only power the device for a short time, so it would need charging very often / the voltage would decrease as the capacitor discharged, so it would be difficult to produce a constant output.

Q4 $W = \frac{1}{2}\frac{Q^2}{C}$, so

$C = \frac{Q^2}{2W} = \frac{(2.25 \times 10^{-3})^2}{2 \times 1.30}$

$= 1.9471... \times 10^{-6}$ F

= 1.95 × 10⁻⁶ F (to 3 s.f.)

Page 350 — Fact Recall Questions

Q1 The capacitance of an object is the amount of charge it is able to store per unit potential difference (p.d.) across it.

Q2 Electrons flow from the power supply to one of the plates of the capacitor. The electrons cannot move across the gap between the plates, as they are separated by an electrical insulator, so a negative charge builds up on the plate. The negative charge on this plate repels electrons in the opposite plate. This causes electrons to flow away from the opposite plate, and a positive charge builds up on this plate.

Q3 Find the area under the *V*-*Q* graph.

Q4 $W = \frac{1}{2}QV$, $W = \frac{1}{2}V^2C$, $W = \frac{1}{2}\frac{Q^2}{C}$

Where W is energy stored by the capacitor, Q is the charge on the capacitor, V is the potential difference across the capacitor and C is the capacitance of the capacitor.

2. Capacitors in Circuits

Page 355 — Application Questions

Q1 In series, $\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2}$.

If $C_1 = C_2 = C$, then $\frac{1}{C_{total}} = \frac{2}{C}$, so $C_{total} = \frac{C}{2}$

So the total capacitance of the circuit is half the capacitance of one capacitor. Potential difference, V, is fixed, so the total charge stored by the capacitors, Q, is also half the total charge stored by a single capacitor if it were connected on its own in the same circuit.

So, $Q = 1.2 \times 10^{-3} \div 2 = 6.0 \times 10^{-4}$ C

$Q = It$, so $I = Q \div t$

$t = 2 \times 60 = 120$ s

So, $I = 6.0 \times 10^{-4} \div 120$

= 5.0 × 10⁻⁶ A

Q2 a) Total capacitance is given by the gradient of the graph.

gradient $= (3.6 \times 10^{-3} - 0) \div (0.8 - 0)$

$= 4.5 \times 10^{-3}$ F

So the total capacitance is 4.5×10^{-3} F.

b) For capacitors connected in parallel, $C_{total} = C_1 + C_2 + ...$

Since all the capacitors have the same capacitance,

C_{total} = number of capacitors × C

So, number of capacitors = $C_{total} \div C$

$= 4.5 \times 10^{-3} \div 1.5 \times 10^{-3}$

= 3

Page 355 — Fact Recall Questions

Q1 $\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + ...$

Q2 The total capacitance of the capacitors.

3. Investigating Charging and Discharging Capacitors

Page 359 — Application Question

Q1 $\frac{\Delta Q}{\Delta t} = -\frac{Q}{CR}$, so $Q = -\frac{CR\,\Delta Q}{\Delta t}$

Capacitor is discharging, so $\Delta Q = -85$ nC $= -8.5 \times 10^{-8}$ C

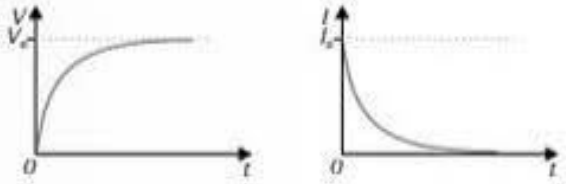
$Q = -\frac{4.2 \times 10^{3} \times 5.5 \times 10^{3} \times (-8.5 \times 10^{-8})}{4.5 \times 10^{-5}}$

$= 0.4363...$ C

= 0.44 C (to 2 s.f.)

Page 359 — Fact Recall Questions

Q1



Q2 E.g. remove the power source and reconnect the circuit.

Q3 E.g. Attach a fully charged capacitor in a circuit containing an ammeter and a fixed resistor, and with a voltmeter/voltage sensor connected across the capacitor. Complete the circuit to allow the capacitor to discharge. Use a data logger connected to the ammeter and voltmeter/voltage sensor to measure the current through the resistor and the potential difference across the capacitor at different times. Once the capacitor has fully discharged, connect the data logger to a computer and use the results obtained by the data logger to plot graphs of current through the circuit and potential difference across the capacitor against time.

4. Charging and Discharging Calculations

Page 364 — Application Questions

Q1 $V = \frac{1}{2}e^{-\frac{t}{RC}} = 15.0 \times e^{-\frac{1.1 \times 10^{-4}}{(6.0 \times 10^{3}) \times (2.4 \times 10^{-8})}} = 5.7345...$
$= \mathbf{5.7\ V}$ **(to 2 s.f.)**
Don't forget to convert all the values you've been given into SI units.

Q2 When the capacitor is fully charged, the charge is $Q = CV = 15 \times 10^{-9} \times 230 = 3.45 \times 10^{-6}$ C
Then after 0.01 seconds,

$Q = Q_0 e^{-\frac{t}{RC}} = 3.45 \times 10^{-6} \times e^{-\frac{0.01}{(15 \times 10^{-9}) \times (400 \times 10^{3})}}$
$= 5.587... \times 10^{-13} = \mathbf{6 \times 10^{-12}\ C}$ **(to 1 s.f.)**

Q3 a) The capacitor losing 63% of its charge is the same as the capacitor discharging to 37% of its original charge, which takes a time equal to the time constant, $\tau = CR$.
$\tau = CR = 15 \times 10^{-6} \times 400 \times 10^{3} = \mathbf{6\ s}$

b) $C = \frac{\tau}{R} = \frac{60}{400 \times 10^{3}} = \mathbf{1.5 \times 10^{-4}\ F}$

c) The capacitor would have to be a lot bigger than the original one. The resistance of the resistor could be increased instead.

Q4 $\frac{Q}{Q_0} = 0.3$

$\frac{Q}{Q_0} = e^{-\frac{t}{RC}}$ so $0.3 = e^{-\frac{20.0}{RC}} \Rightarrow \ln 0.3 = \ln\left(e^{-\frac{20.0}{RC}}\right)$

$\Rightarrow \ln 0.3 = -\frac{20.0}{CR} \Rightarrow CR = -\frac{20.0}{\ln 0.3} = \mathbf{16.6\ s}$ **(to 3 s.f.)**

Q5 a) $\ln(V_0)$, where V_0 is the initial potential difference across the capacitor.

b) Vertical axis intercept $= \ln V_0 = 2.708$
Rearranging to find V_0:
$V_0 = e^{2.708} = 14.99... = \mathbf{15.0\ V}$ **(to 3 s.f.)**

c) The gradient of the graph is:

$-\frac{2.708 - 1.001}{2.00 \times 10^{-4} - 0} = -853\,500$

The gradient $= -\frac{1}{CR}$ so $CR = -1 \div -853\,500$
$= 1.171... \times 10^{-6}$
$= 1.17\ \mu s$ (to 3 s.f.)
So the time constant $\tau = \mathbf{1.17\ \mu s}$ **(to 3 s.f.)**

Page 364 — Fact Recall Questions

Q1 $Q = Q_0(1 - e^{-\frac{t}{RC}})$

Q2 $V = \frac{1}{2}e^{-\frac{t}{RC}}$

Q3 The resistance of the circuit and the capacitance of the capacitor.

Q4 The time taken for the capacitor to discharge to $\frac{1}{e}$ (about 37%) of its original charge. It is given by $\tau = CR$.

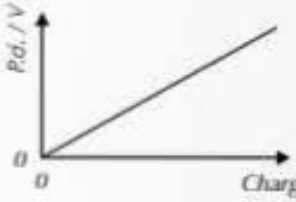
Exam-style Questions — pages 366-367

1 A ***(1 mark)***.
The charge increases quickly and then levels off as it charges, and decreases quickly and then levels off as it discharges.

2 C ***(1 mark)***.
$Q = Q_0 e^{-\frac{t}{RC}}$ so $\frac{Q}{Q_0} = e^{-\frac{t}{RC}}$ where $\frac{Q}{Q_0}$ is 0.65.
So $\ln(0.65) = -\frac{1}{100 \times 10^{3} \times C}$
so $C = -\frac{1}{100 \times 10^{3} \times \ln(0.65)} = 2.3 \times 10^{-6}$ F (to 2 s.f.)

3 B ***(1 mark)***.
First find the total capacitance of the three capacitors in series, $\frac{1}{C_{tot}} = \frac{1}{20} + \frac{1}{20} + \frac{1}{20} = \frac{3}{20}$, so $C_{series} = 6.66...\ \mu F$.
Then find the total capacitance of the circuit by adding this value to the capacitance of the capacitor in parallel,
$C_{total} = 12.0 + 6.66... = 18.66... = 18.7\ \mu F$.

4 a) i)



(1 mark for straight line through the origin)

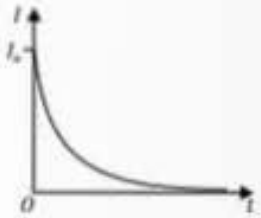
ii) Energy stored by the capacitor ***(1 mark)***.

b) $C = \frac{Q}{V} = \frac{18 \times 10^{-6}}{12} = \mathbf{1.5 \times 10^{-6}\ F}$
(2 marks for correct answer, 1 mark for correct method if answer incorrect)

Remember, when a capacitor is fully charged, the potential difference across it is equal to the p.d. of the source used to charge it.

c) $V = \frac{1}{2}e^{-\frac{t}{RC}} = 12 \times e^{-\frac{0.03}{(1.5 \times 10^{-6}) \times (29 \times 10^{3})}}$
$= 6.020... = \mathbf{6.0\ V}$ **(to 2 s.f.)**
(2 marks for correct answer, 1 mark for correct method if answer incorrect)

d)



(1 mark for correct shape)

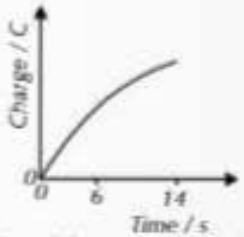
e) $I = Ie^{-\frac{t}{RC}}$, so $t = -CR\ln\left(\frac{I}{I_0}\right)$
Looking for when the current has fallen to 10% of its initial value, so $\frac{I}{I_0} = 0.1$
$t = -(1.5 \times 10^{-6} \times 29 \times 10^{3}) \times \ln(0.1)$
$= 0.1001...\ s = \mathbf{0.10\ s}$ **(to 2 s.f.)**
(2 marks for correct answer, 1 mark for correct method if answer incorrect)

5 a) $W = \frac{1}{2}V^2C = \frac{1}{2} \times 50.0^2 \times 3.0 \times 10^{-3}$
$= 3.75\ J = \mathbf{3.8\ J}$ **(to 2 s.f.)**
(2 marks for correct answer, 1 mark for correct method if answer incorrect)

b) $\tau = CR = 2.0 \times 10^{3} \times 3.0 \times 10^{-3} = \mathbf{6\ s}$ ***(1 mark)***

c) After 14 s, the charge on the capacitor is:
$Q = Q_0(1 - e^{-\frac{t}{RC}}) = CV(1 - e^{-\frac{t}{RC}})$
$Q = 3.0 \times 10^{-3} \times 50.0(1 - e^{-\frac{14}{6}})$
$= 0.135... = \mathbf{0.14\ C}$ **(to 2 s.f.)**
(2 marks for correct answer, 1 mark for correct method if answer incorrect)

d)



(1 mark for correct shape)
The charge should increase to around 63% at 6 s and 90% at 14 s.

Section 2 — Electric Fields

1. Electric Fields

Page 371 — Application Questions

Q1 $F = \frac{Qq}{4\pi\varepsilon_0 r^2}$

$= \frac{(-1.60 \times 10^{-19})^2}{4\pi \times (8.85 \times 10^{-12}) \times (5.22 \times 10^{-11})^2}$

$= 8.44784... \times 10^{-8}$

$= \mathbf{8.45 \times 10^{-8}\ N}$ **(to 3 s.f.)**

Q2 Start by rearranging the formula for the magnitude of E to make r the subject:

$E = \frac{Q}{4\pi\varepsilon_0 r^2} \Rightarrow r^2 = \frac{Q}{4\pi\varepsilon_0 E}$

$\Rightarrow r = \sqrt{\frac{Q}{4\pi\varepsilon_0 E}}$

Then put the numbers in:

$r = \sqrt{\frac{4.15 \times 10^{-8}}{4\pi \times (8.85 \times 10^{-12}) \times 15000}}$

$= 1.57725...$

$= \mathbf{1.6\ m}$ **(to 2 s.f.)**

Q3 $E = \frac{F}{Q} = \frac{0.080}{5.0 \times 10^{-5}} = \mathbf{1600\ NC^{-1}}$

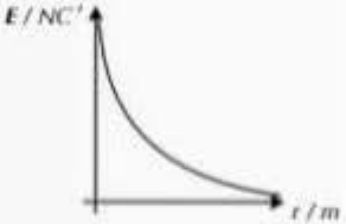
Page 371 — Fact Recall Questions

Q1 E is a measure of the force per unit positive charge (in an electric field).

Q2

Q3 $F = \frac{Qq}{4\pi\varepsilon_0 r^2}$

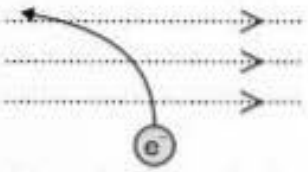
Q4



2. Uniform Electric Fields

Page 373 — Application Questions

Q1



Q2 If the particle isn't moving, the upwards force from the electric field must balance the particle's weight, i.e.

$mg = EQ = \frac{VQ}{d}$

This comes from weight = mg, $E = \frac{F}{Q}$ and $E = \frac{V}{d}$.

Rearrange this to make d the subject, then put in the numbers:

$mg = \frac{VQ}{d} \Rightarrow d = \frac{VQ}{mg}$

$= \frac{(5.00 \times 10^{-5}) \times 2(1.60 \times 10^{-19})}{(6.64 \times 10^{-27}) \times 9.81}$

$= 0.02456...\ m = \mathbf{2.46\ cm}$ **(to 3 s.f.)**

Q3 $C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 7.0 \times 10^{-4}}{8.5 \times 10^{-4}} = 7.28823... \times 10^{-12}$

$= \mathbf{7.3 \times 10^{-12}\ F}$ **(to 2 s.f.)**

$(= 0.73\ nF)$

Page 373 — Fact Recall Question

Q1 $\varepsilon = \varepsilon_r \varepsilon_0$ where ε is the permittivity of the material, ε_r is the relative permittivity and ε_0 is the permittivity of free space.

3. Electric Potential

Page 376 — Application Question

Q1 a) $V = \frac{Q}{4\pi\varepsilon_0 r} = \frac{12.6 \times 10^{-6}}{4\pi \times 8.85 \times 10^{-12} \times 5.19 \times 10^{-2}}$

$= 2.18298... \times 10^{6}$

$= \mathbf{2.18 \times 10^{6}\ V}$ **(to 3 s.f.)**

b) First find the new potential at the centre of the smaller sphere:

$V_2 = \frac{Q}{4\pi\varepsilon_0 r}$

$= \frac{12.6 \times 10^{-6}}{4\pi \times 8.85 \times 10^{-12} \times ((5.19 \times 10^{-2}) + (12.9 \times 10^{-2}))}$

$= 6.26... \times 10^{5}\ V$

The electric potential energy at a point = Vq, and work done is the difference in electric potential energy between two points, so:

Work done

$= V_2 q - V_1 q = (V_2 - V_1)q$

$= (2.18298... \times 10^{6} - 6.26... \times 10^{5}) \times 0.152 \times 10^{-6}$

$= 0.2366163...$

$= \mathbf{0.237\ J}$ **(to 3 s.f.)**

You could also have worked out the electric potential energy at each point using $E = (Qq) \div (4\pi\varepsilon_0 r)$ and then found the difference between the two values.

c) $C = 4\pi\varepsilon_0 R = 4\pi \times 8.85 \times 10^{-12} \times 2.11 \times 10^{-2}$

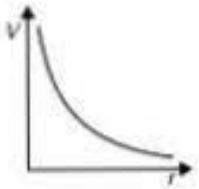
$= 2.346... \times 10^{-12}$

$= \mathbf{2.35 \times 10^{-12}\ F}$ **(to 3 s.f.)**

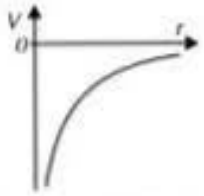
Page 376 — Fact Recall Questions

Q1 The electric potential at a point is the work done to move a unit positive charge from infinity to that point in an electric field.

Q2 a)



b)



Q3 The equation for electric potential, $V = \frac{Q}{4\pi\varepsilon_0 r}$, and the charge on a capacitor $Q = CV$.

4. Comparing Electric and Gravitational Fields

Page 377 — Fact Recall Questions

Q1 Any three from: E.g.

1. Gravitational field strength g is the force a unit mass would experience in a gravitational field. Electric field strength E is the force a unit positive charge would experience in an electric field.
2. Newton's law and Coulomb's law are the same but with G switched for $\frac{1}{4\pi\varepsilon_0}$ and M and m switched for Q and q.
3. Field lines for a radial gravitational field and a radial electric field have the same shape.
4. Gravitational potential V_g and absolute electric potential V give the energy a unit mass or charge would have at a point.

Q2 Any two from: E.g.

1. Gravitational forces are always attractive, but electric forces can be attractive or repulsive.
2. Objects can be shielded from electric fields, but not from gravitational fields.
3. The size of an electric force is dependent on the medium between the charges, but for gravitational fields it makes no difference.

Exam-style Questions — pages 379-380

1 A ***(1 mark)***

There will be a uniform electric field pointing from the 100 V plate to the 0 V plate. The alpha particle is positively charged, so there will be a force on it in this direction due to the electric field.

2 B ***(1 mark)***

Capacitance of an isolated charged sphere is $C = 4\pi\varepsilon_0 R$, where R is the radius of the sphere.

3 C ***(1 mark)***

The magnitude of the force decreases as you move away from the charged sphere by the inverse square law.

4 A ***(1 mark)***

First find an equation for the electric field strength between the lithium nucleus and electron: E_1 = electric field strength due to lithium nucleus electric field − electric field strength due to electron electric field. Remember, electric field strength is a vector, so you take away the electric field strength due to the electron. Charge of proton = $+Q$ and charge of electron = $-Q$, so $E_1 = \frac{3(+Q)}{4\pi\varepsilon_0 r^2} - \frac{(-Q)}{4\pi\varepsilon_0 r^2} = \frac{4Q}{4\pi\varepsilon_0 r^2}$. Then find the electric field strength between the helium nucleus and the electron:

$E_2 = \frac{2(+Q)}{4\pi\varepsilon_0 r^2} - \frac{(-Q)}{4\pi\varepsilon_0 r^2} = \frac{3Q}{4\pi\varepsilon_0 r^2}$. From the two equations you can see that $\frac{E_1}{4} = \frac{E_2}{3}$, so $E_2 = \frac{3E_1}{4}$.

5 a) Sphere A is stationary, so forces acting on it must be equal, so force due to gravity = force due to electric field.

$$F = \frac{Qq}{4\pi\varepsilon_0 r^2} \Rightarrow r = \sqrt{\frac{Qq}{4\pi\varepsilon_0 F}}$$

$$= \sqrt{\frac{(-92.5 \times 10^{-9})(-34.7 \times 10^{-9})}{4\pi \times 8.85 \times 10^{-12} \times 1.99}}$$

$$= 3.8083... = \mathbf{3.81\ m\ (to\ 3\ s.f.)}$$

(2 marks for correct answer, otherwise 1 mark for correct working.)

b) A uniform electric field is produced ***(1 mark)***, pointing from the plate with the more positive potential (the top plate) to the plate with the less positive potential (the bottom plate) ***(1 mark)***.

c) First calculate the electric field strength:

$E = \frac{F}{Q} = \frac{0.18}{92.5 \times 10^{-9}} = 1945.9...\ NC^{-1}$

Then calculate the electric potential:

$E = \frac{V}{d}$ so $V = Ed = 1945.9... \times 0.104$

$= 202.378...$

$= 200$ V (to 2 s.f.)

This is the potential difference between the plates, so the electric potential of the top plate is **200 V (to 2 s.f.)**.

(3 marks for the correct answer, otherwise 1 mark for calculating the electric field strength and 1 mark for correct working for calculating the electric potential.)

d) Rearrange $C = \frac{\varepsilon A}{d}$ for ε:

$\varepsilon = \frac{Cd}{A} = \frac{34.5 \times 10^{-12} \times 0.104}{0.31}$

$= 1.157... \times 10^{-11}\ Fm^{-1}$ ***(1 mark)***

This is larger than ε_0, so the space between the plates is not a vacuum ***(1 mark)***.

Section 3 — Electromagnetism

1. Magnetic Fields

Page 383 — Application Questions

Q1

Q2 The force will act upwards.

Page 383 — Fact Recall Questions

Q1 Magnetic fields are found around permanent magnets and moving charges.

Q2 From north to south.

Q3 The strength of the magnetic field.

Q4 You can use your left hand. The first (index) finger represents the direction of the magnetic field, the second (middle) finger represents the direction of the current, and the thumb represents the direction of the force (or motion).

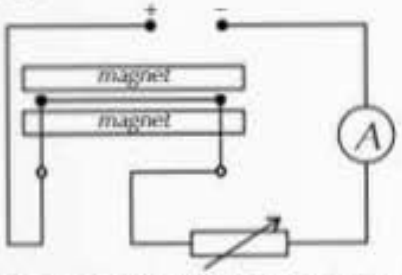
2. Magnetic Flux Density

Page 388 — Application Questions

Q1 a) $F = BIL\sin\theta = 0.20 \times 4.0 \times (25 \times 10^{-2}) \times \sin 53°$
$= 0.1597... =$ **0.16 N (to 2 s.f.)**

b) There is no force acting on the wire.

Q2 a) E.g.



b) A variable resistor should be used so that the resistance, and therefore the current, can be changed (since $V = IR$) in order to determine a value for B.

c) Only that length of wire will experience a vertical force, which will contribute to the reading on the mass balance.

d) Convert the mass readings into force using $F = mg$, then plot your data on a graph of force F against current I. Because $F = BIl$, the gradient of the line of best fit is equal to Bl. You know the length, l, so just divide the gradient by l to get the magnetic flux density, B.

Page 388 — Fact Recall Questions

Q1 The current-carrying wire must be perpendicular to the magnetic field.

Q2 The force on a wire is $F = BIL\sin\theta$. At 90°, $\sin\theta = 1$, so the force is at a maximum.

3. Forces on Charged Particles

Page 393 — Application Questions

Q1 Only charged particles experience a force from magnetic fields (neutrons have no charge).

Q2 The force will act upwards (using the left-hand rule). Because the particle is negatively charged, you need to point your second finger in the *opposite* direction to its actual motion.

Q3 $F = BQv$
$= (640 \times 10^{-3}) \times (3.2 \times 10^{-19}) \times (5.5 \times 10^{3})$
$= 1.1264 \times 10^{-15} =$ **1.1×10^{-15} N (to 2 s.f.)**

Q4 $v = \frac{E}{B} \Rightarrow E = Bv = (20.0 \times 10^{-3}) \times 5.2 \times 10^{6}$
$= 1.04 \times 10^{5} =$ **1.0×10^{5} NC^{-1} (to 2 s.f.)**

Q5 a) $F = \frac{mv^2}{r}$, $F = BQv$

$\Rightarrow \frac{mv^2}{r} = BQv$

$\Rightarrow B = \frac{mv^2}{Qvr}$

Cancelling the 'v's:

$B = \frac{mv}{Qr}$

b) $B = \frac{mv}{Qr} = \frac{(1.673 \times 10^{-27}) \times (1.99 \times 10^{7})}{(1.60 \times 10^{-19}) \times 5.49}$
$= 0.03790... =$ **0.0379 T (to 3 s.f.)**

Page 393 — Fact Recall Question

Q1 It will follow a circular path.

4. Magnetic Flux and Flux Linkage

Page 397 — Application Question

Q1 a) Flux linkage is at its greatest when the area is normal to the magnetic field. If the angle of the field with the normal to the area is increased, the flux linkage decreases.

This is because the number of field lines passing through the area decreases.

b) At highest value of flux linkage $\cos\theta = \cos 0 = 1$, so:

$N\phi = BAN \Rightarrow B = \frac{N\phi}{AN} = \frac{1.3 \times 10^{-6}}{(25 \times 10^{-4}) \times 10}$
$= 5.2 \times 10^{-5}$ T $=$ **52 μT**

Don't forget to convert the area to m^2.

Q2 $N\phi = BAN\cos\theta \Rightarrow \theta = \cos^{-1}\left(\frac{N\phi}{BAN}\right)$

So $\theta = \cos^{-1}\left(\frac{(10.0 \times 10^{-3})}{(2.06 \times 10^{-4}) \times (145 \times 10^{-4}) \times (355)}\right)$
$= 19.429... =$ **19.4° (to 3 s.f.)**

Page 397 — Fact Recall Questions

Q1 Magnetic flux is measured in webers (Wb).

Q2 An e.m.f. (and a current if the conductor is part of a circuit).

Q3 The number of turns on the coil cutting the flux (N).

5. Faraday's Law and Lenz's Law

Page 402 — Application Questions

Q1 The plane's wings act as a conductor, and the Earth has a magnetic field that the plane cuts through, so electromagnetic induction occurs.

Q2 $\varepsilon = -\frac{\Delta(N\phi)}{\Delta t} = -\frac{N\Delta(BA)}{\Delta t} = -\frac{50 \times (-1.5) \times 0.24}{0.32} = 56.25$

So magnitude of e.m.f. = **56 V (to 2 s.f.)**

Here the magnetic flux density decreases, so ΔB is negative.

Q3 a) The current is going away from the reader.

By Lenz's law, the e.m.f. produces a force that opposes the motion that caused it, so the force produced acts downwards.

The direction of the field is to the right, so using Fleming's left-hand rule, the current induced must be in the direction going away from the reader.

b) Distance travelled by wire, $s = v\Delta t$.

So area of flux cut $A = lv\Delta t$ where l = length of wire.

So total magnetic flux cut $\phi = BA = Blv\Delta t$

$\varepsilon = -\frac{\Delta(N\phi)}{\Delta t} = -\frac{\Delta\phi}{\Delta t}$ (since $N = 1$), so

$\varepsilon = -\frac{Blv\Delta t}{\Delta t} = -Blv$

$= -(5.4 \times 10^{-3}) \times 1.2 \times 0.50 = -0.00324$ V

So magnitude of e.m.f. = **0.0032 V (to 2 s.f.)**

Page 402 — Fact Recall Questions

Q1 Faraday's law states that the induced e.m.f. is directly proportional to the rate of change of flux linkage (induced e.m.f., $\varepsilon = -\frac{\Delta N\phi}{\Delta t}$).

Q2 The negative of the induced e.m.f.

Q3 Calculate the negative of the area under the graph to get the total flux linkage change and divide this by the number of turns on the coil.

Q4 Lenz's law states that the induced e.m.f. is always in such a direction as to oppose the change that caused it.

6. Uses of Electromagnetic Induction

Page 407 — Application Questions

Q1 a) $\frac{n_s}{n_p} = \frac{V_s}{V_p}$, so $V_s = V_p\frac{n_s}{n_p} = 190 \times \frac{420}{250} = 319.2$

= 320 V (to 2 s.f.)

b) $\frac{I_p}{I_s} = \frac{V_s}{V_p}$, so $I_p = \frac{I_sV_s}{V_p} = \frac{13 \times 75}{120} = 8.125$

= 8.1 A (to 2 s.f.)

Q2 a) E.g. By using a laminated core.

b) i) A step-down transformer.

ii) $\frac{n_s}{n_p} = \frac{V_s}{V_p} \Rightarrow n_s = n_p\frac{V_s}{V_p}$

$n_s = 110 \times \frac{19}{230} = 9.086...$

= 9 turns (to the nearest whole number)

Page 407 — Fact Recall Questions

Q1 Generators convert kinetic energy into electrical energy. They induce an electric current by rotating a coil in a magnetic field. The output voltage and current change direction with every half rotation of the coil, producing an alternating current.

Q2 An alternating current flowing in the primary (or input) coil produces a changing magnetic field in the iron core. The changing magnetic field is passed through the iron core to the secondary (or output) coil, where it induces an alternating voltage (e.m.f.) of the same frequency as the input voltage. The more turns there are in the secondary coil compared to the primary coil, the bigger the induced e.m.f. is compared to the input e.m.f..

Remember, e.m.f. is just another way of saying voltage.

Q3 Any one from: the current through the primary coil / the voltage across the primary coil.

Exam-style Questions — pages 409-410

1 C ***(1 mark)***

Magnetic flux cut through, $\Delta\phi = BA = BLv\Delta t$, where L is the length of the rod and v is the velocity of the rod.

Faraday's law: $\varepsilon = -\Delta(N\phi) \div \Delta t = -\Delta\phi \div \Delta t$ (as N = 1)

So $\varepsilon = -BLv\Delta t \div \Delta t = -BLv$

Rearranging: $v = -\varepsilon \div (BL)$

$= -(4.5 \times 10^{-3}) \div ((2.1 \times 10^{-2}) \times (3.3 \times 10^{-2}))$

$= -0.649...$

So the velocity of the bar = 0.65 ms^{-1} (to 2 s.f.)

2 A ***(1 mark)***

Convert v to ms^{-1}: 42 kms^{-1} = 42×10^3 ms^{-1}

$v = E \div B \Rightarrow B = E \div v = (37 \times 10^3) \div (42 \times 10^3)$

$= 0.8809... = 0.88$ T (to 2 s.f.)

3 a) The particle will follow a circular path ***(1 mark)***.

b) $F = BQv$

$= 0.93 \times (1.60 \times 10^{-19}) \times (8.1 \times 10^7) = 1.20528 \times 10^{-11}$

$= \mathbf{1.2 \times 10^{-11}}$ **N (to 2 s.f.)**

The force would act downwards.

(1 mark for correct force and 1 mark for direction.)

The magnitude of the charge on an electron is given in the exam data and formulae booklet. Find the direction of the force with Fleming's left-hand rule — because electrons have a negative charge, point your second finger in the opposite direction to its velocity.

c) $F_{alpha} = 2 \times F_e = 2 \times 1.20528 \times 10^{-11} = 2.41056 \times 10^{-11}$

The force will be **2.4×10^{-11} N (to 2 s.f.) upwards.**

(1 mark for correct force and 1 mark for direction.)

If you didn't get the correct answer for b), you'll get the marks for c) as long as the force is double the force for the electron and in the opposite direction.

4 a) Downwards ***(1 mark)***

Fleming's left-hand rule can be used to find the direction of the force on the wire. The force on the cradle is in the opposite direction.

b) E.g. The magnetic flux density can be found by varying the current using the variable d.c. power supply and noting the mass shown on the top pan balance for a range of currents ***(1 mark)***. After converting the masses to weights using $W = mg$ ***(1 mark)***, a graph of force against current can be plotted ***(1 mark)***. Drawing a line of best fit and finding the gradient gives you BL, so dividing the gradient by L gives the magnetic flux density B ***(1 mark)***.

c) $F = BIL \Rightarrow I = \frac{F}{BL}$

$I = \frac{68 \times 10^{-3}}{44 \times 10^{-3} \times 52 \times 10^{-3}} = 2.972... =$ **3.0 A (to 2 s.f.)**

(2 marks for correct answer, otherwise 1 mark for correct working)

5 a) The magnetic field produced by a primary coil carrying direct current is constant after being switched on, so will not produce a changing magnetic field in the core ***(1 mark)***. Faraday's law dictates that a changing magnetic field is required in the core for an e.m.f. to be generated in the secondary coil ***(1 mark)***.

b) $\frac{V_s}{V_p} = \frac{n_s}{n_p} \Rightarrow n_s = n_p \times \frac{V_s}{V_p}$

$n_s + n_p = 250$ so $n_p = 250 - n_s$

So $n_s = (250 - n_s) \times \frac{V_s}{V_p} = \left(250 \times \frac{V_s}{V_p}\right) - \left(n_s \times \frac{V_s}{V_p}\right)$

Bringing the n_s's to one side:

$$n_s \times \left(1 + \frac{V_s}{V_p}\right) = \left(250 \times \frac{V_s}{V_p}\right) \Rightarrow n_s = \frac{\left(250 \times \frac{V_s}{V_p}\right)}{\left(1 + \frac{V_s}{V_p}\right)}$$

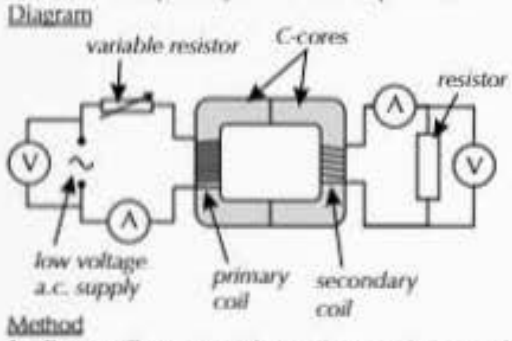
$$n_s = \frac{\left(250 \times \frac{7}{230}\right)}{\left(1 + \frac{7}{230}\right)} = 7.383... = \textbf{7 turns (to 1 s.f.)}$$

(2 marks for correct answer, otherwise 1 mark for correct working.)

c) How to grade your answer (pick the description that best matches your answer):

0 marks: There is no relevant information.
1-2 marks: A diagram has been drawn, and method points 1, 3 and 4 have been covered. One accuracy point has been mentioned. Answer is basic and has lack of structure and information is missing.
3-4 marks: A labelled diagram has been drawn and method points 1, 3, 4, 5 and 6 have been covered. Two accuracy points have been mentioned. Answer has some structure, and information is presented fairly clearly.
5-6 marks: A labelled diagram has been drawn, all of the method points have been covered. All accuracy points have been mentioned. The answer is well structured and information is clearly presented.
Here are some points your answer may include:

Diagram



Method
1. Put two C-cores together and wrap wire around each to make the coils.
2. Make sure the two coils have different numbers of turns.
3. Turn on the a.c. supply to the primary coil.
4. Adjust the variable resistor to change the input current.
5. Record the current through and voltage across each coil for a range of input currents.
6. Keep numbers of turns on the coils constant throughout.
7. Use the results to see how the ratio of the currents through the coils affects the ratio of the voltages across the coils.
8. Results won't quite give $\frac{V_s}{V_p} = \frac{I_p}{I_s}$ due to energy losses in the transformer.

Accuracy
1. Use low currents.
2. Use a laminated core.
3. Use low-resistance wire.

Section 4 — Nuclear and Particle Physics

1. Atomic Structure

Page 414 — Application Questions

Q1 a) 8
b) 8 + 8 = **16**
c) $^{16}_{8}O$

Q2 a) $^{45}_{21}X$ (the nucleon number is 21 + 24 = 45)
b) E.g. $^{45}_{20}X$
Isotopes have the same number of protons but a different number of neutrons. So the proton number must be the same as in part a), but the nucleon number must be different.

Q3 a) 2
b) 2
Isotopes have the same proton number.

Page 414 — Fact Recall Questions

Q1 The fact that most alpha particles passed straight through the foil showed that most of the atom is empty space.
Some positively-charged alpha particles were deflected by the atom by a large angle, showing that there must be a nucleus with a large positive charge.
Very few alpha particles bounced back, showing that the nucleus is very small.
High momentum alpha particles were deflected backwards by the nucleus, showing that most of the mass of the atom must be in the nucleus.

Q2 Inside an atom there is a nucleus which contains neutrons and protons. Electrons orbit the nucleus. Most of the atom is empty space, as the electrons orbit at relatively large distances.

Q3 The number of protons in the nucleus of an atom.

Q4 The total number of protons and neutrons in the nucleus of an atom.

Q5 Atoms with the same number of protons but different numbers of neutrons.

2. The Nucleus

Page 418 — Application Questions

Q1 Rearrange $R = r_0 A^{1/3}$ for A,

$A = \left(\frac{R}{R_0}\right)^3 = \left(\frac{7}{1.4}\right)^3 = 5^3 = 125$

number of protons = A – number of neutrons

$= 125 - 73$

$= \mathbf{52}$

Q2 a) For separations less than around 0.5 fm the strong nuclear force is repulsive. So at 0.4 fm it would be repulsive.

b) For separations between about 0.5 fm and 3 fm the strong nuclear force is attractive. So at 1.5 fm it would be attractive.

c) For separations bigger than about 3 fm the strong nuclear force has little effect. So at 4.2 fm, the strong interaction is small enough that it can be ignored.

Q3 a) $\rho = \frac{m}{V}$,

Model the atom as a sphere, so $V = \frac{4}{3}\pi r^3$

So, $\rho = \frac{3m}{4\pi r^3}$

Rearrange for r:

$r = \left(\frac{3m}{4\pi\rho}\right)^{\frac{1}{3}}$

$= \left(\frac{3 \times 8.5 \times 10^{-26}}{4\pi \times 8.4 \times 10^{17}}\right)^{\frac{1}{3}}$

$= 6.22802... \times 10^{-11}$ m

$= \mathbf{6.2 \times 10^{-11}}$ **m (to 2 s.f.)**

b) $\rho = \frac{3m}{4\pi r^3}$

$= \frac{3 \times 8.5 \times 10^{-26}}{4\pi \times (5.2 \times 10^{-15})^3}$

$= 1.44317... \times 10^{17}$ kg m^{-3}

$= \mathbf{1.4 \times 10^{17}}$ **kg m^{-3} (to 2 s.f.)**

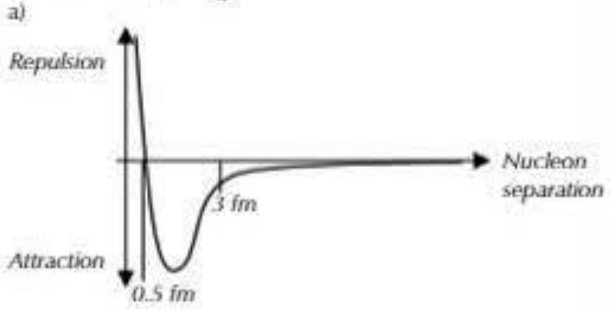
Page 418 — Fact Recall Questions

Q1 Approximate diameter of an atom = 0.1 nm (1×10^{-10} m)
Approximate diameter of a small nucleus = a few fm

Q2 Nuclear radius increases proportionally with the cube root of the nucleon number ($R = r_0 A^{1/3}$).

Q3 The nucleus is assumed to be a sphere and its volume is calculated using $V = \frac{4}{3}\pi r^3$

Q4 a)



b) Between 0 and about 0.5 fm.

c) Between about 0.5 and 3 fm.

3. Particles and Antiparticles

Page 423 — Application Questions

Q1 $E_\gamma = 2mc^2$

$= 2 \times 1.673 \times 10^{-27} \times (3.00 \times 10^8)^2$

$= 3.0114 \times 10^{-10}$ J

$= \mathbf{3.01 \times 10^{-10}}$ **J (to 3 s.f.)**

Q2 a) $E_\gamma = mc^2$

$= 1.983 \times 10^{-27} \times (3.00 \times 10^8)^2$

$= 1.7847 \times 10^{-10}$ J

$= \mathbf{1.78 \times 10^{-10}}$ **J (to 3 s.f.)**

b) $E_\gamma = hf$, so:

$f = \frac{E_\gamma}{h} = \frac{1.7847 \times 10^{-10}}{6.63 \times 10^{-34}}$

$= 2.6918... \times 10^{23}$ Hz = $\mathbf{2.69 \times 10^{23}}$ **Hz (to 3 s.f.)**

$E_\gamma = \frac{hc}{\lambda}$, so

$\lambda = \frac{hc}{E} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^8}{1.7847 \times 10^{-10}}$

$= 1.1144... \times 10^{-15}$ m = $\mathbf{1.11 \times 10^{-15}}$ **m (to 3 s.f.)**

Q3 $E_\gamma = 2mc^2$, and $E_\gamma = hf$, so

$hf = 2mc^2$, so:

$f = \frac{2mc^2}{h} = \frac{2 \times 1.88 \times 10^{-28} \times (3.00 \times 10^8)^2}{6.63 \times 10^{-34}}$

$= 5.10407... \times 10^{22}$ Hz = $\mathbf{5.10 \times 10^{22}}$ **Hz (to 3 s.f.)**

Page 423 — Fact Recall Questions

Q1 Hadrons

Q2 Any one from: the weak nuclear force / gravity / the electrostatic force.

Q3 The neutrino has zero charge and (almost) zero mass. It is a lepton.

Q4 a) An antiparticle has the opposite charge to its corresponding particle.

b) An antiparticle has the same mass as its corresponding particle.

Q5 The positron

Q6 a) –1

b) 0

c) 0

Q7 $\Delta E = \Delta mc^2$, where ΔE is the change in energy, Δm is the change in mass, and c is the speed of light.

Q8 Energy can be converted into mass and produce particles, if there is enough energy. The mass is always produced in a particle-antiparticle pair.

Q9 Two photons

4. Quarks and Anti-quarks

Page 426 — Application Questions

Q1 a) Charge $= \frac{2}{3} + \left(-\frac{2}{3}\right) = 0$

b) Charge $= \frac{2}{3} + \frac{2}{3} + \left(-\frac{1}{3}\right) = 1$

c) Charge $= \left(-\frac{1}{3}\right) + \left(-\frac{1}{3}\right) + \left(-\frac{1}{3}\right) = -1$

d) Charge $= \left(-\frac{1}{3}\right) + \left(-\frac{1}{3}\right) + \left(-\frac{1}{3}\right) = -1$

Q2 The particle has a charge of −1, and contains 3 quarks, only one of which is a strange quark. The only two quarks which can be added to a strange quark to make −1 are 2 down quarks. Therefore, $\Sigma^- = dds$.

Q3

d	s	s	→	X	d	s	+	d	$\bar{u}$
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$		X	$\frac{1}{3}$	$\frac{1}{3}$	+	$\frac{1}{3}$	$\frac{2}{3}$

So, $-1 = X + \left(-\frac{2}{3}\right) + (-1) \Rightarrow X - \frac{2}{3} = 0$

$X = +\frac{2}{3}$

The only quark with a charge of $+\frac{2}{3}$ is the up quark. Therefore, X is an up quark.

Page 426 — Fact Recall Questions

Q1 Quarks: up (u), down (d) and strange (s). Anti-quarks: anti-up ($\bar{u}$), anti-down ($\bar{d}$) and anti-strange ($\bar{s}$).

Q2 $\bar{u} = -\frac{2}{3}e$ $\quad \bar{d} = +\frac{1}{3}e$ $\quad \bar{s} = +\frac{1}{3}e$

Q3 proton = uud
neutron = udd

Q4 In beta-minus decay, a down quark decays into an up-quark via the weak interaction, emitting an electron and an antineutrino.

Q5 $u \rightarrow d + {}^{0}_{-1}e + \nu$

Q6 charge

5. Radioactive Decay

Page 430 — Application Question

Q1 a) Since the radiation is blocked by aluminium, but not by paper, it must be beta-minus radiation.

b) The Geiger-Muller tube is still detecting counts from background radiation, so the count rate does not drop to zero (even though no radiation from the source can reach it).

Page 430 — Fact Recall Questions

Q1 When an unstable atomic nucleus breaks down to become more stable, by releasing energy and/or particles.

Q2 a) alpha radiation
b) gamma radiation
c) beta-minus radiation

Q3 Alpha radiation, beta-minus radiation, beta-plus radiation and gamma radiation.

6. Nuclear Decay Equations

Page 432 — Application Questions

Q1 Beta-minus decay involves the conversion of a neutron to a proton when an electron is emitted (along with the release of an antineutrino). So nucleon number A remains constant, and proton number increases by 1:
${}^{137}_{55}Cs \longrightarrow {}^{137}_{56}Ba + {}^{0}_{-1}\beta + {}^{0}_{0}\bar{\nu}$
You can check your answer by making sure that the total charge (proton number) on the left of the equation balances the total charge on the right. Do the same for the nucleon number.

Q2 ${}^{211}_{85}At \longrightarrow {}^{207}_{83}Bi + {}^{4}_{2}\alpha$

Q3 ${}^{25}_{13}Al \longrightarrow {}^{25}_{12}Mg + {}^{0}_{+1}\beta + {}^{0}_{0}\nu$

7. Exponential Law of Decay

Page 436 — Application Questions

Q1 $A = \lambda N = (1.1 \times 10^{-13}) \times (4.5 \times 10^{18})$
$= 4.95 \times 10^{5} =$ **5.0×10^{5} Bq (to 2 s.f.)**

Q2 Time in seconds $= t = 6.5 \times 3600 = 23\,400$ s
Activity at time t:
$A = A_0 e^{-\lambda t} = (3.2 \times 10^{3}) \times e^{-(1.3 \times 10^{-4}) \times 23400}$
$= 152.76... =$ **150 Bq (to 2 s.f.)**

Q3 Time in seconds $= t = 35 \times 60 = 2100$ s
Number of unstable nuclei in sample at time t:
$N = N_0 e^{-\lambda t} = (2.5 \times 10^{15}) \times e^{-(9.87 \times 10^{-3}) \times 2100}$
$= 2.49... \times 10^{6}$
$=$ **2.5×10^{6} (to 2 s.f.)**

Page 436 — Fact Recall Questions

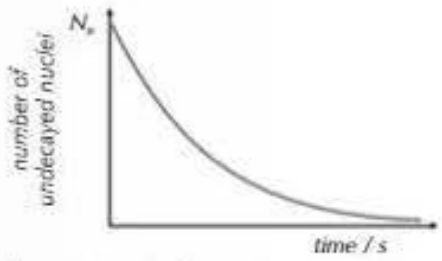
Q1 a) The probability of an atomic nucleus decaying per unit time.

b) The number of atomic nuclei that decay per second.

Q2 a) $\frac{\Delta N}{\Delta t} = -\lambda N$,
where $\frac{\Delta N}{\Delta t}$ is the rate of change of the number of undecayed nuclei, λ is the decay constant and N is the number of undecayed nuclei in sample.
You could also define ΔN and Δt individually as ΔN is the change in number of undecayed nuclei, Δt is the change in time.

b) E.g. spreadsheet modelling

Q3



Q4 Rolling a 6 on a dice is a random event with a constant probability, similar to the event of an undecayed nucleus decaying.

8. Half-life and Radioactive Dating

Page 441 — Application Questions

Q1 Find the number of times the activity halved in 24 hours — count the number of times, n, you have to divide 2400 by 2 to reach 75.
$n = 5$, therefore $t_{1/2} = 24 \div 5 =$ **4.8 hours**

Q2 Decay constant $\lambda = \frac{\ln 2}{t_{1/2}} = \frac{0.693...}{483} = 1.435... \times 10^{-3}$
$=$ **1.44×10^{-3} s^{-1} (to 3 s.f.)**

Q3 Half-life of carbon-14 in seconds:
$t_{1/2} = 5730 \times (365 \times 24 \times 60 \times 60) = 1.8070... \times 10^{11}$ s
Decay constant $\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{1.8070... \times 10^{11}}$
$= 3.8358... \times 10^{-12}$ s^{-1}

Activity when animal died $= A_0 = 1.2$ Bq
Activity now $= A = 0.45$ Bq
Activity decreases exponentially according to: $A = A_0 e^{-\lambda t}$
Rearranging this for time $t = -\frac{1}{\lambda} \times \ln\left(\frac{A}{A_0}\right)$
$= 2.5569... \times 10^{11}$ s $=$ **8100 years (to 2 s.f.)**

Page 441 — Fact Recall Questions

Q1 The time it takes for the number of undecayed nuclei in a sample of an isotope (or the sample's activity or count rate) to halve.

Q2 Read off the activity at $t = 0$. Halve this value, draw a horizontal line to the curve, then a vertical line down to the x-axis. Read off the time — this the half-life. Find the time taken for the activity to fall to a quarter of the original value and divide by two to check.

Q3 E.g. Set up a Geiger-Müller tube, held steady in a clamp stand, and attached to a Geiger counter. Find the background count by recording the total background counts in e.g. 30 s. Repeat this measurement twice, and calculate an average. Divide this average by the number of seconds to find the background count rate. This value should be subtracted from all subsequent measurements of count rate. Place the sample of Nobelium-255 close to the detector of the Geiger-Müller tube, within the range of an alpha-particle to ensure that the radiation reaches the detector. Record the number of counts in 10 seconds, divide this by 10 and subtract the background count rate to find the count rate due to the Nobelium. Repeat this every 30 s for around 5 minutes. Plot your results on a graph of count rate against time, and draw a line of best fit. Find the half-life from your graph by reading off how long it takes for the count rate to drop to half its initial value.

Q4 E.g. carbon-14.

9. Binding Energy

Page 444 — Application Questions

Q1 Mass defect is equivalent to binding energy.
Convert the mass defect from u to kg:
mass defect $= 0.0989 \times 1.661 \times 10^{-27} = 1.6427... \times 10^{-28}$ kg
$\Delta E = \Delta mc^2 = 1.6427... \times 10^{-28} \times (3.00 \times 10^8)^2$
$= 1.4784... \times 10^{-11}$ J
$= ((1.4784... \times 10^{-11} \div 1.60 \times 10^{-19}) \div 10^6)$ MeV
$= 92.4035...$ MeV
$=$ **92.4 MeV (to 3 s.f.)**

Q2 Mass of nucleus = 15.994915 u
Number of protons = 8, number of neutrons = 8
Mass of proton = 1.00728 u, Mass of neutron = 1.00867 u
Total mass of nucleons = (8 × 1.00728 u) + (8 × 1.00867 u)
$= 16.1276$ u
So mass defect = mass of nucleons − mass of nucleus
$= 16.1276$ u $- 15.994915$ u
$=$ **0.132685 u**

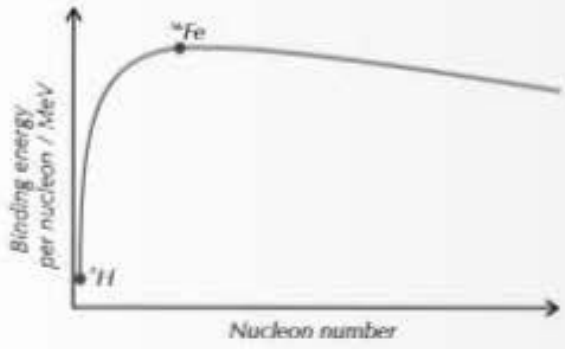
Q3 Binding energy = binding energy per nucleon × nucleon number $= 8.79$ MeV $\times 56 = 492.24$ MeV
Convert from binding energy to mass defect using
$\Delta E = \Delta mc^2$
First, convert binding energy from MeV to J
$\Delta E = (492.24 \times 10^6) \times 1.6 \times 10^{-19} = 7.875... \times 10^{-11}$ kg
mass defect $= \Delta m = \frac{\Delta E}{c^2} = \frac{7.875... \times 10^{-11}}{(3.00 \times 10^8)^2}$
$= 8.750... \times 10^{-28}$ kg
$= (8.750... \times 10^{-28} \div 1.661 \times 10^{-27})$ u
$= 0.52684...$ u
$=$ **0.527 u (to 3 s.f.)**

Q4 a) 8.6 MeV
Just read the value of the binding energy per nucleon at a nucleon number of 100 from the graph.
b) Multiply the binding energy per nucleon by the nucleon number.
Total binding energy $= 8.6 \times 100 =$ **860 MeV**

Page 444 — Fact Recall Questions

Q1 The energy needed to separate all the nucleons in a nucleus. It is equivalent to the mass defect.

Q2



10. Nuclear Fission and Fusion

Page 448 — Application Questions

Q1 a) fusion
b) fission
c) fusion
d) fission
If the nucleon number is less than 56, fusion is more energetically favourable. If it's more than 56, fission is more energetically favourable.

Q2 ${}^{3}_{1}\text{H} + {}^{1}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + {}^{1}_{0}\text{X}$
X has a nucleon number of 1 and a proton number of 0. So particle X is a neutron.

Q3 $\Delta m = m_p + m_p - m_{He} - m_n - m_n$
$= 1.00728 + 1.00728 - 2.01355 - 0.00055 - 0$
$= 0.00046$ u
$= (0.00046 \times 1.661 \times 10^{-27})$ kg
$= 7.6406 \times 10^{-31}$ kg
$\Delta E = \Delta mc^2$
$= 7.6406 \times 10^{-31} \times (3.00 \times 10^8)^2$
$= 6.87654 \times 10^{-14}$ J
In MeV: $\Delta E = ((6.87654 \times 10^{-14}) \div (1.60 \times 10^{-19})) \div 10^6$ MeV
$= 0.4297... =$ **0.430 MeV (to 3 s.f.)**

Q4 First, calculate the number of neutrons produced by balancing the nucleon number A:
$A_{before} = 1 + 235$, $A_{after} = 94 + 139 + x$
Therefore number of neutrons produced $= x = 3$
So $\Delta m = m_n + m_{U\,235} - m_{Sr\,94} - m_{Xe\,139} - 3m_n$
$= 1.00867 + 234.99333 - 93.88431 - 138.90613 - (3 \times 1.00867)$
$= 0.18555$ u
$= (0.18555 \times 1.661 \times 10^{-27})$ kg
$= 3.08198... \times 10^{-28}$ kg
$\Delta E = \Delta mc^2$
$= 3.08198... \times 10^{-28} \times (3.00 \times 10^8)^2$
$= 2.77378... \times 10^{-11}$ J
In MeV: $\Delta E = ((2.77378... \times 10^{-11}) \div (1.60 \times 10^{-19})) \div 10^6$
$= 173.361... =$ **173 MeV (to 3 s.f.)**

Page 448 — Fact Recall Questions

Q1 Nuclear fission is when a larger nucleus splits into two smaller nuclei. It can be induced by firing low energy neutrons at the nucleus. When the nucleus absorbs one of these neutrons, it becomes unstable and will undergo fission.

Q2 The fusing of two smaller nuclei to form one larger nucleus.

Q3 Nuclei are positively charged, so they repel each other via the electrostatic interaction. For fusion to take place, the temperature must be very high so that the nuclei can travel fast enough (and so have enough energy) to overcome this repulsion (so that the strong nuclear interaction can take over and attract them together).

11. Fission Reactors

Page 451 — Fact Recall Questions

Q1 When the neutrons released by nuclear fission cause other nuclei to fission and release more neutrons — and so on.

Q2 The neutrons need to be slowed down in order for uranium nuclei to absorb them, to induce fission.

Q3 The control rods are made of a material that absorbs neutrons (e.g. boron). By inserting them into the reactor, the number of neutrons in the reactor can be limited. The rate of reaction can therefore be controlled by changing how far the control rods are inserted into the reactor.

Q4 The material used for the coolant needs to be a liquid or a gas at room temperature so it can be pumped around the reactor. It also needs to be efficient at transferring heat from the reactor. Water could be used as a coolant.

Q5 Any one of: e.g. facilities to store nuclear waste must be built deep underground, which can permanently change the landscape. / Leaked nuclear waste can contaminate water supplies and damage organisms.

Exam-style Questions — pages 453-455

1 B ***(1 mark)***

2 D ***(1 mark)***
A, B and C cannot occur because charge is not conserved.

3 A ***(1 mark)***
The number of atoms drops to a quarter of its initial value in 10.4 years, so the half-life of cobalt-60 is 5.2 years. Convert this to seconds, and substitute into the equation $\lambda t_{\frac{1}{2}} = \ln(2)$ to find the decay constant.

4 a) Any one from: e.g. both hadrons and leptons feel and interact with the weak nuclear force / both hadrons and leptons can feel and interact with the electrostatic force (if they are charged) ***(1 mark)***.
Any one from: e.g. hadrons can feel and interact with the strong nuclear force, but leptons cannot / leptons are fundamental particles, but hadrons are not (they are made up of quarks) ***(1 mark)***.

b) Total charge of the Δ^{++} hadron $= +2e$.
Charge on an up quark $= +\frac{2}{3}e$
So, $2e = \frac{2}{3}e + \frac{2}{3}e + x$
$x = 2e - (\frac{2}{3}e + \frac{2}{3}e) = +\frac{2}{3}e$
The only quark with a charge of $+\frac{2}{3}e$ is the up quark, so quark x is an up quark ***(1 mark)***.

c) mass $= 2.19 \times 10^{-27}$ kg
charge $= -2e$
(1 mark)

d) $\Delta E = \Delta mc^2$
For minimum energy in pair production,
Δm = mass of particle + mass of antiparticle $= 2m_{electron}$
So, $\Delta E = 2m_{electron}c^2$
$= 2 \times 1.71 \times 10^{-28} \times (3.00 \times 10^8)^2$
$= 3.078 \times 10^{-11}$ J
$= ((3.078 \times 10^{-11}) \div (1.60 \times 10^{-19})) \div 10^6$
$= 192.375$ MeV
$=$ **192 MeV (to 3 s.f.)**
(3 marks for correct answer, otherwise 1 mark for correct method and 1 mark for correct conversion to MeV)

5 a) There are 94 protons.
There are 240 − 94 = 146 neutrons. ***(1 mark)***

b) The strong nuclear force ***(1 mark)***.

c) $^{240}_{94}Pu \rightarrow {}^{236}_{92}U + {}^{4}_{2}\alpha$
(2 marks — 1 mark for U numbers correct and 1 mark for α numbers correct)

d) With nine α decays, the proton number will decrease from 94 by 18 to 76. The proton number of Tl is 81, so you need **five β⁻ decays**.
(2 marks for correct answer, otherwise 1 mark for correct working if answer incorrect)

e) In beta-minus decay, a down quark emits an electron and an antineutrino ***(1 mark)*** and decays into an up quark via the weak interaction ***(1 mark)***.

6 a) Initial activity = 72 Bq.
72 ÷ 2 = 36 Bq
Time when the activity has dropped to 36 Bq = 10 mins.
So, half life = **10 mins *(1 mark)***

b) $\lambda t_{\frac{1}{2}} = \ln(2)$, so $\lambda = \frac{\ln(2)}{t_{\frac{1}{2}}}$
$t_{\frac{1}{2}} = 10 \text{ mins} = 10 \times 60 = 600 \text{ s}$
$\lambda = \frac{\ln(2)}{600} = 0.011552... \text{ s}^{-1}$
Activity at 17 mins = 22 Bq
Rearrange the activity equation, $A = \lambda N$, for number of undecayed nuclei:
$N = \frac{A}{\lambda} = \frac{22}{0.011552...} = 19043.57... =$ **20 000 (to 1 s.f.)**
(3 marks for correct answer, otherwise 1 mark for correct calculation of the decay constant and 1 mark for correct method)
If you got a different answer to part a) and used that answer in this question, you'll still get all the marks, as long as your method is correct.

c) Minimum energy of photon produced by annihilation is given by the mass of one of the annihilated particles converted into energy, $E_p = mc^2$.
The energy of a photon is related to the frequency by $E = hf$. So:
$hf = mc^2$
Rearrange for f:
$f = \frac{mc^2}{h} = \frac{9.11 \times 10^{-31} \times (3.00 \times 10^8)^2}{6.63 \times 10^{-34}}$
$= 1.23665... \times 10^{20}$ Hz
$=$ **1.24×10^{20} Hz (to 3 s.f.)**
(3 marks for correct answer, otherwise 1 mark for correctly equating the two equations and 1 mark for correct substitution)

7 a) $R = r_0 A^{1/3}$
$= 1.4 \times 10^{-15} \times 33^{1/3}$
$= 4.4905... \times 10^{-15}$ m
$=$ **4.5×10^{-15} m (to 2 s.f.)**
(2 marks for correct answer, otherwise 1 mark for correct working if answer incorrect)

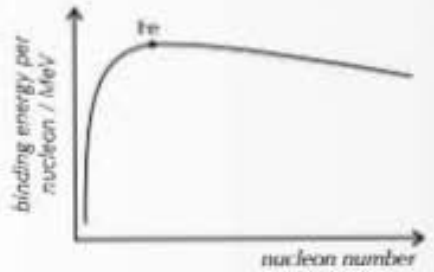
b) $\rho = \frac{m}{V}$
Volume of a sphere, $V = \frac{4}{3}\pi r^3$, so:
$\rho = \frac{3m}{4\pi R^3}$, where R is the radius of the nucleus.
Convert mass to kg, $m = (32.97 \times 1.661 \times 10^{-27})$ kg
$= 5.476... \times 10^{-26}$ kg
$\rho = \frac{3 \times 5.476... \times 10^{-26}}{4\pi \times (4.4905... \times 10^{-15})^3}$
$= 1.44378... \times 10^{17}$ kg m^{-3}
$=$ **1.4×10^{17} kg m^{-3} (to 2 s.f.)**
(3 marks for correct answer, otherwise 1 mark for correct conversion of mass and 1 mark for correct working)

c) $^{32}_{15}P \longrightarrow {}^{32}_{16}S + {}^{0}_{-1}\beta + {}^{0}_{0}\bar{\nu}$
(2 marks — 1 mark for correctly labelled sulfur isotope, 1 mark for correctly labelled beta-minus particle and antineutrino.)

d) Half-life in s = 25.4 × 24 × 3600 = 2 194 560 s
$\lambda t_{\frac{1}{2}} = \ln(2)$
So rearrange:
$\lambda = \ln(2) \div t_{\frac{1}{2}} = \ln(2) \div 2\,194\,560$
$= 3.158... \times 10^{-7} =$ **3.16×10^{-7} s^{-1} (to 3 s.f.)**
(2 marks for the correct answer, otherwise 1 mark for correct working if answer incorrect)

e) Number of undecayed atoms given by: $N = N_0 e^{-\lambda t}$
Rearrange for time: $t = -\ln\left(\frac{N}{N_0}\right) \times \frac{1}{\lambda}$
So $t = -\ln\left(\frac{7.0 \times 10^{11}}{1.6 \times 10^{12}}\right) \times \frac{1}{3.158... \times 10^{-7}}$
$= 9.907... \times 10^6$ s
$= 114.67...$ days
$=$ **110 days (to 2 s.f.)**
(2 marks for the correct answer, otherwise 1 mark for correct working if answer incorrect)

8 a) The energy needed to pull all the nucleons in a nucleus apart/the energy released when a nucleus forms ***(1 mark)***.

b)


(2 marks — 1 mark for the overall shape, 1 mark for indicating Fe is at the peak of the curve)

c) Binding energy is equivalent to the mass defect.
$\Delta E = \Delta m c^2$
So binding energy per nucleon = $\frac{\Delta m c^2}{\text{nucleon number}}$
Convert mass defect to kg:
0.62065 u = (0.62065 × 1.661 × 10^{-27}) kg
$= 1.0308... \times 10^{-27}$ kg
Binding energy per nucleon
$= \frac{1.0308... \times 10^{-27} \times (3.00 \times 10^8)^2}{56}$
$= 1.4057... \times 10^{-12}$ J
$= ((1.4057... \times 10^{-12}) \div (1.60 \times 10^{-19})) \div 10^6$
$= 8.7860...$ MeV = **8.79 MeV (to 3 s.f.)**
(2 marks for correct answer, otherwise 1 mark for correct method if answer incorrect)

d) Use conservation of charge to find proton number of $^{a}_{b}Sr$:
$a = 92 - 54 =$ **38**
Use conservation of nucleon number to find number of neutrons: $b = (235 + 1) - (140 + 94) =$ **2**
(2 marks — 1 mark for correctly calculating each of a and b.)

e) The total binding energy of the final nuclei is greater than the binding energy of the initial nucleus ***(1 mark)***. An increase in binding energy means an increase in the total mass defect, so some of the mass from the initial nucleus is converted into energy ***(1 mark)***.

f) $\Delta m = (234.99333 + 1.00867) - (139.89194 + 93.89446 + (2 \times 1.00867))$
$= 0.19826$ u
$= (0.19826 \times 1.661 \times 10^{-27})$ kg $= 3.293... \times 10^{-28}$ kg
$\Delta E = \Delta m c^2 = 3.293... \times 10^{-28} \times (3.00 \times 10^8)^2$
$= 2.9637... \times 10^{-11}$ J
$= ((2.9637... \times 10^{-11}) \div (1.60 \times 10^{-19})) \div 10^6$ MeV
$= 185.23... =$ **185 MeV (to 3 s.f.)**
(3 marks for the correct answer, otherwise 1 mark for correctly calculating the mass defect and 1 mark for attempting to convert it to binding energy.)
If you used your answer from 8d), but got 8d) wrong, you'd still get all the marks for this question if your method is correct.

Section 5 — Medical Imaging

1. X-ray Imaging

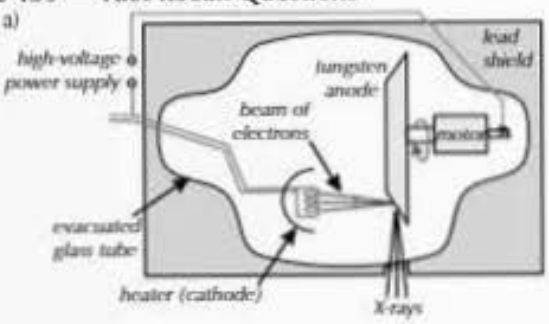
Page 459 — Application Questions

Q1 $E_{max} = e \times V = 1.60 \times 10^{-19} \times 85.0 \times 10^{3} =$ **1.36×10^{-14} J**
Assume here that all of the kinetic energy of the electrons is converted to X-rays.

Q2 $I = I_0 e^{-\mu x} = 30.0 \times e^{-0.5 \times 0.439} = 5.345...$ **5.3 Wm^{-2} (to 2 s.f.)**

Page 459 — Fact Recall Questions

Q1 a)



b) A lot of the electrons' energy is converted to heat, so rotating the anode helps spread the heat and prevents it from overheating.

Q2 Simple scattering, the photoelectric effect, Compton effect, pair production.

Q3 A barium meal contains atoms with a high atomic number and so it shows up clearly in X-ray images compared to soft tissue, which is made up of material with lower atomic numbers. A patient can be given a barium meal to swallow. This helps improve the contrast between tissues from the digestive tract and surrounding tissues on an X-ray image.

Q4 A computed axial tomography (CAT) scan is a scan in which an X-ray tube is rotated around the body and the emitted X-ray beam in the shape of a fan is picked up by detectors. These feed a signal to a computer, which works out how much attenuation has been caused by each part of the body and produces a high quality image.

Q5 Any two from: e.g. CAT scans can be processed by a computer to produce more detailed images than regular X-rays / CAT scans are better at imaging soft tissues / data from CAT scanners can be manipulated to generate a 3D image.

2. Medical Uses of Nuclear Radiation

Page 462 — Fact Recall Questions

Q1 The tracer is bound to a compound that is naturally used by the tissue or organ that is being investigated. The tracer is injected into or swallowed by the patient and moves through the body to the region of interest. The radiation emitted due to the isotope in the tracer is then detected by a gamma camera/PET scanner and sent to a computer to produce an image.

Q2 E.g. Technetium-99m. The (gamma) radiation it emits can easily pass outside the body and be detected. It has a half-life of 6 hours, which is long enough for data to be recorded, but short enough to limit the patient's exposure to radiation to an acceptable level. It also decays to a much more stable isotope.

Q3 PET scans.

Q4 Lead shielding — prevents radiation from other sources being detected by the camera.
Lead collimator — only lets gamma rays travelling parallel to the holes in the collimator pass through.
Scintillator — A sodium iodide crystal that produces a flash of light when hit by a gamma ray.
Photomultiplier tubes — detect the flashes of light and turn them into electrical signals.
Electronic circuit — collects the electrical signals and sends the information to a computer which forms an image.

Q5 The patient is injected with glucose that is bound to a radiotracer. The radiotracer emits positrons that annihilate with electrons in the body, producing gamma photons which are detected outside the body. Tissues in the body with a high rate of metabolic activity have a high rate of uptake of glucose, so these areas will have accumulated more radiotracer than areas of low metabolic activity. This means the PET scanner will detect more gamma radiation in areas of high metabolic activity. This information can be sent to a computer to build up an image of metabolic activity in the body.

3. Medical Uses of Ultrasound

Page 467 — Application Questions

Q1 a) $Z = \rho c \Rightarrow c = \frac{Z}{\rho} = \frac{1.5 \times 10^6}{1000} = 1500\ \text{ms}^{-1}$

b) $\frac{I_r}{I_0} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2} = \frac{((1.5 \times 10^6) - (8.0 \times 10^6))^2}{((1.5 \times 10^6) + (8.0 \times 10^6))^2}$

$= 0.4681...$

So the percentage $= 0.4681... \times 100 =$ **47% (to 2 s.f.)**

Q2 $\frac{\Delta f}{f} = \frac{2v\cos\theta}{c} \Rightarrow \Delta f = \frac{2vf\cos\theta}{c}$

$\Delta f = \frac{2 \times 0.31 \times 12 \times 10^6 \times \cos 34^\circ}{1550} = 3979.38...$

$=$ **4.0 kHz (to 2 s.f.)**

Page 467 — Fact Recall Questions

Q1 Ultrasound is a longitudinal wave with a frequency higher than humans can hear (> 20 000 Hz).

Q2 $Z = \rho c$, where Z is the acoustic impedance in kgm^{-2}s^{-1}, ρ is the density of the material in kgm^{-3} and c is the speed of sound in the medium in ms^{-1}.

Q3 The piezoelectric effect is when a material produces a potential difference when it's deformed, or vice versa.

Q4 Ultrasound waves are generated and detected by an ultrasound transducer. Inside the transducer, an alternating potential difference is applied to piezoelectric crystals, which vibrate to create a sound wave. When the wave is reflected and returns, it causes the crystals to vibrate, which in turn creates an alternating potential difference that is detected in an adjoining circuit.

Q5 Ultrasound imaging doesn't work if there are any air gaps between the transducer and body tissue because the difference in acoustic impedance between the air and the soft tissue would cause the majority of an ultrasound signal to be reflected back. Because of this, a coupling medium with similar acoustic impedance to the skin is needed between the transducer and the body, because this allows ultrasound to be transmitted into the body.

Q6 a) Short pulses of ultrasound are sent into the body while an electron beam inside a cathode ray oscilloscope sweeps across its screen. Reflected pulses are detected and show up as vertical deflections on the oscilloscope screen. The difference in time between transmission and detection of the reflected pulse allows a computer to calculate the depth of an object. E.g. A-scans can be used to measure the depth of an eyeball.

b) In a B-scan the amplitude of reflected pulses is shown as the brightness of spots on a screen. A linear array of transducers allows a 2D image to be formed of inside the body. E.g. B-scans can be used to form a 2D image of a fetus.

Exam-style Questions — pages 469-470

1 D ***(1 mark)***

$I_r \div I_0 = (Z_2 - Z_1)^2 \div (Z_2 + Z_1)^2$

$= ((1.70 \times 10^6) - (1.58 \times 10^6))^2$

$\div ((1.70 \times 10^6) + (1.58 \times 10^6))^2$

$= 0.001358... = 0.00134$ (to 3 s.f.)

2 D ***(1 mark)***

Taking the natural logarithm of $(I \div I_0) = e^{-\mu x}$: $\ln(I \div I_0) = -\mu x$

Rearrange to: $\mu = (-\ln(I \div I_0)) \div x$

If the intensity is reduced by 95%, then the intensity of the attenuated beam (I) is $0.05\ I_0$. So $I \div I_0 = 0.05$.

$\mu = (-\ln(0.05)) \div 4.5 = 0.6657... = 0.67\ \text{cm}^{-1}$ (to 2 s.f.)

3 B ***(1 mark)***

PET scanning is the only imaging technique listed which can make use of a medical tracer bound to glucose.

4 a) The gamma rays travel through a lead collimator so that only rays parallel to holes in the lead can get through ***(1 mark)***. The gamma rays hit the scintillator, which emits a flash of light whenever a gamma ray hits it ***(1 mark)***. Photomultiplier tubes detect the flashes of light from the scintillator and turn them into pulses of electricity ***(1 mark)***.

b) How to grade your answer (pick the description that best matches your answer):

0 marks: There is no relevant information.

1-2 marks: *I* points 3 and 4 have been covered. One *S* point has been mentioned. Answer is basic and has lack of structure, with information missing.

3-4 marks: *I* points 1, 3 and 4 have been mentioned. Two *S* points have been mentioned. Answer has some structure, and information is generally relevant.

5-6 marks: All *I* and *S* points have been covered. The answer is well structured, and information is relevant and presented clearly.

Here are some points your answer may include:

Identifying areas of infection (*I*)

1. White blood cells tagged with technetium-99m are injected into the patient.
2. The white blood cells move through the body and accumulate in areas of infection.
3. Gamma radiation is emitted by the technetium-99m attached to the white blood cells.
4. A gamma camera can be used to detect radiation emitted from technetium-99m.
5. Image is formed by mapping the detected radiation, where high concentrations of radiation correspond to areas of infection.

Suitability of technetium-99m (*S*)

1. Technetium-99m emits gamma radiation which readily passes through the body to be detected.
2. The half-life of technetium-99m is long enough for data to be recorded, but short enough to limit the patient's exposure to radiation to an acceptable level.
3. Technetium-99m decays to a much more stable isotope, which limits the patient's exposure to radiation.

5 a) $E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E}$

The X-rays with the smallest wavelength will be produced by the electrons with the highest energy. The maximum energy of the X-ray photons is equal to the potential difference across the X-ray tube multiplied by the charge of an electron, so:

$$\lambda = \frac{(6.63 \times 10^{-34}) \times (3.00 \times 10^{8})}{(31 \times 10^{3} \times 1.60 \times 10^{-19})} = 4.010... \times 10^{-11}\ \text{m}$$
$$= \mathbf{4.0 \times 10^{-11}\ m\ (to\ 2\ s.f.)}$$

(3 marks total, or 1 mark for saying that minimum wavelength X-rays will be produced by maximum energy electrons and 1 mark for correct working)

b) Any one from: the photoelectric effect, the Compton effect, pair production ***(1 mark)***.

c) Iodine has a high atomic number, so it shows up clearly in X-ray images compared to body tissues ***(1 mark)***. Soft tissues have similar attenuation coefficients, so are difficult to distinguish in X-ray images. By injecting iodine into the blood, the contrast between the blood vessels and the surrounding tissue is improved ***(1 mark)***.

6 a) In an A-scan, a short pulse of ultrasound is sent into the eyeball ***(1 mark)***. Part of the pulse is reflected at the boundary at the back of the eye and is detected by an ultrasound transducer in front of the eye ***(1 mark)***. The time it takes for this echo to come back can be used to calculate the depth of the eyeball ***(1 mark)***.

b) $Z = \rho c = 1.23 \times 340 = 418.2 = \mathbf{420\ kgm^{-2}s^{-1}}$ **(to 2 s.f.)**
(1 mark)

c) The lungs contain large volumes of air. The acoustic impedance of human tissue is much higher than the acoustic impedance of air ***(1 mark)***. This means most of the ultrasound's energy will be reflected back from the boundary between the lungs and the surrounding tissue because there is a large impedance mismatch, so very little of the ultrasound's energy will be transmitted past the lungs to produce an image ***(1 mark)***.

d) Rearranging $\frac{\Delta f}{f} = \frac{2v\cos\theta}{c}$ into $y = mx + c$ format:

$$\Delta f = \left(\frac{2vf}{c}\right)\cos\theta$$

So the gradient of the graph is equal to $\frac{2vf}{c}$ ***(1 mark)***.

e) $\text{gradient} = \frac{2vf}{c} \Rightarrow v = \frac{\text{gradient} \times c}{2f}$

$$\text{gradient} = \frac{\text{change in y}}{\text{change in x}} = \frac{(2.2 \times 10^{3} - 0)}{(0.95 - 0)} = 2.31... \times 10^{3}$$

$$\text{So } v = \frac{(2.31... \times 10^{3}) \times 1550}{2 \times (8.1 \times 10^{6})} = 0.221...$$
$$= \mathbf{0.22\ ms^{-1}}\ \textbf{(to 2 s.f.)}$$

(3 marks for correct answer, or 1 mark for calculating the gradient of the graph and 1 mark for correct working)

You can calculate the gradient between any two points on the graph, but it's best to choose two that are far apart.

Glossary

A

Absolute scale of temperature
A temperature scale that does not depend on the properties of any particular substance. The unit of this scale is the kelvin (K).

Absolute uncertainty
The total uncertainty of a measurement.

Absolute zero
The lowest possible temperature, 0 K (or –273 °C), that a substance can have.

Absorption line spectrum
A spectrum produced from light passed through a substance, which contains dark lines corresponding to different wavelengths of light that have been absorbed by the substance.

Acceleration
The rate of change of velocity.

Accurate result
A result that is really close to the true answer.

Acoustic impedance
The density of a medium multiplied by the speed of sound in that medium.

Activity
The number of unstable nuclei in a radioactive sample that decay per second.

Air bag
Safety devices used in vehicles that inflate quickly on collision to 'cushion' passengers.

Alpha particle
A particle formed of two protons and two neutrons (the same as a helium nucleus).

Alpha (α) radiation
Nuclear radiation made up of alpha particles.

Ammeter
A component used to measure the current flowing through a circuit.

Amplitude
The maximum magnitude of the displacement of a wave, i.e. the distance from the undisturbed position to a crest or trough.

Angle of incidence
The angle that incoming light makes with the normal of a boundary.

Angle of refraction
The angle that refracted light makes with the normal of a boundary.

Angular frequency
The equivalent of angular velocity for an object moving with simple harmonic motion.

Angular velocity
The angle a rotating object moves through per unit time.

Annihilation
When a particle and its antiparticle meet and their mass gets converted to energy in the form of a pair of photons.

Anode
A positively-charged electrode.

Anomalous result
A result that doesn't fit in with the pattern of the other results in a set of data.

Antinode
A point of maximum amplitude on a stationary wave.

Antiparticle
A particle with the same mass as its corresponding particle, but equal and opposite charge.

Archimedes' principle
When an object is fully or partially immersed in a fluid, it experiences an upthrust equal to the weight of the fluid it has displaced.

Asteroid
A chunk of rock and mineral that orbits a star.

Astronomical unit
A unit of distance defined as the mean distance between the Earth and the Sun.

Attenuation (absorption) coefficient
A measure of the fraction of an X-ray beam that is attenuated as it passes through a material.

Average speed
The total distance covered by an object divided by the total time elapsed.

Avogadro's constant
The number of particles contained in one mole of a substance. It is equal to 6.02×10^{23} mol^{-1}.

B

Background radiation
The weak level of nuclear radiation found everywhere.

Base unit
A fundamental unit that cannot be derived from other units.

Beta-minus (β^-) radiation
Nuclear radiation made up of electrons.

Beta-plus (β^+) radiation
Nuclear radiation made up of positrons.

Big Bang theory
The idea that the universe began from a small, very hot and dense region of space, which exploded and has been expanding ever since.

Binding energy
The energy released when a nucleus forms, or the energy required to separate all the nucleons in that nucleus. Equivalent to the mass defect of the nucleus.

Black hole
An object whose escape velocity is greater than the speed of light.

Blue shift
The shift in wavelength and frequency of an EM wave towards (or beyond) the blue end of the electromagnetic spectrum due to the source moving towards the observer.

Boyle's law
At a constant temperature the pressure p and volume V of an ideal gas are inversely proportional.

Braking distance
The distance travelled by a vehicle after the brakes are applied until it comes to a complete stop.

Breaking stress
The stress experienced by a material at the point that it breaks.

Brittle
A brittle material doesn't deform plastically, but snaps when the stress on it reaches a certain point.

Brownian motion
The zigzag, random motion of small particles suspended in a fluid.

C

Calibration
Checking a scale on a measuring instrument by measuring a known value.

Capacitance
The amount of charge an object is able to store per unit potential difference (p.d.) across it.

Capacitor
An electrical component that can store charge, usually made up of two conducting plates separated by a dielectric.

Cathode
A negatively-charged electrode.

Centre of mass (centre of gravity)
The point which you can consider all of an object's weight to act through.

Centripetal acceleration
The acceleration of an object moving with circular motion. It's directed towards the centre of the circle.

Centripetal force
The force on an object moving with circular motion. It's directed towards the centre of the circle, and is responsible for the object's curved path.

Chain reaction (nuclear)
When the neutrons released by a nuclear fission cause other nuclei to fission and release more neutrons — and so on.

Chandrasekhar limit
The maximum mass of a star core for which the electron degeneracy pressure can counteract the gravitational force.

Charge carrier
An electrically charged particle that is free to move within a material.

Citation
A citation is included in the main text of a report to show that a particular bit of information was taken from a source.

Coherent
Sources (or waves) that have the same wavelength and frequency and a fixed phase difference between them are coherent.

Comet
A ball of ice, dust and rock which orbits a star in a highly elliptical orbit.

Compressive deformation
Deformation of an object caused by balanced inwards forces, which leads to compression (squashing).

Compressive force
A force which squashes something.

Constructive interference
When two waves interfere to make a wave with a larger displacement.

Contrast medium
Substance with a high atomic number that shows up clearly in X-ray images and can be followed as it moves through a patient's body
e.g. a barium meal or iodine.

Control rod
A rod inserted into a nuclear reactor to control the rate of fission by absorbing neutrons.

Control variable
A variable that is kept constant in an experiment.

Conventional current
Standard current that is said to flow from a positive terminal to a negative terminal.

Correlation
A relationship between two variables.

Cosmic microwave background radiation (CMBR)
A continuous spectrum of microwave radiation, corresponding to a temperature of 2.7 K, that exists throughout the universe.

Cosmological principle
On a large scale, the universe is homogeneous and isotropic, and the laws of physics are the same everywhere.

Coulomb
A unit of charge. One coulomb (C) is the amount of charge that passes in 1 second when the current is 1 ampere.

Couple
A pair of forces of equal size which act parallel to each other but in opposite directions.

Coupling medium
A substance applied during ultrasound scans to avoid air gaps between the ultrasound transducer and the body.

Critical angle
The angle of incidence at which the angle of refraction is 90°.

Critical damping
Damping such that the amplitude of an oscillation is reduced in the shortest possible time.

Critical mass
The amount of fuel needed for a fission chain reaction to continue at a steady rate on its own.

Crumple zone
Part of a car or other vehicle designed to crumple on impact.

Current
The rate of flow of charge in a circuit. Measured in amperes (A).

D

Damping
A force which causes an oscillating object to lose energy and so causes the amplitude of the object's oscillation to decrease.

Dark energy
A type of energy that fills the whole of space, and might explain the accelerating expansion of the universe.

Dark matter
A source of mass in the universe that cannot be seen, but makes up a large amount of the known of the universe.

Decay constant
The probability of an unstable nucleus decaying per unit time. A measure of how quickly an isotope will decay.

Density
The mass per unit volume of a material or object.

Dependent variable
The variable that you measure in an experiment.

Destructive interference
When two waves interfere to make a wave with a reduced displacement.

Diffraction
When waves spread out as they pass through a narrow gap or go round obstacles.

Diffraction grating
A slide or other thin object that contains lots of equally spaced slits very close together, used to show diffraction patterns of waves.

Diode
A component designed to allow current flow in one direction only.

Directly proportional
Two variables that are directly proportional will produce a straight line that goes through the origin when plotted against each other.

Displacement
How far an object has travelled from its starting point in a given direction.

Displacement (waves)
The distance a point on a wave has moved from its undisturbed position.

Displacement-time graph
A graph showing how the displacement of an object changes over a period of time.

Doppler effect
The change in the frequency and wavelength of a wave emitted (or reflected) from a source moving towards or away from an observer.

Drag
Friction caused by a fluid (gas or liquid).

Driving frequency
The frequency of a periodic external driving force which causes an object to oscillate.

Ductile
A material that can be drawn into wires or other shapes without losing its strength.

Dwarf planet
A planet-like object that orbits a star, but does not meet all the conditions to be considered a planet.

E

Efficiency
The ratio of useful energy given out by a mechanical system to the amount of energy put into the system.

Elastic collision
A collision in which momentum and kinetic energy are conserved.

Elastic deformation
If a deformation is elastic, the material returns to its original shape once the forces are removed.

Elastic limit
The force (or stress) beyond which a material will be permanently stretched.

Elastic potential energy
The energy stored in a stretched material.

Electric field strength
The force per unit positive charge experienced by a body in an electric field.

Electric potential
The work done in bringing a unit positive charge from a point infinitely far away to a specific point in the electric field. / The electric potential energy that a unit charge would have at a specific point.

Electric potential energy
The energy stored by a charge due to its position in an electric field.

Electromagnetic induction
The induction of an electromotive force (e.m.f.) across the ends of a conductor due to a changing external magnetic field.

Electromagnetic spectrum
A continuous spectrum of all the possible frequencies of electromagnetic radiation.

Electromotive force (e.m.f.)
The amount of electrical energy a power supply transfers to each coulomb of charge.

Electron degeneracy pressure
The pressure that stops electrons being forced into the atomic nucleus, e.g. in the dense core of a star.

Elementary charge, *e*
The smallest unit of charge, equal to 1.60×10^{-19} C.

Emission line spectrum
A spectrum of bright lines on a dark background corresponding to different wavelengths of light that have been emitted from a light source.

Equation of state of an ideal gas
A combination of the three gas laws, given by $pV = nRT$ or $pV = NkT$.

Equilibrium
An object is in equilibrium if all the forces acting on it cancel each other out.

Escape velocity
The velocity that an object would need to travel at to have just enough kinetic energy to escape a gravitational field.

Event horizon
The boundary of the region around an object inside which the escape velocity is greater than c.

Exponential relationship
A relationship in which two variables are related by the exponential function, e.g. $y = Ae^{kx}$.

F

Faraday's law
The induced e.m.f. is directly proportional to the rate of change of flux linkage.

Field lines
A way of representing a force field.

Force constant
The force needed to extend an object per unit extension. The units are Nm^{-1}. Each object has its own force constant.

Force field
A region where an object will experience a non-contact force.

Forced vibration
The oscillation of an object due to an external driving force.

Free-body diagram
A diagram showing all the forces acting on an object, but none of the forces the object exerts on its surroundings.

Free fall
The motion of an object undergoing an acceleration of $\boldsymbol{g}$.

Free vibration
The oscillation of an object with no transfer of energy to or from the surroundings.

Frequency (of a wave)
The number of whole wave cycles (oscillations) per second passing a given point.

Frequency (of circular motion)
The number of complete revolutions or cycles that a rotating or oscillating object makes per second.

Friction
A force that opposes motion. It acts in the opposite direction to the motion. It arises when two objects are moving past each other, or an object is moving through a fluid.

Fundamental mode of vibration
The lowest frequency at which a stationary wave is formed.

Fundamental particle
A particle which cannot be split up into smaller particles.

G

Gamma (γ) radiation
Nuclear radiation made up of high-frequency electromagnetic waves/photons (known as gamma rays).

Generator
A device that converts kinetic energy into electrical energy by rotating a coil in a magnetic field.

Geostationary satellite
A satellite that orbits directly over the equator and is always above the same point on Earth. Its orbit takes exactly one day.

Gravitational field
A force field generated by any object with mass which causes any other object with mass within the field to experience an attractive force.

Gravitational field line
Arrows showing the direction of the force that masses would feel in a gravitational field.

Gravitational field strength
The force per unit mass, $\boldsymbol{g}$, experienced by a body in a gravitational field.

Gravitational potential
The gravitational potential at a point is the work done in moving a unit mass from infinity to that point.

Gravitational potential energy
The energy an object gains when lifted up in a gravitational field, due to its position.

H

Hadron
A particle made up of quarks that is subject to the strong nuclear force.

Half-life
The average time it takes for the number of undecayed nuclei (or the activity or count rate) in a sample of a radioactive isotope to halve.

Hertzsprung-Russell (HR) diagram
A graph showing a luminosity-temperature plot of many stars. It shows distinct areas corresponding to main sequence stars, red giant stars, super red giant stars and white dwarf stars.

Homogeneity of units
When both sides of an equation have the same units.

Homogenous
Every part of something is the same as every other part.

Hooke's law
The extension of a stretched object is proportional to the load or force applied to it. This applies up to the limit of proportionality.

Hooke's law limit
See limit of proportionality.

Hubble's law
The recessional velocity of a distant object in space is proportional to its distance away from an observer, $v \approx H_0 d$.

Hypothesis
A specific testable statement, based on a theory, about what will happen in a test situation.

I

Ideal gas
A (theoretical) gas that obeys the three gas laws at all temperatures.

Impulse
The impulse of a force acting on an object is the product of average force and time.

Independent variable
The variable you change in an experiment.

Inelastic collision
A collision in which momentum is conserved but kinetic energy is not.

Instantaneous speed
The speed of an object at a given point in time.

Insulator
A substance which does not conduct electricity. Perfect insulators contain no charge carriers.

Intensity (of a wave)
The rate of flow of energy per unit area at right angles to the direction of travel of a wave.

Interference
The superposition of two or more waves.

Internal energy
The sum of the random distribution of kinetic and potential energies associated with the molecules of a system.

Internal resistance
The resistance created in a power source when electrons collide with atoms inside the power source and lose energy.

Inverse square law
Any law in which a physical quantity is inversely proportional to the square of the distance.

Ionisation
Ionisation occurs when electromagnetic waves transfer enough energy to an atomic electron to remove the electron from the atom.

Ionising radiation
Radiation which, when it hits an atom, can cause the atom to lose an electron.

Isotope
One of two or more forms of an element with the same number of protons but a different number of neutrons.

Isotropic
Everything looks the same in every direction.

***I-V* characteristic**
A graph which shows how the current (I) flowing through a component changes as the potential difference (V) across it is increased.

J

Joule
One joule is the work done when a force of 1 newton moves an object through a distance of 1 metre.

K

Kepler's first law
Each planet moves in an ellipse around the Sun, with the Sun at one focus.

Kepler's second law
A line joining the Sun to a planet will sweep out equal areas in equal times.

Kepler's third law
The period of a planet's orbit and the mean distance between the Sun and the planet are related by $T^2 \propto r^3$.

Kinetic energy
The energy possessed by a moving object due to its movement.

Kinetic model of matter
The idea that solids, liquids and gases are made up of tiny moving or vibrating particles.

Kinetic theory
See kinetic model of matter.

Kirchhoff's first law
The total current entering a junction in a circuit must equal the total current leaving the junction.

Kirchhoff's second law
The total e.m.f. around a series circuit is equal to the sum of the potential differences across each component in the circuit.

L

Laser
A source of coherent, monochromatic light.

Lenz's law
The induced e.m.f. is always in such a direction as to oppose the change that caused it.

Lepton
A fundamental particle that is not subject to the strong nuclear force.

Lever
A structure made of a rigid object rotating around a pivot, in which an effort force works against a load force.

Light-dependent resistor (LDR)
A resistor with a resistance that depends on the intensity of light falling on it. The resistance decreases with increasing light intensity.

Light year
The distance that electromagnetic waves travel through a vacuum in one year.

Limit of proportionality
The point beyond which force is no longer proportional to extension.

Linear momentum
The linear momentum of an object is the product of its mass and velocity.

Longitudinal wave
A wave in which the oscillations of the particles/fields (i.e. the displacement) is in the direction of energy propagation.

Lost volts
The energy wasted per coulomb overcoming the internal resistance of a power source.

Luminosity
The total amount of energy emitted by an object in the form of electromagnetic radiation each second. Also known as power output.

M

Magnetic field
Force field in which a force is exerted on magnetic materials.

Magnetic flux
The total magnetic flux, ϕ, passing through an area, A, perpendicular to a magnetic field, $\boldsymbol{B}$, is equal to $\boldsymbol{B}A$.

Magnetic flux density
The force on one metre of wire carrying a current of one amp at right angles to the magnetic field.

Magnetic flux linkage
The product of the magnetic flux passing through a coil and the number of turns on the coil cutting the flux.

Main sequence
A phase of a star's evolution in which the star is fusing hydrogen in its core.

Margin of error
The maximum difference between a recorded data value and the true value.

Mass
The amount of matter in an object.

Mass defect
The difference between the mass of a nucleus and the sum of the individual masses of the nucleons. Equivalent to the binding energy of the nucleus.

Maxima
Constructive interference that occurs at any point where the path difference is a whole number of wavelengths.

Mean
The average of the values collected in repeated measurements, obtained by adding all the values together and dividing by the total number of values.

Mean drift velocity
The average velocity of the net movement of charge carriers towards a terminal of the power supply.

Mean square speed
Represents the mean of the squared speeds of all the particles. It has units of m^2s^{-2}.

Medical tracer
A radioactive substance that is used in medicine for imaging inside the body.

Minima
Destructive interference that occurs at points where the path difference is half a wavelength, one and a half wavelengths, two and a half wavelengths, etc.

Model
A simplified picture of what's physically going on.

Moderator
A material (often water) in a nuclear reactor that slows down neutrons so they can be captured by uranium nuclei (or other fissionable nuclei).

Mole
An amount of substance containing N_A particles, all of which are identical. N_A is the Avogadro constant.

Moment
The turning effect of a force around a turning point.

Momentum
See linear momentum.

Monochromatic
A light source that is all of the same wavelength (or frequency).

N

Natural frequency
The frequency of an object oscillating freely.

Net force
A single force that is equivalent to all the other forces acting on an object added together.

Neutrino
A lepton with zero charge and (almost) zero mass.

Neutron
A neutral hadron with a relative mass of 1, and quark composition udd.

Neutron star
A star mainly made up of neutrons, formed by the collapse of a red giant with a high core mass.

Newton's 1st law of motion
The velocity of an object will not change unless a resultant force acts on it.

Newton's 2nd law of motion
The rate of change of momentum of an object is equal to the net force which acts on the object.

Newton's 3rd law of motion
If object A exerts a force on object B, then object B exerts an equal but opposite force on object A.

Newton's law of gravitation
The force acting between two point masses is proportional to the product of their masses and inversely proportional to the square of the distance between their centres of mass, or $F = -\frac{GMm}{r^2}$.

Node
A point of zero amplitude on a stationary wave.

Non-ohmic conductor
A conductor that doesn't obey Ohm's law — under constant physical conditions, the resistance of the conductor changes as the current through it changes.

Normal contact force
The perpendicular force that a surface exerts on an object in contact with that surface. The force is equal and opposite to the force applied to the surface by the object.

Nuclear fission
The spontaneous or induced splitting of a larger nucleus into two smaller nuclei.

Nuclear fusion
The fusing of two smaller nuclei to form one larger nucleus.

Nuclear model of the atom
A model of the structure of the atom that states that the atom consists of a small, positively charged nucleus containing protons and neutrons, orbited by electrons. The atom is mostly empty space, and the nucleus contains most of the mass of the atom.

Nuclear radiation
Particles or energy released by an unstable nucleus as it decays, e.g. alpha, beta-minus or beta-plus particles, or gamma rays.

Nucleon
A particle in the nucleus of an atom (which can be a proton or a neutron).

Nucleon number
The number of nucleons in the nucleus of an atom.

O

Ohm
A unit of resistance. A component has a resistance of 1 Ω if a potential difference of 1 V makes a current of 1 A flow through it.

Ohmic conductor
A conductor that obeys Ohm's law — under constant physical conditions, the resistance of an ohmic conductor is constant for any current passing through it.

One-to-one interaction
Where a particle interacts with a single other particle.

Optical density
The property of a medium that describes how fast light travels through it. The more optically dense a material is, the more light slows down when it enters it. The optical density of a material is measured by its refractive index.

Orbital period
The time taken for a satellite to complete a full orbit.

Orbital speed
The speed at which a satellite travels.

Oscilloscope
Displays waves from a signal generator as a function of voltage over time.

Overdamping
Heavy damping such that the system takes longer to return to equilibrium than a critically damped system.

P

Pair production
The process of converting energy to mass to produce a particle-antiparticle pair.

Parallax
A measure of how much a nearby object (e.g. a star) appears to move in relation to a distant background due to the observer's motion (e.g. as the Earth orbits the Sun). Measured as an angle of parallax.

Parsec
The distance of an object if its angle of parallax, as measured from Earth, is equal to 1 arcsecond = $(1/3600)^o$.

Path difference
The amount by which the path travelled by one wave is longer than the path travelled by another wave.

Peer review
Where a scientific report is sent out to peers (other scientists) who examine the methods, data and results, and if they think that the conclusion is reasonable, it's published.

Percentage difference
The difference between a measured value and its true value, as a percentage of the true value.

Percentage error
The uncertainty given as a percentage of the measurement taken.

Period
The time taken for a rotating or oscillating object to complete one revolution or cycle.

Period (waves)
The time taken for one whole wave cycle to pass a given point.

Permittivity
The permittivity of a material is a measure of how difficult it is to create an electric field in that material.

Phase
A measurement of the position of a certain point on a wave cycle, measured as an angle (in degrees or radians) or in fractions of a cycle.

Phase difference
The amount by which one wave lags behind another, or the difference in phase between two points on a wave. It is measured as an angle (in degrees or radians) or in fractions of a cycle.

Photoelectric effect
The emission of electrons from a metal when light of a high enough frequency is shone on it.

Photoelectron
An electron released through the photoelectric effect.

Photon
A quantum of energy of electromagnetic radiation.

Physical quantity
A quantity that can be measured, that has both a numerical value and a unit.

Piezoelectric effect
The production of potential difference across piezoelectric crystals when they are deformed (and vice versa).

Pilot experiment
An experiment carried out before the actual experiment. The same set-up as the actual experiment is used to vary the independent variable to see (roughly) how much the dependent variable changes.

Pivot
The point about which an object will turn when it experiences a moment.

Plane polarised wave
A wave in which all the vibrations are in one direction or plane.

Planet
A natural object in space which orbits a star and satisfies certain conditions.

Planetary nebula
The remnants of the outer layers of a red giant that are ejected as the star becomes a white dwarf.

Planetary satellite
Objects which orbit a planet, such as moons and artificial satellites.

Plastic deformation
If a deformation is plastic, the material is permanently deformed.

Polarised wave
See plane polarised wave.

Polarising filter
A filter that only transmits vibrations of a wave in one direction or plane, called the plane of transmission.

Polymeric
A polymeric material is made up of molecules that are arranged in long chains.

Positron
The antiparticle of an electron. Sometimes called a b^+ particle.

Potential difference
The work done moving a unit charge between two points in a circuit.

Potential divider
A circuit containing a voltage source and a pair of resistors. The voltage across one of the resistors is used as an output voltage. If the resistors aren't fixed, the circuit will be capable of producing a variable output voltage.

Potential energy
Energy that is stored (e.g. elastic potential energy is energy stored in something that has been stretched or compressed, like a spring).

Power
The rate of transfer of energy or the rate of doing work. It's measured in watts (W), where 1 watt is equivalent to 1 joule per second.

Precise result
The smaller the amount of spread of your data from the mean, the more precise it is.

Prediction
See hypothesis.

Prefix
A scaling factor used before a unit.

Pressure
The force applied to a surface per unit area.

Pressure law
At constant volume, the pressure p of an ideal gas is directly proportional to its absolute temperature T.

Principle of conservation of energy
Energy cannot be created or destroyed. Energy can be transferred from one form to another but the total amount of energy in a closed system will not change.

Principle of conservation of momentum
Assuming no external forces act, the total linear momentum before a collision is equal to the total linear momentum after a collision.

Principle of moments
For a body to be in equilibrium, the sum of the clockwise moments acting about any point must equal the sum of the anticlockwise moments about the same point.

Progressive wave
A moving wave that carries energy from one place to another without transferring any material.

Projectile motion
Motion with a constant horizontal velocity and a vertical velocity affected by acceleration due to gravity.

Proton
A positively charged hadron with a relative mass of +1, and quark composition uud.

Proton number
The number of protons in the nucleus of an atom.

Protostar
The earliest stage in the life cycle of a star. Protostars are formed when the force of gravity causes clouds of dust and gas to spiral together.

Q

Qualitative data
Non-numerical data.

Quantitative data
Numerical data.

Quark
A fundamental particle that makes up hadrons.

R

Radial electric field
An electric field where the field lines all point towards or away from the central point.

Radian
A unit of measurement for angles. There are 2π radians in a complete circle.

Radioactive dating
A means of dating objects by analysing the activity of an isotope with a known half-life within the object.

Radioactive decay
When an unstable nucleus breaks down to become more stable, by releasing energy and/or particles.

Random error
An error that occurs by chance. Its effect on experimental results cannot be predicted.

Reaction force
See normal contact force.

Reaction time
The time taken for a person to react after an event (e.g. seeing a hazard).

Recessional velocity
The speed at which an object is receding from Earth.

Red giant
A phase of a star's evolution in which the star is fusing larger elements than hydrogen in its core.

Red shift
The shift in wavelength and frequency of EM radiation towards (or beyond) the red end of the electromagnetic spectrum due to the source moving away from the observer.

Red super giant (or super red giant)
A phase of a massive star's evolution during which fusion of large nuclei (up to iron) takes place.

Reference
A detailed description of where a source of information can be found, usually at the end of a document.

Reflection
When a wave is bounced back when it hits a boundary.

Refraction
The way a wave changes direction as it enters a different medium.

Refractive index
The ratio between the speed of light in a vacuum and the speed of light in a material.

Repeatable result
A result that can be repeated by the same person using the same method and equipment.

Reproducible result
A result that can be consistently reproduced in an independent experiment.

Resistance
A component has a resistance of 1 Ω if a potential difference of 1 V across it makes a current of 1 A flow through it. Resistance is measured in ohms (Ω).

Resistivity
The resistance of a 1 m length of a material with a 1 m^2 cross-sectional area. It is measured in ohm-metres (Ωm).

Resolution
The smallest change in what's being measured that can be detected by the equipment.

Resonance
When an object, driven by a periodic external force at a frequency close to its natural frequency, begins to oscillate with a rapidly increasing amplitude.

Resonant frequency (in free vibrations)
The frequency of an object oscillating freely.

Resonant frequency (in stationary waves)
A frequency at which a stationary wave is formed because an exact number of waves are produced in the time it takes for a wave to get to the end of the vibrating medium and back again.

Resultant force
See net force.

Resultant vector
The vector that's formed when two or more vectors are added together.

Ripple tank
A shallow tank of water in which water waves are created by a vibrating dipper.

Root mean square speed
The square root of the mean square speed.

S

Satellite
Any smaller mass which orbits a much larger mass.

Scalar
A quantity with a size but no direction.

Scale drawing
A drawing that is proportional to the real-life situation it represents.

Seat belt
A belt used in vehicles that is designed to stretch slightly to protect the wearer in a crash.

Semiconductor
A material which conducts electricity (but not as well as a metal). When the temperature of a semiconductor rises, it can release more charge carriers and its resistance decreases.

S.I. derived units
Units that can be derived from the S.I. base units.

Simple harmonic motion
The oscillation of an object where the object's acceleration is directly proportional to its displacement from the midpoint, and is always directed towards the midpoint.

Solenoid
A wire wound into a long coil.

Specific heat capacity
The amount of energy needed to raise the temperature of 1 kg of a substance by 1 K (or 1 °C).

Specific latent heat
The quantity of thermal energy required to change the state of 1 kg of a substance (e.g. to melt or vaporise it) without changing its temperature.

Speed
How fast something is moving, regardless of direction.

Standard form
A number written in the form $A \times 10^n$, where A is a number between 1 and 10 and n is a whole number.

Standard notation
Summarises all the information about the nucleus of an element in the form ${}^{A}_{Z}X$, where A is the nucleon number, Z is the proton number and X is the element's chemical symbol.

Stationary (standing) wave
A wave created by the superposition of two progressive waves with the same frequency (or wavelength), moving in opposite directions.

Stefan's law
The luminosity of a star is directly proportional to the fourth power of the star's surface temperature and to its surface area: $L = 4\pi r^2 \sigma T^4$.

Stiffness
Stiffness is a measure of the extent to which a material resists deformation in response to stress.

Stopping distance
The distance covered by a vehicle in the time between the driver spotting a hazard and the vehicle coming to a complete stop. It's the sum of the thinking distance and the braking distance.

Strain
The change in length divided by the original length of the material.

Strength
The strength of a material is a measure of its ability to withstand stress without breaking.

Stress
The force applied divided by the cross-sectional area.

Strong nuclear force
A force with a short range which is attractive at small separations and repulsive at very small separations.

Supernova
The explosion of a high-mass star after its red super giant phase, caused by the core collapsing and the outer layers of the star falling in and rebounding, creating huge shockwaves.

Superposition
When two or more waves cross, the resultant displacement equals the vector sum of the individual displacements.

Systematic error
An error that usually causes measurements to be shifted from their true value by a fixed amount.

T

Tangent
A line drawn on a graph that is parallel to the curve at the point that it meets it. Used to calculate the gradient of a curve at a point.

Tensile deformation
Deformation of an object caused by balanced outwards forces, which leads to extension (stretching).

Tensile force
A force which stretches something.

Tensile strain
See strain.

Tensile stress
See stress.

Tension
The force that acts equally on either end of an object that is pulled tight.

Terminal potential difference
The potential difference between the two terminals of a power supply. This is equal to e.m.f. when there is no internal resistance.

Terminal velocity
The velocity at which the driving force(s) match the frictional force(s).

Theory
A possible explanation for an observation.

Thermal equilibrium
When two objects are at the same temperature with no net flow of thermal energy between them.

Thermal neutron
A neutron in a nuclear reactor that has been slowed down enough by a moderator that it can be captured by uranium nuclei (or other fissionable nuclei).

Thermistor
A resistor with a resistance that depends on its temperature — it is a type of semiconductor.

Thinking distance
The distance travelled by a vehicle between the driver seeing a hazard and applying the brakes.

Threshold frequency
The lowest frequency of light that when shone on a metal will cause electrons to be released from it (by the photoelectric effect).

Threshold voltage (of an LED)
The lowest voltage that will cause a current to flow through a light-emitting diode (LED).

Time constant
The time taken for the charge on a discharging capacitor to fall to $\frac{1}{e}$ (about 37%) of its initial value, or for the charge of a charging capacitor to rise to about 63% of the full charge.

Torque
The turning effect caused by a force couple.

Total internal reflection (TIR)
When all light is completely reflected back into a medium at a boundary with another less optically dense medium, instead of being refracted. It only happens at angles of incidence greater than the critical angle.

Transformer
A device that makes use of electromagnetic induction to change the size of the voltage for an alternating current.

Transverse wave
A wave in which the oscillations of the particles/fields (i.e. the displacement) is at right angles to the direction of energy propagation.

U

Ultimate tensile strength
The maximum stress that a material can withstand before breaking.

Ultrasound waves
Longitudinal waves with higher frequencies than humans can hear (> 20 kHz).

Uniform electric field
An electric field with the same electric field strength everywhere.

Upthrust
The upwards force experienced by an object partially or fully immersed in a fluid.

V

Valid conclusion
A conclusion that answers the original question.

Valid result
A valid result is a result that is precise and answers the original question.

Validation (of a theory)
The process of testing a theory by repeating experiments, or by using the theory to make new predictions and then testing these new predictions with new experiments.

Variable
A quantity that has the potential to change, e.g. temperature, time, mass and volume.

Vector
A quantity with a size and a direction.

Velocity
The rate of change of displacement.

Velocity selector
A device used to separate out charged particles of a certain velocity from a stream of accelerated charged particles moving at a range of speeds.

Velocity-time graph
A graph showing how the velocity of an object changes over a period of time.

Viscosity
A measure of how thick a fluid is.

Volt
The unit of potential difference. The potential difference across a component is 1 volt when you convert 1 joule of energy moving 1 coulomb of charge through the component.

Voltmeter
A component used to measure the potential difference across another component in a circuit.

W

Watt
A rate of energy transfer equal to 1 joule per second.

Wave-particle duality
All particles have both particle and wave properties. Waves can also show particle properties.

Wave speed
The speed that a wave travels at.

Wavelength
The length of one whole wave oscillation or wave cycle, e.g. the distance between two crests (or troughs) of a wave.

Weak nuclear force
A force that has a short range and can change the flavour of a quark.

Weight
The force experienced by a mass due to a gravitational field.

White dwarf
A star with a low luminosity and a high temperature, left behind when a low-mass star stops fusing elements and its core contracts.

Wien's displacement law
The wavelength corresponding to the peak intensity from the continuous emission spectrum of a star is inversely proportional to its surface temperature: $\lambda_{max} \propto \frac{1}{T}$

Work
The amount of energy transferred from one form to another when a force moves an object through a distance or when a charged particle moves through a potential difference.

Work function
The minimum amount of energy required for an electron to escape a metal's surface.

Worst line
A line of best fit which has either the maximum or minimum possible slope for the data and goes through all of the error bars.

X

X-ray tube
An electrical circuit contained in an evacuated tube that is used to produce X-rays.

Y

Yield point (or yield stress)
The stress at which a large amount of plastic deformation takes place with a constant or reduced load.

Young modulus
The stress divided by strain for a material, up to its limit of proportionality.

Z

Zero error
When a measuring instrument falsely reads a non-zero value when the true value being measured is zero.

Zero order line
The line of maximum brightness at the centre of a diffraction pattern.

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Data Tables

This page summarises some of the constants and values that you might need to refer to when answering questions in this book. Everything here will be provided in your exam data and formulae booklet somewhere... so you need to get used to looking them up and using them correctly. If a number isn't given on this sheet — unlucky... you'll need to remember it as it won't be given to you in the exam.

Physical constants

Quantity	Value
acceleration of free fall, g	$9.81\ ms^{-2}$
elementary charge, e	$1.60 \times 10^{-19}\ C$
speed of light in a vacuum, c	$3.00 \times 10^{8}\ ms^{-1}$
Planck constant, h	$6.63 \times 10^{-34}\ Js$
Avogadro constant, N_A	$6.02 \times 10^{23}\ mol^{-1}$
molar gas constant, R	$8.31\ Jmol^{-1}K^{-1}$
Boltzmann constant, k	$1.38 \times 10^{-23}\ JK^{-1}$
gravitational constant, G	$6.67 \times 10^{-11}\ Nm^2kg^{-2}$
permittivity of free space, ε_0	$8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}\ (Fm^{-1})$
electron rest mass, m_e	$9.11 \times 10^{-31}\ kg$
proton rest mass, m_p	$1.673 \times 10^{-27}\ kg$
neutron rest mass, m_n	$1.675 \times 10^{-27}\ kg$
alpha particle rest mass, m_α	$6.646 \times 10^{-27}\ kg$
Stefan constant, σ	$5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$

Quark charges

Quark	Charge
up	$+\frac{2}{3}e$
down	$-\frac{1}{3}e$
strange	$-\frac{1}{3}e$

Conversion factors

Unit	Conversion
unified atomic mass unit	$1\ u = 1.661 \times 10^{-27}\ kg$
electronvolt	$1\ eV = 1.60 \times 10^{-19}\ J$
day	$1\ day = 8.64 \times 10^{4}\ s$
year	$1\ year \approx 3.16 \times 10^{7}\ s$
light year	$1\ light\ year \approx 9.5 \times 10^{15}\ m$
parsec	$1\ parsec \approx 3.1 \times 10^{16}\ m$

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